Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/69-4.1.12-e-x-^m-a+b-sin-c+d-x^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [357]. This is test number [69].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (357)	0.00 (0)
Mathematica	96.92 (346)	3.08 (11)
Fricas	85.43 (305)	14.57 (52)
Maxima	75.63 (270)	24.37 (87)
Maple	68.63 (245)	31.37 (112)
Giac	52.66 (188)	47.34 (169)
Mupad	36.13 (129)	63.87 (228)
Sympy	31.65 (113)	68.35 (244)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

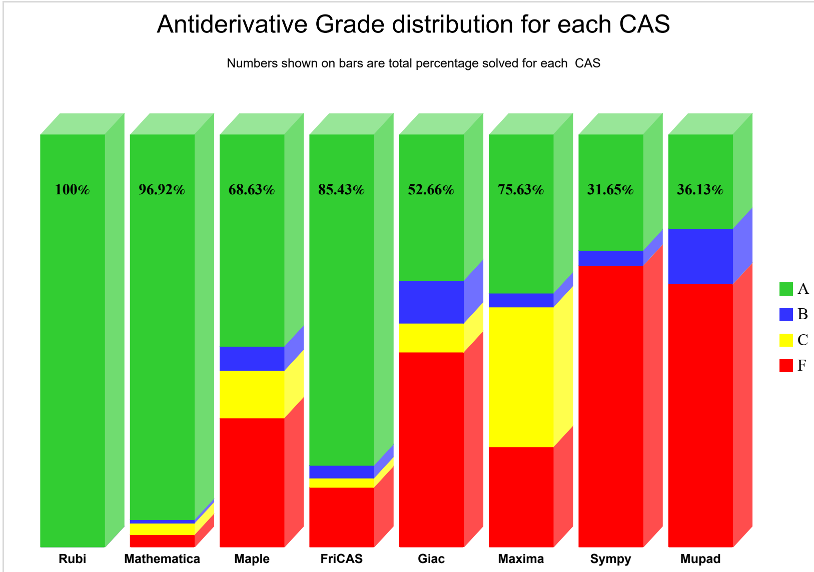
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

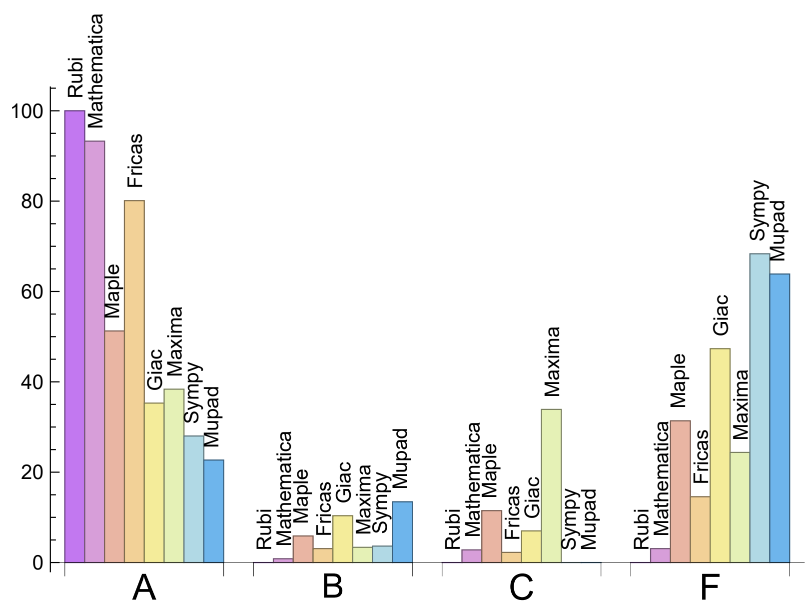
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.28	0.84	2.80	3.08
Fricas	80.11	3.08	2.24	14.57
Maple	51.26	5.88	11.48	31.37
Maxima	38.38	3.36	33.89	24.37
Giac	35.29	10.36	7.00	47.34
Sympy	28.01	3.64	0.00	68.35
Mupad	N/A	13.45	0.00	63.87

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	11	54.55 %	45.45 %	0.00 %
Maple	112	100.00 %	0.00 %	0.00 %
Fricas	52	100.00 %	0.00 %	0.00 %
Giac	169	99.41 %	0.00 %	0.59 %
Maxima	87	93.10 %	2.30 %	4.60 %
Sympy	244	82.79 %	11.07 %	6.15 %
Mupad	228	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

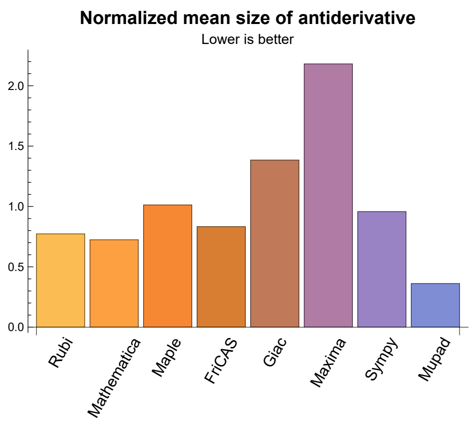
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

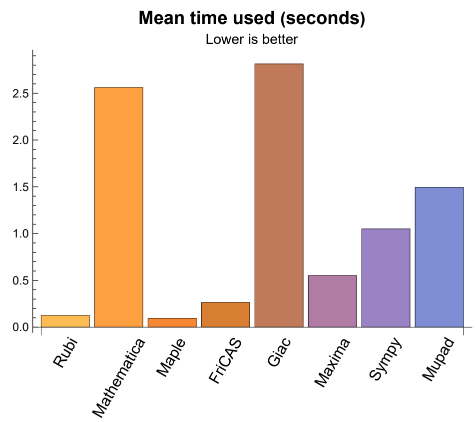
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	131.66	0.77	96.00	1.00
Mathematica	2.56	120.93	0.72	80.50	0.84
Maple	0.09	160.90	1.01	62.00	0.86
Maxima	0.55	212.64	2.18	68.00	0.82
Fricas	0.26	138.11	0.83	74.00	0.82
Sympy	1.05	68.89	0.96	0.00	0.00
Giac	2.81	291.60	1.38	37.00	0.94
Mupad	1.49	24.60	0.36	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 101, 102, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 195, 196, 205, 206, 215, 216, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

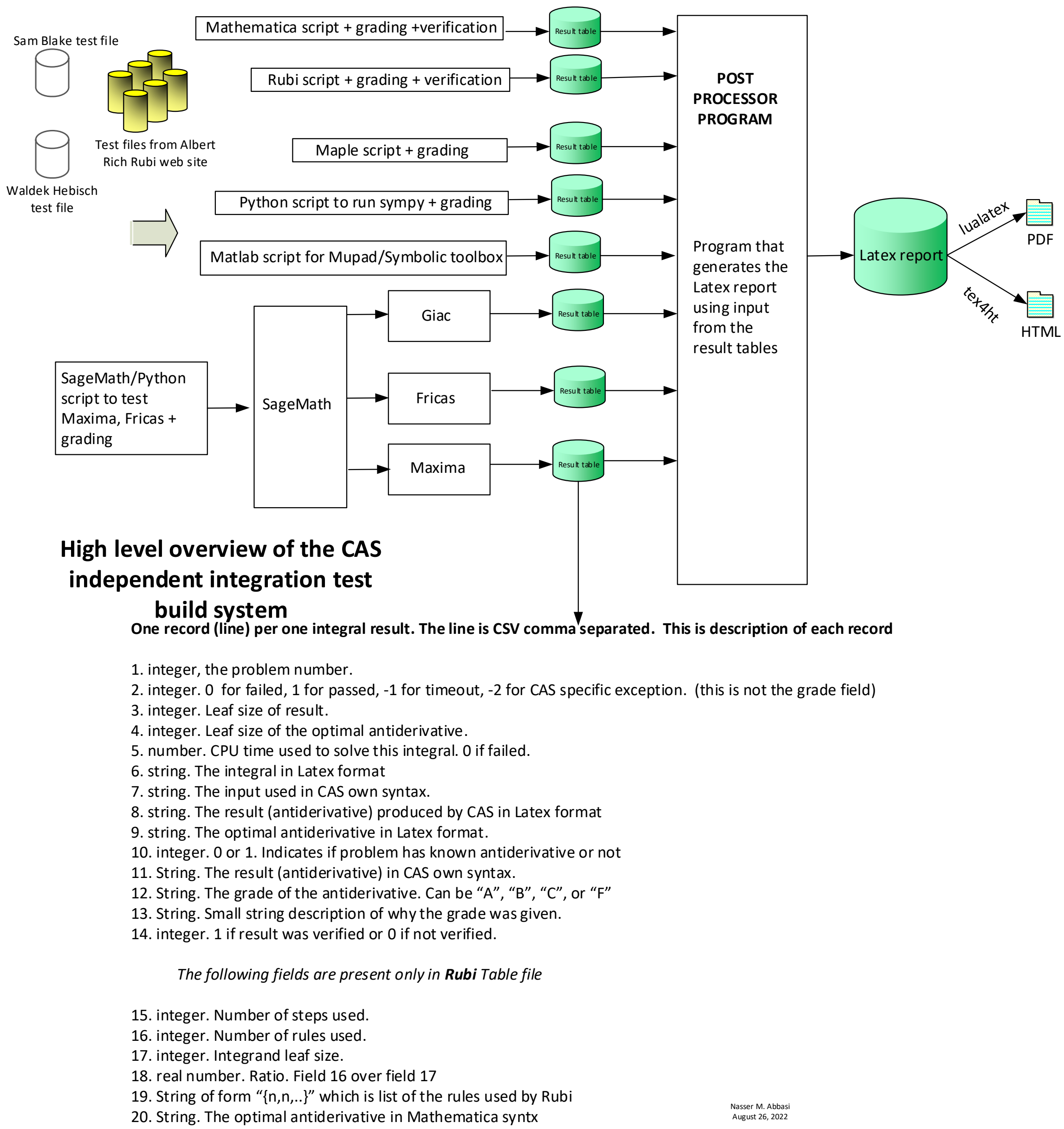
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 192, 194, 195, 196, 198, 199, 202, 204, 205, 206, 207, 208, 209, 213, 214, 215, 216, 218, 219, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade: { 183, 193, 203 }

C grade: { 190, 191, 197, 210, 211, 212, 217, 220, 221, 222 }

F grade: { 171, 172, 173, 200, 201, 226, 260, 261, 282, 283, 286 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 69, 70, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 101, 102, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 162, 163, 164, 169, 170, 175, 176, 177, 178, 179, 180, 181, 185, 186, 189, 195, 196, 197, 198, 199, 200, 205, 206, 209, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

B grade: { 109, 110, 116, 117, 118, 153, 154, 155, 165, 166, 167, 168, 187, 188, 190, 191, 201, 207, 208, 293, 298 }

C grade: { 15, 16, 17, 26, 27, 139, 142, 210, 211, 220, 221, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 331, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 355 }

F grade: { 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 131, 133, 140, 141, 143, 144, 171, 172, 173, 174, 182, 183, 184, 192, 193, 194, 202, 203, 204, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

2.1.4 Maxima

A grade: { 1, 2, 3, 12, 13, 14, 23, 24, 25, 33, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 91, 92, 95, 96, 97, 101, 102, 107, 115, 122, 124, 125, 126, 127, 128, 129, 130, 132, 134, 145, 146, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 189, 193, 194, 195, 196, 204, 205, 206, 209, 215, 216, 225, 226, 231, 238, 245, 255, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 319, 321, 336, 343, 345 }

B grade: { 37, 82, 187, 188, 192, 202, 203, 207, 208, 335, 337, 338 }

C grade: { 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 59, 60, 71, 72, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 135, 136, 137, 138, 153, 154, 155, 156, 165, 166, 167, 168, 197, 198, 199, 212, 213, 214, 217, 218, 219, 222, 223, 224, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 288, 289, 290, 294, 295, 315, 316, 317, 320, 322, 323, 324, 325, 331, 339, 340, 341, 344, 346, 347, 348, 349, 355 }

F grade: { 35, 36, 43, 44, 45, 52, 53, 54, 81, 89, 90, 93, 94, 98, 99, 100, 131, 133, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 200, 201, 210, 211, 220, 221, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 291, 292, 293, 296, 297, 298, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 192, 193, 194, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 237, 238, 246, 247, 248, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 331, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 355 }

B grade: { 35, 36, 43, 44, 81, 89, 184, 245, 255, 256, 298 }

C grade: { 190, 191, 200, 201, 210, 211, 220, 221 }

F grade: { 131, 133, 139, 140, 141, 142, 143, 144, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 254, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 326, 327, 328, 329, 330, 332, 333, 350, 351, 352, 353, 354, 356, 357 }

2.1.6 Sympy

A grade: { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 29, 31, 33, 34, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 69, 70, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 101, 102, 106, 107, 108, 109, 110, 114, 122, 124, 125, 126, 128, 129, 130, 132, 145, 146, 157, 158, 163, 169, 170, 175, 176, 180, 185, 187, 188, 189, 195, 196, 205, 206, 209, 215, 216, 225, 226, 259, 264, 265, 270, 271, 275, 276, 278, 279, 280, 300, 301, 302, 311, 312, 313, 319 }

B grade: { 7, 8, 28, 32, 37, 82, 115, 116, 117, 118, 127, 314, 321 }

C grade: { }

F grade: { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 30, 35, 36, 43, 44, 45, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 90, 98, 99, 100, 103, 104, 105, 111, 112, 113, 119, 120, 121, 123, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 164, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 181, 182, 183, 184, 186, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 277, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.7 Giac

A grade: { 2, 3, 4, 12, 13, 14, 15, 23, 24, 25, 26, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 69, 70, 71, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 101, 102, 106, 107, 108, 115, 116, 117, 122, 124, 125, 126, 127, 128, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 187, 188, 189, 195, 196, 205, 206, 208, 209, 215, 216, 225, 226, 229, 230, 231, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 296, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

B grade: { 1, 5, 6, 16, 17, 27, 60, 72, 103, 104, 105, 109, 110, 111, 112, 113, 114, 118, 197, 198, 199, 207, 217, 218, 219, 227, 228, 238, 288, 289, 290, 292, 293, 294, 295, 297, 298 }

C grade: { 7, 8, 9, 18, 19, 20, 28, 29, 31, 32, 153, 154, 155, 156, 165, 166, 167, 168, 212, 213, 214, 235, 236, 237, 239 }

F grade: { 10, 11, 21, 22, 30, 35, 36, 43, 44, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 119, 120, 121, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 200, 201, 202, 203, 204, 210, 211, 220, 221, 222, 223, 224, 232, 233, 234, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.8 Mupad

A grade: { 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 101, 102, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 195, 196, 205, 206, 215, 216, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

B grade: { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 33, 37, 45, 57, 58, 69, 70, 82, 90, 107, 108, 109, 110, 115, 116, 117, 118, 121, 122, 124, 125, 126, 127, 128, 153, 154, 155, 156, 162, 168, 189, 209, 311, 312, 313, 314, 319, 321 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 106, 111, 112, 113, 114, 119, 120, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 165, 166, 167, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 187, 188, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	62	47	51	65	128	53
N.S.	1	1.00	0.89	1.09	0.82	0.89	1.14	2.25	0.93
time (sec)	N/A	0.050	0.060	0.037	0.294	0.360	0.403	3.685	0.192

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	40	37	40	49	75	38
N.S.	1	1.00	1.00	0.91	0.84	0.91	1.11	1.70	0.86
time (sec)	N/A	0.030	0.005	0.033	0.282	0.370	0.184	6.207	0.088

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	41	27	21	23	31	26	21
N.S.	1	1.00	1.64	1.08	0.84	0.92	1.24	1.04	0.84
time (sec)	N/A	0.016	0.012	0.026	0.283	0.381	0.086	6.173	4.611

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	29	50	38	0	32	-1
N.S.	1	1.00	0.94	0.94	1.61	1.23	0.00	1.03	-0.03
time (sec)	N/A	0.024	0.029	0.041	0.358	0.360	0.000	4.917	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	47	57	65	0	99	-1
N.S.	1	1.00	0.91	0.89	1.08	1.23	0.00	1.87	-0.02
time (sec)	N/A	0.063	0.054	0.042	0.358	0.347	0.000	4.927	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	86	65	58	85	0	204	-1
N.S.	1	1.00	1.16	0.88	0.78	1.15	0.00	2.76	-0.01
time (sec)	N/A	0.083	0.060	0.044	0.363	0.379	0.000	3.697	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	125	89	92	103	488	165	-1
N.S.	1	1.00	1.03	0.74	0.76	0.85	4.03	1.36	-0.01
time (sec)	N/A	0.098	0.173	0.059	0.302	0.362	2.397	3.952	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	68	75	86	223	145	-1
N.S.	1	1.00	1.02	0.67	0.74	0.84	2.19	1.42	-0.01
time (sec)	N/A	0.048	0.134	0.043	0.314	0.385	1.867	3.639	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	61	48	53	67	66	102	56
N.S.	1	1.00	0.82	0.65	0.72	0.91	0.89	1.38	0.76
time (sec)	N/A	0.030	0.100	0.030	0.298	0.368	0.247	5.291	4.750

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	66	81	78	0	0	-1
N.S.	1	1.00	1.03	0.75	0.92	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.130	0.042	0.572	0.363	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	83	82	98	0	0	-1
N.S.	1	1.00	1.04	0.73	0.72	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.153	0.060	0.610	0.368	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	122	140	106	121	209	284	149
N.S.	1	1.00	0.75	0.86	0.65	0.74	1.28	1.74	0.91
time (sec)	N/A	0.169	0.228	0.076	0.313	0.358	0.651	6.256	0.393

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	92	93	87	84	136	165	95
N.S.	1	1.00	0.90	0.91	0.85	0.82	1.33	1.62	0.93
time (sec)	N/A	0.099	0.133	0.065	0.326	0.373	0.291	4.354	4.699

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	52	62	52	53	95	57	51
N.S.	1	1.00	0.90	1.07	0.90	0.91	1.64	0.98	0.88
time (sec)	N/A	0.035	0.093	0.056	0.340	0.356	0.148	3.316	4.675

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	157	108	94	0	77	-1
N.S.	1	1.00	0.96	2.12	1.46	1.27	0.00	1.04	-0.01
time (sec)	N/A	0.072	0.093	0.266	0.410	0.383	0.000	2.763	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	116	203	124	147	0	226	-1
N.S.	1	1.00	1.01	1.77	1.08	1.28	0.00	1.97	-0.01
time (sec)	N/A	0.155	0.153	0.280	0.413	0.370	0.000	3.633	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	158	255	128	189	0	448	-1
N.S.	1	1.00	0.93	1.51	0.76	1.12	0.00	2.65	-0.01
time (sec)	N/A	0.194	0.263	0.302	0.428	0.369	0.000	3.374	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	234	189	207	216	0	329	-1
N.S.	1	1.00	0.95	0.77	0.84	0.87	0.00	1.33	-0.00
time (sec)	N/A	0.169	0.359	0.108	0.593	0.396	0.000	4.606	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	191	142	171	176	0	283	-1
N.S.	1	1.00	0.96	0.72	0.86	0.89	0.00	1.43	-0.01
time (sec)	N/A	0.113	0.324	0.079	0.514	0.451	0.000	6.190	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	99	129	134	0	195	-1
N.S.	1	1.00	0.96	0.65	0.84	0.88	0.00	1.27	-0.01
time (sec)	N/A	0.077	0.208	0.059	0.536	0.377	0.000	5.848	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	184	137	170	159	0	0	-1
N.S.	1	1.00	0.98	0.73	0.91	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.314	0.089	0.688	0.390	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	226	175	176	206	0	0	-1
N.S.	1	1.00	0.95	0.73	0.74	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.417	0.093	0.656	0.402	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	75	113	79	79	143	138	94
N.S.	1	1.00	0.64	0.97	0.68	0.68	1.22	1.18	0.80
time (sec)	N/A	0.086	0.189	0.049	0.298	0.362	0.924	4.166	4.971

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	58	66	60	58	92	94	66
N.S.	1	1.00	0.73	0.84	0.76	0.73	1.16	1.19	0.84
time (sec)	N/A	0.051	0.109	0.056	0.328	0.376	0.406	4.024	4.752

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	27	26	46	26	28
N.S.	1	1.00	1.00	0.79	0.82	0.79	1.39	0.79	0.85
time (sec)	N/A	0.020	0.023	0.037	0.289	0.366	0.188	3.474	4.666

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	125	89	63	0	47	-1
N.S.	1	1.00	0.93	2.27	1.62	1.15	0.00	0.85	-0.02
time (sec)	N/A	0.063	0.051	0.308	0.372	0.356	0.000	3.871	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	162	97	118	0	186	-1
N.S.	1	1.00	0.99	1.78	1.07	1.30	0.00	2.04	-0.01
time (sec)	N/A	0.147	0.094	0.391	0.402	0.370	0.000	5.721	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	159	132	143	147	439	259	-1
N.S.	1	1.00	0.85	0.70	0.76	0.78	2.34	1.38	-0.01
time (sec)	N/A	0.160	0.296	0.105	0.517	0.375	2.398	5.883	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	117	99	112	120	129	185	-1
N.S.	1	1.00	0.76	0.65	0.73	0.78	0.84	1.21	-0.01
time (sec)	N/A	0.061	0.159	0.051	0.516	0.406	0.683	5.553	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	167	130	152	147	0	0	-1
N.S.	1	1.00	0.99	0.77	0.90	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.291	0.073	0.607	0.367	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	58	97	51	116	97	-1
N.S.	1	1.00	0.89	0.82	1.37	0.72	1.63	1.37	-0.01
time (sec)	N/A	0.036	0.046	0.076	0.529	0.368	2.146	3.710	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	78	117	73	291	125	-1
N.S.	1	1.00	0.89	0.93	1.39	0.87	3.46	1.49	-0.01
time (sec)	N/A	0.048	0.103	0.073	0.492	0.405	2.200	3.076	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	50	55	52	95	52	55
N.S.	1	1.00	1.00	0.75	0.82	0.78	1.42	0.78	0.82
time (sec)	N/A	0.028	0.031	0.046	0.346	0.357	0.894	4.812	4.988

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	54	47	51	39	-1
N.S.	1	1.00	0.93	0.89	1.23	1.07	1.16	0.89	-0.02
time (sec)	N/A	0.063	0.073	0.063	0.341	0.365	2.519	6.045	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	289	0	0	1435	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.557	0.139	0.039	0.000	0.509	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	188	0	0	1041	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	4.25	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.040	0.024	0.000	0.579	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	48	8078	208	202	63	128
N.S.	1	1.00	1.00	1.00	168.29	4.33	4.21	1.31	2.67
time (sec)	N/A	0.046	0.063	0.065	28.310	0.359	5.723	5.442	6.633

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.275	0.020	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.251	0.027	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.252	0.023	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.019	0.020	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.188	0.023	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	513	0	0	2469	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	3.72	0.00	0.00	-0.00
time (sec)	N/A	0.844	1.555	0.220	0.000	0.597	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	302	0	0	1509	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	4.66	0.00	0.00	-0.00
time (sec)	N/A	0.382	0.644	0.250	0.000	0.696	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	131	0	366	0	144	178
N.S.	1	1.00	1.00	1.44	0.00	4.02	0.00	1.58	1.96
time (sec)	N/A	0.067	0.147	0.135	0.000	0.409	0.000	5.347	5.098

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	4.031	0.190	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	5.446	0.230	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	2.806	0.184	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	3.206	0.214	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	4.854	0.202	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.588	0.017	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	451	0	0	302	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.312	8.333	0.184	0.000	0.136	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	551	0	0	187	0	0	-1
N.S.	1	1.00	1.97	0.00	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.155	4.622	0.158	0.000	0.156	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	93	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.068	1.048	0.015	0.000	0.106	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.250	0.033	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.607	0.152	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	37	40	49	75	38
N.S.	1	1.00	1.00	0.89	0.84	0.91	1.11	1.70	0.86
time (sec)	N/A	0.031	0.008	0.045	0.287	0.361	0.395	4.904	0.175

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	41	27	21	23	31	26	21
N.S.	1	1.00	1.64	1.08	0.84	0.92	1.24	1.04	0.84
time (sec)	N/A	0.017	0.018	0.025	0.281	0.366	0.109	3.855	4.658

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	0	50	38	0	32	-1
N.S.	1	1.00	0.94	0.00	1.61	1.23	0.00	1.03	-0.03
time (sec)	N/A	0.024	0.029	0.016	0.372	0.391	0.000	3.688	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	0	57	65	0	99	-1
N.S.	1	1.00	0.91	0.00	1.08	1.23	0.00	1.87	-0.02
time (sec)	N/A	0.064	0.057	0.023	0.343	0.361	0.000	7.442	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	124	0	109	70	0	0	-1
N.S.	1	1.00	1.11	0.00	0.97	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.160	0.021	0.311	0.102	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	108	0	93	53	0	0	-1
N.S.	1	1.00	1.19	0.00	1.02	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.086	0.019	0.333	0.127	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	89	62	0	0	-1
N.S.	1	1.00	1.19	0.00	0.88	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.145	0.027	0.319	0.108	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	143	0	91	83	0	0	-1
N.S.	1	1.00	1.10	0.00	0.70	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.251	0.028	0.323	0.131	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	124	0	110	68	0	0	-1
N.S.	1	1.00	1.17	0.00	1.04	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.131	0.020	0.303	0.106	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	138	0	85	49	0	0	-1
N.S.	1	1.00	1.68	0.00	1.04	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.071	0.013	0.310	0.102	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	90	66	0	0	-1
N.S.	1	1.00	1.19	0.00	0.89	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.135	0.020	0.311	0.113	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	146	0	91	83	0	0	-1
N.S.	1	1.00	1.16	0.00	0.72	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.309	0.029	0.322	0.107	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	92	137	87	84	143	165	95
N.S.	1	1.00	0.86	1.28	0.81	0.79	1.34	1.54	0.89
time (sec)	N/A	0.084	0.164	0.220	0.292	0.361	0.611	7.243	0.266

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	62	52	53	99	57	51
N.S.	1	1.00	0.87	1.03	0.87	0.88	1.65	0.95	0.85
time (sec)	N/A	0.037	0.120	0.058	0.348	0.395	0.183	4.205	4.717

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	108	95	0	79	-1
N.S.	1	1.00	0.89	0.00	1.35	1.19	0.00	0.99	-0.01
time (sec)	N/A	0.059	0.098	0.166	0.430	0.367	0.000	4.161	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	116	0	124	147	0	226	-1
N.S.	1	1.00	0.95	0.00	1.02	1.20	0.00	1.85	-0.01
time (sec)	N/A	0.143	0.162	0.191	0.405	0.405	0.000	5.304	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	339	0	234	150	0	0	-1
N.S.	1	1.00	1.36	0.00	0.94	0.60	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.384	0.220	0.360	0.141	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	283	0	199	107	0	0	-1
N.S.	1	1.00	1.47	0.00	1.03	0.55	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.225	0.175	0.351	0.142	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	229	332	0	187	131	0	0	-1
N.S.	1	0.99	1.44	0.00	0.81	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.369	0.171	0.360	0.126	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	283	292	0	194	180	0	0	-1
N.S.	1	0.99	1.02	0.00	0.68	0.63	0.00	0.00	-0.00
time (sec)	N/A	0.156	1.574	0.208	0.346	0.125	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	339	0	240	146	0	0	-1
N.S.	1	1.00	1.43	0.00	1.01	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.353	0.195	0.343	0.117	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	281	0	192	105	0	0	-1
N.S.	1	1.00	1.54	0.00	1.05	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.180	0.106	0.375	0.118	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	225	332	0	188	139	0	0	-1
N.S.	1	0.99	1.46	0.00	0.83	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.087	0.347	0.174	0.354	0.117	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	275	294	0	193	181	0	0	-1
N.S.	1	0.99	1.06	0.00	0.70	0.65	0.00	0.00	-0.00
time (sec)	N/A	0.114	1.579	0.164	0.361	0.133	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	188	0	0	1041	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	4.25	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.103	0.032	0.000	0.564	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	49	8078	208	212	64	136
N.S.	1	1.00	1.00	0.96	158.39	4.08	4.16	1.25	2.67
time (sec)	N/A	0.050	0.075	0.076	28.806	0.378	7.880	4.848	6.516

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.286	0.024	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.301	0.025	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.406	0.023	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.217	0.024	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.020	0.020	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.237	0.027	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	302	0	0	1509	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	4.66	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.645	0.224	0.000	0.615	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	131	0	366	0	146	186
N.S.	1	1.00	0.97	1.39	0.00	3.89	0.00	1.55	1.98
time (sec)	N/A	0.075	0.143	0.132	0.000	0.396	0.000	6.218	4.949

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	6.693	0.196	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	8.084	0.210	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	4.517	0.167	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	7.088	0.196	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	5.563	0.238	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	7.741	0.155	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.591	0.014	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	451	0	0	302	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.278	10.859	0.191	0.000	0.126	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	556	0	0	187	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.159	5.219	0.169	0.000	0.126	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	93	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.012	0.019	0.000	0.113	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.253	0.036	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.661	0.189	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	73	86	79	0	400	-1
N.S.	1	1.00	0.90	0.94	1.10	1.01	0.00	5.13	-0.01
time (sec)	N/A	0.083	0.053	0.091	0.361	0.380	0.000	6.684	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	57	76	69	0	251	-1
N.S.	1	1.00	0.87	0.95	1.27	1.15	0.00	4.18	-0.02
time (sec)	N/A	0.064	0.038	0.055	0.349	0.374	0.000	5.251	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	38	58	45	0	132	-1
N.S.	1	1.00	1.00	1.19	1.81	1.41	0.00	4.12	-0.03
time (sec)	N/A	0.050	0.018	0.048	0.343	0.377	0.000	7.818	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	43	29	17	42	-1
N.S.	1	1.00	1.00	1.05	2.05	1.38	0.81	2.00	-0.05
time (sec)	N/A	0.019	0.027	0.039	0.329	0.359	0.614	6.514	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	14	14	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.17	1.17	1.17	1.00
time (sec)	N/A	0.010	0.012	0.025	0.283	0.386	0.326	3.489	4.528

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	42	50	33	29	48	29
N.S.	1	1.00	1.00	1.45	1.72	1.14	1.00	1.66	1.00
time (sec)	N/A	0.017	0.004	0.043	0.333	0.355	0.512	2.669	4.539

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	95	51	44	46	106	46
N.S.	1	1.00	0.84	2.11	1.13	0.98	1.02	2.36	1.02
time (sec)	N/A	0.032	0.038	0.044	0.337	0.344	0.768	3.639	4.631

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	165	50	52	61	191	64
N.S.	1	1.00	1.00	2.70	0.82	0.85	1.00	3.13	1.05
time (sec)	N/A	0.046	0.007	0.062	0.347	0.355	1.044	4.060	4.774

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	96	99	109	0	442	-1
N.S.	1	1.00	0.89	0.99	1.02	1.12	0.00	4.56	-0.01
time (sec)	N/A	0.125	0.120	0.096	0.363	0.391	0.000	4.933	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	76	87	90	0	283	-1
N.S.	1	1.00	1.00	1.17	1.34	1.38	0.00	4.35	-0.02
time (sec)	N/A	0.073	0.117	0.081	0.345	0.354	0.000	4.916	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	52	66	56	0	153	-1
N.S.	1	1.00	1.00	1.27	1.61	1.37	0.00	3.73	-0.02
time (sec)	N/A	0.062	0.061	0.057	0.366	0.359	0.000	5.271	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	36	51	39	31	65	-1
N.S.	1	1.00	0.86	0.97	1.38	1.05	0.84	1.76	-0.03
time (sec)	N/A	0.033	0.043	0.048	0.336	0.361	1.354	4.772	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	34	25	34	262	29	22
N.S.	1	1.00	1.03	1.10	0.81	1.10	8.45	0.94	0.71
time (sec)	N/A	0.019	0.040	0.044	0.292	0.366	1.251	6.406	4.571

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	97	68	60	391	77	41
N.S.	1	1.00	0.84	1.90	1.33	1.18	7.67	1.51	0.80
time (sec)	N/A	0.027	0.054	0.054	0.372	0.358	1.609	6.717	4.615

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	54	197	69	72	654	153	64
N.S.	1	1.00	0.62	2.26	0.79	0.83	7.52	1.76	0.74
time (sec)	N/A	0.044	0.090	0.058	0.397	0.367	2.255	3.550	4.698

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	65	334	68	90	726	255	84
N.S.	1	1.00	0.61	3.12	0.64	0.84	6.79	2.38	0.79
time (sec)	N/A	0.054	0.131	0.073	0.334	0.366	3.434	3.761	4.720

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	59	127	74	0	0	-1
N.S.	1	1.00	1.01	0.74	1.59	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.091	0.072	0.330	0.368	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	43	29	0	0	-1
N.S.	1	1.00	1.00	0.88	1.72	1.16	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.028	0.045	0.347	0.439	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	47	98	64	0	0	55
N.S.	1	1.00	0.81	0.63	1.31	0.85	0.00	0.00	0.73
time (sec)	N/A	0.024	0.069	0.039	0.336	0.375	0.000	0.000	4.801

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	20	17	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.33	1.13	0.87
time (sec)	N/A	0.011	0.012	0.031	0.304	0.365	0.633	4.681	4.671

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	89	65	74	85	0	0	-1
N.S.	1	1.00	0.92	0.67	0.76	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.110	0.053	0.342	0.391	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	1.00	0.75	0.75
time (sec)	N/A	0.007	0.010	0.011	0.280	0.371	0.099	4.488	4.568

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	15	15	15	29	15	14
N.S.	1	1.00	1.10	0.71	0.71	0.71	1.38	0.71	0.67
time (sec)	N/A	0.012	0.018	0.030	0.310	0.384	0.183	4.019	4.736

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.008	0.012	0.015	0.338	0.413	0.086	4.509	4.637

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	52	30	37	379	30	34
N.S.	1	1.00	0.59	0.75	0.43	0.54	5.49	0.43	0.49
time (sec)	N/A	0.026	0.034	0.030	0.314	0.367	0.522	5.457	4.771

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	62	59	47	51	80	47	58
N.S.	1	1.00	0.71	0.68	0.54	0.59	0.92	0.54	0.67
time (sec)	N/A	0.044	0.039	0.022	0.298	0.390	2.965	5.366	4.779

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.670	0.033	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.967	0.037	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.119	0.043	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.607	0.042	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	148	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.306	0.046	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.892	0.044	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	91	35	0	0	-1
N.S.	1	1.00	0.92	0.96	3.64	1.40	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.038	0.066	0.424	0.383	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	100	48	0	0	-1
N.S.	1	1.00	0.86	0.93	2.33	1.12	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.058	0.060	0.437	0.390	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	181	74	0	0	-1
N.S.	1	1.00	0.81	0.78	2.70	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.075	0.056	0.502	0.354	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	189	87	0	0	-1
N.S.	1	1.00	0.84	0.84	2.39	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.077	0.058	0.518	0.379	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	95	74	0	0	0	0	-1
N.S.	1	1.00	1.09	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.061	0.066	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.167	0.046	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	177	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.204	0.073	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	118	110	0	0	0	0	-1
N.S.	1	1.00	1.08	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.144	0.075	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	129	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.377	0.058	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	225	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.414	0.123	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	44	32	32	53	0	-1
N.S.	1	1.00	0.86	1.26	0.91	0.91	1.51	0.00	-0.03
time (sec)	N/A	0.020	0.048	0.049	0.318	0.371	29.247	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	44	29	29	53	0	-1
N.S.	1	1.00	0.85	1.29	0.85	0.85	1.56	0.00	-0.03
time (sec)	N/A	0.020	0.044	0.056	0.294	0.360	29.005	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	44	0	62	0	0	-1
N.S.	1	1.00	1.02	0.96	0.00	1.35	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.056	0.078	0.000	0.367	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	66	0	73	0	0	-1
N.S.	1	1.00	0.87	0.99	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.091	0.070	0.000	0.516	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	99	0	127	0	0	-1
N.S.	1	1.00	0.84	0.88	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.133	0.081	0.000	0.586	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	65	0	90	0	0	-1
N.S.	1	1.00	0.87	0.83	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.089	0.060	0.000	0.528	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	89	0	107	0	0	-1
N.S.	1	1.00	0.86	0.94	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.120	0.069	0.000	0.431	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	141	144	0	183	0	0	-1
N.S.	1	1.00	0.85	0.87	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.194	0.085	0.000	0.555	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	173	586	973	259	0	1021	231
N.S.	1	1.00	0.78	2.63	4.36	1.16	0.00	4.58	1.04
time (sec)	N/A	0.216	0.656	0.152	1.508	0.380	0.000	5.624	4.706

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	117	291	564	165	0	669	136
N.S.	1	1.00	0.78	1.94	3.76	1.10	0.00	4.46	0.91
time (sec)	N/A	0.113	0.410	0.083	1.083	0.484	0.000	4.593	0.187

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	120	272	81	0	367	58
N.S.	1	1.00	0.96	1.74	3.94	1.17	0.00	5.32	0.84
time (sec)	N/A	0.050	0.131	0.043	0.757	0.415	0.000	2.725	0.106

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	42	53	45	0	143	41
N.S.	1	1.00	1.00	1.08	1.36	1.15	0.00	3.67	1.05
time (sec)	N/A	0.006	0.013	0.031	0.300	0.387	0.000	3.284	0.084

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	3.417	0.032	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.008	5.937	0.033	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	440	365	0	448	0	0	-1
N.S.	1	1.00	1.31	1.08	0.00	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.511	0.132	0.000	0.424	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	265	225	0	303	0	0	-1
N.S.	1	1.00	1.14	0.97	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.304	0.088	0.000	0.400	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	101	0	158	0	0	-1
N.S.	1	1.00	0.79	0.84	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.199	0.049	0.000	0.370	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	52	0	73	0	0	52
N.S.	1	1.00	1.00	0.87	0.00	1.22	0.00	0.00	0.87
time (sec)	N/A	0.023	0.027	0.036	0.000	0.371	0.000	0.000	5.196

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.008	1.054	0.046	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.008	14.508	0.046	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	218	1248	1823	338	0	1071	-1
N.S.	1	1.00	0.64	3.66	5.35	0.99	0.00	3.14	-0.00
time (sec)	N/A	0.410	1.839	0.122	2.306	0.376	0.000	6.832	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	151	669	1038	213	0	705	-1
N.S.	1	1.00	0.59	2.61	4.05	0.83	0.00	2.75	-0.00
time (sec)	N/A	0.252	1.138	0.086	1.336	0.364	0.000	3.001	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	309	484	134	0	389	-1
N.S.	1	1.00	0.93	2.53	3.97	1.10	0.00	3.19	-0.01
time (sec)	N/A	0.132	0.407	0.046	0.914	0.367	0.000	3.828	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	136	69	89	0	151	95
N.S.	1	1.00	0.81	1.64	0.83	1.07	0.00	1.82	1.14
time (sec)	N/A	0.030	0.041	0.042	0.317	0.354	0.000	4.384	0.046

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	9.113	0.036	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	13.753	0.035	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	0	0	0	424	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.316	101.476	0.060	0.000	0.115	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	0	0	0	321	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.183	42.134	0.051	0.000	0.111	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	0	0	0	229	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	0.129	33.863	0.041	0.000	0.103	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	0	0	107	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.011	0.026	0.000	0.094	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	45.137	0.037	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	93.287	0.043	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	467	302	0	436	0	0	-1
N.S.	1	1.00	1.26	0.81	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.912	0.126	0.000	0.410	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	242	150	0	266	0	0	-1
N.S.	1	1.00	1.22	0.76	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.500	0.058	0.000	0.372	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	100	80	0	137	0	0	-1
N.S.	1	1.00	0.95	0.76	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.116	0.038	0.000	0.374	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.010	3.817	0.044	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	17.985	0.046	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	405	0	0	489	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.204	1.589	0.051	0.000	0.133	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	700	0	0	326	0	0	-1
N.S.	1	1.00	2.98	0.00	0.00	1.39	0.00	0.00	-0.00
time (sec)	N/A	0.097	1.495	0.036	0.000	0.117	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	203	0	0	175	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.296	0.014	0.000	0.099	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	3.942	0.044	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	20.459	0.058	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	138	1246	1105	196	529	701	-1
N.S.	1	1.00	0.34	3.04	2.70	0.48	1.29	1.71	-0.00
time (sec)	N/A	0.276	1.001	0.056	0.368	0.349	0.385	2.464	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	85	366	350	88	221	219	-1
N.S.	1	1.00	0.46	1.98	1.89	0.48	1.19	1.18	-0.01
time (sec)	N/A	0.114	0.277	0.022	0.294	0.369	0.224	5.960	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	61	62	44	65	44	43
N.S.	1	1.00	0.93	1.13	1.15	0.81	1.20	0.81	0.80
time (sec)	N/A	0.019	0.056	0.014	0.278	0.373	0.151	6.440	4.730

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	238	793	0	266	0	0	-1
N.S.	1	1.00	1.00	3.33	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.516	0.636	0.058	0.000	0.497	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	397	1831	0	446	0	0	-1
N.S.	1	1.00	1.17	5.40	0.00	1.32	0.00	0.00	-0.00
time (sec)	N/A	0.678	2.244	0.067	0.000	0.469	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	383	0	695	281	0	0	-1
N.S.	1	1.00	1.00	0.00	1.82	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.213	2.996	0.024	0.647	0.134	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	705	0	377	187	0	0	-1
N.S.	1	1.00	2.42	0.00	1.30	0.64	0.00	0.00	-0.00
time (sec)	N/A	0.132	1.654	0.016	0.633	0.132	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	0	112	69	0	0	-1
N.S.	1	1.00	1.07	0.00	0.97	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.103	0.000	0.352	0.102	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	5.887	0.019	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	8.434	0.017	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	557	696	878	457	0	6606	-1
N.S.	1	1.00	0.91	1.14	1.44	0.75	0.00	10.81	-0.00
time (sec)	N/A	0.546	1.356	0.507	0.777	0.427	0.000	5.785	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	367	295	409	243	0	2159	-1
N.S.	1	1.00	1.22	0.98	1.36	0.81	0.00	7.17	-0.00
time (sec)	N/A	0.274	0.412	0.030	0.531	0.375	0.000	5.167	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	84	124	125	0	413	-1
N.S.	1	1.00	1.05	0.89	1.32	1.33	0.00	4.39	-0.01
time (sec)	N/A	0.077	0.056	0.023	0.389	0.412	0.000	4.286	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	0	441	0	330	0	0	-1
N.S.	1	1.00	0.00	1.60	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.847	9.708	0.056	0.000	0.376	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	0	2734	0	477	0	0	-1
N.S.	1	1.00	0.00	7.81	0.00	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.736	180.041	0.060	0.000	0.476	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	518	0	994	510	0	0	-1
N.S.	1	1.00	1.33	0.00	2.55	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.290	3.104	0.036	0.781	0.172	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	835	0	505	333	0	0	-1
N.S.	1	1.00	3.33	0.00	2.01	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.152	1.744	0.027	0.626	0.113	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	166	0	151	144	0	0	-1
N.S.	1	1.00	1.44	0.00	1.31	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.281	0.006	0.358	0.115	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	9.358	0.035	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	11.725	0.034	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	256	2704	2157	336	0	1558	-1
N.S.	1	1.00	0.40	4.27	3.41	0.53	0.00	2.46	-0.00
time (sec)	N/A	0.436	1.519	0.060	0.426	0.365	0.000	5.082	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	147	801	684	145	0	454	-1
N.S.	1	1.00	0.51	2.78	2.38	0.50	0.00	1.58	-0.00
time (sec)	N/A	0.184	0.391	0.016	0.337	0.369	0.000	5.118	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	134	120	58	94	82	69
N.S.	1	1.00	0.76	1.58	1.41	0.68	1.11	0.96	0.81
time (sec)	N/A	0.037	0.077	0.014	0.320	0.347	0.266	4.028	4.606

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	118	327	0	460	0	0	-1
N.S.	1	1.00	0.30	0.83	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.956	0.360	0.062	0.000	0.417	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	180	1176	0	760	0	0	-1
N.S.	1	1.00	0.32	2.12	0.00	1.37	0.00	0.00	-0.00
time (sec)	N/A	1.446	0.697	0.073	0.000	0.426	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	510	395	562	314	0	777	-1
N.S.	1	1.00	0.99	0.77	1.10	0.61	0.00	1.51	-0.00
time (sec)	N/A	0.389	1.784	0.066	0.357	0.399	0.000	4.476	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	213	175	249	164	0	407	-1
N.S.	1	1.00	0.88	0.72	1.02	0.67	0.00	1.67	-0.00
time (sec)	N/A	0.181	0.550	0.012	0.317	0.374	0.000	4.380	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	86	92	98	0	170	-1
N.S.	1	1.00	0.88	0.66	0.71	0.75	0.00	1.31	-0.01
time (sec)	N/A	0.053	0.106	0.010	0.328	0.372	0.000	3.692	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	14.038	0.018	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	15.075	0.017	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	929	936	1004	647	0	11930	-1
N.S.	1	1.00	1.09	1.09	1.17	0.76	0.00	13.95	-0.00
time (sec)	N/A	0.708	2.832	0.480	0.812	0.402	0.000	6.991	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	540	391	460	305	0	3728	-1
N.S.	1	1.00	1.29	0.93	1.10	0.73	0.00	8.90	-0.00
time (sec)	N/A	0.345	0.564	0.031	0.559	0.362	0.000	4.928	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	108	138	139	0	663	-1
N.S.	1	1.00	0.98	0.79	1.01	1.02	0.00	4.88	-0.01
time (sec)	N/A	0.099	0.076	0.020	0.382	0.377	0.000	5.082	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	170	156	0	566	0	0	-1
N.S.	1	1.00	0.39	0.36	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	1.324	0.483	0.065	0.000	0.423	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	313	1554	0	827	0	0	-1
N.S.	1	1.00	0.55	2.75	0.00	1.46	0.00	0.00	-0.00
time (sec)	N/A	1.665	0.763	0.086	0.000	0.444	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	613	452	1261	501	0	0	-1
N.S.	1	1.00	0.97	0.72	2.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.518	1.784	0.064	0.797	0.415	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	378	225	586	273	0	0	-1
N.S.	1	1.00	1.19	0.71	1.84	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.260	0.725	0.017	0.543	0.394	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	146	105	219	143	0	0	-1
N.S.	1	1.00	1.04	0.74	1.55	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.109	0.014	0.408	0.377	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	18.729	0.032	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	180.031	0.031	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	226	0	195	229	0	566	-1
N.S.	1	1.00	0.78	0.00	0.67	0.79	0.00	1.96	-0.00
time (sec)	N/A	0.176	0.380	0.018	0.495	0.757	0.000	4.271	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	111	0	191	141	0	310	-1
N.S.	1	1.00	0.55	0.00	0.95	0.70	0.00	1.53	-0.00
time (sec)	N/A	0.120	0.202	0.017	0.482	0.799	0.000	3.497	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	97	0	155	126	0	196	-1
N.S.	1	1.00	0.61	0.00	0.97	0.79	0.00	1.22	-0.01
time (sec)	N/A	0.091	0.153	0.017	0.494	0.739	0.000	3.703	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	70	0	128	66	0	83	-1
N.S.	1	1.00	0.82	0.00	1.51	0.78	0.00	0.98	-0.01
time (sec)	N/A	0.046	0.055	0.017	0.465	0.747	0.000	5.917	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	22	34	0	35	-1
N.S.	1	1.00	1.00	0.00	0.52	0.81	0.00	0.83	-0.02
time (sec)	N/A	0.031	0.051	0.015	0.317	0.353	0.000	6.507	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	85	0	125	0	0	0	-1
N.S.	1	1.00	0.71	0.00	1.04	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.103	0.017	0.489	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	115	0	128	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.73	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.119	0.014	0.507	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	184	0	128	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.48	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.240	0.017	0.484	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	425	0	0	713	-1
N.S.	1	1.00	0.66	0.00	1.59	0.00	0.00	2.67	-0.00
time (sec)	N/A	0.198	0.511	0.016	0.699	0.000	0.000	3.489	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	160	0	422	0	0	321	-1
N.S.	1	1.00	0.70	0.00	1.86	0.00	0.00	1.41	-0.00
time (sec)	N/A	0.141	0.311	0.016	0.678	0.000	0.000	5.865	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	0	128	71	0	265	-1
N.S.	1	1.00	0.81	0.00	1.44	0.80	0.00	2.98	-0.01
time (sec)	N/A	0.057	0.043	0.017	0.453	0.749	0.000	6.941	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	22	34	0	52	-1
N.S.	1	1.00	1.00	0.00	0.50	0.77	0.00	1.18	-0.02
time (sec)	N/A	0.042	0.053	0.016	0.301	0.363	0.000	6.306	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	96	0	486	0	0	84	-1
N.S.	1	1.00	0.72	0.00	3.65	0.00	0.00	0.63	-0.01
time (sec)	N/A	0.089	0.104	0.016	0.582	0.000	0.000	6.297	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	133	0	379	0	0	0	-1
N.S.	1	1.00	0.79	0.00	2.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.168	0.014	0.581	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	87	0	125	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.99	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.107	0.016	0.496	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	208	0	128	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.52	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.235	0.025	0.464	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	131	0	171	0	0	0	-1
N.S.	1	1.00	0.78	0.00	1.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.121	0.023	0.450	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	88	0	152	0	0	0	-1
N.S.	1	1.00	0.76	0.00	1.31	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.123	0.018	0.570	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	30	48	0	0	-1
N.S.	1	1.00	0.93	0.00	0.67	1.07	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.049	0.021	0.361	0.352	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	0	170	94	0	0	-1
N.S.	1	1.00	0.79	0.00	1.87	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.062	0.020	0.499	0.799	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	107	0	1400	151	0	0	-1
N.S.	1	1.00	0.62	0.00	8.14	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.115	0.020	1.389	0.857	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	112	0	935	172	0	0	-1
N.S.	1	1.00	0.52	0.00	4.31	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.193	0.022	1.080	0.897	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	237	0	1231	0	0	0	-1
N.S.	1	1.00	0.79	0.00	4.12	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.609	0.020	1.076	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	228	0	823	0	0	0	-1
N.S.	1	1.00	0.87	0.00	3.14	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.235	0.020	0.832	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	113	0	128	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.199	0.020	0.511	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	90	0	125	0	0	0	-1
N.S.	1	1.00	0.74	0.00	1.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.132	0.026	0.456	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	136	0	382	0	0	0	-1
N.S.	1	1.00	0.83	0.00	2.33	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.211	0.026	0.571	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	96	0	486	0	0	0	-1
N.S.	1	1.00	0.68	0.00	3.45	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.111	0.025	0.615	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	30	48	0	0	-1
N.S.	1	1.00	0.94	0.00	0.64	1.02	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.049	0.020	0.298	0.368	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	72	0	128	110	0	0	-1
N.S.	1	1.00	0.76	0.00	1.35	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.071	0.020	0.460	0.808	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	165	0	442	0	0	0	-1
N.S.	1	1.00	0.70	0.00	1.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.616	0.022	0.607	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	192	0	402	0	0	0	-1
N.S.	1	1.00	0.69	0.00	1.45	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.849	0.020	0.660	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.006	5.750	0.012	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	9.178	0.006	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	5.202	0.006	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	192	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.096	0.592	0.020	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.029	0.013	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.006	1.139	0.008	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.006	1.100	0.007	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	539	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	10.906	0.023	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	403	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	5.690	0.016	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	215	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.326	0.049	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.175	0.035	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	1.859	0.022	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.013	1.399	0.017	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	856	786	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.653	16.953	0.151	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	552	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	27.062	0.198	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	247	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.533	0.080	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	2.461	0.190	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	2.087	0.141	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.945	0.037	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.010	0.893	0.055	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.004	0.190	0.045	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.449	0.055	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.565	0.027	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.020	180.091	0.268	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.010	180.072	0.160	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.004	7.469	0.264	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	137.981	0.163	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	180.046	0.171	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	1.399	0.036	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	150	209	258	223	0	1264	-1
N.S.	1	1.00	0.67	0.93	1.15	1.00	0.00	5.64	-0.00
time (sec)	N/A	0.297	0.380	0.174	0.382	0.356	0.000	7.740	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	79	115	155	138	0	530	-1
N.S.	1	1.00	0.67	0.97	1.31	1.17	0.00	4.49	-0.01
time (sec)	N/A	0.156	0.154	0.062	0.348	0.370	0.000	5.748	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	50	43	65	52	0	137	-1
N.S.	1	1.00	1.32	1.13	1.71	1.37	0.00	3.61	-0.03
time (sec)	N/A	0.053	0.023	0.055	0.318	0.393	0.000	4.055	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	83	164	0	135	0	172	-1
N.S.	1	1.00	0.81	1.59	0.00	1.31	0.00	1.67	-0.01
time (sec)	N/A	0.183	0.126	0.088	0.000	0.361	0.000	3.207	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	149	0	165	0	347	-1
N.S.	1	1.00	0.90	1.59	0.00	1.76	0.00	3.69	-0.01
time (sec)	N/A	0.132	0.483	0.100	0.000	0.373	0.000	5.634	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	151	542	0	419	0	1502	-1
N.S.	1	1.00	0.65	2.33	0.00	1.80	0.00	6.45	-0.00
time (sec)	N/A	0.297	1.222	0.122	0.000	0.387	0.000	7.646	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	252	265	324	309	0	1145	-1
N.S.	1	1.00	0.99	1.04	1.28	1.22	0.00	4.51	-0.00
time (sec)	N/A	0.406	0.289	0.119	0.427	0.417	0.000	5.357	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	105	110	137	130	0	305	-1
N.S.	1	1.00	1.12	1.17	1.46	1.38	0.00	3.24	-0.01
time (sec)	N/A	0.152	0.095	0.072	0.337	0.376	0.000	4.835	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	195	354	0	289	0	368	-1
N.S.	1	1.00	0.76	1.39	0.00	1.13	0.00	1.44	-0.00
time (sec)	N/A	0.447	0.235	0.132	0.000	0.385	0.000	3.882	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	263	313	0	348	0	700	-1
N.S.	1	1.00	1.35	1.61	0.00	1.78	0.00	3.59	-0.01
time (sec)	N/A	0.274	0.942	0.140	0.000	0.418	0.000	7.175	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	740	1139	0	910	0	3062	-1
N.S.	1	1.00	1.57	2.42	0.00	1.94	0.00	6.51	-0.00
time (sec)	N/A	0.641	2.216	0.198	0.000	0.444	0.000	7.917	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.504	0.108	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.286	0.082	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.028	0.024	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	74.513	0.603	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	16.550	1.158	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	2.211	0.124	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	1.674	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	66.460	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	1.003	0.034	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	0	0	80	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.085	0.115	0.000	0.097	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	47	151	146	74	129	0	109
N.S.	1	1.00	0.49	1.57	1.52	0.77	1.34	0.00	1.14
time (sec)	N/A	0.143	0.144	0.109	0.564	0.347	2.494	0.000	5.706

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	40	133	99	64	107	0	88
N.S.	1	1.00	0.54	1.80	1.34	0.86	1.45	0.00	1.19
time (sec)	N/A	0.117	0.156	0.129	0.545	0.349	1.315	0.000	5.489

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	117	60	55	70	0	63
N.S.	1	1.00	0.67	2.60	1.33	1.22	1.56	0.00	1.40
time (sec)	N/A	0.084	0.096	0.140	0.546	0.348	0.774	0.000	5.139

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	105	31	43	49	0	49
N.S.	1	1.00	1.00	4.20	1.24	1.72	1.96	0.00	1.96
time (sec)	N/A	0.012	0.045	0.197	0.742	0.356	0.465	0.000	4.847

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	228	42	80	0	0	-1
N.S.	1	1.00	0.65	4.15	0.76	1.45	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.034	0.125	0.581	0.355	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	155	229	112	0	0	-1
N.S.	1	1.00	0.66	2.01	2.97	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.131	0.134	0.692	0.357	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	69	183	256	140	0	0	-1
N.S.	1	1.00	0.59	1.58	2.21	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.095	0.109	0.583	0.360	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	138	0	0	98	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.203	0.112	0.000	0.104	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	38	135	32	67	85	0	71
N.S.	1	1.00	0.66	2.33	0.55	1.16	1.47	0.00	1.22
time (sec)	N/A	0.126	0.065	0.155	0.531	0.353	2.373	0.000	5.186

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	105	240	73	156	0	0	-1
N.S.	1	1.00	0.68	1.55	0.47	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.184	0.172	0.556	0.355	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	119	16	51	63	0	53
N.S.	1	1.00	1.00	3.84	0.52	1.65	2.03	0.00	1.71
time (sec)	N/A	0.062	0.038	0.215	0.532	0.375	0.808	0.000	4.822

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	80	157	51	128	0	0	-1
N.S.	1	1.00	0.68	1.34	0.44	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.081	0.170	0.553	0.369	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	268	47	94	0	0	-1
N.S.	1	1.00	0.64	3.67	0.64	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.036	0.158	0.601	0.347	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	105	232	76	150	0	0	-1
N.S.	1	1.00	0.78	1.72	0.56	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.192	0.175	0.610	0.367	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	214	52	138	0	0	-1
N.S.	1	1.00	0.68	2.18	0.53	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.086	0.158	0.605	0.371	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	142	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.216	0.190	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.133	0.177	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.128	0.201	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.124	0.161	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	119	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.092	0.153	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	280	144	98	0	0	-1
N.S.	1	1.00	0.64	3.84	1.97	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.048	0.303	0.660	0.354	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.112	0.155	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	114	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.115	0.172	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	142	0	0	112	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.365	0.101	0.000	0.104	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	79	208	286	111	0	0	-1
N.S.	1	1.00	0.48	1.26	1.73	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.202	0.110	0.573	0.360	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	69	190	219	95	0	0	-1
N.S.	1	1.00	0.50	1.37	1.58	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.192	0.119	0.565	0.348	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	55	174	162	82	0	0	-1
N.S.	1	1.00	0.70	2.20	2.05	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.123	0.161	0.533	0.356	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	158	116	60	0	0	-1
N.S.	1	1.00	0.85	2.87	2.11	1.09	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.066	0.232	0.528	0.348	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	50	283	52	88	0	0	-1
N.S.	1	1.00	0.51	2.86	0.53	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.061	0.125	0.572	0.380	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	65	211	265	108	0	0	-1
N.S.	1	1.00	0.76	2.45	3.08	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.104	0.118	0.574	0.366	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	85	238	296	142	0	0	-1
N.S.	1	1.00	0.71	2.00	2.49	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.139	0.107	0.633	0.393	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	189	0	0	130	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.596	0.131	0.000	0.111	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	200	47	96	0	0	-1
N.S.	1	1.00	0.74	2.20	0.52	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.158	0.172	0.569	0.362	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	113	309	99	146	0	0	-1
N.S.	1	1.00	0.58	1.58	0.51	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.196	0.203	0.562	0.393	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	182	28	72	0	0	-1
N.S.	1	1.00	0.85	2.80	0.43	1.11	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.082	0.200	0.575	0.346	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	93	224	76	114	0	0	-1
N.S.	1	1.00	0.63	1.51	0.51	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.070	0.172	0.622	0.380	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	60	331	55	100	0	0	-1
N.S.	1	1.00	0.52	2.88	0.48	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.069	0.168	0.664	0.382	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	107	301	90	127	0	0	-1
N.S.	1	1.00	0.81	2.28	0.68	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.137	0.184	0.620	0.361	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	79	277	64	132	0	0	-1
N.S.	1	1.00	0.49	1.72	0.40	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.101	0.171	0.610	0.379	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.527	0.157	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	161	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.382	0.157	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	168	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.381	0.137	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	160	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.382	0.138	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	149	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.178	0.163	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	343	153	106	0	0	-1
N.S.	1	1.00	0.52	2.83	1.26	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.100	0.296	0.686	0.378	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	125	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.247	0.142	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	129	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.255	0.142	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [227] had the largest ratio of [27]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	16	0.250
2	A	5	4	1.00	16	0.250
3	A	4	3	1.00	14	0.214
4	A	5	4	1.00	16	0.250
5	A	7	6	1.00	16	0.375
6	A	8	6	1.00	16	0.375
7	A	7	6	1.00	16	0.375
8	A	6	5	1.00	16	0.312
9	A	4	3	1.00	12	0.250
10	A	6	5	1.00	16	0.312
11	A	7	6	1.00	16	0.375
12	A	10	8	1.00	18	0.444
13	A	7	6	1.00	18	0.333
14	A	2	2	1.00	16	0.125
15	A	9	6	1.00	18	0.333
16	A	13	8	1.00	18	0.444
17	A	15	8	1.00	18	0.444
18	A	13	8	1.00	18	0.444
19	A	11	8	1.00	18	0.444
20	A	8	5	1.00	14	0.357
21	A	11	8	1.00	18	0.444
22	A	13	8	1.00	18	0.444
23	A	7	5	1.00	14	0.357
24	A	4	4	1.00	14	0.286
25	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	4	1.00	14	0.286
27	A	12	6	1.00	14	0.429
28	A	10	5	1.00	14	0.357
29	A	8	4	1.00	10	0.400
30	A	9	5	1.00	14	0.357
31	A	6	3	1.00	10	0.300
32	A	7	4	1.00	14	0.286
33	A	3	2	1.00	12	0.167
34	A	8	6	1.00	12	0.500
35	A	11	7	1.00	18	0.389
36	A	9	6	1.00	18	0.333
37	A	4	4	1.00	16	0.250
38	A	0	0	0.00	0	0.000
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	0	0	0.00	0	0.000
43	A	19	11	1.00	18	0.611
44	A	12	9	1.00	18	0.500
45	A	6	6	1.00	16	0.375
46	A	0	0	0.00	0	0.000
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	13	5	1.00	20	0.250
53	A	9	5	1.00	20	0.250
54	A	5	3	1.00	18	0.167
55	A	0	0	0.00	0	0.000
56	A	0	0	0.00	0	0.000
57	A	5	4	1.00	16	0.250
58	A	4	3	1.00	16	0.188
59	A	5	4	1.00	16	0.250
60	A	7	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	6	4	1.00	16	0.250
62	A	5	3	1.00	14	0.214
63	A	6	4	1.00	16	0.250
64	A	7	5	1.00	16	0.312
65	A	6	4	1.00	16	0.250
66	A	4	2	1.00	12	0.167
67	A	6	4	1.00	16	0.250
68	A	7	5	1.00	16	0.312
69	A	7	6	1.00	18	0.333
70	A	2	2	1.00	18	0.111
71	A	9	6	1.00	18	0.333
72	A	13	8	1.00	18	0.444
73	A	11	7	1.00	18	0.389
74	A	9	5	1.00	16	0.312
75	A	11	7	0.99	18	0.389
76	A	13	7	0.99	18	0.389
77	A	11	7	1.00	18	0.389
78	A	8	4	1.00	14	0.286
79	A	11	7	0.99	18	0.389
80	A	13	7	0.99	18	0.389
81	A	9	6	1.00	18	0.333
82	A	4	4	1.00	18	0.222
83	A	0	0	0.00	0	0.000
84	A	0	0	0.00	0	0.000
85	A	0	0	0.00	0	0.000
86	A	0	0	0.00	0	0.000
87	A	0	0	0.00	0	0.000
88	A	0	0	0.00	0	0.000
89	A	12	9	1.00	18	0.500
90	A	6	6	1.00	18	0.333
91	A	0	0	0.00	0	0.000
92	A	0	0	0.00	0	0.000
93	A	0	0	0.00	0	0.000
94	A	0	0	0.00	0	0.000
95	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	0	0	0.00	0	0.000
97	A	0	0	0.00	0	0.000
98	A	13	5	1.00	20	0.250
99	A	9	5	1.00	20	0.250
100	A	5	3	1.00	18	0.167
101	A	0	0	0.00	0	0.000
102	A	0	0	0.00	0	0.000
103	A	7	5	1.00	12	0.417
104	A	6	5	1.00	10	0.500
105	A	5	5	1.00	8	0.625
106	A	3	3	1.00	12	0.250
107	A	2	2	1.00	12	0.167
108	A	3	3	1.00	12	0.250
109	A	4	3	1.00	12	0.250
110	A	5	3	1.00	12	0.250
111	A	9	6	1.00	14	0.429
112	A	8	8	1.00	12	0.667
113	A	6	6	1.00	10	0.600
114	A	5	4	1.00	14	0.286
115	A	3	3	1.00	14	0.214
116	A	3	3	1.00	14	0.214
117	A	5	5	1.00	14	0.357
118	A	5	4	1.00	14	0.286
119	A	5	5	1.00	8	0.625
120	A	3	3	1.00	12	0.250
121	A	4	4	1.00	12	0.333
122	A	2	2	1.00	12	0.167
123	A	5	5	1.00	12	0.417
124	A	2	2	1.00	12	0.167
125	A	3	2	1.00	14	0.143
126	A	3	3	1.00	6	0.500
127	A	5	5	1.00	8	0.625
128	A	7	5	1.00	8	0.625
129	A	0	0	0.00	0	0.000
130	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	1.00	20	0.150
132	A	0	0	0.00	0	0.000
133	A	5	5	1.00	22	0.227
134	A	0	0	0.00	0	0.000
135	A	3	3	1.00	12	0.250
136	A	5	4	1.00	14	0.286
137	A	8	4	1.00	14	0.286
138	A	8	4	1.00	14	0.286
139	A	3	2	1.00	8	0.250
140	A	5	3	1.00	10	0.300
141	A	8	3	1.00	10	0.300
142	A	3	2	1.00	12	0.167
143	A	5	3	1.00	14	0.214
144	A	8	3	1.00	14	0.214
145	A	3	3	1.00	16	0.188
146	A	3	3	1.00	16	0.188
147	A	5	5	1.00	16	0.312
148	A	7	6	1.00	18	0.333
149	A	12	6	1.00	18	0.333
150	A	6	5	1.00	16	0.312
151	A	8	6	1.00	18	0.333
152	A	14	6	1.00	18	0.333
153	A	10	8	1.00	18	0.444
154	A	7	6	1.00	18	0.333
155	A	5	4	1.00	16	0.250
156	A	1	1	1.00	10	0.100
157	A	0	0	0.00	0	0.000
158	A	0	0	0.00	0	0.000
159	A	16	11	1.00	18	0.611
160	A	12	10	1.00	18	0.556
161	A	8	7	1.00	16	0.438
162	A	3	3	1.00	10	0.300
163	A	0	0	0.00	0	0.000
164	A	0	0	0.00	0	0.000
165	A	14	10	1.00	20	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	11	8	1.00	20	0.400
167	A	7	6	1.00	18	0.333
168	A	3	3	1.00	12	0.250
169	A	0	0	0.00	0	0.000
170	A	0	0	0.00	0	0.000
171	A	14	9	1.00	20	0.450
172	A	10	7	1.00	20	0.350
173	A	8	5	1.00	18	0.278
174	A	3	2	1.00	12	0.167
175	A	0	0	0.00	0	0.000
176	A	0	0	0.00	0	0.000
177	A	18	14	1.00	20	0.700
178	A	12	11	1.00	18	0.611
179	A	5	5	1.00	12	0.417
180	A	0	0	0.00	0	0.000
181	A	0	0	0.00	0	0.000
182	A	13	10	1.00	20	0.500
183	A	8	5	1.00	18	0.278
184	A	3	2	1.00	12	0.167
185	A	0	0	0.00	0	0.000
186	A	0	0	0.00	0	0.000
187	A	14	3	1.00	22	0.136
188	A	8	3	1.00	20	0.150
189	A	3	3	1.00	14	0.214
190	A	8	4	1.00	22	0.182
191	A	10	6	1.00	22	0.273
192	A	12	9	1.00	22	0.409
193	A	9	6	1.00	20	0.300
194	A	4	3	1.00	14	0.214
195	A	0	0	0.00	0	0.000
196	A	0	0	0.00	0	0.000
197	A	23	5	1.00	22	0.227
198	A	14	5	1.00	20	0.250
199	A	6	5	1.00	14	0.357
200	A	13	5	1.00	22	0.227

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	10	6	1.00	22	0.273
202	A	14	8	1.00	22	0.364
203	A	8	3	1.00	20	0.150
204	A	4	3	1.00	14	0.214
205	A	0	0	0.00	0	0.000
206	A	0	0	0.00	0	0.000
207	A	20	4	1.00	22	0.182
208	A	11	4	1.00	20	0.200
209	A	4	3	1.00	14	0.214
210	A	11	4	1.00	22	0.182
211	A	13	6	1.00	22	0.273
212	A	17	10	1.00	22	0.454
213	A	10	8	1.00	20	0.400
214	A	5	5	1.00	14	0.357
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	29	5	1.00	22	0.227
218	A	17	5	1.00	20	0.250
219	A	7	5	1.00	14	0.357
220	A	16	5	1.00	22	0.227
221	A	13	6	1.00	22	0.273
222	A	24	13	1.00	22	0.591
223	A	15	12	1.00	20	0.600
224	A	7	7	1.00	14	0.500
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	9	4	1.00	27	0.148
228	A	7	4	1.00	27	0.148
229	A	6	4	1.00	27	0.148
230	A	4	4	1.00	27	0.148
231	A	3	3	1.00	27	0.111
232	A	6	6	1.00	27	0.222
233	A	7	6	1.00	27	0.222
234	A	9	6	1.00	27	0.222
235	A	9	8	1.00	27	0.296

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	8	8	1.00	27	0.296
237	A	5	5	1.00	27	0.185
238	A	4	4	1.00	27	0.148
239	A	6	6	1.00	27	0.222
240	A	7	7	1.00	27	0.259
241	A	7	7	1.00	27	0.259
242	A	9	6	1.00	27	0.222
243	A	7	6	1.00	27	0.222
244	A	6	6	1.00	27	0.222
245	A	3	3	1.00	27	0.111
246	A	4	4	1.00	27	0.148
247	A	6	4	1.00	27	0.148
248	A	7	4	1.00	27	0.148
249	A	11	9	1.00	27	0.333
250	A	10	9	1.00	27	0.333
251	A	8	7	1.00	27	0.259
252	A	7	7	1.00	27	0.259
253	A	8	8	1.00	27	0.296
254	A	6	6	1.00	27	0.222
255	A	4	4	1.00	27	0.148
256	A	5	5	1.00	27	0.185
257	A	9	9	1.00	27	0.333
258	A	10	9	1.00	27	0.333
259	A	0	0	0.00	0	0.000
260	A	14	5	1.00	16	0.312
261	A	11	5	1.00	16	0.312
262	A	8	5	1.00	14	0.357
263	A	3	2	1.00	12	0.167
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	16	6	1.00	20	0.300
267	A	13	6	1.00	20	0.300
268	A	10	6	1.00	18	0.333
269	A	4	2	1.00	16	0.125
270	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	0	0	0.00	0	0.000
272	A	28	10	1.00	22	0.454
273	A	19	10	1.00	20	0.500
274	A	8	4	1.00	18	0.222
275	A	0	0	0.00	0	0.000
276	A	0	0	0.00	0	0.000
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	0	0	0.00	0	0.000
280	A	0	0	0.00	0	0.000
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	0	0	0.00	0	0.000
285	A	0	0	0.00	0	0.000
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	23	6	1.00	20	0.300
289	A	15	6	1.00	18	0.333
290	A	6	5	1.00	12	0.417
291	A	12	6	1.00	20	0.300
292	A	7	6	1.00	20	0.300
293	A	15	6	1.00	20	0.300
294	A	27	11	1.00	20	0.550
295	A	12	8	1.00	14	0.571
296	A	22	6	1.00	22	0.273
297	A	12	8	1.00	22	0.364
298	A	27	11	1.00	22	0.500
299	A	0	0	0.00	0	0.000
300	A	0	0	0.00	0	0.000
301	A	0	0	0.00	0	0.000
302	A	0	0	0.00	0	0.000
303	A	0	0	0.00	0	0.000
304	A	0	0	0.00	0	0.000
305	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	0	0	0.00	0	0.000
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	4	3	1.00	18	0.167
311	A	5	3	1.00	18	0.167
312	A	4	3	1.00	18	0.167
313	A	3	3	1.00	16	0.188
314	A	2	2	1.00	14	0.143
315	A	4	4	1.00	18	0.222
316	A	5	5	1.00	18	0.278
317	A	6	5	1.00	18	0.278
318	A	4	3	1.00	20	0.150
319	A	4	4	1.00	20	0.200
320	A	5	5	1.00	20	0.250
321	A	3	3	1.00	18	0.167
322	A	4	4	1.00	16	0.250
323	A	4	4	1.00	20	0.200
324	A	5	5	1.00	20	0.250
325	A	6	6	1.00	20	0.300
326	A	4	3	1.00	20	0.150
327	A	4	3	1.00	20	0.150
328	A	4	3	1.00	20	0.150
329	A	4	3	1.00	18	0.167
330	A	4	3	1.00	16	0.188
331	A	4	4	1.00	20	0.200
332	A	4	3	1.00	20	0.150
333	A	4	3	1.00	20	0.150
334	A	6	4	1.00	18	0.222
335	A	5	4	1.00	18	0.222
336	A	5	5	1.00	18	0.278
337	A	3	3	1.00	16	0.188
338	A	3	3	1.00	14	0.214
339	A	6	5	1.00	18	0.278
340	A	6	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	8	7	1.00	18	0.389
342	A	6	4	1.00	20	0.200
343	A	4	4	1.00	20	0.200
344	A	7	6	1.00	20	0.300
345	A	4	4	1.00	18	0.222
346	A	6	5	1.00	16	0.312
347	A	6	5	1.00	20	0.250
348	A	7	7	1.00	20	0.350
349	A	8	7	1.00	20	0.350
350	A	6	4	1.00	20	0.200
351	A	6	4	1.00	20	0.200
352	A	6	4	1.00	20	0.200
353	A	6	4	1.00	18	0.222
354	A	6	4	1.00	16	0.250
355	A	6	5	1.00	20	0.250
356	A	6	4	1.00	20	0.200
357	A	6	4	1.00	20	0.200

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^5(a+b\sin(c+dx^2))\,dx$

Optimal. Leaf size=57

$$\frac{ax^6}{6}+\frac{b\cos(c+dx^2)}{d^3}-\frac{bx^4\cos(c+dx^2)}{2d}+\frac{bx^2\sin(c+dx^2)}{d^2}$$

```
[Out] 1/6*a*x^6+b*cos(d*x^2+c)/d^3-1/2*b*x^4*cos(d*x^2+c)/d+b*x^2*sin(d*x^2+c)/d^
2
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {14, 3460, 3377, 2718}

$$\frac{ax^6}{6}+\frac{b\cos(c+dx^2)}{d^3}+\frac{bx^2\sin(c+dx^2)}{d^2}-\frac{bx^4\cos(c+dx^2)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 + (b*Cos[c + d*x^2])/d^3 - (b*x^4*Cos[c + d*x^2])/(2*d) + (b*x^2*
Sin[c + d*x^2])/d^2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int\left(ax^5+bx^5\sin\left(c+dx^2\right)\right)\,dx\\
&= \frac{ax^6}{6}+b\int x^5\sin\left(c+dx^2\right)\,dx\\
&= \frac{ax^6}{6}+\frac{1}{2}b\text{Subst}\left(\int x^2\sin(c+dx)\,dx,x,x^2\right)\\
&= \frac{ax^6}{6}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{b\text{Subst}\left(\int x\cos(c+dx)\,dx,x,x^2\right)}{d}\\
&= \frac{ax^6}{6}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}-\frac{b\text{Subst}\left(\int\sin(c+dx)\,dx,x,x^2\right)}{d^2}\\
&= \frac{ax^6}{6}+\frac{b\cos\left(c+dx^2\right)}{d^3}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.89

$$\frac{ad^3x^6-3b(-2+d^2x^4)\cos\left(c+dx^2\right)+6bdx^2\sin\left(c+dx^2\right)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^2]),x]
[Out] (a*d^3*x^6 - 3*b*(-2 + d^2*x^4)*Cos[c + d*x^2] + 6*b*d*x^2*Sin[c + d*x^2])/
(6*d^3)
```

Maple [A]
time = 0.04, size = 62, normalized size = 1.09

method	result	size
risch	$\frac{a\,x^6}{6}-\frac{b\left(x^4d^2-2\right)\cos\left(d\,x^2+c\right)}{2d^3}+\frac{b\,x^2\sin\left(d\,x^2+c\right)}{d^2}$	47
default	$\frac{a\,x^6}{6}+b\left(-\frac{x^4\cos\left(d\,x^2+c\right)}{2d}+\frac{\frac{x^2\sin\left(d\,x^2+c\right)}{d}+\frac{\cos\left(d\,x^2+c\right)}{d^2}}{d}\right)$	62
norman	$\frac{\frac{2b}{d^3}+\frac{a\,x^6}{6}+\frac{a\,x^6\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{6}-\frac{b\,x^4}{2d}+\frac{2b\,x^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{d^2}+\frac{b\,x^4\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{2d}}{1+\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x^6+b*(-1/2/d*x^4*cos(d*x^2+c)+2/d*(1/2/d*x^2*sin(d*x^2+c)+1/2/d^2*co
s(d*x^2+c)))
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.82

$$\frac{1}{6}\,ax^6+\frac{(2\,dx^2\sin\left(dx^2+c\right)-(d^2x^4-2)\cos\left(dx^2+c\right))b}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/2*(2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*b/d
^3
```

Fricas [A]
time = 0.36, size = 51, normalized size = 0.89

$$\frac{ad^3x^6+6\,bdx^2\sin\left(dx^2+c\right)-3\,(bd^2x^4-2\,b)\cos\left(dx^2+c\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^3*x^6 + 6*b*d*x^2*sin(d*x^2 + c) - 3*(b*d^2*x^4 - 2*b)*cos(d*x^2 +
 c))/d^3
```

Sympy [A]
time = 0.40, size = 65, normalized size = 1.14

$$\begin{cases}\frac{ax^6}{6}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b\cos\left(c+dx^2\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{x^6(a+b\sin\left(c\right))}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**6/6 - b*x**4*cos(c + d*x**2)/(2*d) + b*x**2*sin(c + d*x**2)
/d**2 + b*cos(c + d*x**2)/d**3, Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(53) = 106.
time = 3.68, size = 128, normalized size = 2.25

$$-\frac{\left((dx^2+c)^2b-2\,(dx^2+c)bc-2\,b\right)\cos\left(dx^2+c\right)}{2\,d^3}+\frac{((dx^2+c)b-bc)\sin\left(dx^2+c\right)}{d^3}+\frac{(dx^2+c)^3a-3\,(dx^2+c)^2ac}{6\,d^3}+\frac{(dx^2+c)ac^2-bc^2\cos\left(dx^2+c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c - 2*b)*cos(d*x^2 + c)/d^3 + ((d*x
^2 + c)*b - b*c)*sin(d*x^2 + c)/d^3 + 1/6*((d*x^2 + c)^3*a - 3*(d*x^2 + c)^
2*a*c)/d^3 + 1/2*((d*x^2 + c)*a*c^2 - b*c^2*cos(d*x^2 + c))/d^3
```

Mupad [B]
time = 0.19, size = 53, normalized size = 0.93

$$\frac{a\,x^6}{6}+\frac{b\,\cos\left(d\,x^2+c\right)-\frac{b\,d^2\,x^4\,\cos\left(d\,x^2+c\right)}{2}+b\,d\,x^2\,\sin\left(d\,x^2+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*sin(c + d*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*cos(c + d*x^2) - (b*d^2*x^4*cos(c + d*x^2))/2 + b*d*x^2*sin(
c + d*x^2))/d^3
```

3.2 $\int x^3(a+b\sin(c+dx^2))\,dx$

Optimal. Leaf size=44

$$\frac{ax^4}{4}-\frac{bx^2\cos(c+dx^2)}{2d}+\frac{b\sin(c+dx^2)}{2d^2}$$

```
[Out] 1/4*a*x^4-1/2*b*x^2*cos(d*x^2+c)/d+1/2*b*sin(d*x^2+c)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {14, 3460, 3377, 2717}

$$\frac{ax^4}{4}+\frac{b\sin(c+dx^2)}{2d^2}-\frac{bx^2\cos(c+dx^2)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int \left(ax^3+bx^3\sin\left(c+dx^2\right)\right)\,dx\\ &= \frac{ax^4}{4}+b\int x^3\sin\left(c+dx^2\right)\,dx\\ &= \frac{ax^4}{4}+\frac{1}{2}b\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^2\right)\\ &= \frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\text{Subst}\left(\int\cos(c+dx)\,dx,x,x^2\right)}{2d}\\ &= \frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$\frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^2]),x]
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Maple [A]
time = 0.03, size = 40, normalized size = 0.91

method	result	size
risch	$\frac{a\,x^4}{4}-\frac{b\,x^2\cos\left(d\,x^2+c\right)}{2d}+\frac{b\sin\left(d\,x^2+c\right)}{2d^2}$	39
default	$\frac{a\,x^4}{4}+b\left(-\frac{x^2\cos\left(d\,x^2+c\right)}{2d}+\frac{\sin\left(d\,x^2+c\right)}{2d^2}\right)$	40
norman	$\frac{\frac{b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{d^2}+\frac{a\,x^4}{4}+\frac{a\,x^4\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{4}-\frac{b\,x^2}{2d}+\frac{b\,x^2\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{2d}}{1+\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+b*(-1/2/d*x^2*cos(d*x^2+c)+1/2/d^2*sin(d*x^2+c))
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.84

$$\frac{1}{4}\,ax^4-\frac{\left(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right)\right)b}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 - 1/2*(d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*b/d^2
```

Fricas [A]
time = 0.37, size = 40, normalized size = 0.91

$$\frac{ad^2x^4 - 2\,bdx^2\cos\left(dx^2+c\right) + 2\,b\sin\left(dx^2+c\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^2*x^4 - 2*b*d*x^2*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c))/d^2
```

Sympy [A]
time = 0.18, size = 49, normalized size = 1.11

$$\begin{cases} \frac{ax^4}{4} - \frac{bx^2\cos\left(c+dx^2\right)}{2d} + \frac{b\sin\left(c+dx^2\right)}{2d^2} & \text{for } d \neq 0 \\ \frac{x^4(a+b\sin\left(c\right))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**4/4 - b*x**2*cos(c + d*x**2)/(2*d) + b*sin(c + d*x**2)/(2*d
**2), Ne(d, 0)), (x**4*(a + b*sin(c))/4, True))
```

Giac [A]
time = 6.21, size = 75, normalized size = 1.70

$$\frac{\left(dx^2+c\right)^2a - 2\left(dx^2+c\right)b\cos\left(dx^2+c\right) + 2\,b\sin\left(dx^2+c\right)}{4\,d^2} - \frac{\left(dx^2+c\right)ac - bc\cos\left(dx^2+c\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/4*((d*x^2 + c)^2*a - 2*(d*x^2 + c)*b*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c))
/d^2 - 1/2*((d*x^2 + c)*a*c - b*c*cos(d*x^2 + c))/d^2
```

Mupad [B]
time = 0.09, size = 38, normalized size = 0.86

$$\frac{a\,x^4}{4} + \frac{\frac{b\,\sin\left(d\,x^2+c\right)}{2} - \frac{b\,d\,x^2\,\cos\left(d\,x^2+c\right)}{2}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*sin(c + d*x^2)),x)
```

```
[Out] (a*x^4)/4 + ((b*sin(c + d*x^2))/2 - (b*d*x^2*cos(c + d*x^2))/2)/d^2
```

3.3 $\int x(a+b\sin(c+dx^2))\,dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}-\frac{b\cos(c+dx^2)}{2d}$$

```
[Out] 1/2*a*x^2-1/2*b*cos(d*x^2+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {14, 3460, 2718}

$$\frac{ax^2}{2}-\frac{b\cos(c+dx^2)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 - (b*Cos[c + d*x^2])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int \left(ax+bx\sin\left(c+dx^2\right)\right)\,dx\\ &= \frac{ax^2}{2}+b\int x\sin\left(c+dx^2\right)\,dx\\ &= \frac{ax^2}{2}+\frac{1}{2}b\text{Subst}\left(\int \sin(c+dx)\,dx,x,x^2\right)\\ &= \frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 41, normalized size = 1.64

$$\frac{ax^2}{2}-\frac{b\cos(c)\cos\left(dx^2\right)}{2d}+\frac{b\sin(c)\sin\left(dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^2]),x]
[Out] (a*x^2)/2 - (b*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sin[c]*Sin[d*x^2])/(2*d)
```

Maple [**A**]
time = 0.03, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{a\,x^2}{2}-\frac{b\cos(d\,x^2+c)}{2d}$	22
derivativedivides	$\frac{(d\,x^2+c)a-b\cos(d\,x^2+c)}{2d}$	27
default	$\frac{(d\,x^2+c)a-b\cos(d\,x^2+c)}{2d}$	27
norman	$\frac{\frac{b\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\,x^2}{2}+\frac{a\,x^2\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2/d*((d*x^2+c)*a-b*cos(d*x^2+c))
```

Maxima [**A**]
time = 0.28, size = 21, normalized size = 0.84

$$\frac{1}{2}\,ax^2-\frac{b\cos\left(dx^2+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 - 1/2*b*cos(d*x^2 + c)/d
```

Fricas [A]
time = 0.38, size = 23, normalized size = 0.92

$$\frac{adx^2 - b\cos(dx^2 + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x^2 - b*cos(d*x^2 + c))/d
```

Sympy [A]
time = 0.09, size = 31, normalized size = 1.24

$$\begin{cases} \frac{ax^2}{2} - \frac{b\cos(c+dx^2)}{2d} & \text{for } d \neq 0 \\ \frac{x^2(a+b\sin(c))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**2/2 - b*cos(c + d*x**2)/(2*d), Ne(d, 0)), (x**2*(a + b*sin(
c))/2, True))
```

Giac [A]
time = 6.17, size = 26, normalized size = 1.04

$$\frac{(dx^2 + c)a - b\cos(dx^2 + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x^2 + c)*a - b*cos(d*x^2 + c))/d
```

Mupad [B]
time = 4.61, size = 21, normalized size = 0.84

$$\frac{a\,x^2}{2} - \frac{b\,\cos(d\,x^2 + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*sin(c + d*x^2)),x)
```

```
[Out] (a*x^2)/2 - (b*cos(c + d*x^2))/(2*d)
```

3.4 $\int \frac{a+b\sin(c+dx^2)}{x}\,dx$

Optimal. Leaf size=31

$$a\log(x)+\frac{1}{2}b\mathrm{Ci}(dx^2)\sin(c)+\frac{1}{2}b\cos(c)\mathrm{Si}(dx^2)$$

```
[Out] a*ln(x)+1/2*b*cos(c)*Si(d*x^2)+1/2*b*Ci(d*x^2)*sin(c)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3458, 3457, 3456}

$$a\log(x)+\frac{1}{2}b\sin(c)\mathrm{CosIntegral}(dx^2)+\frac{1}{2}b\cos(c)\mathrm{Si}(dx^2)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x,x]
```

```
[Out] a*Log[x] + (b*CosIntegral[d*x^2]*Sin[c])/2 + (b*Cos[c]*SinIntegral[d*x^2])/
2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(c+dx^2)}{x}\,dx &= \int \left(\frac{a}{x}+\frac{b\sin(c+dx^2)}{x}\right)dx \\
&= a\log(x)+b\int \frac{\sin(c+dx^2)}{x}\,dx \\
&= a\log(x)+(b\cos(c))\int \frac{\sin(dx^2)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx^2)}{x}\,dx \\
&= a\log(x)+\frac{1}{2}b\mathrm{Ci}\left(dx^2\right)\sin(c)+\frac{1}{2}b\cos(c)\mathrm{Si}\left(dx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.94

$$a\log(x)+\frac{1}{2}b\left(\mathrm{Ci}\left(dx^2\right)\sin(c)+\cos(c)\mathrm{Si}\left(dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x,x]
[Out] a*Log[x] + (b*(CosIntegral[d*x^2]*Sin[c] + Cos[c]*SinIntegral[d*x^2]))/2
```

Maple [A]
time = 0.04, size = 29, normalized size = 0.94

method	result	size
default	$a\ln(x)+b\left(\frac{\cos(c)\,\mathrm{sinIntegral}(d\,x^2)}{2}+\frac{\sin(c)\,\mathrm{cosineIntegral}(d\,x^2)}{2}\right)$	29
risch	$a\ln(x)-\frac{\mathrm{e}^{-ic}\mathrm{csgn}(d\,x^2)\pi b}{4}+\frac{\mathrm{e}^{-ic}\,\mathrm{sinIntegral}(d\,x^2)b}{2}-\frac{i\mathrm{e}^{-ic}\,\mathrm{expIntegral}(1,-id\,x^2)b}{4}+\frac{ib\,\mathrm{e}^{ic}\,\mathrm{expIntegral}(1,-id\,x^2)}{4}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*(1/2*cos(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 50, normalized size = 1.61

$$-\frac{1}{4}\left(\left(i\,\mathrm{Ei}\left(i\,dx^2\right)-i\,\mathrm{Ei}\left(-i\,dx^2\right)\right)\cos(c)-\left(\mathrm{Ei}\left(i\,dx^2\right)+\mathrm{Ei}\left(-i\,dx^2\right)\right)\sin(c)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="maxima")
```

```
[Out] -1/4*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2)
)*sin(c))*b + a*log(x)
```

Fricas [A]
time = 0.36, size = 38, normalized size = 1.23

$$\frac{1}{2} b \cos(c) \operatorname{Si}\left(dx^2\right) + a \log(x) + \frac{1}{4} \left(b \operatorname{Ci}\left(dx^2\right) + b \operatorname{Ci}\left(-dx^2\right)\right) \sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*cos(c)*sin_integral(d*x^2) + a*log(x) + 1/4*(b*cos_integral(d*x^2) +
b*cos_integral(-d*x^2))*sin(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sin(c + dx^2)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x, x)
```

Giac [A]
time = 4.92, size = 32, normalized size = 1.03

$$\frac{1}{2} b \operatorname{Ci}\left(dx^2\right) \sin(c) + \frac{1}{2} b \cos(c) \operatorname{Si}\left(dx^2\right) + \frac{1}{2} a \log\left(dx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*cos_integral(d*x^2)*sin(c) + 1/2*b*cos(c)*sin_integral(d*x^2) + 1/2*a
*log(d*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$a\ \ln(x) + \frac{b\ \sin(c)\ \mathrm{cosint}(d\,x^2)}{2} + \frac{b\ \cos(c)\ \mathrm{sinint}(d\,x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))/x,x)
```

```
[Out] a*log(x) + (b*sin(c)*cosint(d*x^2))/2 + (b*cos(c)*sinint(d*x^2))/2
```

3.5 $\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx$

Optimal. Leaf size=53

$$-\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\mathrm{Ci}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{2x^2}-\frac{1}{2}bd\sin(c)\mathrm{Si}\left(dx^2\right)$$

```
[Out] -1/2*a/x^2+1/2*b*d*Ci(d*x^2)*cos(c)-1/2*b*d*Si(d*x^2)*sin(c)-1/2*b*sin(d*x^
2+c)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {14, 3460, 3378, 3384, 3380, 3383}

$$-\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\mathrm{CosIntegral}\left(dx^2\right)-\frac{1}{2}bd\sin(c)\mathrm{Si}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*d*Cos[c]*CosIntegral[d*x^2])/2 - (b*Sin[c + d*x^2])/(2*x^2)
 - (b*d*Sin[c]*SinIntegral[d*x^2])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b\sin(c+dx^2)}{x^3}\right)dx \\
&= -\frac{a}{2x^2}+b\int \frac{\sin(c+dx^2)}{x^3}\,dx \\
&= -\frac{a}{2x^2}+\frac{1}{2}b\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{a}{2x^2}-\frac{b\sin(c+dx^2)}{2x^2}+\frac{1}{2}(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,x^2\right) \\
&= -\frac{a}{2x^2}-\frac{b\sin(c+dx^2)}{2x^2}+\frac{1}{2}(bd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,x^2\right)-\frac{1}{2}(bd\sin(c \\
&= -\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\text{Ci}(dx^2)-\frac{b\sin(c+dx^2)}{2x^2}-\frac{1}{2}bd\sin(c)\text{Si}(dx^2)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.91

$$-\frac{a-bdx^2\cos(c)\text{Ci}(dx^2)+b\sin(c+dx^2)+bdx^2\sin(c)\text{Si}(dx^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^3,x]
```

```
[Out] -1/2*(a - b*d*x^2*Cos[c]*CosIntegral[d*x^2] + b*Sin[c + d*x^2] + b*d*x^2*Si
n[c]*SinIntegral[d*x^2])/x^2
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.89

method	result
default	$-\frac{a}{2x^2}+b\left(-\frac{\sin(d\,x^2+c)}{2x^2}+d\left(\frac{\cos(c)\,\mathrm{cosineIntegral}(d\,x^2)}{2}-\frac{\sin(c)\,\mathrm{sinIntegral}(d\,x^2)}{2}\right)\right)$
risch	$-\frac{bd\,\mathrm{expIntegral}(1,-id\,x^2)\mathrm{e}^{ic}}{4}+\frac{i\pi\,\mathrm{csgn}(d\,x^2)\mathrm{e}^{-ic}bd}{4}-\frac{i\,\mathrm{sinIntegral}(d\,x^2)\mathrm{e}^{-ic}bd}{2}-\frac{\mathrm{expIntegral}(1,-id\,x^2)\mathrm{e}^{-ic}bd}{4}-\frac{a}{2x^2}-\frac{b\,\mathrm{s}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2+b*(-1/2/x^2*sin(d*x^2+c)+d*(1/2*cos(c)*Ci(d*x^2)-1/2*sin(c)*Si(d
*x^2)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 57, normalized size = 1.08

$$\frac{1}{4}\left(\left(\Gamma\left(-1,i\,dx^2\right)+\Gamma\left(-1,-i\,dx^2\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1,i\,dx^2\right)-i\,\Gamma\left(-1,-i\,dx^2\right)\right)\sin\left(c\right)\right)bd-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*x
^2) - I*gamma(-1, -I*d*x^2))*sin(c))*b*d - 1/2*a/x^2
```

Fricas [A]
time = 0.35, size = 65, normalized size = 1.23

$$-\frac{2\,bdx^2\sin\left(c\right)\mathrm{Si}\left(dx^2\right)-\left(bdx^2\,\mathrm{Ci}\left(dx^2\right)+bdx^2\,\mathrm{Ci}\left(-dx^2\right)\right)\cos\left(c\right)+2\,b\sin\left(dx^2+c\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d*x^2*sin(c)*sin_integral(d*x^2) - (b*d*x^2*cos_integral(d*x^2) +
 b*d*x^2*cos_integral(-d*x^2))*cos(c) + 2*b*sin(d*x^2 + c) + 2*a)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+dx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(45) = 90$.
time = 4.93, size = 99, normalized size = 1.87

$$\frac{(dx^2+c)bd^2\cos(c)\,\mathrm{Ci}\,(dx^2) - bcd^2\cos(c)\,\mathrm{Ci}\,(dx^2) - (dx^2+c)bd^2\sin(c)\,\mathrm{Si}\,(dx^2) + bcd^2\sin(c)\,\mathrm{Si}\,(dx^2) - bd^2\sin(dx^2+c) - ad^2}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((d*x^2 + c)*b*d^2*cos(c)*cos_integral(d*x^2) - b*c*d^2*cos(c)*cos_inte
gral(d*x^2) - (d*x^2 + c)*b*d^2*sin(c)*sin_integral(d*x^2) + b*c*d^2*sin(c)
*sin_integral(d*x^2) - b*d^2*sin(d*x^2 + c) - a*d^2)/(d^2*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\sin\left(d\,x^2+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^2))/x^3, x)
```

3.6 $\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx$

Optimal. Leaf size=74

$$-\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{1}{4}bd^2\text{Ci}\left(dx^2\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)$$

```
[Out] -1/4*a/x^4-1/4*b*d*cos(d*x^2+c)/x^2-1/4*b*d^2*cos(c)*Si(d*x^2)-1/4*b*d^2*Ci
(d*x^2)*sin(c)-1/4*b*sin(d*x^2+c)/x^4
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {14, 3460, 3378, 3384, 3380, 3383}

$$-\frac{a}{4x^4}-\frac{1}{4}bd^2\sin(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{b\sin\left(c+dx^2\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*d*Cos[c + d*x^2])/(4*x^2) - (b*d^2*CosIntegral[d*x^2]*Sin[c
])/4 - (b*Sin[c + d*x^2])/(4*x^4) - (b*d^2*Cos[c]*SinIntegral[d*x^2])/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^2\right)}{x^5}\right)\,dx \\
&= -\frac{a}{4x^4}+b\int \frac{\sin\left(c+dx^2\right)}{x^5}\,dx \\
&= -\frac{a}{4x^4}+\frac{1}{2}b\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,x^2\right) \\
&= -\frac{a}{4x^4}-\frac{b\sin\left(c+dx^2\right)}{4x^4}+\frac{1}{4}(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx,x \right. \\
&= -\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}\left(bd^2\cos(c)\right)\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx \right. \\
&= -\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{1}{4}bd^2\text{Ci}\left(dx^2\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}bd^2\cos(c)\text{S}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 1.16

$$-\frac{a}{4x^4}-\frac{b\cos\left(dx^2\right)\left(dx^2\cos(c)+\sin(c)\right)}{4x^4}+\frac{b\left(-\cos(c)+dx^2\sin(c)\right)\sin\left(dx^2\right)}{4x^4}-\frac{1}{4}bd^2\left(\text{Ci}\left(dx^2\right)\sin(c)+\cos(c)\text{Si}\left(dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*Cos[d*x^2]*(d*x^2*Cos[c] + Sin[c]))/(4*x^4) + (b*(-Cos[c] +
 d*x^2*Sin[c])*Sin[d*x^2])/(4*x^4) - (b*d^2*(CosIntegral[d*x^2]*Sin[c] + Co
s[c]*SinIntegral[d*x^2]))/4
```

Maple [A]
time = 0.04, size = 65, normalized size = 0.88

method	result
default	$-\frac{a}{4x^4}+b\left(-\frac{\sin(d\,x^2+c)}{4x^4}+\frac{d\left(-\frac{\cos(d\,x^2+c)}{2x^2}-d\left(\frac{\cos(c)\,\mathrm{sinIntegral}(d\,x^2)}{2}+\frac{\sin(c)\,\mathrm{cosineIntegral}(d\,x^2)}{2}\right)\right)}{2}\right)$
risch	$-\frac{a}{4x^4}+\frac{\pi\,\mathrm{csgn}(d\,x^2)\mathrm{e}^{-ic}b\,d^2}{8}-\frac{\mathrm{sinIntegral}(d\,x^2)\mathrm{e}^{-ic}b\,d^2}{4}+\frac{i\,\mathrm{expIntegral}(1,-id\,x^2)\mathrm{e}^{-ic}b\,d^2}{8}-\frac{ib\,d^2\,\mathrm{expIntegral}(1,-id\,x^2)\mathrm{e}^{ic}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4+b*(-1/4/x^4*sin(d*x^2+c)+1/2*d*(-1/2/x^2*cos(d*x^2+c)-d*(1/2*cos
(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 58, normalized size = 0.78

$$\frac{1}{4}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos\left(c\right)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin\left(c\right)\right)bd^2-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*d
*x^2) + gamma(-2, -I*d*x^2))*sin(c))*b*d^2 - 1/4*a/x^4
```

Fricas [A]
time = 0.38, size = 85, normalized size = 1.15

$$-\frac{2\,bd^2x^4\cos\left(c\right)\mathrm{Si}\left(dx^2\right)+2\,bdx^2\cos\left(dx^2+c\right)+2\,b\sin\left(dx^2+c\right)+\left(bd^2x^4\,\mathrm{Ci}\left(dx^2\right)+bd^2x^4\,\mathrm{Ci}\left(-dx^2\right)\right)\sin\left(c\right)+2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*d^2*x^4*cos(c)*sin_integral(d*x^2) + 2*b*d*x^2*cos(d*x^2 + c) + 2
*b*sin(d*x^2 + c) + (b*d^2*x^4*cos_integral(d*x^2) + b*d^2*x^4*cos_integral
(-d*x^2))*sin(c) + 2*a)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(64) = 128.
time = 3.70, size = 204, normalized size = 2.76

$$-\frac{(dx^2+c)^2bd^3\operatorname{Ci}(dx^2)\sin(c)-2(dx^2+c)bcd^3\operatorname{Ci}(dx^2)\sin(c)+bc^2d^3\operatorname{Ci}(dx^2)\sin(c)+(dx^2+c)^2bd^3\cos(c)\operatorname{Si}(dx^2)-2(dx^2+c)bcd^3\cos(c)\operatorname{Si}(dx^2)+bc^2d^3\cos(c)\operatorname{Si}(dx^2)+(dx^2+c)bd^3\cos(dx^2+c)-bcd^3\cos(dx^2+c)+bd^3\sin(dx^2+c)+ad^3}{4\left((dx^2+c)^2-2(dx^2+c)c+c^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((d*x^2 + c)^2*b*d^3*cos_integral(d*x^2)*sin(c) - 2*(d*x^2 + c)*b*c*d^
3*cos_integral(d*x^2)*sin(c) + b*c^2*d^3*cos_integral(d*x^2)*sin(c) + (d*x^
2 + c)^2*b*d^3*cos(c)*sin_integral(d*x^2) - 2*(d*x^2 + c)*b*c*d^3*cos(c)*si
n_integral(d*x^2) + b*c^2*d^3*cos(c)*sin_integral(d*x^2) + (d*x^2 + c)*b*d^
3*cos(d*x^2 + c) - b*c*d^3*cos(d*x^2 + c) + b*d^3*sin(d*x^2 + c) + a*d^3)/(
((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(d\,x^2+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))/x^5,x)
```

```
[Out] int((a + b*sin(c + d*x^2))/x^5, x)
```

3.7 $\int x^4(a+b\sin(c+dx^2))\,dx$

Optimal. Leaf size=121

$$\frac{ax^5}{5}-\frac{bx^3\cos(c+dx^2)}{2d}-\frac{3b\sqrt{\frac{\pi}{2}}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3b\sqrt{\frac{\pi}{2}}C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{4d^{5/2}}+\frac{3bx\sin(c+dx^2)}{4d^2}$$

```
[Out] 1/5*a*x^5-1/2*b*x^3*cos(d*x^2+c)/d+3/4*b*x*sin(d*x^2+c)/d^2-3/8*b*cos(c)*Fr
esnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-3/8*b*FresnelC(
x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {14, 3466, 3467, 3434, 3433, 3432}

$$\frac{ax^5}{5}-\frac{3\sqrt{\frac{\pi}{2}}\,b\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}\,x\right)}{4d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,b\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}+\frac{3bx\sin(c+dx^2)}{4d^2}-\frac{bx^3\cos(c+dx^2)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 - (b*x^3*Cos[c + d*x^2])/(2*d) - (3*b*Sqrt[Pi/2]*Cos[c]*FresnelS[
Sqrt[d]*Sqrt[2/Pi]*x])/(4*d^(5/2)) - (3*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[
2/Pi]*x]*Sin[c])/(4*d^(5/2)) + (3*b*x*Sin[c + d*x^2])/(4*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int\left(ax^4+bx^4\sin\left(c+dx^2\right)\right)\,dx\\
&= \frac{ax^5}{5}+b\int x^4\sin\left(c+dx^2\right)\,dx\\
&= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}+\frac{(3b)\int x^2\cos\left(c+dx^2\right)\,dx}{2d}\\
&= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{(3b)\int\sin\left(c+dx^2\right)\,dx}{4d^2}\\
&= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{(3b\cos(c))\int\sin\left(dx^2\right)\,dx}{4d^2}-\\
&= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}-\frac{3b\sqrt{\frac{\pi}{2}}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3b\sqrt{\frac{\pi}{2}}C\left(\sqrt{d}\right.}{4d}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 125, normalized size = 1.03

$$\frac{ax^5}{5}-\frac{bx\cos\left(dx^2\right)\left(2dx^2\cos(c)-3\sin(c)\right)}{4d^2}-\frac{3b\sqrt{\frac{\pi}{2}}\left(\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{4d^{5/2}}+\frac{bx\left(3\cos(c)+2dx^2\sin(c)\right)\sin\left(dx^2\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 - (b*x*Cos[d*x^2]*(2*d*x^2*Cos[c] - 3*Sin[c]))/(4*d^2) - (3*b*Sqr
t[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt[2/Pi
]*x]*Sin[c]))/(4*d^(5/2)) + (b*x*(3*Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(4
*d^2)
```

Maple [A]
time = 0.06, size = 89, normalized size = 0.74

method	result	size
default	$\frac{a\,x^{5}}{5}+b\left(-\frac{x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{\frac{3x\sin\left(d\,x^{2}+c\right)}{4d}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(c\right)\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin\left(c\right)\,\mathrm{FresnelC}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}}{d}\right)$	89
risch	$\frac{a\,x^{5}}{5}-\frac{3ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{16d^{2}\sqrt{id}}+\frac{3ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{16d^{2}\sqrt{-id}}-\frac{b\,x^{3}\cos\left(d\,x^{2}+c\right)}{2d}+\frac{3bx\sin\left(d\,x^{2}+c\right)}{4d^{2}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+b*(-1/2/d*x^3*cos(d*x^2+c)+3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2
)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*Fres
nelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 92, normalized size = 0.76

$$\frac{1}{5}\,ax^{5}-\frac{\left(16\,d^{3}x^{3}\cos\left(dx^{2}+c\right)-24\,d^{2}x\sin\left(dx^{2}+c\right)+3\,\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i+1\right)\,\cos\left(c\right)-\left(i-1\right)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}\,x\right)+\left(-\left(i-1\right)\,\cos\left(c\right)+\left(i+1\right)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{-i\,d}\,x\right)\right)d^{\frac{3}{2}}\right)b}{32\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 - 1/32*(16*d^3*x^3*cos(d*x^2 + c) - 24*d^2*x*sin(d*x^2 + c) + 3*s
qrt(2)*sqrt(pi)*(((I + 1)*cos(c) - (I - 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I
- 1)*cos(c) + (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*d^(3/2))*b/d^4
```

Fricas [A]
time = 0.36, size = 103, normalized size = 0.85

$$\frac{8\,ad^{3}x^{5}-20\,bd^{2}x^{3}\cos\left(dx^{2}+c\right)-15\,\sqrt{2}\,\pi b\sqrt{\frac{d}{\pi}}\,\cos\left(c\right)\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)-15\,\sqrt{2}\,\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+30\,bdx\sin\left(dx^{2}+c\right)}{40\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*d^3*x^5 - 20*b*d^2*x^3*cos(d*x^2 + c) - 15*sqrt(2)*pi*b*sqrt(d/pi
)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) - 15*sqrt(2)*pi*b*sqrt(d/pi)*fre
snel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 30*b*d*x*sin(d*x^2 + c))/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(126) = 252$.
time = 2.40, size = 488, normalized size = 4.03

$$\frac{ax^5}{5} - \frac{5\sqrt{2}\sqrt{\pi}\,bx^4\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}\,bx^4\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)}{2} - \frac{21\sqrt{2}\sqrt{\pi}\,bx^4\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{32\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}\,bx^4\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)}{2} - \frac{15\sqrt{2}\sqrt{\pi}\,b\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{128d^2\Gamma\left(\frac{9}{4}\right)} - \frac{63\sqrt{2}\sqrt{\pi}\,b\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{128d^2\Gamma\left(\frac{11}{4}\right)} + \frac{5bx^3\sqrt{\frac{1}{d}}\sin(c)\sin(dx^2)\Gamma\left(\frac{1}{4}\right)}{32\sqrt{d}\,\Gamma\left(\frac{9}{4}\right)} - \frac{21bx^3\sqrt{\frac{1}{d}}\cos(c)\cos(dx^2)\Gamma\left(\frac{3}{4}\right)}{32\sqrt{d}\,\Gamma\left(\frac{11}{4}\right)} + \frac{15bx\sqrt{\frac{1}{d}}\sin(c)\cos(dx^2)\Gamma\left(\frac{1}{4}\right)}{64d^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{63bx\sqrt{\frac{1}{d}}\sin(dx^2)\cos(c)\Gamma\left(\frac{3}{4}\right)}{64d^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**2+c)),x)
```

```
[Out] a*x**5/5 - 5*sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt
(d)*x/sqrt(pi))*gamma(1/4)/(32*gamma(9/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1
/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 - 21*sqrt(2)*sqrt(pi)*b*x
**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(32*ga
mma(11/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt
(d)*x/sqrt(pi))/2 - 15*sqrt(2)*sqrt(pi)*b*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)
*sqrt(d)*x/sqrt(pi))*gamma(1/4)/(128*d**2*gamma(9/4)) - 63*sqrt(2)*sqrt(pi)
*b*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(128*d*
*2*gamma(11/4)) + 5*b*x**3*sqrt(1/d)*sin(c)*sin(d*x**2)*gamma(1/4)/(32*sqrt
(d)*gamma(9/4)) - 21*b*x**3*sqrt(1/d)*cos(c)*cos(d*x**2)*gamma(3/4)/(32*sqr
t(d)*gamma(11/4)) + 15*b*x*sqrt(1/d)*sin(c)*cos(d*x**2)*gamma(1/4)/(64*d**(
3/2)*gamma(9/4)) + 63*b*x*sqrt(1/d)*sin(d*x**2)*cos(c)*gamma(3/4)/(64*d**(3
/2)*gamma(11/4))
```

Giac [C] Result contains complex when optimal does not.
time = 3.95, size = 165, normalized size = 1.36

$$\frac{1}{5}\,ax^5 - \frac{3i\,\sqrt{2}\,\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{16\,d^2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} + \frac{3i\,\sqrt{2}\,\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{16\,d^2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} + \frac{i\,(2i\,bdx^3 - 3\,bx)e^{(i\,dx^2+i\,c)}}{8\,d^2} + \frac{i\,(2i\,bdx^3 + 3\,bx)e^{(-i\,dx^2-i\,c)}}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/5*a*x^5 - 3/16*I*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*
sqrt(abs(d)))*e^(I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d))) + 3/16*I*sqrt(2)
*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d^2
*(I*d/abs(d) + 1)*sqrt(abs(d))) + 1/8*I*(2*I*b*d*x^3 - 3*b*x)*e^(I*d*x^2 +
I*c)/d^2 + 1/8*I*(2*I*b*d*x^3 + 3*b*x)*e^(-I*d*x^2 - I*c)/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*sin(c + d*x^2)),x)
[Out] int(x^4*(a + b*sin(c + d*x^2)), x)
```

3.8 $\int x^2(a+b\sin(c+dx^2))\,dx$

Optimal. Leaf size=102

$$\frac{ax^3}{3}-\frac{bx\cos(c+dx^2)}{2d}+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)}{2d^{3/2}}-\frac{b\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)}{2d^{3/2}}$$

```
[Out] 1/3*a*x^3-1/2*b*x*cos(d*x^2+c)/d+1/4*b*cos(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi
^(1/2))*2^(1/2)*Pi^(1/2)/d^(3/2)-1/4*b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))
*sin(c)*2^(1/2)*Pi^(1/2)/d^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {14, 3466, 3435, 3433, 3432}

$$\frac{ax^3}{3}+\frac{\sqrt{\frac{\pi}{2}}\,b\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}\,x\right)}{2d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,b\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)}{2d^{3/2}}-\frac{bx\cos(c+dx^2)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 - (b*x*Cos[c + d*x^2])/(2*d) + (b*Sqrt[Pi/2]*Cos[c]*FresnelC[Sqrt
[d]*Sqrt[2/Pi]*x])/(2*d^(3/2)) - (b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x]*Sin[c])/(2*d^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int\left(ax^2+bx^2\sin\left(c+dx^2\right)\right)\,dx\\
&= \frac{ax^3}{3}+b\int x^2\sin\left(c+dx^2\right)\,dx\\
&= \frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{b\int\cos\left(c+dx^2\right)\,dx}{2d}\\
&= \frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{(b\cos(c))\int\cos\left(dx^2\right)\,dx}{2d}-\frac{(b\sin(c))\int\sin\left(dx^2\right)\,dx}{2d}\\
&= \frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{b\sqrt{\frac{\pi}{2}}S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 104, normalized size = 1.02

$$\frac{ax^3}{3}-\frac{bx\cos(c)\cos\left(dx^2\right)}{2d}+\frac{b\sqrt{\frac{\pi}{2}}\left(\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)}{2d^{3/2}}+\frac{bx\sin(c)\sin\left(dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 - (b*x*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sqrt[Pi/2]*(Cos[c]*FresnelC[
Sqrt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]))/(2*d^(3/2))
 + (b*x*Sin[c]*Sin[d*x^2])/(2*d)
```

Maple [A]
time = 0.04, size = 68, normalized size = 0.67

method	result	size
default	$\frac{a x^{3}}{3}+b \left(-\frac{x \cos \left(d \,x^{2}+c \right)}{2 d}+\frac{\sqrt{2}\, \sqrt{\pi}\, \left(\cos \left(c \right) \operatorname{FresnelC}\left(\frac{x \sqrt{d}\, \sqrt{2}}{\sqrt{\pi}}\right)-\sin \left(c \right) \operatorname{S}\left(\frac{x \sqrt{d}\, \sqrt{2}}{\sqrt{\pi}}\right)\right)}{4 d^{\frac{3}{2}}}\right)$	68
risch	$\frac{b \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-i d}\, x \right) {\mathrm e}^{i c}}{8 d \sqrt{-i d}}+\frac{b \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{i d}\, x \right) {\mathrm e}^{-i c}}{8 d \sqrt{i d}}+\frac{a \,x^{3}}{3}-\frac{b x \cos \left(d \,x^{2}+c \right)}{2 d}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fre
snelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2
))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 75, normalized size = 0.74

$$\frac{1}{3}\,ax^{3}-\frac{\left(8\,d^{2}x\cos\left(dx^{2}+c\right)+\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i-1\right)\,\cos\left(c\right)+\left(i+1\right)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}\,x\right)+\left(-\left(i+1\right)\,\cos\left(c\right)-\left(i-1\right)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{-i\,d}\,x\right)\right)d^{\frac{3}{2}}\right)b}{16\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 - 1/16*(8*d^2*x*cos(d*x^2 + c) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(c
) + (I + 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I + 1)*cos(c) - (I - 1)*sin(c))*e
rf(sqrt(-I*d)*x))*d^(3/2))*b/d^3
```

Fricas [A]
time = 0.39, size = 86, normalized size = 0.84

$$\frac{4\,ad^{2}x^{3}+3\,\sqrt{2}\,\pi b\sqrt{\frac{d}{\pi}}\,\cos\left(c\right)\operatorname{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)-3\,\sqrt{2}\,\pi b\sqrt{\frac{d}{\pi}}\,\operatorname{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-6\,bdx\cos\left(dx^{2}+c\right)}{12\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*x^3 + 3*sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*
sqrt(d/pi)) - 3*sqrt(2)*pi*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*s
in(c) - 6*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(102) = 204$.

time = 1.87, size = 223, normalized size = 2.19

$$\frac{ax^3}{3} - \frac{bd^{\frac{3}{2}}x^5\sqrt{\frac{1}{d}}\cos(c)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle| -\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{b\sqrt{d}\,x^3\sqrt{\frac{1}{d}}\sin(c)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle| -\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}\,bx^2\sqrt{\frac{1}{d}}\sin(c)C\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)}{2} + \frac{\sqrt{2}\sqrt{\pi}\,bx^2\sqrt{\frac{1}{d}}\cos(c)S\left(\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{\pi}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**2+c)),x)
```

```
[Out] a*x**3/3 - b*d**(3/2)*x**5*sqrt(1/d)*cos(c)*gamma(3/4)*gamma(5/4)*hyper((3/
4, 5/4), (3/2, 7/4, 9/4), -d**2*x**4/4)/(8*gamma(7/4)*gamma(9/4)) - b*sqrt(
d)*x**3*sqrt(1/d)*sin(c)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4,
 7/4), -d**2*x**4/4)/(8*gamma(5/4)*gamma(7/4)) + sqrt(2)*sqrt(pi)*b*x**2*sq
rt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 + sqrt(2)*sqrt(pi)*b*
x**2*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))/2
```

Giac [C] Result contains complex when optimal does not.
time = 3.64, size = 145, normalized size = 1.42

$$\frac{1}{3}ax^3 - \frac{bxe^{(i\,dx^2+i\,c)}}{4\,d} - \frac{bxe^{(-i\,dx^2-i\,c)}}{4\,d} - \frac{\sqrt{2}\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{8\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}} - \frac{\sqrt{2}\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{8\,d\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 - 1/4*b*x*e^(I*d*x^2 + I*c)/d - 1/4*b*x*e^(-I*d*x^2 - I*c)/d - 1/
8*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(
I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/8*sqrt(2)*sqrt(pi)*b*erf(-1/2*s
qrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d*(I*d/abs(d) + 1)*sqrt(a
bs(d)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\sin\left(d\,x^2+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*sin(c + d*x^2)),x)
```

```
[Out] int(x^2*(a + b*sin(c + d*x^2)), x)
```

3.9 $\int (a+b\sin(c+dx^2))\,dx$

Optimal. Leaf size=74

$$ax+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{b\sqrt{\frac{\pi}{2}}C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}$$

```
[Out] a*x+1/2*b*cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(1
/2)+1/2*b*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3434, 3433, 3432}

$$ax+\frac{\sqrt{\frac{\pi}{2}}\,b\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}\,x\right)}{\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}\,b\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x^2],x]
```

```
[Out] a*x + (b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sqrt[d] + (b*Sqr
t[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx = ax + b\int \sin\left(c+dx^2\right)\,dx$$
$$= ax + (b\cos(c))\int \sin\left(dx^2\right)\,dx + (b\sin(c))\int \cos\left(dx^2\right)\,dx$$
$$= ax + \frac{b\sqrt{\frac{\pi}{2}}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)}{\sqrt{d}}$$

Mathematica [A]
time = 0.10, size = 61, normalized size = 0.82

$$ax + \frac{b\sqrt{\frac{\pi}{2}}\left(\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right) + C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x^2],x]
```

```
[Out] a*x + (b*Sqrt[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c]))/Sqrt[d]
```

Maple [A]
time = 0.03, size = 48, normalized size = 0.65

method	result	size
default	$ax + \frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos(c)\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{FresnelC}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2\sqrt{d}}$	48
risch	$ax + \frac{ib\,\mathrm{e}^{-ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)}{4\sqrt{id}} - \frac{ib\,\mathrm{e}^{ic}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)}{4\sqrt{-id}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*b*2^(1/2)*Pi^(1/2)/d^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1
/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 53, normalized size = 0.72

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\,\cos\left(c\right)+(i-1)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}\,x\right)+\left((i-1)\,\cos\left(c\right)-(i+1)\,\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{-i\,d}\,x\right)\right)b}{8\,\sqrt{d}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(c) + (I - 1)*sin(c))*erf(sqrt(I*d)*x)
+ ((I - 1)*cos(c) - (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*b/sqrt(d) + a*x
```

Fricas [A]
time = 0.37, size = 67, normalized size = 0.91

$$\frac{\sqrt{2}\,\pi b\sqrt{\frac{d}{\pi}}\,\cos\left(c\right)\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)+\sqrt{2}\,\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+2\,adx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) + sqr
t(2)*pi*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 2*a*d*x)/d
```

Sympy [A]
time = 0.25, size = 66, normalized size = 0.89

$$ax+\frac{\sqrt{2}\,\sqrt{\pi}\,b\left(\sin\left(c\right)C\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi}}\right)+\cos\left(c\right)S\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{d}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x**2+c),x)
```

```
[Out] a*x + sqrt(2)*sqrt(pi)*b*(sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi)) + cos
(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi)))*sqrt(1/d)/2
```

Giac [C] Result contains complex when optimal does not.
time = 5.29, size = 102, normalized size = 1.38

$$-\frac{1}{4}\left(-\frac{i\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}+\frac{i\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d))
)*e^(I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))) + I*sqrt(2)*sqrt(pi)*erf(-1/2*sq
rt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((I*d/abs(d) + 1)*sqrt(abs(
d)))*b + a*x
```

Mupad [B]
time = 4.75, size = 56, normalized size = 0.76

$$a\,x+\frac{\sqrt{2}\,b\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi}}\right)\,\cos\left(c\right)}{2\,\sqrt{d}}+\frac{\sqrt{2}\,b\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{d}\,x}{\sqrt{\pi}}\right)\,\sin\left(c\right)}{2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(c + d*x^2),x)
```

```
[Out] a*x + (2^(1/2)*b*pi^(1/2)*fresnels((2^(1/2)*d^(1/2)*x)/pi^(1/2))*cos(c))/(2
*d^(1/2)) + (2^(1/2)*b*pi^(1/2)*fresnelc((2^(1/2)*d^(1/2)*x)/pi^(1/2))*sin(
c))/(2*d^(1/2))
```

3.10 $\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx$

Optimal. Leaf size=88

$$-\frac{a}{x}+b\sqrt{d}\,\sqrt{2\pi}\,\cos(c)C\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-b\sqrt{d}\,\sqrt{2\pi}\,S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{x}$$

```
[Out] -a/x-b*sin(d*x^2+c)/x+b*cos(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*d^(1/2)
*2^(1/2)*Pi^(1/2)-b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*d^(1/2)*2^(
1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {14, 3468, 3435, 3433, 3432}

$$-\frac{a}{x}+\sqrt{2\pi}\,b\sqrt{d}\,\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{d}\,x\right)-\sqrt{2\pi}\,b\sqrt{d}\,\sin(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-\frac{b\sin\left(c+dx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Sqrt[d]*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] - b*Sqr
t[d]*Sqrt[2*Pi]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - (b*Sin[c + d*x^2])/
x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx &= \int\left(\frac{a}{x^2}+\frac{b\sin(c+dx^2)}{x^2}\right)dx\\ &= -\frac{a}{x}+b\int\frac{\sin(c+dx^2)}{x^2}\,dx\\ &= -\frac{a}{x}-\frac{b\sin(c+dx^2)}{x}+(2bd)\int\cos\left(c+dx^2\right)\,dx\\ &= -\frac{a}{x}-\frac{b\sin(c+dx^2)}{x}+(2bd\cos(c))\int\cos\left(dx^2\right)\,dx-(2bd\sin(c))\int\sin\left(dx^2\right)\,dx\\ &= -\frac{a}{x}+b\sqrt{d}\sqrt{2\pi}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-b\sqrt{d}\sqrt{2\pi}S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{b}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 91, normalized size = 1.03

$$-\frac{a}{x}-\frac{b\cos(dx^2)\sin(c)}{x}+b\sqrt{d}\sqrt{2\pi}\left(\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)\right)-\frac{b\cos(c)\sin(dx^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*Cos[d*x^2]*Sin[c])/x + b*Sqrt[d]*Sqrt[2*Pi]*(Cos[c]*FresnelC[Sq
rt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]) - (b*Cos[c]*Si
n[d*x^2])/x
```

Maple [A]
time = 0.04, size = 66, normalized size = 0.75

method	result
default	$-\frac{a}{x}+b\left(-\frac{\sin(dx^2+c)}{x}+\sqrt{d}\sqrt{2}\sqrt{\pi}\left(\cos(c)\,\mathrm{FresnelC}\left(\frac{x\sqrt{d}\sqrt{2}}{\sqrt{\pi}}\right)-\sin(c)\,\mathrm{S}\left(\frac{x\sqrt{d}\sqrt{2}}{\sqrt{\pi}}\right)\right)\right)$

risch	$\frac{bd\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)e^{ic}}{2\sqrt{-id}}+\frac{bd\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)e^{-ic}}{2\sqrt{id}}-\frac{a}{x}-\frac{b\sin\left(dx^{2}+c\right)}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x+b*(-sin(d*x^2+c)/x+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(x*d^(1/2)
*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.57, size = 81, normalized size = 0.92

$$-\frac{\sqrt{dx^{2}}\left(\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},idx^{2}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-idx^{2}\right)\right)\cos\left(c\right)+\left((i+1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},idx^{2}\right)-(i-1)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-idx^{2}\right)\right)\sin\left(c\right)\right)b}{8x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(d*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I + 1)*sqrt(2)*g
amma(-1/2, -I*d*x^2))*cos(c) + ((I + 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I -
 1)*sqrt(2)*gamma(-1/2, -I*d*x^2))*sin(c))*b/x - a/x
```

Fricas [**A**]
time = 0.36, size = 78, normalized size = 0.89

$$\frac{\sqrt{2}\,\pi bx\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)-\sqrt{2}\,\pi bx\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-b\sin\left(dx^{2}+c\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - sqrt(
2)*pi*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) - b*sin(d*x^2
 + c) - a)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+dx^{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**2, x)
```

<u>**Giac**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)/x^2, x)
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\sin\left(d\,x^2 + c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))/x^2,x)
```

```
[Out] int((a + b*sin(c + d*x^2))/x^2, x)
```

3.11 $\int \frac{a+b\sin(c+dx^2)}{x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\,\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\,C\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)-\frac{b\sin\left(c\right.}{3x}$$

```
[Out] -1/3*a/x^3-2/3*b*d*cos(d*x^2+c)/x-1/3*b*sin(d*x^2+c)/x^3-2/3*b*d^(3/2)*cos(
c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-2/3*b*d^(3/2)*Fres
nelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {14, 3468, 3469, 3434, 3433, 3432}

$$-\frac{a}{3x^3}-\frac{2}{3}\sqrt{2\pi}\,bd^{3/2}\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{d}\,x\right)-\frac{2}{3}\sqrt{2\pi}\,bd^{3/2}\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{b\sin\left(c+dx^2\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*d*Cos[c + d*x^2])/(3*x) - (2*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*
FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/3 - (2*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d
]*Sqrt[2/Pi]*x]*Sin[c])/3 - (b*Sin[c + d*x^2])/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b\sin\left(c+dx^2\right)}{x^4}\right)\,dx \\
&= -\frac{a}{3x^3}+b\int \frac{\sin\left(c+dx^2\right)}{x^4}\,dx \\
&= -\frac{a}{3x^3}-\frac{b\sin\left(c+dx^2\right)}{3x^3}+\frac{1}{3}(2bd)\int \frac{\cos\left(c+dx^2\right)}{x^2}\,dx \\
&= -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{b\sin\left(c+dx^2\right)}{3x^3}-\frac{1}{3}\left(4bd^2\right)\int \sin\left(c+dx^2\right)\,dx \\
&= -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{b\sin\left(c+dx^2\right)}{3x^3}-\frac{1}{3}\left(4bd^2\cos(c)\right)\int \sin\left(dx^2\right)\,dx-\frac{1}{3} \\
&= -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\,\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\,C
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 119, normalized size = 1.04

$$-\frac{a}{3x^3}-\frac{b\cos\left(dx^2\right)\left(2dx^2\cos(c)+\sin(c)\right)}{3x^3}-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\left(\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)+C\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)\right)+\frac{b\left(-\cos(c)+2dx^2\sin(c)\right)\sin\left(dx^2\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Cos[d*x^2]*(2*d*x^2*Cos[c] + Sin[c]))/(3*x^3) - (2*b*d^(3/2
)*Sqrt[2*Pi]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c]))/3 + (b*(-Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(3*x^3)
```

Maple [A]
time = 0.06, size = 83, normalized size = 0.73

method	result
default	$-\frac{a}{3x^3}+b\left(-\frac{\sin(d\,x^2+c)}{3x^3}+\frac{2d\left(-\frac{\cos(d\,x^2+c)}{x}-\sqrt{d}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(c)\,\mathrm{FresnelC}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right.\right.\right.}{3}\right.$
risch	$-\frac{a}{3x^3}-\frac{ib\,d^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{3\sqrt{id}}+\frac{ib\,d^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{3\sqrt{-id}}-\frac{2bd\cos(d\,x^2+c)}{3x}-\frac{b\sin(d\,x^2+c)}{3x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*(-1/3*sin(d*x^2+c)/x^3+2/3*d*(-1/x*cos(d*x^2+c)-d^(1/2)*2^(1/2
)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d
^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.61, size = 82, normalized size = 0.72

$$-\frac{\sqrt{dx^2}\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},i\,dx^2\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\cos(c)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},i\,dx^2\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\sin(c)\right)bd}{8\,x}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(d*x^2)*((-(I + 1)*sqrt(2)*gamma(-3/2, I*d*x^2) + (I - 1)*sqrt(2)*
gamma(-3/2, -I*d*x^2))*cos(c) + ((I - 1)*sqrt(2)*gamma(-3/2, I*d*x^2) - (I
+ 1)*sqrt(2)*gamma(-3/2, -I*d*x^2))*sin(c))*b*d/x - 1/3*a/x^3
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.86

$$-\frac{2\sqrt{2}\,\pi bdx^3\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)+2\sqrt{2}\,\pi bdx^3\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin(c)+2\,bdx^2\cos(dx^2+c)+b\sin(dx^2+c)+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/p
i)) + 2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin
(c) + 2*b*d*x^2*cos(d*x^2 + c) + b*sin(d*x^2 + c) + a)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sin \left(c + d x^{2}\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\sin\left(d\,x^2 + c\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))/x^4,x)
```

```
[Out] int((a + b*sin(c + d*x^2))/x^4, x)
```

3.12 $\int x^5 (a + b \sin(c + dx^2))^2 \, dx$

Optimal. Leaf size=163

$$-\frac{b^2x^2}{8d^2}+\frac{a^2x^6}{6}+\frac{b^2x^6}{12}+\frac{2ab\cos(c+dx^2)}{d^3}-\frac{abx^4\cos(c+dx^2)}{d}+\frac{2abx^2\sin(c+dx^2)}{d^2}+\frac{b^2\cos(c+dx^2)\sin(c+d}{8d^3}$$

```
[Out] -1/8*b^2*x^2/d^2+1/6*a^2*x^6+1/12*b^2*x^6+2*a*b*cos(d*x^2+c)/d^3-a*b*x^4*co
s(d*x^2+c)/d+2*a*b*x^2*sin(d*x^2+c)/d^2+1/8*b^2*cos(d*x^2+c)*sin(d*x^2+c)/d
^3-1/4*b^2*x^4*cos(d*x^2+c)*sin(d*x^2+c)/d+1/4*b^2*x^2*sin(d*x^2+c)^2/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3460, 3398, 3377, 2718, 3392, 30, 2715, 8}

$$\frac{a^2x^6}{6}+\frac{2ab\cos(c+dx^2)}{d^3}+\frac{2abx^2\sin(c+dx^2)}{d^2}-\frac{abx^4\cos(c+dx^2)}{d}+\frac{b^2\sin(c+dx^2)\cos(c+dx^2)}{8d^3}+\frac{b^2x^2\sin^2(c+dx^2)}{4d^2}-\frac{b^2x^4\sin(c+dx^2)\cos(c+dx^2)}{4d}-\frac{b^2x^2}{8d^2}+\frac{b^2x^6}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] -1/8*(b^2*x^2)/d^2 + (a^2*x^6)/6 + (b^2*x^6)/12 + (2*a*b*Cos[c + d*x^2])/d^
3 - (a*b*x^4*Cos[c + d*x^2])/d + (2*a*b*x^2*Sin[c + d*x^2])/d^2 + (b^2*Cos[
c + d*x^2]*Sin[c + d*x^2])/(8*d^3) - (b^2*x^4*Cos[c + d*x^2]*Sin[c + d*x^2]
)/(4*d) + (b^2*x^2*Sin[c + d*x^2]^2)/(4*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+b\sin(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^2\sin(c+dx)+b^2x^2\sin^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^6}{6}+(ab)\text{Subst}\left(\int x^2\sin(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x^2\sin^2(c+
\end{aligned}
$$

$$= \frac{a^2x^6}{6}-\frac{abx^4\cos\left(c+dx^2\right)}{d}-\frac{b^2x^4\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2x^2\sin^2\left(c+\right.}{4d^2}$$

$$= \frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b^2\cos\left(c+dx^2\right)\text{s}}{8d^3}$$

$$= -\frac{b^2x^2}{8d^2}+\frac{a^2x^6}{6}+\frac{b^2x^6}{12}+\frac{2ab\cos\left(c+dx^2\right)}{d^3}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin}{d}$$

Mathematica [A]
time = 0.23, size = 122, normalized size = 0.75

$$\frac{8a^2d^3x^6+4b^2d^3x^6-48ab(-2+d^2x^4)\cos(c+dx^2)-6b^2dx^2\cos(2(c+dx^2))+96abdx^2\sin(c+dx^2)+3b^2\sin(2(c+dx^2))-6b^2d^2x^4\sin(2(c+dx^2))}{48d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (8*a^2*d^3*x^6 + 4*b^2*d^3*x^6 - 48*a*b*(-2 + d^2*x^4)*Cos[c + d*x^2] - 6*b
^2*d*x^2*Cos[2*(c + d*x^2)] + 96*a*b*d*x^2*Sin[c + d*x^2] + 3*b^2*Sin[2*(c
+ d*x^2)] - 6*b^2*d^2*x^4*Sin[2*(c + d*x^2)])/(48*d^3)
```

Maple [A]
time = 0.08, size = 140, normalized size = 0.86

method	result
risch	$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{ab(x^4d^2-2)\cos(dx^2+c)}{d^3}+\frac{2abx^2\sin(dx^2+c)}{d^2}-\frac{b^2x^2\cos(2dx^2+2c)}{8d^2}-\frac{b^2(2x^4d^2-1)\sin(2dx^2+2c)}{16d^3}$
default	$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{b^2\left(\frac{x^4\sin(2dx^2+2c)}{4d}-\frac{-\frac{x^2\cos(2dx^2+2c)}{4d}+\frac{\sin(2dx^2+2c)}{8d^2}}{d}\right)}{2}+2ab\left(-\frac{x^4\cos(dx^2+c)}{2d}+\frac{\frac{x^2\sin(dx^2+c)}{d}+\frac{\cos(}{}}{d}\right.$
norman	$\frac{\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^6\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\frac{abx^4\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{4d^3}-\frac{b^2\left(\tan^3\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x^6+1/12*b^2*x^6-1/2*b^2*(1/4/d*x^4*sin(2*d*x^2+2*c)-1/d*(-1/4/d*x^
2*cos(2*d*x^2+2*c)+1/8/d^2*sin(2*d*x^2+2*c)))+2*a*b*(-1/2/d*x^4*cos(d*x^2+c
)+2/d*(1/2/d*x^2*sin(d*x^2+c)+1/2/d^2*cos(d*x^2+c)))
```

Maxima [A]
time = 0.31, size = 106, normalized size = 0.65

$$\frac{1}{6}a^2x^6+\frac{(2\,dx^2\sin(dx^2+c)-(d^2x^4-2)\cos(dx^2+c))ab}{d^3}+\frac{(4\,d^3x^6-6\,dx^2\cos(2\,dx^2+2\,c)-3\,(2\,d^2x^4-1)\sin(2\,dx^2+2\,c))b^2}{48\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + (2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*a*b/d
^3 + 1/48*(4*d^3*x^6 - 6*d*x^2*cos(2*d*x^2 + 2*c) - 3*(2*d^2*x^4 - 1)*sin(2
*d*x^2 + 2*c))*b^2/d^3
```

Fricas [A]
time = 0.36, size = 121, normalized size = 0.74

$$\frac{2\,(2\,a^2+b^2)d^3x^6-6\,b^2dx^2\cos\left(dx^2+c\right)^2+3\,b^2dx^2-24\,(abd^2x^4-2\,ab)\cos\left(dx^2+c\right)+3\,(16\,abdx^2-(2\,b^2d^2x^4-b^2)\cos\left(dx^2+c\right))\sin\left(dx^2+c\right)}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(2*(2*a^2 + b^2)*d^3*x^6 - 6*b^2*d*x^2*cos(d*x^2 + c)^2 + 3*b^2*d*x^2
- 24*(a*b*d^2*x^4 - 2*a*b)*cos(d*x^2 + c) + 3*(16*a*b*d*x^2 - (2*b^2*d^2*x^
4 - b^2)*cos(d*x^2 + c))*sin(d*x^2 + c))/d^3
```

Sympy [A]
time = 0.65, size = 209, normalized size = 1.28

$$\begin{cases} \frac{a^2x^6}{6}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}+\frac{2ab\cos\left(c+dx^2\right)}{d^3}+\frac{b^2x^6\sin^2\left(c+dx^2\right)}{12}+\frac{b^2x^6\cos^2\left(c+dx^2\right)}{12}-\frac{b^2x^4\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{8d^2}-\frac{b^2x^2\cos^2\left(c+dx^2\right)}{8d^2}+\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{8d^3} & \text{for}\ d\neq 0 \\ \frac{x^6(a+b\sin\left(c\right))^2}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 - a*b*x**4*cos(c + d*x**2)/d + 2*a*b*x**2*sin(c + d*
x**2)/d**2 + 2*a*b*cos(c + d*x**2)/d**3 + b**2*x**6*sin(c + d*x**2)**2/12 +
 b**2*x**6*cos(c + d*x**2)**2/12 - b**2*x**4*sin(c + d*x**2)*cos(c + d*x**2
)/(4*d) + b**2*x**2*sin(c + d*x**2)**2/(8*d**2) - b**2*x**2*cos(c + d*x**2)
**2/(8*d**2) + b**2*sin(c + d*x**2)*cos(c + d*x**2)/(8*d**3), Ne(d, 0)), (x
**6*(a + b*sin(c))**2/6, True))
```

Giac [A]
time = 6.26, size = 284, normalized size = 1.74

$$-\frac{((dx^2+c)b^2-b^2c)\cos\left(2\,dx^2+2\,c\right)}{8\,d^3}-\frac{\left((dx^2+c)^2ab-2\,(dx^2+c)abc-2\,ab\right)\cos\left(dx^2+c\right)}{d^3}-\frac{\left(2\,(dx^2+c)^2b^2-4\,(dx^2+c)b^2c-b^2\right)\sin\left(2\,dx^2+2\,c\right)}{16\,d^3}+\frac{2\,((dx^2+c)ab-abc)\sin\left(dx^2+c\right)}{d^3}+\frac{2\,(dx^2+c)^3a^2+(dx^2+c)^3b^2-6\,(dx^2+c)^2a^2c-3\,(dx^2+c)^2b^2c}{12\,d^3}+\frac{4\,(dx^2+c)a^2c^2+(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right))b^2c^2-8\,abc^2\cos\left(dx^2+c\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*((d*x^2 + c)*b^2 - b^2*c)*cos(2*d*x^2 + 2*c)/d^3 - ((d*x^2 + c)^2*a*b
- 2*(d*x^2 + c)*a*b*c - 2*a*b)*cos(d*x^2 + c)/d^3 - 1/16*(2*(d*x^2 + c)^2*b
^2 - 4*(d*x^2 + c)*b^2*c - b^2)*sin(2*d*x^2 + 2*c)/d^3 + 2*((d*x^2 + c)*a*b
 - a*b*c)*sin(d*x^2 + c)/d^3 + 1/12*(2*(d*x^2 + c)^3*a^2 + (d*x^2 + c)^3*b^
2 - 6*(d*x^2 + c)^2*a^2*c - 3*(d*x^2 + c)^2*b^2*c)/d^3 + 1/8*(4*(d*x^2 + c)
*a^2*c^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2*c^2 - 8*a*b*c^2*cos(d*x
^2 + c))/d^3
```

Mupad [B]
time = 0.39, size = 149, normalized size = 0.91

$$\frac{\frac{3\,b^2\,\sin\left(2\,d\,x^2+2\,c\right)}{2}-96\,a\,b\,\sin\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2+4\,a^2\,d^3\,x^6+2\,b^2\,d^3\,x^6+3\,b^2\,d\,x^2\,\left(2\,\sin\left(d\,x^2+c\right)^2-1\right)-3\,b^2\,d^2\,x^4\,\sin\left(2\,d\,x^2+2\,c\right)+24\,a\,b\,d^2\,x^4\,\left(2\,\sin\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2-1\right)+48\,a\,b\,d\,x^2\,\sin\left(d\,x^2+c\right)}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] ((3*b^2*sin(2*c + 2*d*x^2))/2 - 96*a*b*sin(c/2 + (d*x^2)/2)^2 + 4*a^2*d^3*x
^6 + 2*b^2*d^3*x^6 + 3*b^2*d*x^2*(2*sin(c + d*x^2)^2 - 1) - 3*b^2*d^2*x^4*s
in(2*c + 2*d*x^2) + 24*a*b*d^2*x^4*(2*sin(c/2 + (d*x^2)/2)^2 - 1) + 48*a*b*
d*x^2*sin(c + d*x^2))/(24*d^3)
```

3.13 $\int x^3(a+b\sin(c+dx^2))^2\,dx$

Optimal. Leaf size=102

$$\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{abx^2\cos(c+dx^2)}{d}+\frac{ab\sin(c+dx^2)}{d^2}-\frac{b^2x^2\cos(c+dx^2)\sin(c+dx^2)}{4d}+\frac{b^2\sin^2(c+dx^2)}{8d^2}$$

```
[Out] 1/4*a^2*x^4+1/8*b^2*x^4-a*b*x^2*cos(d*x^2+c)/d+a*b*sin(d*x^2+c)/d^2-1/4*b^2
*x^2*cos(d*x^2+c)*sin(d*x^2+c)/d+1/8*b^2*sin(d*x^2+c)^2/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3460, 3398, 3377, 2717, 3391, 30}

$$\frac{a^2x^4}{4}+\frac{ab\sin(c+dx^2)}{d^2}-\frac{abx^2\cos(c+dx^2)}{d}+\frac{b^2\sin^2(c+dx^2)}{8d^2}-\frac{b^2x^2\sin(c+dx^2)\cos(c+dx^2)}{4d}+\frac{b^2x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4)/8 - (a*b*x^2*Cos[c + d*x^2])/d + (a*b*Sin[c + d*x^2
])/d^2 - (b^2*x^2*Cos[c + d*x^2]*Sin[c + d*x^2])/(4*d) + (b^2*Sin[c + d*x^2
]^2)/(8*d^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+b\sin(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx\sin(c+dx)+b^2x\sin^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+(ab)\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x\sin^2(c+d\right.\\
&= \frac{a^2x^4}{4}-\frac{abx^2\cos\left(c+dx^2\right)}{d}-\frac{b^2x^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2\sin^2\left(c+\right.}{8d^2}\\
&= \frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{ab\sin\left(c+dx^2\right)}{d^2}-\frac{b^2x^2\cos\left(c+dx^2\right)\text{s}}{4d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.90

$$\frac{4a^2d^2x^4+2b^2d^2x^4-16abdx^2\cos\left(c+dx^2\right)-b^2\cos\left(2\left(c+dx^2\right)\right)+16ab\sin\left(c+dx^2\right)-2b^2dx^2\sin\left(2\left(c+dx^2\right)\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (4*a^2*d^2*x^4 + 2*b^2*d^2*x^4 - 16*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c
 + d*x^2)] + 16*a*b*Sin[c + d*x^2] - 2*b^2*d*x^2*Sin[2*(c + d*x^2)])/(16*d^
2)
```

Maple [A]
time = 0.06, size = 93, normalized size = 0.91

method	result
risch	$\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{ab\,x^2\cos(d\,x^2+c)}{d}+\frac{ab\sin(d\,x^2+c)}{d^2}-\frac{b^2\cos(2d\,x^2+2c)}{16d^2}-\frac{b^2x^2\sin(2d\,x^2+2c)}{8d}$
default	$\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{b^2\left(\frac{x^2\sin\left(2d\,x^2+2c\right)}{4d}+\frac{\cos\left(2d\,x^2+2c\right)}{8d^2}\right)}{2}+2ab\left(-\frac{x^2\cos(d\,x^2+c)}{2d}+\frac{\sin(d\,x^2+c)}{2d^2}\right)$
norman	$\frac{\left(\frac{a^2}{4}+\frac{b^2}{8}\right)x^4+\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^4\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{4}+\frac{b^2}{8}\right)x^4\left(\tan^4\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)+\frac{ab\,x^2\left(\tan^4\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b^2\left(\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{2d^2}-\frac{b^2x^2\tan}{}}{\left(1+\tan^2\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+1/8*b^2*x^4-1/2*b^2*(1/4/d*x^2*sin(2*d*x^2+2*c)+1/8/d^2*cos(2*d
*x^2+2*c))+2*a*b*(-1/2/d*x^2*cos(d*x^2+c)+1/2/d^2*sin(d*x^2+c))
```

Maxima [A]
time = 0.33, size = 87, normalized size = 0.85

$$\frac{1}{4}a^2x^4-\frac{(dx^2\cos(dx^2+c)-\sin(dx^2+c))ab}{d^2}+\frac{(2\,d^2x^4-2\,dx^2\sin(2\,dx^2+2\,c)-\cos(2\,dx^2+2\,c))b^2}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 - (d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*a*b/d^2 + 1/16*(2*d^2
*x^4 - 2*d*x^2*sin(2*d*x^2 + 2*c) - cos(2*d*x^2 + 2*c))*b^2/d^2
```

Fricas [A]
time = 0.37, size = 84, normalized size = 0.82

$$\frac{(2\,a^2+b^2)d^2x^4-8\,abdx^2\cos(dx^2+c)-b^2\cos(dx^2+c)^2-2\,(b^2dx^2\cos(dx^2+c)-4\,ab)\sin(dx^2+c)}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^2 + b^2)*d^2*x^4 - 8*a*b*d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c
)^2 - 2*(b^2*d*x^2*cos(d*x^2 + c) - 4*a*b)*sin(d*x^2 + c))/d^2
```

Sympy [A]
time = 0.29, size = 136, normalized size = 1.33

$$\begin{cases} \frac{a^2x^4}{4}-\frac{abx^2\cos(c+dx^2)}{d}+\frac{ab\sin(c+dx^2)}{d^2}+\frac{b^2x^4\sin^2(c+dx^2)}{8}+\frac{b^2x^4\cos^2(c+dx^2)}{8}-\frac{b^2x^2\sin(c+dx^2)\cos(c+dx^2)}{4d}+\frac{b^2\sin^2(c+dx^2)}{8d^2} & \text{for } d\neq 0 \\ \frac{x^4(a+b\sin(c))^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 - a*b*x**2*cos(c + d*x**2)/d + a*b*sin(c + d*x**2)/d
**2 + b**2*x**4*sin(c + d*x**2)**2/8 + b**2*x**4*cos(c + d*x**2)**2/8 - b**
2*x**2*sin(c + d*x**2)*cos(c + d*x**2)/(4*d) + b**2*sin(c + d*x**2)**2/(8*d
**2), Ne(d, 0)), (x**4*(a + b*sin(c))**2/4, True))
```

Giac [A]
time = 4.35, size = 165, normalized size = 1.62

$$\frac{4\,{\left(dx^{2}+c\right)}^{2}a^{2}+2\,{\left(dx^{2}+c\right)}^{2}b^{2}-16\,{\left(dx^{2}+c\right)}ab\cos\left(dx^{2}+c\right)-2\,{\left(dx^{2}+c\right)}b^{2}\sin\left(2\,dx^{2}+2\,c\right)-b^{2}\cos\left(2\,dx^{2}+2\,c\right)+16\,ab\sin\left(dx^{2}+c\right)}{16\,d^{2}}-\frac{4\,{\left(dx^{2}+c\right)}a^{2}c+{\left(2\,dx^{2}+2\,c-\sin\left(2\,dx^{2}+2\,c\right)\right)}b^{2}c-8\,abc\cos\left(dx^{2}+c\right)}{8\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*(d*x^2 + c)^2*a^2 + 2*(d*x^2 + c)^2*b^2 - 16*(d*x^2 + c)*a*b*cos(d*
x^2 + c) - 2*(d*x^2 + c)*b^2*sin(2*d*x^2 + 2*c) - b^2*cos(2*d*x^2 + 2*c) +
16*a*b*sin(d*x^2 + c))/d^2 - 1/8*(4*(d*x^2 + c)*a^2*c + (2*d*x^2 + 2*c - si
n(2*d*x^2 + 2*c))*b^2*c - 8*a*b*c*cos(d*x^2 + c))/d^2
```

Mupad [B]
time = 4.70, size = 95, normalized size = 0.93

$$-\frac{b^2\,{\cos\left(d\,x^2+c\right)}^2-2\,a^2\,d^2\,x^4-b^2\,d^2\,x^4-8\,a\,b\,\sin\left(d\,x^2+c\right)+8\,a\,b\,d\,x^2\,\cos\left(d\,x^2+c\right)+2\,b^2\,d\,x^2\,\cos\left(d\,x^2+c\right)\,\sin\left(d\,x^2+c\right)}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] -(b^2*cos(c + d*x^2)^2 - 2*a^2*d^2*x^4 - b^2*d^2*x^4 - 8*a*b*sin(c + d*x^2)
 + 8*a*b*d*x^2*cos(c + d*x^2) + 2*b^2*d*x^2*cos(c + d*x^2)*sin(c + d*x^2))/
(8*d^2)
```

3.14 $\int x(a+b\sin(c+dx^2))^2\,dx$

Optimal. Leaf size=58

$$\frac{1}{4}\left(2a^2+b^2\right)x^2-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}$$

```
[Out] 1/4*(2*a^2+b^2)*x^2-a*b*cos(d*x^2+c)/d-1/4*b^2*cos(d*x^2+c)*sin(d*x^2+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3460, 2723}

$$\frac{1}{4}x^2\left(2a^2+b^2\right)-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^2)/4 - (a*b*Cos[c + d*x^2])/d - (b^2*Cos[c + d*x^2]*Sin[c
+ d*x^2])/(4*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx&=\frac{1}{2}\text{Subst}\left(\int(a+b\sin(c+dx))^2\,dx,x,x^2\right)\\&=\frac{1}{4}\left(2a^2+b^2\right)x^2-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.90

$$-\frac{-2(2a^2+b^2)\left(c+dx^2\right)+8ab\cos\left(c+dx^2\right)+b^2\sin\left(2(c+dx^2)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] -1/8*(-2*(2*a^2 + b^2)*(c + d*x^2) + 8*a*b*Cos[c + d*x^2] + b^2*Sin[2*(c +
d*x^2)])/d
```

Maple [A]
time = 0.06, size = 62, normalized size = 1.07

method	result
risch	$\frac{x^2a^2}{2}+\frac{x^2b^2}{4}-\frac{ab\cos\left(dx^2+c\right)}{d}-\frac{b^2\sin\left(2dx^2+2c\right)}{8d}$
derivativedivides	$\frac{b^2\left(-\frac{\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)}{2}+\frac{dx^2}{2}+\frac{c}{2}\right)-2ab\cos\left(dx^2+c\right)+a^2\left(dx^2+c\right)}{2d}$
default	$\frac{b^2\left(-\frac{\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)}{2}+\frac{dx^2}{2}+\frac{c}{2}\right)-2ab\cos\left(dx^2+c\right)+a^2\left(dx^2+c\right)}{2d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^2+\left(a^2+\frac{b^2}{2}\right)x^2\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}+\frac{b^2}{4}\right)x^2\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+\frac{2ab\left(\tan^4\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{2d}}{\left(1+\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b^2*(-1/2*cos(d*x^2+c)*sin(d*x^2+c)+1/2*d*x^2+1/2*c)-2*a*b*cos(d*x^2
+c)+a^2*(d*x^2+c))
```

Maxima [A]
time = 0.34, size = 52, normalized size = 0.90

$$\frac{1}{2}a^2x^2+\frac{\left(2\,dx^2-\sin\left(2\,dx^2+2\,c\right)\right)b^2}{8\,d}-\frac{ab\cos\left(dx^2+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(2*d*x^2 - sin(2*d*x^2 + 2*c))*b^2/d - a*b*cos(d*x^2 + c)
/d
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.91

$$\frac{(2\,a^2+b^2)dx^2-b^2\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)-4\,ab\cos\left(dx^2+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d*x^2 - b^2*cos(d*x^2 + c)*sin(d*x^2 + c) - 4*a*b*cos(d*
x^2 + c))/d
```

Sympy [A]
time = 0.15, size = 95, normalized size = 1.64

$$\begin{cases} \frac{a^2x^2}{2}-\frac{ab\cos\left(c+dx^2\right)}{d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4}+\frac{b^2x^2\cos^2\left(c+dx^2\right)}{4}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d} & \text{for}\ d\neq 0 \\ \frac{x^2(a+b\sin\left(c\right))^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 - a*b*cos(c + d*x**2)/d + b**2*x**2*sin(c + d*x**2)*
*2/4 + b**2*x**2*cos(c + d*x**2)**2/4 - b**2*sin(c + d*x**2)*cos(c + d*x**2
)/(4*d), Ne(d, 0)), (x**2*(a + b*sin(c))**2/2, True))
```

Giac [A]
time = 3.32, size = 57, normalized size = 0.98

$$\frac{4\,(dx^2+c)a^2+(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right))b^2-8\,ab\cos\left(dx^2+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x^2 + c)*a^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2 - 8*a*b*c
os(d*x^2 + c))/d
```

Mupad [B]
time = 4.67, size = 51, normalized size = 0.88

$$\frac{a^2\,x^2}{2}+\frac{b^2\,x^2}{4}-\frac{b^2\,\sin\left(2\,d\,x^2+2\,c\right)}{8\,d}-\frac{a\,b\,\cos\left(d\,x^2+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^2)/4 - (b^2*sin(2*c + 2*d*x^2))/(8*d) - (a*b*cos(c + d
*x^2))/d
```

3.15 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{4}b^2\cos(2c)\text{Ci}\left(2dx^2\right)+\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\text{Ci}\left(dx^2\right)\sin(c)+ab\cos(c)\text{Si}\left(dx^2\right)+\frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)$$

```
[Out] -1/4*b^2*Ci(2*d*x^2)*cos(2*c)+1/2*(2*a^2+b^2)*ln(x)+a*b*cos(c)*Si(d*x^2)+a*
b*Ci(d*x^2)*sin(c)+1/4*b^2*Si(2*d*x^2)*sin(2*c)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3484, 6, 3459, 3457, 3456, 3458}

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\sin(c)\text{CosIntegral}\left(dx^2\right)+ab\cos(c)\text{Si}\left(dx^2\right)-\frac{1}{4}b^2\cos(2c)\text{CosIntegral}\left(2dx^2\right)+\frac{1}{4}b^2\sin(2c)\text{Si}\left(2dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x,x]
```

```
[Out] -1/4*(b^2*Cos[2*c]*CosIntegral[2*d*x^2]) + ((2*a^2 + b^2)*Log[x])/2 + a*b*C
osIntegral[d*x^2]*Sin[c] + a*b*Cos[c]*SinIntegral[d*x^2] + (b^2*Sin[2*c]*Si
nIntegral[2*d*x^2])/4
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx^2))^2}{x}\,dx = \int \left(\frac{a^2}{x}+\frac{b^2}{2x}-\frac{b^2\cos(2c+2dx^2)}{2x}+\frac{2ab\sin(c+dx^2)}{x}\right)dx$$
$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\cos(2c+2dx^2)}{2x}+\frac{2ab\sin(c+dx^2)}{x}\right)dx$$
$$= \frac{1}{2}(2a^2+b^2)\log(x)+(2ab)\int \frac{\sin(c+dx^2)}{x}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^2)}{x}\,dx$$
$$= \frac{1}{2}(2a^2+b^2)\log(x)+(2ab\cos(c))\int \frac{\sin(dx^2)}{x}\,dx-\frac{1}{2}(b^2\cos(2c))\int \frac{\cos(2dx^2)}{x}$$
$$= -\frac{1}{4}b^2\cos(2c)\mathrm{Ci}(2dx^2)+\frac{1}{2}(2a^2+b^2)\log(x)+ab\mathrm{Ci}(dx^2)\sin(c)+ab\cos(c)\mathrm{Si}($$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.96

$$\frac{1}{2}(2a^2+b^2)\log(x)-\frac{1}{4}b\left(b\cos(2c)\mathrm{Ci}(2dx^2)-4a\mathrm{Ci}(dx^2)\sin(c)-4a\cos(c)\mathrm{Si}(dx^2)-b\sin(2c)\mathrm{Si}(2dx^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x,x]
[Out] ((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^2] - 4*a*CosInt
egral[d*x^2]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^2] - b*Sin[2*c]*SinIntegra
l[2*d*x^2]))/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 157, normalized size = 2.12

method	result
risch	$-\frac{e^{-ic}\pi\,\mathrm{csgn}(d\,x^2)ab}{2}+e^{-ic}\,\mathrm{sinIntegral}\,(d\,x^2)\,ab-\frac{ie^{-ic}\,\mathrm{expIntegral}(1,-id\,x^2)ab}{2}+\ln(x)\,a^2+\frac{\ln(x)b^2}{2}-\frac{i\pi\,\mathrm{csgn}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-I*c)*Pi*csgn(d*x^2)*a*b+exp(-I*c)*Si(d*x^2)*a*b-1/2*I*exp(-I*c)*E
i(1,-I*d*x^2)*a*b+ln(x)*a^2+1/2*ln(x)*b^2-1/8*I*Pi*csgn(d*x^2)*exp(-2*I*c)*
b^2+1/4*I*exp(-2*I*c)*Si(2*d*x^2)*b^2+1/8*exp(-2*I*c)*Ei(1,-2*I*d*x^2)*b^2+
1/8*b^2*exp(2*I*c)*Ei(1,-2*I*d*x^2)+1/2*I*a*b*exp(I*c)*Ei(1,-I*d*x^2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.41, size = 108, normalized size = 1.46

$$-\frac{1}{2}\left(\left(i\operatorname{Ei}\left(i\,dx^{2}\right)-i\operatorname{Ei}\left(-i\,dx^{2}\right)\right)\cos\left(c\right)-\left(\operatorname{Ei}\left(i\,dx^{2}\right)+\operatorname{Ei}\left(-i\,dx^{2}\right)\right)\sin\left(c\right)\right)ab-\frac{1}{8}\left(\left(\operatorname{Ei}\left(2i\,dx^{2}\right)+\operatorname{Ei}\left(-2i\,dx^{2}\right)\right)\cos\left(2\,c\right)-\left(-i\operatorname{Ei}\left(2i\,dx^{2}\right)+i\operatorname{Ei}\left(-2i\,dx^{2}\right)\right)\sin\left(2\,c\right)-4\,\log\left(x\right)\right)b^{2}+a^{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2)
)*sin(c))*a*b - 1/8*((Ei(2*I*d*x^2) + Ei(-2*I*d*x^2))*cos(2*c) - (-I*Ei(2*I
*d*x^2) + I*Ei(-2*I*d*x^2))*sin(2*c) - 4*log(x))*b^2 + a^2*log(x)
```

Fricas [A]
time = 0.38, size = 94, normalized size = 1.27

$$\frac{1}{4}\,b^{2}\sin\left(2\,c\right)\operatorname{Si}\left(2\,dx^{2}\right)+ab\cos\left(c\right)\operatorname{Si}\left(dx^{2}\right)-\frac{1}{8}\left(b^{2}\operatorname{Ci}\left(2\,dx^{2}\right)+b^{2}\operatorname{Ci}\left(-2\,dx^{2}\right)\right)\cos\left(2\,c\right)+\frac{1}{2}\left(2\,a^{2}+b^{2}\right)\log\left(x\right)+\frac{1}{2}\left(ab\operatorname{Ci}\left(dx^{2}\right)+ab\operatorname{Ci}\left(-dx^{2}\right)\right)\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*sin(2*c)*sin_integral(2*d*x^2) + a*b*cos(c)*sin_integral(d*x^2) - 1
/8*(b^2*cos_integral(2*d*x^2) + b^2*cos_integral(-2*d*x^2))*cos(2*c) + 1/2*
(2*a^2 + b^2)*log(x) + 1/2*(a*b*cos_integral(d*x^2) + a*b*cos_integral(-d*x
^2))*sin(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\sin\left(c+dx^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x, x)
```

Giac [A]
time = 2.76, size = 77, normalized size = 1.04

$$-\frac{1}{4}\,b^{2}\cos\left(2\,c\right)\operatorname{Ci}\left(2\,dx^{2}\right)+ab\operatorname{Ci}\left(dx^{2}\right)\sin\left(c\right)+ab\cos\left(c\right)\operatorname{Si}\left(dx^{2}\right)-\frac{1}{4}\,b^{2}\sin\left(2\,c\right)\operatorname{Si}\left(-2\,dx^{2}\right)+\frac{1}{2}\,a^{2}\log\left(dx^{2}\right)+\frac{1}{4}\,b^{2}\log\left(dx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] -1/4*b^2*cos(2*c)*cos_integral(2*d*x^2) + a*b*cos_integral(d*x^2)*sin(c) +
a*b*cos(c)*sin_integral(d*x^2) - 1/4*b^2*sin(2*c)*sin_integral(-2*d*x^2) +
1/2*a^2*log(d*x^2) + 1/4*b^2*log(d*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))^2/x,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x, x)
```

3.16 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=115

$$-\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2(c+dx^2)\right)}{4x^2}+abd\cos(c)\text{Ci}\left(dx^2\right)+\frac{1}{2}b^2d\text{Ci}\left(2dx^2\right)\sin(2c)-\frac{ab\sin\left(c+dx^2\right)}{x^2}-abd\sin(c)\text{Si}\left(d\right.$$

```
[Out] 1/4*(-2*a^2-b^2)/x^2+a*b*d*Ci(d*x^2)*cos(c)+1/4*b^2*cos(2*d*x^2+2*c)/x^2+1/
2*b^2*d*cos(2*c)*Si(2*d*x^2)-a*b*d*Si(d*x^2)*sin(c)+1/2*b^2*d*Ci(2*d*x^2)*s
in(2*c)-a*b*sin(d*x^2+c)/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3461, 3378, 3384, 3380, 3383, 3460}

$$-\frac{2a^2+b^2}{4x^2}+abd\cos(c)\text{CosIntegral}\left(dx^2\right)-abd\sin(c)\text{Si}\left(dx^2\right)-\frac{ab\sin\left(c+dx^2\right)}{x^2}+\frac{1}{2}b^2d\sin(2c)\text{CosIntegral}\left(2dx^2\right)+\frac{1}{2}b^2d\cos(2c)\text{Si}\left(2dx^2\right)+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

```
[Out] -1/4*(2*a^2 + b^2)/x^2 + (b^2*Cos[2*(c + d*x^2)])/(4*x^2) + a*b*d*Cos[c]*Co
sIntegral[d*x^2] + (b^2*d*CosIntegral[2*d*x^2]*Sin[2*c])/2 - (a*b*Sin[c + d
*x^2])/x^2 - a*b*d*Sin[c]*SinIntegral[d*x^2] + (b^2*d*Cos[2*c]*SinIntegral[
2*d*x^2])/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx = \int \left(\frac{a^2}{x^3}+\frac{b^2}{2x^3}-\frac{b^2\cos(2c+2dx^2)}{2x^3}+\frac{2ab\sin(c+dx^2)}{x^3}\right)dx$$
$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x^3}-\frac{b^2\cos(2c+2dx^2)}{2x^3}+\frac{2ab\sin(c+dx^2)}{x^3}\right)dx$$
$$= -\frac{2a^2+b^2}{4x^2}+(2ab)\int \frac{\sin(c+dx^2)}{x^3}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^2)}{x^3}\,dx$$
$$= -\frac{2a^2+b^2}{4x^2}+(ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^2\right)-\frac{1}{4}b^2\text{Subst}\left(\int \frac{\cos(2c+2}{x^2}\right.$$
$$= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2(c+dx^2))}{4x^2}-\frac{ab\sin(c+dx^2)}{x^2}+(abd)\text{Subst}\left(\int \frac{\cos(c+}{x}\right.$$
$$= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2(c+dx^2))}{4x^2}-\frac{ab\sin(c+dx^2)}{x^2}+(abd\cos(c))\text{Subst}\left(\int \frac{c}{}\right.$$
$$= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2(c+dx^2))}{4x^2}+abd\cos(c)\text{Ci}(dx^2)+\frac{1}{2}b^2d\text{Ci}(2dx^2)\sin(2c$$

Mathematica [A]
time = 0.15, size = 116, normalized size = 1.01

$$\frac{-2a^2-b^2+b^2\cos(2(c+dx^2))+4abdx^2\cos(c)\text{Ci}(dx^2)+2b^2dx^2\text{Ci}(2dx^2)\sin(2c)-4ab\sin(c+dx^2)-4abdx^2\sin(c)\text{Si}(dx^2)+2b^2dx^2\cos(2c)\text{Si}(2dx^2)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*d*x^2*Cos[c]*CosIntegral[d*x
^2] + 2*b^2*d*x^2*CosIntegral[2*d*x^2]*Sin[2*c] - 4*a*b*Sin[c + d*x^2] - 4*
a*b*d*x^2*Sin[c]*SinIntegral[d*x^2] + 2*b^2*d*x^2*Cos[2*c]*SinIntegral[2*d*
x^2])/(4*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 203, normalized size = 1.77

method	result
risch	$-\frac{\text{csgn}(d\,x^2)\text{e}^{-2ic}\pi\, b^2 d}{4}+\frac{\text{sinIntegral}(2d\,x^2)\text{e}^{-2ic}b^2 d}{2}-\frac{i\,\text{expIntegral}(1,-2id\,x^2)\text{e}^{-2ic}b^2 d}{4}+\frac{ib^2 d\,\text{expIntegral}(1,-2id\,x^2)\text{e}^{2ic}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*csgn(d*x^2)*exp(-2*I*c)*Pi*b^2*d+1/2*Si(2*d*x^2)*exp(-2*I*c)*b^2*d-1/4
*I*Ei(1,-2*I*d*x^2)*exp(-2*I*c)*b^2*d+1/4*I*b^2*d*Ei(1,-2*I*d*x^2)*exp(2*I*
```

```
c)-1/2*a*b*d*Ei(1,-I*d*x^2)*exp(I*c)+1/2*I*csgn(d*x^2)*Pi*exp(-I*c)*a*b*d-I
*Si(d*x^2)*exp(-I*c)*a*b*d-1/2*Ei(1,-I*d*x^2)*exp(-I*c)*a*b*d-1/2*a^2/x^2-1
/4*b^2/x^2-a*b*sin(d*x^2+c)/x^2+1/4*b^2*cos(2*d*x^2+2*c)/x^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.41, size = 124, normalized size = 1.08

$$\frac{1}{2}\left(\left(\Gamma\left(-1, i\,dx^2\right)+\Gamma\left(-1, -i\,dx^2\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1, i\,dx^2\right)-i\,\Gamma\left(-1, -i\,dx^2\right)\right)\sin\left(c\right)\right)abd+\frac{\left(\left(\left(i\,\Gamma\left(-1, 2i\,dx^2\right)-i\,\Gamma\left(-1, -2i\,dx^2\right)\right)\cos\left(2\,c\right)+\left(\Gamma\left(-1, 2i\,dx^2\right)+\Gamma\left(-1, -2i\,dx^2\right)\right)\sin\left(2\,c\right)\right)dx^2-1\right)b^2}{4\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*x
^2) - I*gamma(-1, -I*d*x^2))*sin(c))*a*b*d + 1/4*(((I*gamma(-1, 2*I*d*x^2)
- I*gamma(-1, -2*I*d*x^2))*cos(2*c) + (gamma(-1, 2*I*d*x^2) + gamma(-1, -2*
I*d*x^2))*sin(2*c))*d*x^2 - 1)*b^2/x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.37, size = 147, normalized size = 1.28

$$\frac{2\,b^2dx^2\cos\left(2\,c\right)\operatorname{Si}\left(2\,dx^2\right)-4\,abdx^2\sin\left(c\right)\operatorname{Si}\left(dx^2\right)+2\,b^2\cos\left(dx^2+c\right)^2-4\,ab\sin\left(dx^2+c\right)-2\,a^2-2\,b^2+2\left(abdx^2\operatorname{Ci}\left(dx^2\right)+abdx^2\operatorname{Ci}\left(-dx^2\right)\right)\cos\left(c\right)+\left(b^2dx^2\operatorname{Ci}\left(2\,dx^2\right)+b^2dx^2\operatorname{Ci}\left(-2\,dx^2\right)\right)\sin\left(2\,c\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d*x^2*cos(2*c)*sin_integral(2*d*x^2) - 4*a*b*d*x^2*sin(c)*sin_in
tegral(d*x^2) + 2*b^2*cos(d*x^2 + c)^2 - 4*a*b*sin(d*x^2 + c) - 2*a^2 - 2*b
^2 + 2*(a*b*d*x^2*cos_integral(d*x^2) + a*b*d*x^2*cos_integral(-d*x^2))*cos
(c) + (b^2*d*x^2*cos_integral(2*d*x^2) + b^2*d*x^2*cos_integral(-2*d*x^2))*
sin(2*c))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(108) = 216.
time = 3.63, size = 226, normalized size = 1.97

$$\frac{4\left(dx^2+c\right)abd^2\cos\left(c\right)\operatorname{Ci}\left(dx^2\right)-4\,abcd^2\cos\left(c\right)\operatorname{Ci}\left(dx^2\right)+2\left(dx^2+c\right)b^2d^2\operatorname{Ci}\left(2\,dx^2\right)\sin\left(2\,c\right)-2\,b^2cd^2\operatorname{Ci}\left(2\,dx^2\right)\sin\left(2\,c\right)-4\left(dx^2+c\right)abd^2\sin\left(c\right)\operatorname{Si}\left(dx^2\right)+4\,abcd^2\sin\left(c\right)\operatorname{Si}\left(dx^2\right)-2\left(dx^2+c\right)b^2d^2\cos\left(2\,c\right)\operatorname{Si}\left(-2\,dx^2\right)+2\,b^2cd^2\cos\left(2\,c\right)\operatorname{Si}\left(-2\,dx^2\right)+b^2d^2\cos\left(2\,dx^2+2\,c\right)-4\,abd^2\sin\left(dx^2+c\right)-2\,a^2d^2-b^2d^2}{4\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x^2 + c)*a*b*d^2*cos(c)*cos_integral(d*x^2) - 4*a*b*c*d^2*cos(c)*
cos_integral(d*x^2) + 2*(d*x^2 + c)*b^2*d^2*cos_integral(2*d*x^2)*sin(2*c)
- 2*b^2*c*d^2*cos_integral(2*d*x^2)*sin(2*c) - 4*(d*x^2 + c)*a*b*d^2*sin(c)
*sin_integral(d*x^2) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^2) - 2*(d*x^2 +
c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^2) + 2*b^2*c*d^2*cos(2*c)*sin_integ
ral(-2*d*x^2) + b^2*d^2*cos(2*d*x^2 + 2*c) - 4*a*b*d^2*sin(d*x^2 + c) - 2*a
^2*d^2 - b^2*d^2)/(d^2*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))^2/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x^3, x)
```

3.17 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=169

$$-\frac{2a^2+b^2}{8x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}+\frac{1}{2}b^2d^2\cos(2c)\mathrm{Ci}\left(2dx^2\right)-\frac{1}{2}abd^2\mathrm{Ci}\left(dx^2\right)\sin(c)-\frac{ab\sin\left(c+\right.}{2x^4}$$

```
[Out] 1/8*(-2*a^2-b^2)/x^4+1/2*b^2*d^2*Ci(2*d*x^2)*cos(2*c)-1/2*a*b*d*cos(d*x^2+c
)/x^2+1/8*b^2*cos(2*d*x^2+2*c)/x^4-1/2*a*b*d^2*cos(c)*Si(d*x^2)-1/2*a*b*d^2
*Ci(d*x^2)*sin(c)-1/2*b^2*d^2*Si(2*d*x^2)*sin(2*c)-1/2*a*b*sin(d*x^2+c)/x^4
-1/4*b^2*d*sin(2*d*x^2+2*c)/x^2
```

Rubi [A]
time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3461, 3378, 3384, 3380, 3383, 3460}

$$-\frac{2a^2+b^2}{8x^4}-\frac{1}{2}abd^2\sin(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{2}abd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{abd\cos\left(c+dx^2\right)}{2x^2}-\frac{ab\sin\left(c+dx^2\right)}{2x^4}+\frac{1}{2}b^2d^2\cos(2c)\text{CosIntegral}\left(2dx^2\right)-\frac{1}{2}b^2d^2\sin(2c)\text{Si}\left(2dx^2\right)-\frac{b^2d\sin\left(2\left(c+dx^2\right)\right)}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2)/x^4 - (a*b*d*Cos[c + d*x^2])/(2*x^2) + (b^2*Cos[2*(c + d
*x^2)])/(8*x^4) + (b^2*d^2*Cos[2*c]*CosIntegral[2*d*x^2])/2 - (a*b*d^2*CosI
ntegral[d*x^2]*Sin[c])/2 - (a*b*Sin[c + d*x^2])/(2*x^4) - (b^2*d*Sin[2*(c +
 d*x^2)])/(4*x^2) - (a*b*d^2*Cos[c]*SinIntegral[d*x^2])/2 - (b^2*d^2*Sin[2*
c]*SinIntegral[2*d*x^2])/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx = \int \left(\frac{a^2}{x^5}+\frac{b^2}{2x^5}-\frac{b^2\cos(2c+2dx^2)}{2x^5}+\frac{2ab\sin(c+dx^2)}{x^5}\right)dx$$

$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x^5}-\frac{b^2\cos(2c+2dx^2)}{2x^5}+\frac{2ab\sin(c+dx^2)}{x^5}\right)dx$$

$$= -\frac{2a^2+b^2}{8x^4}+(2ab)\int \frac{\sin(c+dx^2)}{x^5}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^2)}{x^5}\,dx$$

$$= -\frac{2a^2+b^2}{8x^4}+(ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,x^2\right)-\frac{1}{4}b^2\text{Subst}\left(\int \frac{\cos(2c+2a}{x^3}\right.$$

$$= -\frac{2a^2+b^2}{8x^4}+\frac{b^2\cos(2(c+dx^2))}{8x^4}-\frac{ab\sin(c+dx^2)}{2x^4}+\frac{1}{2}(abd)\text{Subst}\left(\int \frac{\cos(c+}{x^2}\right.$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos(c+dx^2)}{2x^2}+\frac{b^2\cos(2(c+dx^2))}{8x^4}-\frac{ab\sin(c+dx^2)}{2x^4}-\frac{b^2d\,\mathrm{s}}{}$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos(c+dx^2)}{2x^2}+\frac{b^2\cos(2(c+dx^2))}{8x^4}-\frac{ab\sin(c+dx^2)}{2x^4}-\frac{b^2d\,\mathrm{s}}{}$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos(c+dx^2)}{2x^2}+\frac{b^2\cos(2(c+dx^2))}{8x^4}+\frac{1}{2}b^2d^2\cos(2c)\text{Ci}(2dx^2)$$

Mathematica [A]
time = 0.26, size = 158, normalized size = 0.93

$$-\frac{2a^2+b^2+4abdx^2\cos(c+dx^2)-b^2\cos(2(c+dx^2))-4b^2d^2x^4\cos(2c)\text{Ci}(2dx^2)+4abd^2x^4\text{Ci}(dx^2)\sin(c)+4ab\sin(c+dx^2)+2b^2dx^2\sin(2(c+dx^2))+4abd^2x^4\cos(c)\text{Si}(dx^2)+4b^2d^2x^4\sin(2c)\text{Si}(2dx^2)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2 + 4*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] - 4
*b^2*d^2*x^4*Cos[2*c]*CosIntegral[2*d*x^2] + 4*a*b*d^2*x^4*CosIntegral[d*x^
2]*Sin[c] + 4*a*b*Sin[c + d*x^2] + 2*b^2*d*x^2*Sin[2*(c + d*x^2)] + 4*a*b*d
^2*x^4*Cos[c]*SinIntegral[d*x^2] + 4*b^2*d^2*x^4*Sin[2*c]*SinIntegral[2*d*x
^2])/x^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 255, normalized size = 1.51

method	result
risch	$\frac{\pi\,\text{csgn}(d\,x^2)\,e^{-ic}ab\,d^2}{4}-\frac{\text{sinIntegral}(d\,x^2)\,e^{-ic}ab\,d^2}{2}+\frac{i\,\text{expIntegral}(1,-id\,x^2)\,e^{-ic}ab\,d^2}{4}-\frac{a^2}{4x^4}-\frac{b^2}{8x^4}+\frac{i\pi\,\text{csgn}(d\,x^2)\,e^{-2ic}b^2 d}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi*csgn(d*x^2)*exp(-I*c)*a*b*d^2-1/2*Si(d*x^2)*exp(-I*c)*a*b*d^2+1/4*I*
Ei(1,-I*d*x^2)*exp(-I*c)*a*b*d^2-1/4*a^2/x^4-1/8*b^2/x^4+1/4*I*Pi*csgn(d*x^
2)*exp(-2*I*c)*b^2*d^2-1/2*I*Si(2*d*x^2)*exp(-2*I*c)*b^2*d^2-1/4*Ei(1,-2*I*
d*x^2)*exp(-2*I*c)*b^2*d^2-1/4*b^2*d^2*Ei(1,-2*I*d*x^2)*exp(2*I*c)-1/4*I*a*
b*d^2*Ei(1,-I*d*x^2)*exp(I*c)-1/2*a*b*d*cos(d*x^2+c)/x^2-1/2*a*b*sin(d*x^2+
c)/x^4+1/8*b^2*cos(2*d*x^2+2*c)/x^4-1/4*b^2*d*sin(2*d*x^2+2*c)/x^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.43, size = 128, normalized size = 0.76

$$\frac{1}{2}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos\left(c\right)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin\left(c\right)\right)abd^2-\frac{\left(4\left(\left(\Gamma\left(-2,2i\,dx^2\right)+\Gamma\left(-2,-2i\,dx^2\right)\right)\cos\left(2\,c\right)+\left(-i\,\Gamma\left(-2,2i\,dx^2\right)+i\,\Gamma\left(-2,-2i\,dx^2\right)\right)\sin\left(2\,c\right)\right)d^2x^4+1\right)b^2}{8\,x^4}-\frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*d
*x^2) + gamma(-2, -I*d*x^2))*sin(c))*a*b*d^2 - 1/8*(4*((gamma(-2, 2*I*d*x^2
) + gamma(-2, -2*I*d*x^2))*cos(2*c) + (-I*gamma(-2, 2*I*d*x^2) + I*gamma(-2
, -2*I*d*x^2))*sin(2*c))*d^2*x^4 + 1)*b^2/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time = 0.37, size = 189, normalized size = 1.12

$$-\frac{2\,b^2d^2x^4\sin\left(2\,c\right)\mathrm{Si}\left(2\,dx^2\right)+2\,abd^2x^4\cos\left(c\right)\mathrm{Si}\left(dx^2\right)+2\,abdx^2\cos\left(dx^2+c\right)-b^2\cos\left(dx^2+c\right)^2+a^2+b^2-\left(b^2d^2x^4\,\mathrm{Ci}\left(2\,dx^2\right)+b^2d^2x^4\,\mathrm{Ci}\left(-2\,dx^2\right)\right)\cos\left(2\,c\right)+2\left(b^2dx^2\cos\left(dx^2+c\right)+ab\right)\sin\left(dx^2+c\right)+\left(abd^2x^4\,\mathrm{Ci}\left(dx^2\right)+abd^2x^4\,\mathrm{Ci}\left(-dx^2\right)\right)\sin\left(c\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*d^2*x^4*sin(2*c)*sin_integral(2*d*x^2) + 2*a*b*d^2*x^4*cos(c)*s
in_integral(d*x^2) + 2*a*b*d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c)^2 + a^
2 + b^2 - (b^2*d^2*x^4*cos_integral(2*d*x^2) + b^2*d^2*x^4*cos_integral(-2*
d*x^2))*cos(2*c) + 2*(b^2*d*x^2*cos(d*x^2 + c) + a*b)*sin(d*x^2 + c) + (a*b
*d^2*x^4*cos_integral(d*x^2) + a*b*d^2*x^4*cos_integral(-d*x^2))*sin(c))/x^
4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(153) = 306.
time = 3.37, size = 448, normalized size = 2.65

$$\frac{4\left(dx^2+c\right)^2b^2d^3\cos\left(2c\right)\mathrm{Ci}\left(2dx^2\right)-8\left(dx^2+c\right)b^2cd^3\cos\left(2c\right)\mathrm{Ci}\left(2dx^2\right)+4b^2c^2d^3\cos\left(2c\right)\mathrm{Ci}\left(2dx^2\right)-4\left(dx^2+c\right)^2abd^3\mathrm{Ci}\left(dx^2\right)\sin\left(c\right)+8\left(dx^2+c\right)abcd^3\mathrm{Ci}\left(dx^2\right)\sin\left(c\right)-4abc^2d^3\mathrm{Ci}\left(dx^2\right)\sin\left(c\right)-4\left(dx^2+c\right)^2abd^3\cos\left(c\right)\mathrm{Si}\left(dx^2\right)+8\left(dx^2+c\right)abcd^3\cos\left(c\right)\mathrm{Si}\left(dx^2\right)-4abc^2d^3\cos\left(c\right)\mathrm{Si}\left(dx^2\right)+4\left(dx^2+c\right)^2b^2d^3\sin\left(2c\right)\mathrm{Si}\left(-2dx^2\right)-8\left(dx^2+c\right)b^2cd^3\sin\left(2c\right)\mathrm{Si}\left(-2dx^2\right)+4b^2c^2d^3\sin\left(2c\right)\mathrm{Si}\left(-2dx^2\right)-4\left(dx^2+c\right)abd^3\cos\left(dx^2+c\right)+4abcd^3\cos\left(dx^2+c\right)-2\left(dx^2+c\right)b^2d^3\sin\left(2dx^2+2c\right)+2b^2cd^3\sin\left(2dx^2+2c\right)+b^2d^3\cos\left(2dx^2+2c\right)-4abd^3\sin\left(dx^2+c\right)-2a^2d^3-b^2d^3}{8\left(\left(dx^2+c\right)^2-2\left(dx^2+c\right)c+c^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x^2 + c)^2*b^2*d^3*cos(2*c)*cos_integral(2*d*x^2) - 8*(d*x^2 + c)
*b^2*c*d^3*cos(2*c)*cos_integral(2*d*x^2) + 4*b^2*c^2*d^3*cos(2*c)*cos_inte
gral(2*d*x^2) - 4*(d*x^2 + c)^2*a*b*d^3*cos_integral(d*x^2)*sin(c) + 8*(d*x
^2 + c)*a*b*c*d^3*cos_integral(d*x^2)*sin(c) - 4*a*b*c^2*d^3*cos_integral(d
*x^2)*sin(c) - 4*(d*x^2 + c)^2*a*b*d^3*cos(c)*sin_integral(d*x^2) + 8*(d*x^
2 + c)*a*b*c*d^3*cos(c)*sin_integral(d*x^2) - 4*a*b*c^2*d^3*cos(c)*sin_inte
gral(d*x^2) + 4*(d*x^2 + c)^2*b^2*d^3*sin(2*c)*sin_integral(-2*d*x^2) - 8*(
d*x^2 + c)*b^2*c*d^3*sin(2*c)*sin_integral(-2*d*x^2) + 4*b^2*c^2*d^3*sin(2*
c)*sin_integral(-2*d*x^2) - 4*(d*x^2 + c)*a*b*d^3*cos(d*x^2 + c) + 4*a*b*c*
d^3*cos(d*x^2 + c) - 2*(d*x^2 + c)*b^2*d^3*sin(2*d*x^2 + 2*c) + 2*b^2*c*d^3
*sin(2*d*x^2 + 2*c) + b^2*d^3*cos(2*d*x^2 + 2*c) - 4*a*b*d^3*sin(d*x^2 + c)
 - 2*a^2*d^3 - b^2*d^3)/(((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^2}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))^2/x^5,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x^5, x)
```

3.18 $\int x^4(a+b\sin(c+dx^2))^2\,dx$

<u>**Optimal**</u>. Leaf size=247

$$\frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3b^2\sqrt{\pi}\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3ab\sqrt{\frac{\pi}{2}}\cos(c)S\left(\right.}{2d^{5/}}$$

```
[Out] 1/10*(2*a^2+b^2)*x^5-a*b*x^3*cos(d*x^2+c)/d-3/32*b^2*x*cos(2*d*x^2+2*c)/d^2
+3/2*a*b*x*sin(d*x^2+c)/d^2-1/8*b^2*x^3*sin(2*d*x^2+2*c)/d-3/4*a*b*cos(c)*F
resnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)-3/4*a*b*Fresne
lC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(5/2)+3/64*b^2*cos
(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(5/2)-3/64*b^2*FresnelS(2*x
*d^(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)/d^(5/2)
```

<u>**Rubi**</u> [**A**]
time = 0.17, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3467, 3466, 3435, 3433, 3432, 3434}

$$\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{3\sqrt{\frac{\pi}{2}}\,ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}\,x\right)}{2d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)}{2d^{5/2}}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2}-\frac{abx^3\cos\left(c+dx^2\right)}{d}+\frac{3\sqrt{\pi}\,b^2\cos(2c)\text{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3\sqrt{\pi}\,b^2\sin(2c)S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{64d^{5/2}}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^5)/10 - (a*b*x^3*Cos[c + d*x^2])/d - (3*b^2*x*Cos[2*c + 2*
d*x^2])/(32*d^2) + (3*b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
])/(64*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/
(2*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/(2*d
^(5/2)) - (3*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(64*d^
(5/2)) + (3*a*b*x*Sin[c + d*x^2])/(2*d^2) - (b^2*x^3*Sin[2*c + 2*d*x^2])/(8
*d)
```

<u>Rule 6</u>

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

<u>Rule 3432</u>

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

<u>Rule 3433</u>

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int\left(a^2x^4+\frac{b^2x^4}{2}-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+2abx^4\sin\left(c+dx^2\right)\right)\,dx$$
$$=\int\left(\left(a^2+\frac{b^2}{2}\right)x^4-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+2abx^4\sin\left(c+dx^2\right)\right)\,dx$$
$$=\frac{1}{10}\left(2a^2+b^2\right)x^5+(2ab)\int x^4\sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int x^4\cos\left(2c+2dx^2\right)$$
$$=\frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d}+\frac{(3ab)\int x^2 c}{}$$
$$=\frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3abx\sin(c}{2d^2}$$
$$=\frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3abx\sin(c}{2d^2}$$
$$=\frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3b^2\sqrt{\pi}\,\mathrm{co}}{}$$

Mathematica [A]
time = 0.36, size = 234, normalized size = 0.95

$$\frac{64a^2d^{5/2}x^5+32b^2d^{5/2}x^5-320abd^{3/2}x^3\cos\left(c+dx^2\right)-30b^2\sqrt{d}\,x\cos\left(2\left(c+dx^2\right)\right)+15b^2\sqrt{\pi}\,\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-240ab\sqrt{2\pi}\,\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-240ab\sqrt{2\pi}\,C\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)-15b^2\sqrt{\pi}\,S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\sin(2c)+480ab\sqrt{d}\,x\sin\left(c+dx^2\right)-40b^2d^{3/2}x^3\sin\left(2\left(c+dx^2\right)\right)}{320d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (64*a^2*d^(5/2)*x^5 + 32*b^2*d^(5/2)*x^5 - 320*a*b*d^(3/2)*x^3*Cos[c + d*x^
2] - 30*b^2*Sqrt[d]*x*Cos[2*(c + d*x^2)] + 15*b^2*Sqrt[Pi]*Cos[2*c]*Fresnel
C[(2*Sqrt[d]*x)/Sqrt[Pi]] - 240*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x] - 240*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - 15*b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 480*a*b*Sqrt[d]*x*S
in[c + d*x^2] - 40*b^2*d^(3/2)*x^3*Sin[2*(c + d*x^2)])/(320*d^(5/2))
```

Maple [A]
time = 0.11, size = 189, normalized size = 0.77

method	result

default	$\frac{x^5a^2}{5}+\frac{x^5b^2}{10}-\frac{b^2\left(\frac{x^3\sin\left(2d\,x^2+2c\right)}{4d}-\frac{3\left(-\frac{x\cos\left(2d\,x^2+2c\right)}{4d}+\frac{\sqrt{\pi}\left(\cos(2c)\,\mathrm{FresnelC}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}\right)}{4d}\right)}{2}+2ab$
risch	$\frac{x^5a^2}{5}-\frac{3iab\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{8d^2\sqrt{id}}+\frac{x^5b^2}{10}+\frac{3b^2\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{256d^2\sqrt{id}}+\frac{3b^2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2id}\,x\right)\mathrm{e}^{2ic}}{128d^2\sqrt{-2id}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*a^2+1/10*x^5*b^2-1/2*b^2*(1/4/d*x^3*sin(2*d*x^2+2*c)-3/4/d*(-1/4/d*
x*cos(2*d*x^2+2*c)+1/8/d^(3/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(
1/2))-sin(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))))+2*a*b*(-1/2/d*x^3*cos(d*x^
2+c)+3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fresn
elS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))
)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.59, size = 207, normalized size = 0.84

$$\frac{1}{5}\,a^2x^5-\frac{\left(16\,d^3x^3\cos\left(dx^2+c\right)-24\,d^2x\sin\left(dx^2+c\right)+3\,\sqrt{2}\,\sqrt{\pi}\left(\left(\left(i+1\right)\,\cos\left(c\right)-\left(i-1\right)\,\sin\left(c\right)\right)\mathrm{erf}\left(\sqrt{i\,d}\,x\right)+\left(-\left(i-1\right)\,\cos\left(c\right)+\left(i+1\right)\,\sin\left(c\right)\right)\mathrm{erf}\left(\sqrt{-i\,d}\,x\right)\right)d^{\frac{3}{2}}\right)ab}{16\,d^4}+\frac{\left(256\,d^4x^5-320\,d^3x^3\sin\left(2\,dx^2+2\,c\right)-240\,d^2x\cos\left(2\,dx^2+2\,c\right)+15\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(\left(-\left(i-1\right)\,\cos\left(2\,c\right)-\left(i+1\right)\,\sin\left(2\,c\right)\right)\mathrm{erf}\left(\sqrt{2i\,d}\,x\right)+\left(\left(i+1\right)\,\cos\left(2\,c\right)+\left(i-1\right)\,\sin\left(2\,c\right)\right)\mathrm{erf}\left(\sqrt{-2i\,d}\,x\right)\right)d^{\frac{3}{2}}\right)b^2}{2560\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - 1/16*(16*d^3*x^3*cos(d*x^2 + c) - 24*d^2*x*sin(d*x^2 + c) + 3
*sqrt(2)*sqrt(pi)*(((I + 1)*cos(c) - (I - 1)*sin(c))*erf(sqrt(I*d)*x) + (-(
I - 1)*cos(c) + (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*d^(3/2))*a*b/d^4 + 1/256
0*(256*d^4*x^5 - 320*d^3*x^3*sin(2*d*x^2 + 2*c) - 240*d^2*x*cos(2*d*x^2 + 2
*c) + 15*4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(2*c) - (I + 1)*sin(2*c))*e
rf(sqrt(2*I*d)*x) + ((I + 1)*cos(2*c) + (I - 1)*sin(2*c))*erf(sqrt(-2*I*d)*
x))*d^(3/2))*b^2/d^4
```

Fricas [A]
time = 0.40, size = 216, normalized size = 0.87

$$\frac{32\left(2\,a^2+b^2\right)d^3x^5-320\,abd^2x^3\cos\left(dx^2+c\right)-60\,b^2dx\cos\left(dx^2+c\right)^2-240\,\sqrt{2}\,\pi ab\sqrt{\frac{d}{\pi}}\,\cos\left(c\right)\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)-240\,\sqrt{2}\,\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+15\,\pi b^2\sqrt{\frac{d}{\pi}}\,\cos\left(2\,c\right)\mathrm{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)-15\,\pi b^2\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(2\,c\right)+30\,b^2dx-80\left(b^2d^2x^3\cos\left(dx^2+c\right)-6\,abdx\right)\sin\left(dx^2+c\right)}{320\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/320*(32*(2*a^2 + b^2)*d^3*x^5 - 320*a*b*d^2*x^3*cos(d*x^2 + c) - 60*b^2*d
*x*cos(d*x^2 + c)^2 - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt
(2)*x*sqrt(d/pi)) - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqr
t(d/pi))*sin(c) + 15*pi*b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi))
 - 15*pi*b^2*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 30*b^2*d*x -
 80*(b^2*d^2*x^3*cos(d*x^2 + c) - 6*a*b*d*x)*sin(d*x^2 + c))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b\sin\left(c + dx^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**2))**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.61, size = 329, normalized size = 1.33

$$\frac{1}{5}a^2x^5 + \frac{1}{10}b^2x^5 - \frac{3i\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{8d^2\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}} + \frac{3i\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{8d^2\left(\frac{id}{|d|}+1\right)\sqrt{|d|}} - \frac{3\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(-\frac{id}{|d|}+1\right)\right)e^{(2ic)}}{128d^{\frac{5}{2}}\left(-\frac{id}{|d|}+1\right)} - \frac{3\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(\frac{id}{|d|}+1\right)\right)e^{(-2ic)}}{128d^{\frac{5}{2}}\left(\frac{id}{|d|}+1\right)} - \frac{(-4ib^2dx^3+3b^2x)e^{(2idx^2+2ic)}}{64d^2} + \frac{i(2iabdx^3-3abx)e^{(idx^2+ic)}}{4d^2} + \frac{i(2iabdx^3+3abx)e^{(-idx^2-ic)}}{4d^2} - \frac{(4ib^2dx^3+3b^2x)e^{(-2idx^2-2ic)}}{64d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/5*a^2*x^5 + 1/10*b^2*x^5 - 3/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*
(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d))
) + 3/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs
(d)))*e^(-I*c)/(d^2*(I*d/abs(d) + 1)*sqrt(abs(d))) - 3/128*sqrt(pi)*b^2*erf
(-sqrt(d)*x*(-I*d/abs(d) + 1))*e^(2*I*c)/(d^(5/2)*(-I*d/abs(d) + 1)) - 3/12
8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(I*d/abs(d) + 1))*e^(-2*I*c)/(d^(5/2)*(I*d/ab
s(d) + 1)) - 1/64*(-4*I*b^2*d*x^3 + 3*b^2*x)*e^(2*I*d*x^2 + 2*I*c)/d^2 + 1/
4*I*(2*I*a*b*d*x^3 - 3*a*b*x)*e^(I*d*x^2 + I*c)/d^2 + 1/4*I*(2*I*a*b*d*x^3
+ 3*a*b*x)*e^(-I*d*x^2 - I*c)/d^2 - 1/64*(4*I*b^2*d*x^3 + 3*b^2*x)*e^(-2*I*
d*x^2 - 2*I*c)/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int(x^4*(a + b*sin(c + d*x^2))^2, x)
```

3.19 $\int x^2(a+b\sin(c+dx^2))^2\,dx$

Optimal. Leaf size=198

$$\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{ab\sqrt{\frac{\pi}{2}}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}+\frac{b^2\sqrt{\pi}\cos(2c)S\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}-\frac{ab\sqrt{\frac{\pi}{2}}S\left(\sqrt{\right.}\right.}{}$$

```
[Out] 1/6*(2*a^2+b^2)*x^3-a*b*x*cos(d*x^2+c)/d-1/8*b^2*x*sin(2*d*x^2+2*c)/d+1/2*a
*b*cos(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(3/2)-1/2
*a*b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(3/2)+1
/16*b^2*cos(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(3/2)+1/16*b^2*F
resnelC(2*x*d^(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)/d^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3467, 3434, 3433, 3432, 3466, 3435}

$$\frac{1}{6}x^3\left(2a^2+b^2\right)+\frac{\sqrt{\frac{\pi}{2}}ab\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}ab\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{\sqrt{\pi}b^2\sin(2c)\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}+\frac{\sqrt{\pi}b^2\cos(2c)S\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{16d^{3/2}}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (a*b*x*Cos[c + d*x^2])/d + (a*b*Sqrt[Pi/2]*Cos[c]*F
resnelC[Sqrt[d]*Sqrt[2/Pi]*x])/d^(3/2) + (b^2*Sqrt[Pi]*Cos[2*c]*FresnelS[(2
*Sqrt[d]*x)/Sqrt[Pi]])/(16*d^(3/2)) - (a*b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c])/d^(3/2) + (b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*
Sin[2*c])/(16*d^(3/2)) - (b^2*x*Sin[2*c + 2*d*x^2])/(8*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx=\int\left(a^2x^2+\frac{b^2x^2}{2}-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+2abx^2\sin\left(c+dx^2\right)\right)\,dx$$
$$=\int\left(\left(a^2+\frac{b^2}{2}\right)x^2-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+2abx^2\sin\left(c+dx^2\right)\right)\,dx$$
$$=\frac{1}{6}\left(2a^2+b^2\right)x^3+(2ab)\int x^2\sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int x^2\cos\left(2c+2dx^2\right)\,dx$$
$$=\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}+\frac{(ab)\int\cos\left(c+d\right.}{d}$$
$$=\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}+\frac{(ab\cos(c))\int\cos}{d}$$
$$=\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{ab\sqrt{\frac{\pi}{2}}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}+\frac{b^2\sqrt{}}{}$$

Mathematica [A]
time = 0.32, size = 191, normalized size = 0.96

$$\frac{16a^2d^{3/2}x^3+8b^2d^{3/2}x^3-48ab\sqrt{d}\,x\cos\left(c+dx^2\right)+24ab\sqrt{2\pi}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+3b^2\sqrt{\pi}\cos(2c)S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-24ab\sqrt{2\pi}\,S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)+3b^2\sqrt{\pi}\,C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\sin(2c)-6b^2\sqrt{d}\,x\sin\left(2\left(c+dx^2\right)\right)}{48d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (16*a^2*d^(3/2)*x^3 + 8*b^2*d^(3/2)*x^3 - 48*a*b*Sqrt[d]*x*Cos[c + d*x^2] +
 24*a*b*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 3*b^2*Sqrt[Pi]*C
os[2*c]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 24*a*b*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + 3*b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*S
in[2*c] - 6*b^2*Sqrt[d]*x*Sin[2*(c + d*x^2)])/(48*d^(3/2))
```

Maple [A]
time = 0.08, size = 142, normalized size = 0.72

method	result
default	$\frac{x^3a^2}{3}+\frac{x^3b^2}{6}-\frac{b^2\left(\frac{x\sin\left(2d\,x^2+2c\right)}{4d}-\frac{\sqrt{\pi}\left(\cos(2c)\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)+\sin(2c)\,\mathrm{FresnelC}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{8d^{\frac{3}{2}}}\right)}{2}+2ab\left(-\frac{x\cos\left(d\,x^2+c\right)}{2d}+\right.$

risch	$\frac{ib^2\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)e^{-2ic}}{64d\sqrt{id}} - \frac{ib^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2id}\,x\right)e^{2ic}}{32d\sqrt{-2id}} + \frac{ab\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-id}\,x\right)e^{ic}}{4d\sqrt{-id}} + \frac{ab\sqrt{\pi}\,\operatorname{erf}\left(\right.}{4d\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a^2+1/6*x^3*b^2-1/2*b^2*(1/4/d*x*sin(2*d*x^2+2*c)-1/8/d^(3/2)*Pi^(1
/2)*(cos(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))+sin(2*c)*FresnelC(2*x*d^(1/2)/
Pi^(1/2))))+2*a*b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(
c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/P
i^(1/2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 171, normalized size = 0.86

$$\frac{1}{3}a^2x^3 - \frac{\left(8\,d^2x\cos\left(dx^2+c\right) + \sqrt{2}\,\sqrt{\pi}\left(\left(\left(i-1\right)\cos\left(c\right) + \left(i+1\right)\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{i\,d}\,x\right) + \left(-\left(i+1\right)\cos\left(c\right) - \left(i-1\right)\sin\left(c\right)\right)\operatorname{erf}\left(\sqrt{-i\,d}\,x\right)\right)d^{\frac{3}{2}}\right)ab}{8\,d^3} + \frac{\left(64\,d^3x^3 - 48\,d^2x\sin\left(2\,dx^2+2\,c\right) + 3\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(\left(\left(i+1\right)\cos\left(2\,c\right) - \left(i-1\right)\sin\left(2\,c\right)\right)\operatorname{erf}\left(\sqrt{2i\,d}\,x\right) + \left(-\left(i-1\right)\cos\left(2\,c\right) + \left(i+1\right)\sin\left(2\,c\right)\right)\operatorname{erf}\left(\sqrt{-2i\,d}\,x\right)\right)d^{\frac{3}{2}}\right)b^2}{384\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 - 1/8*(8*d^2*x*cos(d*x^2 + c) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(
c) + (I + 1)*sin(c))*erf(sqrt(I*d)*x) + (-(I + 1)*cos(c) - (I - 1)*sin(c))*
erf(sqrt(-I*d)*x))*d^(3/2))*a*b/d^3 + 1/384*(64*d^3*x^3 - 48*d^2*x*sin(2*d*
x^2 + 2*c) + 3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*c) - (I - 1)*sin(2*
c))*erf(sqrt(2*I*d)*x) + (-(I - 1)*cos(2*c) + (I + 1)*sin(2*c))*erf(sqrt(-2
*I*d)*x))*d^(3/2))*b^2/d^3
```

Fricas [A]
time = 0.45, size = 176, normalized size = 0.89

$$\frac{8\left(2\,a^2+b^2\right)d^2x^3 + 24\,\sqrt{2}\,\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right) - 12\,b^2dx\cos\left(dx^2+c\right)\sin\left(dx^2+c\right) - 24\,\sqrt{2}\,\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right) + 3\,\pi b^2\sqrt{\frac{d}{\pi}}\cos\left(2\,c\right)\mathrm{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right) + 3\,\pi b^2\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(2\,c\right) - 48\,abdx\cos\left(dx^2+c\right)}{48\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(8*(2*a^2 + b^2)*d^2*x^3 + 24*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel
_cos(sqrt(2)*x*sqrt(d/pi)) - 12*b^2*d*x*cos(d*x^2 + c)*sin(d*x^2 + c) - 24*
sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + 3*pi*b
^2*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + 3*pi*b^2*sqrt(d/pi)*fr
esnel_cos(2*x*sqrt(d/pi))*sin(2*c) - 48*a*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a + b\sin\left(c + dx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**2*(a + b*sin(c + d*x**2))**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 6.19, size = 283, normalized size = 1.43

$$\frac{1}{3}\,a^2x^3+\frac{1}{6}\,b^2x^3+\frac{i\,b^2xe^{(2i\,dx^2+2i\,c)}}{16\,d}-\frac{abxe^{(i\,dx^2+i\,c)}}{2\,d}-\frac{abxe^{(-i\,dx^2-i\,c)}}{2\,d}-\frac{i\,b^2xe^{(-2i\,dx^2-2i\,c)}}{16\,d}-\frac{\sqrt{2}\,\sqrt{\pi}\,ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{4\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}-\frac{\sqrt{2}\,\sqrt{\pi}\,ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{4\,d\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}+\frac{i\,\sqrt{\pi}\,b^2\,\mathrm{erf}\left(-\sqrt{d}\,x\left(-\frac{i\,d}{|d|}+1\right)\right)e^{(2i\,c)}}{32\,d^{\frac{3}{2}}\left(-\frac{i\,d}{|d|}+1\right)}-\frac{i\,\sqrt{\pi}\,b^2\,\mathrm{erf}\left(-\sqrt{d}\,x\left(\frac{i\,d}{|d|}+1\right)\right)e^{(-2i\,c)}}{32\,d^{\frac{3}{2}}\left(\frac{i\,d}{|d|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*x^3 + 1/6*b^2*x^3 + 1/16*I*b^2*x*e^(2*I*d*x^2 + 2*I*c)/d - 1/2*a*b*
x*e^(I*d*x^2 + I*c)/d - 1/2*a*b*x*e^(-I*d*x^2 - I*c)/d - 1/16*I*b^2*x*e^(-2
*I*d*x^2 - 2*I*c)/d - 1/4*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(-I*d/abs
(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/4*sqr
t(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c
)/(d*(I*d/abs(d) + 1)*sqrt(abs(d))) + 1/32*I*sqrt(pi)*b^2*erf(-sqrt(d)*x*(-
I*d/abs(d) + 1))*e^(2*I*c)/(d^(3/2)*(-I*d/abs(d) + 1)) - 1/32*I*sqrt(pi)*b^
2*erf(-sqrt(d)*x*(I*d/abs(d) + 1))*e^(-2*I*c)/(d^(3/2)*(I*d/abs(d) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int(x^2*(a + b*sin(c + d*x^2))^2, x)
```

3.20 $\int (a+b\sin(c+dx^2))^2\,dx$

Optimal. Leaf size=153

$$\frac{1}{2}\left(2a^2+b^2\right)x-\frac{b^2\sqrt{\pi}\,\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{ab\sqrt{2\pi}\,\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)}{\sqrt{d}}+\frac{ab\sqrt{2\pi}\,C\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)}{\sqrt{d}}$$

```
[Out] 1/2*(2*a^2+b^2)*x-1/4*b^2*cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)/
d^(1/2)+1/4*b^2*FresnelS(2*x*d^(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)/d^(1/2)+a*
b*cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^(1/2)+a*b*
FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3438, 3435, 3433, 3432, 3434}

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{\sqrt{2\pi}\,ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{d}\,x\right)}{\sqrt{d}}+\frac{\sqrt{2\pi}\,ab\cos(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)}{\sqrt{d}}-\frac{\sqrt{\pi}\,b^2\cos(2c)\text{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{\sqrt{\pi}\,b^2\sin(2c)S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{4\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi
]])/(4*Sqrt[d]) + (a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sq
rt[d] + (a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d] + (b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2dx^2\right)+2ab\sin\left(c+dx^2\right)\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int \cos\left(2c+2dx^2\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab\cos(c))\int \sin\left(dx^2\right)\,dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \cos\left(2dx^2\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2\right)x-\frac{b^2\sqrt{\pi}\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{ab\sqrt{2\pi}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 147, normalized size = 0.96

$$\frac{4a^2\sqrt{d}\,x+2b^2\sqrt{d}\,x-b^2\sqrt{\pi}\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)+4ab\sqrt{2\pi}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)+4ab\sqrt{2\pi}\,C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)+b^2\sqrt{\pi}\,S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\sin(2c)}{4\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (4*a^2*Sqrt[d]*x + 2*b^2*Sqrt[d]*x - b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt
[d]*x)/Sqrt[Pi]] + 4*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] +
 4*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[Pi]*Fres
nelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Maple [A]
time = 0.06, size = 99, normalized size = 0.65

method	result

default	$a^2x+\frac{b^2x}{2}-\frac{b^2\sqrt{\pi}\left(\cos(2c)\,\mathrm{FresnelC}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)}{4\sqrt{d}}+\frac{ab\sqrt{2}\,\sqrt{\pi}\,\left(\cos(c)\mathrm{S}\left(\frac{x\sqrt{d}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(\right.}{\sqrt{d}}$
risch	$a^2x+\frac{iab\,\mathrm{e}^{-ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{id}\,x\right)}{2\sqrt{id}}+\frac{b^2x}{2}-\frac{b^2\mathrm{e}^{-2ic}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)}{16\sqrt{id}}-\frac{b^2\mathrm{e}^{2ic}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2id}\,x\right)}{8\sqrt{-2id}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+1/2*b^2*x-1/4*b^2*Pi^(1/2)/d^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^
(1/2))-sin(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))+a*b*2^(1/2)*Pi^(1/2)/d^(1/2
)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^
(1/2)/Pi^(1/2)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 129, normalized size = 0.84

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\left(-(i+1)\,\cos\left(c\right)+(i-1)\,\sin\left(c\right)\right)\mathrm{erf}\left(\sqrt{i\,d}\,x\right)+\left((i-1)\,\cos\left(c\right)-(i+1)\,\sin\left(c\right)\right)\mathrm{erf}\left(\sqrt{-i\,d}\,x\right)\right)ab}{4\,\sqrt{d}}+a^2x+\frac{\left(4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,\left(\left((i-1)\,\cos\left(2\,c\right)+(i+1)\,\sin\left(2\,c\right)\right)\mathrm{erf}\left(\sqrt{2i\,d}\,x\right)+\left(-(i+1)\,\cos\left(2\,c\right)-(i-1)\,\sin\left(2\,c\right)\right)\mathrm{erf}\left(\sqrt{-2i\,d}\,x\right)\right)d^{\frac{3}{2}}+16\,d^2x\right)b^2}{32\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(c) + (I - 1)*sin(c))*erf(sqrt(I*d)*x)
+ ((I - 1)*cos(c) - (I + 1)*sin(c))*erf(sqrt(-I*d)*x))*a*b/sqrt(d) + a^2*x
+ 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*c) + (I + 1)*sin(2*c))*erf
(sqrt(2*I*d)*x) + (-(I + 1)*cos(2*c) - (I - 1)*sin(2*c))*erf(sqrt(-2*I*d)*x
))*d^(3/2) + 16*d^2*x)*b^2/d^2
```

Fricas [A]
time = 0.38, size = 134, normalized size = 0.88

$$\frac{4\,\sqrt{2}\,\pi ab\sqrt{\frac{d}{\pi}}\,\cos\left(c\right)\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)+4\,\sqrt{2}\,\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-\pi b^2\sqrt{\frac{d}{\pi}}\,\cos\left(2\,c\right)\mathrm{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)+\pi b^2\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(2\,c\right)+2\left(2\,a^2+b^2\right)dx}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) +
 4*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) - pi*
b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) + pi*b^2*sqrt(d/pi)*fre
snel_sin(2*x*sqrt(d/pi))*sin(2*c) + 2*(2*a^2 + b^2)*d*x)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 5.85, size = 195, normalized size = 1.27

$$\frac{i\sqrt{2}\sqrt{\pi}\,ab\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{2\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}}-\frac{i\sqrt{2}\sqrt{\pi}\,ab\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{2\left(\frac{id}{|d|}+1\right)\sqrt{|d|}}+\frac{\sqrt{\pi}\,b^2\,\mathrm{erf}\left(-\sqrt{d}\,x\left(-\frac{id}{|d|}+1\right)\right)e^{(2ic)}}{8\sqrt{d}\left(-\frac{id}{|d|}+1\right)}+\frac{\sqrt{\pi}\,b^2\,\mathrm{erf}\left(-\sqrt{d}\,x\left(\frac{id}{|d|}+1\right)\right)e^{(-2ic)}}{8\sqrt{d}\left(\frac{id}{|d|}+1\right)}+\frac{1}{2}\left(2a^2+b^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)
))*e^(I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/2*I*sqrt(2)*sqrt(pi)*a*b*er
f(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((I*d/abs(d) + 1)*
sqrt(abs(d))) + 1/8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(-I*d/abs(d) + 1))*e^(2*I*c
)/(sqrt(d)*(-I*d/abs(d) + 1)) + 1/8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(I*d/abs(d)
 + 1))*e^(-2*I*c)/(sqrt(d)*(I*d/abs(d) + 1)) + 1/2*(2*a^2 + b^2)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\sin\left(d\,x^2+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2, x)
```

3.21 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=187

$$-\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+2ab\sqrt{d}\,\sqrt{2\pi}\,\cos(c)C\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)+b^2\sqrt{d}\,\sqrt{\pi}\,\cos(2c)S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-2ab\sqrt{}$$

```
[Out] 1/2*(-2*a^2-b^2)/x+1/2*b^2*cos(2*d*x^2+2*c)/x-2*a*b*sin(d*x^2+c)/x+b^2*cos(
2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))*d^(1/2)*Pi^(1/2)+b^2*FresnelC(2*x*d^(1/
2)/Pi^(1/2))*sin(2*c)*d^(1/2)*Pi^(1/2)+2*a*b*cos(c)*FresnelC(x*d^(1/2)*2^(1
/2)/Pi^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)-2*a*b*FresnelS(x*d^(1/2)*2^(1/2)/Pi^
(1/2))*sin(c)*d^(1/2)*2^(1/2)*Pi^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3469, 3434, 3433, 3432, 3468, 3435}

$$-\frac{2a^2+b^2}{2x}+2\sqrt{2\pi}\,ab\sqrt{d}\,\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{d}\,x\right)-2\sqrt{2\pi}\,ab\sqrt{d}\,\sin(c)S\left(\sqrt{d}\,\sqrt{\frac{2}{\pi}}\,x\right)-\frac{2ab\sin\left(c+dx^2\right)}{x}+\sqrt{\pi}\,b^2\sqrt{d}\,\sin(2c)\text{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)+\sqrt{\pi}\,b^2\sqrt{d}\,\cos(2c)S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

```
[Out] -1/2*(2*a^2 + b^2)/x + (b^2*Cos[2*c + 2*d*x^2])/(2*x) + 2*a*b*Sqrt[d]*Sqrt[
2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + b^2*Sqrt[d]*Sqrt[Pi]*Cos[2*c]
*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 2*a*b*Sqrt[d]*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[d]*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[
Pi]]*Sin[2*c] - (2*a*b*Sin[c + d*x^2])/x
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx^2))^2}{x^2}\,dx = \int \left(\frac{a^2}{x^2}+\frac{b^2}{2x^2}-\frac{b^2\cos(2c+2dx^2)}{2x^2}+\frac{2ab\sin(c+dx^2)}{x^2}\right)dx$$
$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x^2}-\frac{b^2\cos(2c+2dx^2)}{2x^2}+\frac{2ab\sin(c+dx^2)}{x^2}\right)dx$$
$$= -\frac{2a^2+b^2}{2x}+(2ab)\int \frac{\sin(c+dx^2)}{x^2}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^2)}{x^2}\,dx$$
$$= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^2)}{2x}-\frac{2ab\sin(c+dx^2)}{x}+(4abd)\int \cos(c+dx^2)$$
$$= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^2)}{2x}-\frac{2ab\sin(c+dx^2)}{x}+(4abd\cos(c))\int \cos(dx$$
$$= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^2)}{2x}+2ab\sqrt{d}\sqrt{2\pi}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+b^2\sqrt{}$$

Mathematica [A]
time = 0.31, size = 184, normalized size = 0.98

$$\frac{-2a^2-b^2+b^2\cos(2(c+dx^2))+4ab\sqrt{d}\sqrt{2\pi}\,x\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+2b^2\sqrt{d}\sqrt{\pi}\,x\cos(2c)S\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)-4ab\sqrt{d}\sqrt{2\pi}\,xS\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)+2b^2\sqrt{d}\sqrt{\pi}\,xC\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)\sin(2c)-4ab\sin(c+dx^2)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*Cos[c]*
FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*Cos[2*c]*FresnelS
[(2*Sqrt[d]*x)/Sqrt[Pi]] - 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
]*Sin[2*c] - 4*a*b*Sin[c + d*x^2])/(2*x)
```

Maple [A]
time = 0.09, size = 137, normalized size = 0.73

method	result
default	$-\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\left(-\frac{\cos(2dx^2+2c)}{x}-2\sqrt{d}\sqrt{\pi}\left(\cos(2c)\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)+\sin(2c)\,\mathrm{FresnelC}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)}{2}+2ab\left(-\frac{\sin(dx^2+c}{x}\right.$
risch	$\frac{ib^2d\sqrt{\pi}\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{4\sqrt{id}}-\frac{ib^2d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2id}\,x\right)\mathrm{e}^{2ic}}{2\sqrt{-2id}}+\frac{abd\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-id}\,x\right)\mathrm{e}^{ic}}{\sqrt{-id}}+\frac{abd\sqrt{\pi}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(a^2+1/2*b^2)/x-1/2*b^2*(-1/x*cos(2*d*x^2+2*c)-2*d^(1/2)*Pi^(1/2)*(cos(2*c
)*FresnelS(2*x*d^(1/2)/Pi^(1/2))+sin(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))))+
2*a*b*(-sin(d*x^2+c)/x+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(x*d^(1/2)*
2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.69, size = 170, normalized size = 0.91

$$-\frac{\sqrt{dx^2}\left(\left(\left(i-1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},i\,dx^2\right)-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},-i\,dx^2\right)\right)\cos\left(c\right)+\left(\left(i+1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},i\,dx^2\right)-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},-i\,dx^2\right)\right)\sin\left(c\right)\right)ab}{4\,x}-\frac{\left(\sqrt{2}\,\sqrt{dx^2}\left(\left(-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},2i\,dx^2\right)+\left(i-1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},-2i\,dx^2\right)\right)\cos\left(2\,c\right)+\left(\left(i-1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},2i\,dx^2\right)-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\tfrac{1}{2},-2i\,dx^2\right)\right)\sin\left(2\,c\right)\right)+8\right)b^2}{16\,x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(d*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I + 1)*sqrt(2)*g
amma(-1/2, -I*d*x^2))*cos(c) + ((I + 1)*sqrt(2)*gamma(-1/2, I*d*x^2) - (I -
 1)*sqrt(2)*gamma(-1/2, -I*d*x^2))*sin(c))*a*b/x - 1/16*(sqrt(2)*sqrt(d*x^2
)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*d*x^2) + (I - 1)*sqrt(2)*gamma(-1/2, -
2*I*d*x^2))*cos(2*c) + ((I - 1)*sqrt(2)*gamma(-1/2, 2*I*d*x^2) - (I + 1)*sq
rt(2)*gamma(-1/2, -2*I*d*x^2))*sin(2*c)) + 8)*b^2/x - a^2/x
```

Fricas [A]
time = 0.39, size = 159, normalized size = 0.85

$$\frac{2\sqrt{2}\,\pi abx\sqrt{\frac{d}{\pi}}\cos\left(c\right)\operatorname{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)-2\sqrt{2}\,\pi abx\sqrt{\frac{d}{\pi}}\operatorname{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+\pi b^2x\sqrt{\frac{d}{\pi}}\cos\left(2\,c\right)\operatorname{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right)+\pi b^2x\sqrt{\frac{d}{\pi}}\operatorname{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)\sin\left(2\,c\right)+b^2\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*sqrt(2)*pi*a*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - 2
*sqrt(2)*pi*a*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + pi*
b^2*x*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + pi*b^2*x*sqrt(d/pi)
*fresnel_cos(2*x*sqrt(d/pi))*sin(2*c) + b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*
x^2 + c) - a^2 - b^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))^2/x^2,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x^2, x)
```

3.22 $\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx$

Optimal. Leaf size=239

$$-\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos(c+dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3}+\frac{4}{3}b^2d^{3/2}\sqrt{\pi}\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\frac{4}{3}abd^{3/2}\sqrt{2\pi}\cos(c)S$$

```
[Out] 1/6*(-2*a^2-b^2)/x^3-4/3*a*b*d*cos(d*x^2+c)/x+1/6*b^2*cos(2*d*x^2+2*c)/x^3-
2/3*a*b*sin(d*x^2+c)/x^3-2/3*b^2*d*sin(2*d*x^2+2*c)/x+4/3*b^2*d^(3/2)*cos(2
*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))*Pi^(1/2)-4/3*b^2*d^(3/2)*FresnelS(2*x*d^
(1/2)/Pi^(1/2))*sin(2*c)*Pi^(1/2)-4/3*a*b*d^(3/2)*cos(c)*FresnelS(x*d^(1/2)
*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-4/3*a*b*d^(3/2)*FresnelC(x*d^(1/2)*2^(1
/2)/Pi^(1/2))*sin(c)*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3469, 3468, 3435, 3433, 3432, 3434}

$$-\frac{2a^2+b^2}{6x^3}-\frac{4}{3}\sqrt{2\pi}\,abd^{3/2}\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}\,x\right)-\frac{4}{3}\sqrt{2\pi}\,abd^{3/2}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)-\frac{4abd\cos(c+dx^2)}{3x}-\frac{2ab\sin(c+dx^2)}{3x^3}+\frac{4}{3}\sqrt{\pi}\,b^2d^{3/2}\cos(2c)\text{FresnelC}\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\frac{4}{3}\sqrt{\pi}\,b^2d^{3/2}\sin(2c)S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)-\frac{2b^2d\sin(2c+2dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

```
[Out] -1/6*(2*a^2 + b^2)/x^3 - (4*a*b*d*Cos[c + d*x^2])/(3*x) + (b^2*Cos[2*c + 2*
d*x^2])/(6*x^3) + (4*b^2*d^(3/2)*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/S
qrt[Pi]])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/3
- (4*b^2*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/3 - (2
*a*b*Sin[c + d*x^2])/(3*x^3) - (2*b^2*d*Sin[2*c + 2*d*x^2])/(3*x)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx &= \int \left(\frac{a^2}{x^4}+\frac{b^2}{2x^4}-\frac{b^2\cos(2c+2dx^2)}{2x^4}+\frac{2ab\sin(c+dx^2)}{x^4}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^4}-\frac{b^2\cos(2c+2dx^2)}{2x^4}+\frac{2ab\sin(c+dx^2)}{x^4}\right)dx \\
&= -\frac{2a^2+b^2}{6x^3}+(2ab)\int \frac{\sin(c+dx^2)}{x^4}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^2)}{x^4}\,dx \\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos(2c+2dx^2)}{6x^3}-\frac{2ab\sin(c+dx^2)}{3x^3}+\frac{1}{3}(4abd)\int \frac{\cos(c+dx^2)}{x^2} \\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos(c+dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3}-\frac{2ab\sin(c+dx^2)}{3x^3}-\frac{2b^2}{} \\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos(c+dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3}-\frac{2ab\sin(c+dx^2)}{3x^3}-\frac{2b^2}{} \\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos(c+dx^2)}{3x}+\frac{b^2\cos(2c+2dx^2)}{6x^3}+\frac{4}{3}b^2d^{3/2}\sqrt{\pi}\cos(2c)C\Bigg(
\end{aligned}
$$

Mathematica [A]
time = 0.42, size = 226, normalized size = 0.95

$$
-\frac{2a^2+b^2+8abdx^2\cos(c+dx^2)-b^2\cos(2(c+dx^2))-8b^2d^{3/2}\sqrt{\pi}\,x^3\cos(2c)C\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)+8abd^{3/2}\sqrt{2\pi}\,x^3\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)+8abd^{3/2}\sqrt{2\pi}\,x^3C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}\,x\right)\sin(c)+8b^2d^{3/2}\sqrt{\pi}\,x^3S\left(\frac{2\sqrt{d}\,x}{\sqrt{\pi}}\right)\sin(2c)+4ab\sin(c+dx^2)+4b^2dx^2\sin(2(c+dx^2))}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

```
[Out] -1/6*(2*a^2 + b^2 + 8*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] - 8
*b^2*d^(3/2)*Sqrt[Pi]*x^3*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]] + 8*a*b
*d^(3/2)*Sqrt[2*Pi]*x^3*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + 8*a*b*d^(3/
2)*Sqrt[2*Pi]*x^3*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + 8*b^2*d^(3/2)*Sqr
t[Pi]*x^3*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 4*a*b*Sin[c + d*x^2]
+ 4*b^2*d*x^2*Sin[2*(c + d*x^2)])/x^3
```

Maple [A]
time = 0.09, size = 175, normalized size = 0.73

method	result

default	$-\frac{a^2+\frac{b^2}{2}}{3x^3}-\frac{b^2\left(-\frac{\cos\left(2d\,x^2+2c\right)}{3x^3}-\frac{4d\left(-\frac{\sin\left(2d\,x^2+2c\right)}{x}+2\sqrt{d}\,\sqrt{\pi}\,\left(\cos(2c)\,\mathrm{FresnelC}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\mathrm{S}\left(\frac{2x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)}{3}\right)}{2}+2a$
risch	$-\frac{2iab\,d^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{id}\,x\right)\mathrm{e}^{-ic}}{3\sqrt{id}}-\frac{a^2}{3x^3}-\frac{b^2}{6x^3}+\frac{b^2d^2\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{id}\,x\right)\mathrm{e}^{-2ic}}{3\sqrt{id}}+\frac{2b^2d^2\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2id}\right.}{3\sqrt{-2id}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a^2+1/2*b^2)/x^3-1/2*b^2*(-1/3/x^3*cos(2*d*x^2+2*c)-4/3*d*(-1/x*sin(2
*d*x^2+2*c)+2*d^(1/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))-sin
(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))))+2*a*b*(-1/3*sin(d*x^2+c)/x^3+2/3*d*
(-1/x*cos(d*x^2+c)-d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1
/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.66, size = 176, normalized size = 0.74

$$-\frac{\sqrt{dx^2}\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},i\,dx^2\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\cos(c)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},i\,dx^2\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\sin(c)\right)abd}{4\,x}-\frac{\left(3\sqrt{2}\sqrt{dx^2}\left(\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},2i\,dx^2\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-2i\,dx^2\right)\right)\cos(2c)+\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},2i\,dx^2\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{3}{2},-2i\,dx^2\right)\right)\sin(2c)\right)dx^2+4\right)b^2}{24\,x^3}-\frac{a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(d*x^2)*((-(I + 1)*sqrt(2)*gamma(-3/2, I*d*x^2) + (I - 1)*sqrt(2)*
gamma(-3/2, -I*d*x^2))*cos(c) + ((I - 1)*sqrt(2)*gamma(-3/2, I*d*x^2) - (I
+ 1)*sqrt(2)*gamma(-3/2, -I*d*x^2))*sin(c))*a*b*d/x - 1/24*(3*sqrt(2)*sqrt(
d*x^2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*d*x^2) + (I + 1)*sqrt(2)*gamma(-3
/2, -2*I*d*x^2))*cos(2*c) + (-(I + 1)*sqrt(2)*gamma(-3/2, 2*I*d*x^2) + (I -
 1)*sqrt(2)*gamma(-3/2, -2*I*d*x^2))*sin(2*c))*d*x^2 + 4)*b^2/x^3 - 1/3*a^2
/x^3
```

Fricas [A]
time = 0.40, size = 206, normalized size = 0.86

$$-\frac{4\sqrt{2}\,\pi abdx^3\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\,\pi abdx^3\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{d}{\pi}}\right)\sin(c)-4\,\pi b^2dx^3\sqrt{\frac{d}{\pi}}\cos(2c)\,\mathrm{C}\left(2\,x\sqrt{\frac{d}{\pi}}\right)+4\,\pi b^2dx^3\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right)\sin(2c)+4\,abdx^2\cos(dx^2+c)-b^2\cos(dx^2+c)^2+a^2+b^2+2\left(2\,b^2dx^2\cos(dx^2+c)+ab\right)\sin(dx^2+c)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d
/pi)) + 4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))
```

```
*sin(c) - 4*pi*b^2*d*x^3*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) +
4*pi*b^2*d*x^3*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 4*a*b*d*x^
2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c)^2 + a^2 + b^2 + 2*(2*b^2*d*x^2*cos(d*
x^2 + c) + a*b)*sin(d*x^2 + c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(c + dx^2))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\sin(d\,x^2 + c))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^2))^2/x^4,x)
```

```
[Out] int((a + b*sin(c + d*x^2))^2/x^4, x)
```

3.23 $\int x^5 \sin^3(a+bx^2)\,dx$

Optimal. Leaf size=117

$$\frac{7\cos(a+bx^2)}{9b^3}-\frac{x^4\cos(a+bx^2)}{3b}-\frac{\cos^3(a+bx^2)}{27b^3}+\frac{2x^2\sin(a+bx^2)}{3b^2}-\frac{x^4\cos(a+bx^2)\sin^2(a+bx^2)}{6b}+\frac{x^2\sin$$

```
[Out] 7/9*cos(b*x^2+a)/b^3-1/3*x^4*cos(b*x^2+a)/b-1/27*cos(b*x^2+a)^3/b^3+2/3*x^2
*sin(b*x^2+a)/b^2-1/6*x^4*cos(b*x^2+a)*sin(b*x^2+a)^2/b+1/9*x^2*sin(b*x^2+a
)^3/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3460, 3392, 3377, 2718, 2713}

$$-\frac{\cos^3(a+bx^2)}{27b^3}+\frac{7\cos(a+bx^2)}{9b^3}+\frac{x^2\sin^3(a+bx^2)}{9b^2}+\frac{2x^2\sin(a+bx^2)}{3b^2}-\frac{x^4\cos(a+bx^2)}{3b}-\frac{x^4\sin^2(a+bx^2)\cos(a+bx^2)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sin[a + b*x^2]^3,x]
```

```
[Out] (7*Cos[a + b*x^2])/(9*b^3) - (x^4*Cos[a + b*x^2])/(3*b) - Cos[a + b*x^2]^3/
(27*b^3) + (2*x^2*Sin[a + b*x^2])/(3*b^2) - (x^4*Cos[a + b*x^2]*Sin[a + b*x
^2]^2)/(6*b) + (x^2*Sin[a + b*x^2]^3)/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^5 \sin^3\left(a+bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\sin^3(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{x^4\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{x^2\sin^3\left(a+bx^2\right)}{9b^2} + \frac{1}{3}\text{Subst}\left(\int x^2\sin(a+bx)\,dx\right.$$
$$= -\frac{x^4\cos\left(a+bx^2\right)}{3b} - \frac{x^4\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{x^2\sin^3\left(a+bx^2\right)}{9b^2} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{\cos\left(a+bx^2\right)}{9b^3} - \frac{x^4\cos\left(a+bx^2\right)}{3b} - \frac{\cos^3\left(a+bx^2\right)}{27b^3} + \frac{2x^2\sin\left(a+bx^2\right)}{3b^2} - \frac{x^4\cos(a+}{}$$
$$= \frac{7\cos\left(a+bx^2\right)}{9b^3} - \frac{x^4\cos\left(a+bx^2\right)}{3b} - \frac{\cos^3\left(a+bx^2\right)}{27b^3} + \frac{2x^2\sin\left(a+bx^2\right)}{3b^2} - \frac{x^4\cos(a}{}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.64

$$\frac{-81(-2+b^2x^4)\cos\left(a+bx^2\right)+(-2+9b^2x^4)\cos\left(3(a+bx^2)\right)-6bx^2\left(-27\sin\left(a+bx^2\right)+\sin\left(3(a+bx^2)\right)\right)}{216b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sin[a + b*x^2]^3,x]
```

```
[Out] (-81*(-2 + b^2*x^4)*Cos[a + b*x^2] + (-2 + 9*b^2*x^4)*Cos[3*(a + b*x^2)] -
6*b*x^2*(-27*Sin[a + b*x^2] + Sin[3*(a + b*x^2)]))/(216*b^3)
```

Maple [A]
time = 0.05, size = 113, normalized size = 0.97

method	result	size

risch	$-\frac{3(b^2x^4-2)\cos(b\,x^2+a)}{8b^3}+\frac{3x^2\sin(b\,x^2+a)}{4b^2}+\frac{(9b^2x^4-2)\cos(3b\,x^2+3a)}{216b^3}-\frac{x^2\sin(3b\,x^2+3a)}{36b^2}$	85
default	$-\frac{3x^4\cos(b\,x^2+a)}{8b}+\frac{\frac{3x^2\sin\left(b\,x^2+a\right)}{4b}+\frac{3\cos\left(b\,x^2+a\right)}{4b^2}}{b}+\frac{x^4\cos(3b\,x^2+3a)}{24b}-\frac{\frac{x^2\sin\left(3b\,x^2+3a\right)}{6b}+\frac{\cos\left(3b\,x^2+3a\right)}{18b^2}}{6b}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x^4*cos(b*x^2+a)/b+3/2/b*(1/2/b*x^2*sin(b*x^2+a)+1/2/b^2*cos(b*x^2+a))
+1/24/b*x^4*cos(3*b*x^2+3*a)-1/6/b*(1/6/b*x^2*sin(3*b*x^2+3*a)+1/18/b^2*cos
(3*b*x^2+3*a))
```

Maxima [A]
time = 0.30, size = 79, normalized size = 0.68

$$-\frac{6\,bx^2\sin\left(3\,bx^2+3\,a\right)-162\,bx^2\sin\left(bx^2+a\right)-\left(9\,b^2x^4-2\right)\cos\left(3\,bx^2+3\,a\right)+81\,\left(b^2x^4-2\right)\cos\left(bx^2+a\right)}{216\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/216*(6*b*x^2*sin(3*b*x^2 + 3*a) - 162*b*x^2*sin(b*x^2 + a) - (9*b^2*x^4
- 2)*cos(3*b*x^2 + 3*a) + 81*(b^2*x^4 - 2)*cos(b*x^2 + a))/b^3
```

Fricas [A]
time = 0.36, size = 79, normalized size = 0.68

$$\frac{\left(9\,b^2x^4-2\right)\cos\left(bx^2+a\right)^3-3\,\left(9\,b^2x^4-14\right)\cos\left(bx^2+a\right)-6\,\left(bx^2\cos\left(bx^2+a\right)^2-7\,bx^2\right)\sin\left(bx^2+a\right)}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/54*((9*b^2*x^4 - 2)*cos(b*x^2 + a)^3 - 3*(9*b^2*x^4 - 14)*cos(b*x^2 + a)
- 6*(b*x^2*cos(b*x^2 + a)^2 - 7*b*x^2)*sin(b*x^2 + a))/b^3
```

Sympy [A]
time = 0.92, size = 143, normalized size = 1.22

$$\begin{cases} -\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{x^4\cos^3\left(a+bx^2\right)}{3b}+\frac{7x^2\sin^3\left(a+bx^2\right)}{9b^2}+\frac{2x^2\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{3b^2}+\frac{7\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{9b^3}+\frac{20\cos^3\left(a+bx^2\right)}{27b^3} & \text{for}\ b\neq 0 \\ \frac{x^6\sin^3\left(a\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**4*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**4*cos(a + b*
x**2)**3/(3*b) + 7*x**2*sin(a + b*x**2)**3/(9*b**2) + 2*x**2*sin(a + b*x**2
)*cos(a + b*x**2)**2/(3*b**2) + 7*sin(a + b*x**2)**2*cos(a + b*x**2)/(9*b**
3) + 20*cos(a + b*x**2)**3/(27*b**3), Ne(b, 0)), (x**6*sin(a)**3/6, True))
```

Giac [A]
time = 4.17, size = 138, normalized size = 1.18

$$-\frac{x^2\sin\left(3\,bx^2+3\,a\right)}{36\,b^2}+\frac{3\,x^2\sin\left(bx^2+a\right)}{4\,b^2}+\frac{\left(\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)\right)a^2}{6\,b^3}+\frac{\left(9\,(bx^2+a)^2-18\,(bx^2+a)a-2\right)\cos\left(3\,bx^2+3\,a\right)}{216\,b^3}-\frac{3\left((bx^2+a)^2-2\,(bx^2+a)a-2\right)\cos\left(bx^2+a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/36*x^2*sin(3*b*x^2 + 3*a)/b^2 + 3/4*x^2*sin(b*x^2 + a)/b^2 + 1/6*(cos(b*
x^2 + a)^3 - 3*cos(b*x^2 + a))*a^2/b^3 + 1/216*(9*(b*x^2 + a)^2 - 18*(b*x^2
 + a)*a - 2)*cos(3*b*x^2 + 3*a)/b^3 - 3/8*((b*x^2 + a)^2 - 2*(b*x^2 + a)*a
- 2)*cos(b*x^2 + a)/b^3
```

Mupad [B]
time = 4.97, size = 94, normalized size = 0.80

$$\frac{\frac{3\,\cos\left(b\,x^2+a\right)}{4}-\frac{\cos\left(3\,b\,x^2+3\,a\right)}{108}+b\,\left(\frac{3\,x^2\,\sin\left(b\,x^2+a\right)}{4}-\frac{x^2\,\sin\left(3\,b\,x^2+3\,a\right)}{36}\right)+b^2\,\left(\frac{x^4\,\cos\left(3\,b\,x^2+3\,a\right)}{24}-\frac{3\,x^4\,\cos\left(b\,x^2+a\right)}{8}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*sin(a + b*x^2)^3,x)
```

```
[Out] ((3*cos(a + b*x^2))/4 - cos(3*a + 3*b*x^2)/108 + b*((3*x^2*sin(a + b*x^2))/
4 - (x^2*sin(3*a + 3*b*x^2))/36) + b^2*((x^4*cos(3*a + 3*b*x^2))/24 - (3*x^
4*cos(a + b*x^2))/8))/b^3
```

3.24 $\int x^3 \sin^3(a+bx^2)\,dx$

Optimal. Leaf size=79

$$-\frac{x^2\cos(a+bx^2)}{3b}+\frac{\sin(a+bx^2)}{3b^2}-\frac{x^2\cos(a+bx^2)\sin^2(a+bx^2)}{6b}+\frac{\sin^3(a+bx^2)}{18b^2}$$

```
[Out] -1/3*x^2*cos(b*x^2+a)/b+1/3*sin(b*x^2+a)/b^2-1/6*x^2*cos(b*x^2+a)*sin(b*x^2
+a)^2/b+1/18*sin(b*x^2+a)^3/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3460, 3391, 3377, 2717}

$$\frac{\sin^3(a+bx^2)}{18b^2}+\frac{\sin(a+bx^2)}{3b^2}-\frac{x^2\cos(a+bx^2)}{3b}-\frac{x^2\sin^2(a+bx^2)\cos(a+bx^2)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[a + b*x^2]^3,x]
```

```
[Out] -1/3*(x^2*Cos[a + b*x^2])/b + Sin[a + b*x^2]/(3*b^2) - (x^2*Cos[a + b*x^2]*
Sin[a + b*x^2]^2)/(6*b) + Sin[a + b*x^2]^3/(18*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sin^3\left(a+bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x\sin^3(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{\sin^3\left(a+bx^2\right)}{18b^2} + \frac{1}{3}\text{Subst}\left(\int x\sin(a+bx)\,dx, x,\right.$$
$$= -\frac{x^2\cos\left(a+bx^2\right)}{3b} - \frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{\sin^3\left(a+bx^2\right)}{18b^2} + \frac{\text{Subst}\left(\int \cos\right.}{}$$
$$= -\frac{x^2\cos\left(a+bx^2\right)}{3b} + \frac{\sin\left(a+bx^2\right)}{3b^2} - \frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{\sin^3\left(a+bx^2\right.}{18b^2}$$

Mathematica [A]
time = 0.11, size = 58, normalized size = 0.73

$$-\frac{27bx^2\cos\left(a+bx^2\right) - 3bx^2\cos\left(3\left(a+bx^2\right)\right) - 27\sin\left(a+bx^2\right) + \sin\left(3\left(a+bx^2\right)\right)}{72b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[a + b*x^2]^3,x]
```

```
[Out] -1/72*(27*b*x^2*Cos[a + b*x^2] - 3*b*x^2*Cos[3*(a + b*x^2)] - 27*Sin[a + b*
x^2] + Sin[3*(a + b*x^2)])/b^2
```

Maple [A]
time = 0.06, size = 66, normalized size = 0.84

method	result	s
default	$-\frac{3x^2\cos\left(b\,x^2+a\right)}{8b}+\frac{3\sin\left(b\,x^2+a\right)}{8b^2}+\frac{x^2\cos\left(3b\,x^2+3a\right)}{24b}-\frac{\sin\left(3b\,x^2+3a\right)}{72b^2}$	6
risch	$-\frac{3x^2\cos\left(b\,x^2+a\right)}{8b}+\frac{3\sin\left(b\,x^2+a\right)}{8b^2}+\frac{x^2\cos\left(3b\,x^2+3a\right)}{24b}-\frac{\sin\left(3b\,x^2+3a\right)}{72b^2}$	6
norman	$\frac{\frac{x^2\left(\tan^4\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}+\frac{2\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{3b^2}+\frac{16\left(\tan^3\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{9b^2}+\frac{2\left(\tan^5\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{3b^2}-\frac{x^2}{3b}-\frac{x^2\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}+\frac{x^2\left(\tan^6\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x^2*cos(b*x^2+a)/b+3/8*sin(b*x^2+a)/b^2+1/24/b*x^2*cos(3*b*x^2+3*a)-1/
72/b^2*sin(3*b*x^2+3*a)
```

Maxima [A]
time = 0.33, size = 60, normalized size = 0.76

$$\frac{3\,bx^2\cos\left(3\,bx^2+3\,a\right)-27\,bx^2\cos\left(bx^2+a\right)-\sin\left(3\,bx^2+3\,a\right)+27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x^2*cos(3*b*x^2 + 3*a) - 27*b*x^2*cos(b*x^2 + a) - sin(3*b*x^2 +
3*a) + 27*sin(b*x^2 + a))/b^2
```

Fricas [A]
time = 0.38, size = 58, normalized size = 0.73

$$\frac{3\,bx^2\cos\left(bx^2+a\right)^3-9\,bx^2\cos\left(bx^2+a\right)-\left(\cos\left(bx^2+a\right)^2-7\right)\sin\left(bx^2+a\right)}{18\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b*x^2*cos(b*x^2 + a)^3 - 9*b*x^2*cos(b*x^2 + a) - (cos(b*x^2 + a)^2
 - 7)*sin(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.41, size = 92, normalized size = 1.16

$$\begin{cases} -\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{x^2\cos^3\left(a+bx^2\right)}{3b}+\frac{7\sin^3\left(a+bx^2\right)}{18b^2}+\frac{\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{3b^2} & \text{for } b\neq 0 \\ \frac{x^4\sin^3\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**2*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**2*cos(a + b*
x**2)**3/(3*b) + 7*sin(a + b*x**2)**3/(18*b**2) + sin(a + b*x**2)*cos(a + b
*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sin(a)**3/4, True))
```

Giac [A]
time = 4.02, size = 94, normalized size = 1.19

$$-\frac{\left(\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)\right)a}{6\,b^2}+\frac{3\left(bx^2+a\right)\cos\left(3\,bx^2+3\,a\right)-27\left(bx^2+a\right)\cos\left(bx^2+a\right)-\sin\left(3\,bx^2+3\,a\right)+27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))*a/b^2 + 1/72*(3*(b*x^2 + a)*cos(
3*b*x^2 + 3*a) - 27*(b*x^2 + a)*cos(b*x^2 + a) - sin(3*b*x^2 + 3*a) + 27*si
n(b*x^2 + a))/b^2
```

Mupad [B]
time = 4.75, size = 66, normalized size = 0.84

$$\frac{\frac{7\sin\left(b\,x^2+a\right)}{18}-\frac{{\cos\left(b\,x^2+a\right)}^2\,\sin\left(b\,x^2+a\right)}{18}+b\,\left(\frac{x^2\,{\cos\left(b\,x^2+a\right)}^3}{6}-\frac{x^2\,\cos\left(b\,x^2+a\right)}{2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(a + b*x^2)^3,x)
```

```
[Out] ((7*sin(a + b*x^2))/18 - (cos(a + b*x^2)^2*sin(a + b*x^2))/18 + b*((x^2*cos
(a + b*x^2)^3)/6 - (x^2*cos(a + b*x^2))/2))/b^2
```

3.25 $\int x \sin^3(a+bx^2)\,dx$

Optimal. Leaf size=33

$$-\frac{\cos(a+bx^2)}{2b}+\frac{\cos^3(a+bx^2)}{6b}$$

```
[Out] -1/2*cos(b*x^2+a)/b+1/6*cos(b*x^2+a)^3/b
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3460, 2713}

$$\frac{\cos^3(a+bx^2)}{6b}-\frac{\cos(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x^2]^3,x]
```

```
[Out] -1/2*Cos[a + b*x^2]/b + Cos[a + b*x^2]^3/(6*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\sin^3(a+bx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sin^3(a+bx)\,dx, x, x^2\right)\\ &= -\frac{\text{Subst}\left(\int (1-x^2)\,dx, x, \cos(a+bx^2)\right)}{2b}\\ &= -\frac{\cos(a+bx^2)}{2b}+\frac{\cos^3(a+bx^2)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3\cos(a+bx^2)}{8b}+\frac{\cos(3(a+bx^2))}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x^2]^3,x]
[Out] (-3*Cos[a + b*x^2])/(8*b) + Cos[3*(a + b*x^2)]/(24*b)
```

Maple [A]
time = 0.04, size = 26, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{(2+\sin^2(b\,x^2+a))\cos(b\,x^2+a)}{6b}$	26
default	$-\frac{(2+\sin^2(b\,x^2+a))\cos(b\,x^2+a)}{6b}$	26
risch	$-\frac{3\cos(b\,x^2+a)}{8b}+\frac{\cos(3b\,x^2+3a)}{24b}$	31
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}-\frac{2}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/6/b*(2+sin(b*x^2+a)^2)*cos(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.82

$$\frac{\cos(3\,bx^2+3\,a)-9\,\cos(bx^2+a)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/24*(cos(3*b*x^2 + 3*a) - 9*cos(b*x^2 + a))/b
```

Fricas [A]
time = 0.37, size = 26, normalized size = 0.79

$$\frac{\cos(bx^2+a)^3-3\,\cos(bx^2+a)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Sympy [A]
time = 0.19, size = 46, normalized size = 1.39

$$\begin{cases} -\frac{\sin^2(a+bx^2)\cos(a+bx^2)}{2b} - \frac{\cos^3(a+bx^2)}{3b} & \text{for } b \neq 0 \\ \frac{x^2\sin^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - cos(a + b*x**2)**3/(
3*b), Ne(b, 0)), (x**2*sin(a)**3/2, True))
```

Giac [A]
time = 3.47, size = 26, normalized size = 0.79

$$\frac{\cos(bx^2+a)^3 - 3\cos(bx^2+a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Mupad [B]
time = 4.67, size = 28, normalized size = 0.85

$$-\frac{3\cos(b\,x^2+a) - \cos(b\,x^2+a)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x^2)^3,x)
```

```
[Out] -(3*cos(a + b*x^2) - cos(a + b*x^2)^3)/(6*b)
```

3.26 $\int \frac{\sin^3(a+bx^2)}{x} dx$

Optimal. Leaf size=55

$$\frac{3}{8}\text{Ci}(bx^2)\sin(a) - \frac{1}{8}\text{Ci}(3bx^2)\sin(3a) + \frac{3}{8}\cos(a)\text{Si}(bx^2) - \frac{1}{8}\cos(3a)\text{Si}(3bx^2)$$

```
[Out] 3/8*cos(a)*Si(b*x^2)-1/8*cos(3*a)*Si(3*b*x^2)+3/8*Ci(b*x^2)*sin(a)-1/8*Ci(3
*b*x^2)*sin(3*a)
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3484, 3458, 3457, 3456}

$$\frac{3}{8}\sin(a)\text{CosIntegral}(bx^2) - \frac{1}{8}\sin(3a)\text{CosIntegral}(3bx^2) + \frac{3}{8}\cos(a)\text{Si}(bx^2) - \frac{1}{8}\cos(3a)\text{Si}(3bx^2)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^2]*Sin[a])/8 - (CosIntegral[3*b*x^2]*Sin[3*a])/8 + (3*Co
s[a]*SinIntegral[b*x^2])/8 - (Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx^2)}{x}\,dx = \int \left(\frac{3\sin(a+bx^2)}{4x} - \frac{\sin(3a+3bx^2)}{4x}\right)dx$$
$$= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx^2)}{x}\,dx\right) + \frac{3}{4}\int \frac{\sin(a+bx^2)}{x}\,dx$$
$$= \frac{1}{4}(3\cos(a))\int \frac{\sin(bx^2)}{x}\,dx - \frac{1}{4}\cos(3a)\int \frac{\sin(3bx^2)}{x}\,dx + \frac{1}{4}(3\sin(a))\int \frac{\cos(bx^2)}{x}\,dx$$
$$= \frac{3}{8}\mathrm{Ci}\left(bx^2\right)\sin(a) - \frac{1}{8}\mathrm{Ci}\left(3bx^2\right)\sin(3a) + \frac{3}{8}\cos(a)\mathrm{Si}\left(bx^2\right) - \frac{1}{8}\cos(3a)\mathrm{Si}\left(3bx^2\right)$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.93

$$\frac{1}{8}\left(3\mathrm{Ci}\left(bx^2\right)\sin(a) - \mathrm{Ci}\left(3bx^2\right)\sin(3a) + 3\cos(a)\mathrm{Si}\left(bx^2\right) - \cos(3a)\mathrm{Si}\left(3bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^2]*Sin[a] - CosIntegral[3*b*x^2]*Sin[3*a] + 3*Cos[a]*Sin
Integral[b*x^2] - Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 125, normalized size = 2.27

method	result
risch	$-\frac{i e^{3ia} \operatorname{expIntegral}(1,-3ix^2b)}{16} + \frac{\pi e^{-3ia} \operatorname{csgn}(b x^2)}{16} - \frac{e^{-3ia} \operatorname{sinIntegral}(3b x^2)}{8} + \frac{i \operatorname{expIntegral}(1,-3ix^2b) e^{-3ia}}{16} - \frac{3\pi \operatorname{csgn}(b x}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*exp(3*I*a)*Ei(1,-3*I*x^2*b)+1/16*Pi*exp(-3*I*a)*csgn(b*x^2)-1/8*exp
(-3*I*a)*Si(3*b*x^2)+1/16*I*Ei(1,-3*I*x^2*b)*exp(-3*I*a)-3/16*Pi*csgn(b*x^2
)*exp(-I*a)+3/8*exp(-I*a)*Si(b*x^2)-3/16*I*exp(-I*a)*Ei(1,-I*b*x^2)+3/16*I*
exp(I*a)*Ei(1,-I*b*x^2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 89, normalized size = 1.62

$$\frac{1}{16}\left(i\,\mathrm{Ei}\left(3i\,bx^2\right) - i\,\mathrm{Ei}\left(-3i\,bx^2\right)\right)\cos(3a) - \frac{3}{16}\left(i\,\mathrm{Ei}\left(i\,bx^2\right) - i\,\mathrm{Ei}\left(-i\,bx^2\right)\right)\cos(a) - \frac{1}{16}\left(\mathrm{Ei}\left(3i\,bx^2\right) + \mathrm{Ei}\left(-3i\,bx^2\right)\right)\sin(3a) + \frac{3}{16}\left(\mathrm{Ei}\left(i\,bx^2\right) + \mathrm{Ei}\left(-i\,bx^2\right)\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*(I*Ei(3*I*b*x^2) - I*Ei(-3*I*b*x^2))*cos(3*a) - 3/16*(I*Ei(I*b*x^2) -
I*Ei(-I*b*x^2))*cos(a) - 1/16*(Ei(3*I*b*x^2) + Ei(-3*I*b*x^2))*sin(3*a) + 3
/16*(Ei(I*b*x^2) + Ei(-I*b*x^2))*sin(a)
```

Fricas [A]
time = 0.36, size = 63, normalized size = 1.15

$$-\frac{1}{16}\left(\operatorname{Ci}\left(3\,bx^2\right)+\operatorname{Ci}\left(-3\,bx^2\right)\right)\sin\left(3\,a\right)+\frac{3}{16}\left(\operatorname{Ci}\left(bx^2\right)+\operatorname{Ci}\left(-bx^2\right)\right)\sin\left(a\right)-\frac{1}{8}\cos\left(3\,a\right)\operatorname{Si}\left(3\,bx^2\right)+\frac{3}{8}\cos\left(a\right)\operatorname{Si}\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] -1/16*(cos_integral(3*b*x^2) + cos_integral(-3*b*x^2))*sin(3*a) + 3/16*(cos
_integral(b*x^2) + cos_integral(-b*x^2))*sin(a) - 1/8*cos(3*a)*sin_integral
(3*b*x^2) + 3/8*cos(a)*sin_integral(b*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3/x,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x, x)
```

Giac [A]
time = 3.87, size = 47, normalized size = 0.85

$$-\frac{1}{8}\operatorname{Ci}\left(3\,bx^2\right)\sin\left(3\,a\right)+\frac{3}{8}\operatorname{Ci}\left(bx^2\right)\sin\left(a\right)+\frac{3}{8}\cos\left(a\right)\operatorname{Si}\left(bx^2\right)+\frac{1}{8}\cos\left(3\,a\right)\operatorname{Si}\left(-3\,bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] -1/8*cos_integral(3*b*x^2)*sin(3*a) + 3/8*cos_integral(b*x^2)*sin(a) + 3/8*
cos(a)*sin_integral(b*x^2) + 1/8*cos(3*a)*sin_integral(-3*b*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\sin\left(b\,x^2+a\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^2)^3/x,x)
```

```
[Out] int(sin(a + b*x^2)^3/x, x)
```

3.27 $\int \frac{\sin^3(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=91

$$\frac{3}{8}b\cos(a)\text{Ci}(bx^2)-\frac{3}{8}b\cos(3a)\text{Ci}(3bx^2)-\frac{3\sin(a+bx^2)}{8x^2}+\frac{\sin(3(a+bx^2))}{8x^2}-\frac{3}{8}b\sin(a)\text{Si}(bx^2)+\frac{3}{8}b\sin(3a)\text{Si}(3$$

```
[Out] 3/8*b*Ci(b*x^2)*cos(a)-3/8*b*Ci(3*b*x^2)*cos(3*a)-3/8*b*Si(b*x^2)*sin(a)+3/
8*b*Si(3*b*x^2)*sin(3*a)-3/8*sin(b*x^2+a)/x^2+1/8*sin(3*b*x^2+3*a)/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3484, 3460, 3378, 3384, 3380, 3383}

$$\frac{3}{8}b\cos(a)\text{CosIntegral}(bx^2)-\frac{3}{8}b\cos(3a)\text{CosIntegral}(3bx^2)-\frac{3}{8}b\sin(a)\text{Si}(bx^2)+\frac{3}{8}b\sin(3a)\text{Si}(3bx^2)-\frac{3\sin(a+bx^2)}{8x^2}+\frac{\sin(3(a+bx^2))}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3/x^3,x]
```

```
[Out] (3*b*Cos[a]*CosIntegral[b*x^2])/8 - (3*b*Cos[3*a]*CosIntegral[3*b*x^2])/8 -
 (3*Sin[a + b*x^2])/(8*x^2) + Sin[3*(a + b*x^2)]/(8*x^2) - (3*b*Sin[a]*SinI
ntegral[b*x^2])/8 + (3*b*Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx &= \int \left(\frac{3\sin\left(a+bx^2\right)}{4x^3}-\frac{\sin\left(3a+3bx^2\right)}{4x^3}\right)dx\\ &= -\left(\frac{1}{4}\int \frac{\sin\left(3a+3bx^2\right)}{x^3}\,dx\right)+\frac{3}{4}\int \frac{\sin\left(a+bx^2\right)}{x^3}\,dx\\ &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x^2}\,dx,x,x^2\right)\right)+\frac{3}{8}\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{1}{8}(3b)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,x^2\right)-\frac{1}{8}(3b)\text{S}\\ &= -\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{1}{8}(3b\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,x^2\right)-\frac{1}{8}(3\\ &= \frac{3}{8}b\cos(a)\text{Ci}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\text{Ci}\left(3bx^2\right)-\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{3}{8}b\sin(\end{aligned}$$

Mathematica [A]
time = 0.09, size = 90, normalized size = 0.99

$$\frac{3bx^2\cos(a)\text{Ci}(bx^2)-3bx^2\cos(3a)\text{Ci}(3bx^2)-3\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)-3bx^2\sin(a)\text{Si}(bx^2)+3bx^2\sin(3a)\text{Si}(3bx^2)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3/x^3,x]
```

```
[Out] (3*b*x^2*Cos[a]*CosIntegral[b*x^2] - 3*b*x^2*Cos[3*a]*CosIntegral[3*b*x^2]
- 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)] - 3*b*x^2*Sin[a]*SinIntegral[b*x^2]
 + 3*b*x^2*Sin[3*a]*SinIntegral[3*b*x^2])/(8*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 162, normalized size = 1.78

method	result
risch	$-\frac{3i\mathrm{e}^{-3ia}\operatorname{csgn}(b\,x^2)\pi b}{16}+\frac{3i\mathrm{e}^{-3ia}\operatorname{sinIntegral}(3b\,x^2)b}{8}+\frac{3\,\mathrm{e}^{-3ia}\operatorname{expIntegral}(1,-3ix^2b)b}{16}+\frac{3\,\mathrm{e}^{3ia}b\operatorname{expIntegral}(1,-3ix^2b)}{16}-3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*I*exp(-3*I*a)*csgn(b*x^2)*Pi*b+3/8*I*exp(-3*I*a)*Si(3*b*x^2)*b+3/16*e
xp(-3*I*a)*Ei(1,-3*I*x^2*b)*b+3/16*exp(3*I*a)*b*Ei(1,-3*I*x^2*b)-3/16*exp(I
*a)*b*Ei(1,-I*b*x^2)+3/16*I*csgn(b*x^2)*exp(-I*a)*Pi*b-3/8*I*exp(-I*a)*Si(b
*x^2)*b-3/16*Ei(1,-I*b*x^2)*exp(-I*a)*b-3/8*sin(b*x^2+a)/x^2+1/8*sin(3*b*x^
2+3*a)/x^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.40, size = 97, normalized size = 1.07

$$-\frac{3}{16}\left(\left(\Gamma\left(-1,3i\,bx^2\right)+\Gamma\left(-1,-3i\,bx^2\right)\right)\cos\left(3\,a\right)-\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos\left(a\right)+\left(-i\,\Gamma\left(-1,3i\,bx^2\right)+i\,\Gamma\left(-1,-3i\,bx^2\right)\right)\sin\left(3\,a\right)+\left(i\,\Gamma\left(-1,i\,bx^2\right)-i\,\Gamma\left(-1,-i\,bx^2\right)\right)\sin\left(a\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/16*((gamma(-1, 3*I*b*x^2) + gamma(-1, -3*I*b*x^2))*cos(3*a) - (gamma(-1,
 I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) + (-I*gamma(-1, 3*I*b*x^2) + I*gamm
a(-1, -3*I*b*x^2))*sin(3*a) + (I*gamma(-1, I*b*x^2) - I*gamma(-1, -I*b*x^2)
)*sin(a))*b
```

Fricas [A]
time = 0.37, size = 118, normalized size = 1.30

$$\frac{6\,bx^2\sin\left(3\,a\right)\operatorname{Si}\left(3\,bx^2\right)-6\,bx^2\sin\left(a\right)\operatorname{Si}\left(bx^2\right)-3\left(bx^2\operatorname{Ci}\left(3\,bx^2\right)+bx^2\operatorname{Ci}\left(-3\,bx^2\right)\right)\cos\left(3\,a\right)+3\left(bx^2\operatorname{Ci}\left(bx^2\right)+bx^2\operatorname{Ci}\left(-bx^2\right)\right)\cos\left(a\right)+8\left(\cos\left(bx^2+a\right)^2-1\right)\sin\left(bx^2+a\right)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*b*x^2*sin(3*a)*sin_integral(3*b*x^2) - 6*b*x^2*sin(a)*sin_integral(
b*x^2) - 3*(b*x^2*cos_integral(3*b*x^2) + b*x^2*cos_integral(-3*b*x^2))*cos
(3*a) + 3*(b*x^2*cos_integral(b*x^2) + b*x^2*cos_integral(-b*x^2))*cos(a) +
 8*(cos(b*x^2 + a)^2 - 1)*sin(b*x^2 + a))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx^2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(80) = 160.
time = 5.72, size = 186, normalized size = 2.04

$$-\frac{3(bx^2+a)b^2\cos(3a)\operatorname{Ci}(3bx^2)-3ab^2\cos(3a)\operatorname{Ci}(3bx^2)-3(bx^2+a)b^2\cos(a)\operatorname{Ci}(bx^2)+3ab^2\cos(a)\operatorname{Ci}(bx^2)+3(bx^2+a)b^2\sin(a)\operatorname{Si}(bx^2)-3ab^2\sin(a)\operatorname{Si}(bx^2)+3(bx^2+a)b^2\sin(3a)\operatorname{Si}(-3bx^2)-3ab^2\sin(3a)\operatorname{Si}(-3bx^2)-b^2\sin(3bx^2+3a)+3b^2\sin(bx^2+a)}{8b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(b*x^2 + a)*b^2*cos(3*a)*cos_integral(3*b*x^2) - 3*a*b^2*cos(3*a)*c
os_integral(3*b*x^2) - 3*(b*x^2 + a)*b^2*cos(a)*cos_integral(b*x^2) + 3*a*b
^2*cos(a)*cos_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(a)*sin_integral(b*x^2
) - 3*a*b^2*sin(a)*sin_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(3*a)*sin_int
egral(-3*b*x^2) - 3*a*b^2*sin(3*a)*sin_integral(-3*b*x^2) - b^2*sin(3*b*x^2
 + 3*a) + 3*b^2*sin(b*x^2 + a))/(b^2*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(b\,x^2+a\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^2)^3/x^3,x)
```

```
[Out] int(sin(a + b*x^2)^3/x^3, x)
```

3.28 $\int x^2 \sin^3 (a + bx^2) \, dx$

Optimal. Leaf size=188

$$-\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3\sqrt{\cdots}}{\cdots}$$

```
[Out] -3/8*x*cos(b*x^2+a)/b+1/24*x*cos(3*b*x^2+3*a)/b-1/144*cos(3*a)*FresnelC(x*b
^(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)+1/144*FresnelS(x*b^(1/2)*
6^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(3/2)+3/16*cos(a)*FresnelC(x*
b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)-3/16*FresnelS(x*b^(1/2)*
2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3484, 3466, 3435, 3433, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}\,x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sin(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x^2]^3,x]
```

```
[Out] (-3*x*Cos[a + b*x^2])/(8*b) + (x*Cos[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi/2]
*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[Pi/6]*Cos[3*a]*
FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) + (Sqrt[Pi/6]*FresnelS[Sqrt[b]*Sqrt[6/
Pi]*x]*Sin[3*a])/(24*b^(3/2))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int x^2\sin^3\left(a+bx^2\right)\,dx = \int \left(\frac{3}{4}x^2\sin\left(a+bx^2\right)-\frac{1}{4}x^2\sin\left(3a+3bx^2\right)\right)\,dx$$

$$= -\left(\frac{1}{4}\int x^2\sin\left(3a+3bx^2\right)\,dx\right)+\frac{3}{4}\int x^2\sin\left(a+bx^2\right)\,dx$$

$$= -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}-\frac{\int\cos\left(3a+3bx^2\right)\,dx}{24b}+\frac{3\int\cos\left(a+bx^2\right)}{8b}$$

$$= -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}+\frac{(3\cos(a))\int\cos\left(bx^2\right)\,dx}{8b}-\frac{\cos(3a)\int\cos(}{24b}$$

$$= -\frac{3x\cos\left(a+bx^2\right)}{8b}+\frac{x\cos\left(3a+3bx^2\right)}{24b}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos}{}$$

Mathematica [A]
time = 0.30, size = 159, normalized size = 0.85

$$\frac{-54\sqrt{b}\,x\cos\left(a+bx^2\right)+6\sqrt{b}\,x\cos\left(3\left(a+bx^2\right)\right)+27\sqrt{2\pi}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-27\sqrt{2\pi}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)+\sqrt{6\pi}S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{144b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x^2]^3,x]
```

```
[Out] (-54*Sqrt[b]*x*Cos[a + b*x^2] + 6*Sqrt[b]*x*Cos[3*(a + b*x^2)] + 27*Sqrt[2*
Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]*FresnelC[Sq
rt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] +
 Sqrt[6*Pi]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(144*b^(3/2))
```

Maple [A]
time = 0.10, size = 132, normalized size = 0.70

method	result
default	$-\frac{3x\cos(b\,x^2+a)}{8b}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}}+\frac{x\cos(3b\,x^2+3a)}{24b}-\frac{\sqrt{2}\,\cdots}{\cdots}$
risch	$-\frac{\mathrm{e}^{-3ia}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{288b\sqrt{ib}}-\frac{\mathrm{e}^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{96b\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{32b\sqrt{-ib}}+\frac{3\,\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\cdots\right)}{32b\sqrt{ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x*cos(b*x^2+a)/b+3/16/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x*b^(1
/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*x*c
os(3*b*x^2+3*a)/b-1/144/b^(3/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(
1/2)*b^(1/2)*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 143, normalized size = 0.76

$$\frac{24\,b^2x\cos\left(3\,bx^2+3\,a\right)-216\,b^2x\cos\left(bx^2+a\right)+9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i-1\right)\,\cos\left(3\,a\right)+\left(i+1\right)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{3i\,b}\,x\right)+\left(-\left(i+1\right)\,\cos\left(3\,a\right)-\left(i-1\right)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{-3i\,b}\,x\right)\right)b^{\frac{3}{2}}-27\,\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i-1\right)\,\cos\left(a\right)+\left(i+1\right)\,\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{i\,b}\,x\right)+\left(-\left(i+1\right)\,\cos\left(a\right)-\left(i-1\right)\,\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{-i\,b}\,x\right)\right)b^{\frac{3}{2}}}{576\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(24*b^2*x*cos(3*b*x^2 + 3*a) - 216*b^2*x*cos(b*x^2 + a) + 9^(1/4)*sqr
t(2)*sqrt(pi)*(((I - 1)*cos(3*a) + (I + 1)*sin(3*a))*erf(sqrt(3*I*b)*x) + (
-(I + 1)*cos(3*a) - (I - 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))*b^(3/2) - 27*sqr
t(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(I*b)*x) + (-(I +
1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^3
```

Fricas [A]
time = 0.37, size = 147, normalized size = 0.78

$$\frac{24\,bx\cos\left(bx^2+a\right)^3-\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(3\,a\right)\mathrm{C}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)+27\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(a\right)\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)-27\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)-72\,bx\cos\left(bx^2+a\right)}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/144*(24*b*x*cos(b*x^2 + a)^3 - sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_cos
(sqrt(6)*x*sqrt(b/pi)) + 27*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2
)*x*sqrt(b/pi)) + sqrt(6)*pi*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*s
in(3*a) - 27*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a)
 - 72*b*x*cos(b*x^2 + a))/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(194) = 388$.
time = 2.40, size = 439, normalized size = 2.34

$$-\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}+\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(3a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{3\sqrt{b}\,x^3\sqrt{\frac{1}{b}}\sin(a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{b}\,x^3\sqrt{\frac{1}{b}}\sin(3a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(a)C\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\pi}}\right)}{8}-\frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(3a)C\left(\frac{\sqrt{6}\sqrt{b}\,x}{\sqrt{\pi}}\right)}{24}+\frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(a)S\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\pi}}\right)}{8}-\frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(3a)S\left(\frac{\sqrt{6}\sqrt{b}\,x}{\sqrt{\pi}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(b*x**2+a)**3,x)
```

```
[Out] -3*b**(3/2)*x**5*sqrt(1/b)*cos(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (
3/2, 7/4, 9/4), -b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) + 3*b**(3/2)*x**5*
sqrt(1/b)*cos(3*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4),
 -9*b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) - 3*sqrt(b)*x**3*sqrt(1/b)*sin(
a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4/4)/(
32*gamma(5/4)*gamma(7/4)) + sqrt(b)*x**3*sqrt(1/b)*sin(3*a)*gamma(1/4)*gamm
a(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*gamma(5/4)*ga
mma(7/4)) + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fresnelc(sqrt(2)*sqrt(
b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*sin(3*a)*fresnelc(sqrt(6
)*sqrt(b)*x/sqrt(pi))/24 + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(a)*fresnel
s(sqrt(2)*sqrt(b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*cos(3*a)*
fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi))/24
```

Giac [C] Result contains complex when optimal does not.
time = 5.88, size = 259, normalized size = 1.38

$$\frac{xe^{(3i\,bx^2+3i\,a)}}{48\,b}-\frac{3\,xe^{(i\,bx^2+i\,a)}}{16\,b}-\frac{3\,xe^{(-i\,bx^2-i\,a)}}{16\,b}+\frac{xe^{(-3i\,bx^2-3i\,a)}}{48\,b}+\frac{\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b}\,x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(3i\,a)}}{288\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)}-\frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{32\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}-\frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{32\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b}\,x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(-3i\,a)}}{288\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/48*x*e^(3*I*b*x^2 + 3*I*a)/b - 3/16*x*e^(I*b*x^2 + I*a)/b - 3/16*x*e^(-I*
b*x^2 - I*a)/b + 1/48*x*e^(-3*I*b*x^2 - 3*I*a)/b + 1/288*sqrt(6)*sqrt(pi)*e
rf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3*I*a)/(b^(3/2)*(-I*b/abs(b
) + 1)) - 3/32*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(a
bs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) - 3/32*sqrt(2)*sqrt(pi)*
erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) +
 1)*sqrt(abs(b))) + 1/288*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(I*b/
abs(b) + 1))*e^(-3*I*a)/(b^(3/2)*(I*b/abs(b) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sin\left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*x^2)^3,x)
```

```
[Out] int(x^2*sin(a + b*x^2)^3, x)
```

3.29 $\int \sin^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=153

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}\,x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\sin(a)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\,C\left(\sqrt{b}\right.}{4}$$

```
[Out] -1/24*cos(3*a)*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(1/2
)-1/24*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(1/
2)+3/8*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)
+3/8*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3438, 3434, 3433, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}\,x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}\,x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) - (Sqrt[Pi
/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) + (3*Sqrt[Pi/2]*Fr
esnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelC[Sqr
t[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \sin^3\left(a+bx^2\right)\,dx &= \int \left(\frac{3}{4}\sin\left(a+bx^2\right)-\frac{1}{4}\sin\left(3a+3bx^2\right)\right)\,dx\\ &= -\left(\frac{1}{4}\int \sin\left(3a+3bx^2\right)\,dx\right)+\frac{3}{4}\int \sin\left(a+bx^2\right)\,dx\\ &= \frac{1}{4}(3\cos(a))\int \sin\left(bx^2\right)\,dx-\frac{1}{4}\cos(3a)\int \sin\left(3bx^2\right)\,dx+\frac{1}{4}(3\sin(a))\int \cos\left(bx^2\right)\,dx\\ &= \frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right.}{4\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 117, normalized size = 0.76

$$\frac{\sqrt{\frac{\pi}{6}}\left(3\sqrt{3}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)+3\sqrt{3}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)-C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/6]*(3*Sqrt[3]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Cos[3*a]*Fre
snelS[Sqrt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[3]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[
a] - FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a]))/(4*Sqrt[b])
```

Maple [A]
time = 0.05, size = 99, normalized size = 0.65

method	result
default	$\frac{3\sqrt{2}\sqrt{\pi}\left(\cos(a)\mathrm{S}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8\sqrt{b}}-\frac{\sqrt{2}\sqrt{\pi}\sqrt{3}\left(\cos(3a)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{b}\,x}{\sqrt{\pi}}\right)\right.}{24\sqrt{b}}$
risch	$\frac{i e^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{16\sqrt{-3ib}}-\frac{i e^{-3ia}\sqrt{\pi}\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\sqrt{ib}\,x\right)}{48\sqrt{ib}}+\frac{3i e^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{16\sqrt{ib}}-\frac{3i e^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\right.}{16\sqrt{-ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))+s
in(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/b
^(1/2)*(cos(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 112, normalized size = 0.73

$$\frac{9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\cos\left(3\,a\right)+(i-1)\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{3ib}\,x\right)+\left((i-1)\cos\left(3\,a\right)-(i+1)\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{-3ib}\,x\right)\right)b^{\frac{3}{2}}-9\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\cos\left(a\right)+(i-1)\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{ib}\,x\right)+\left((i-1)\cos\left(a\right)-(i+1)\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{-ib}\,x\right)\right)b^{\frac{3}{2}}}{96\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(3*a) + (I - 1)*sin(3*a))*erf(
sqrt(3*I*b)*x) + ((I - 1)*cos(3*a) - (I + 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))
*b^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(
I*b)*x) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^2
```

Fricas [A]
time = 0.41, size = 120, normalized size = 0.78

$$-\frac{\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{S}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)-9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)-9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) - 9
*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)*p
i*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) - 9*sqrt(2)*pi*sqrt
(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

Sympy [A]
time = 0.68, size = 129, normalized size = 0.84

$$\frac{3\sqrt{2}\,\sqrt{\pi}\left(\sin\left(a\right)C\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\cos\left(a\right)S\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8}-\frac{\sqrt{6}\,\sqrt{\pi}\left(\sin\left(3a\right)C\left(\frac{\sqrt{6}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\cos\left(3a\right)S\left(\frac{\sqrt{6}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*(sin(a)*fresnelc(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*fr
esnels(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/8 - sqrt(6)*sqrt(pi)*(sin(3*a
)*fresnelc(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnels(sqrt(6)*sqrt(b)*
x/sqrt(pi)))*sqrt(1/b)/24
```

Giac [C] Result contains complex when optimal does not.
time = 5.55, size = 185, normalized size = 1.21

$$-\frac{i\,\sqrt{6}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{6}\,\sqrt{b}\,x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(3i\,a)}}{48\,\sqrt{b}\left(-\frac{i\,b}{|b|}+1\right)}+\frac{3i\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{16\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}-\frac{3i\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{16\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{i\,\sqrt{6}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{6}\,\sqrt{b}\,x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(-3i\,a)}}{48\,\sqrt{b}\left(\frac{i\,b}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*I*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3
*I*a)/(sqrt(b)*(-I*b/abs(b) + 1)) + 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2
)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))
) - 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)
))*e^(-I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) + 1/48*I*sqrt(6)*sqrt(pi)*erf(-
1/2*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(I*b/abs(b) + 1
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^2)^3,x)
```

```
[Out] int(sin(a + b*x^2)^3, x)
```

3.30 $\int \frac{\sin^3(a+bx^2)}{x^2} \, dx$

Optimal. Leaf size=168

$$\frac{3}{2}\sqrt{b}\,\sqrt{\frac{\pi}{2}}\,\cos(a)C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)-\frac{1}{2}\sqrt{b}\,\sqrt{\frac{3\pi}{2}}\,\cos(3a)C\left(\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,x\right)-\frac{3}{2}\sqrt{b}\,\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(a)$$

```
[Out] -sin(b*x^2+a)^3/x+3/4*cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)-3/4*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(
1/2)*Pi^(1/2)-1/4*cos(3*a)*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*b^(1/2)*6^(
1/2)*Pi^(1/2)+1/4*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*b^(1/2)*6^(
1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3474, 4670, 3435, 3433, 3432}

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\sqrt{b}\,\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,x\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\,\sqrt{b}\,\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,x\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\sqrt{b}\,\sin(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)+\frac{1}{2}\sqrt{\frac{3\pi}{2}}\,\sqrt{b}\,\sin(3a)S\left(\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,x\right)-\frac{\sin^3(a+bx^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3/x^2,x]
```

```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/2 - (Sqrt[b]*S
qrt[(3*Pi)/2]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/2 - (3*Sqrt[b]*Sqrt[
Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Fr
esnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2 - Sin[a + b*x^2]^3/x
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3474

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sin^3\left(a+bx^2\right)}{x}+(6b)\int \cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)\,dx\\ &= -\frac{\sin^3\left(a+bx^2\right)}{x}+(6b)\int \left(\frac{1}{4}\cos\left(a+bx^2\right)-\frac{1}{4}\cos\left(3a+3bx^2\right)\right)dx\\ &= -\frac{\sin^3\left(a+bx^2\right)}{x}+\frac{1}{2}(3b)\int \cos\left(a+bx^2\right)\,dx-\frac{1}{2}(3b)\int \cos\left(3a+3bx^2\right)\,dx\\ &= -\frac{\sin^3\left(a+bx^2\right)}{x}+\frac{1}{2}(3b\cos(a))\int \cos\left(bx^2\right)\,dx-\frac{1}{2}(3b\cos(3a))\int \cos\left(3bx^2\right)\,dx-\frac{1}{2}(\\ &= \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-\frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 167, normalized size = 0.99

$$\frac{3\sqrt{b}\sqrt{2\pi}\,x\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{b}\sqrt{6\pi}\,x\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-3\sqrt{b}\sqrt{2\pi}\,xS\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)+\sqrt{b}\sqrt{6\pi}\,xS\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)-3\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3/x^2,x]
```

```
[Out] (3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[b]*Sqr
t[6*Pi]*x*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x] - 3*Sqrt[b]*Sqrt[2*Pi]*x*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelS[Sqrt[
b]*Sqrt[6/Pi]*x]*Sin[3*a] - 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)])/(4*x)
```

Maple [A]
time = 0.07, size = 130, normalized size = 0.77

method	result
default	$-\frac{3\sin(b\,x^2+a)}{4x}+\frac{3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4}+\frac{\sin(3b\,x^2+3a)}{4x}-\frac{\sqrt{b}}{}$
risch	$-\frac{\mathrm{e}^{-3ia}b\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{8\sqrt{ib}}-\frac{3\,\mathrm{e}^{3ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{8\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{8\sqrt{-ib}}+\frac{3\,\mathrm{e}^{-ia}b\sqrt{\pi}}{8\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/x*sin(b*x^2+a)+3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x*b^(1/2)
*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/4*sin(3*b
*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC(2^(1/2)
/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(
1/2)*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.61, size = 152, normalized size = 0.90

$$\frac{\sqrt{3}\,\sqrt{bx^2}\,\left(\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},3i\,bx^2\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-3i\,bx^2\right)\right)\cos(3\,a)+\left((i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},3i\,bx^2\right)-(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-3i\,bx^2\right)\right)\sin(3\,a)\right)-3\,\sqrt{bx^2}\,\left(\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,bx^2\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-i\,bx^2\right)\right)\cos(a)+\left((i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,bx^2\right)-(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-i\,bx^2\right)\right)\sin(a)\right)}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(3)*sqrt(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) - (I + 1
)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*cos(3*a) + ((I + 1)*sqrt(2)*gamma(-1/2,
3*I*b*x^2) - (I - 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*sin(3*a)) - 3*sqrt(b*
x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -
I*b*x^2))*cos(a) + ((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I - 1)*sqrt(2)*
gamma(-1/2, -I*b*x^2))*sin(a)))/x
```

Fricas [A]
time = 0.37, size = 147, normalized size = 0.88

$$-\frac{\sqrt{6}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(3\,a)\,\mathrm{C}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)-3\,\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)-\sqrt{6}\,\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)\sin(3\,a)+3\,\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin(a)-4\left(\cos(bx^2+a)^2-1\right)\sin(bx^2+a)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_cos(sqrt(6)*x*sqrt(b/pi)) -
3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(6
)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)*pi
```

```
*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a) - 4*(cos(b*x^2 + a)^
2 - 1)*sin(b*x^2 + a))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3/x**2,x)
[Out] Integral(sin(a + b*x**2)**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sin(b*x^2 + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(b\,x^2+a\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^2)^3/x^2,x)
[Out] int(sin(a + b*x^2)^3/x^2, x)
```

3.31 $\int x^2 \sin^3\left(x^2\right)\,dx$

Optimal. Leaf size=71

$$-\frac{1}{2}x\cos\left(x^2\right)+\frac{1}{6}x\cos^3\left(x^2\right)+\frac{3}{8}\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,x\right)-\frac{1}{24}\sqrt{\frac{\pi}{6}}\,C\left(\sqrt{\frac{6}{\pi}}\,x\right)$$

```
[Out] -1/2*x*cos(x^2)+1/6*x*cos(x^2)^3-1/144*FresnelC(x*6^(1/2)/Pi^(1/2))*6^(1/2)
*Pi^(1/2)+3/16*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3484, 3466, 3433}

$$\frac{3}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,x\right)-\frac{1}{24}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,x\right)-\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{24}x\cos\left(3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x^2]^3,x]
```

```
[Out] (-3*x*Cos[x^2])/8 + (x*Cos[3*x^2])/24 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x
])/8 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/24
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin^3\left(x^2\right)\,dx &= \int \left(\frac{3}{4}x^2\sin\left(x^2\right) - \frac{1}{4}x^2\sin\left(3x^2\right)\right)\,dx \\
&= -\left(\frac{1}{4}\int x^2\sin\left(3x^2\right)\,dx\right) + \frac{3}{4}\int x^2\sin\left(x^2\right)\,dx \\
&= -\frac{3}{8}x\cos\left(x^2\right) + \frac{1}{24}x\cos\left(3x^2\right) - \frac{1}{24}\int \cos\left(3x^2\right)\,dx + \frac{3}{8}\int \cos\left(x^2\right)\,dx \\
&= -\frac{3}{8}x\cos\left(x^2\right) + \frac{1}{24}x\cos\left(3x^2\right) + \frac{3}{8}\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,x\right) - \frac{1}{24}\sqrt{\frac{\pi}{6}}\,C\left(\sqrt{\frac{6}{\pi}}\,x\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.89

$$\frac{1}{144}\left(6x\left(-9\cos\left(x^2\right)+\cos\left(3x^2\right)\right)+27\sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\,x\right)-\sqrt{6\pi}\,C\left(\sqrt{\frac{6}{\pi}}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x^2]^3,x]
[Out] (6*x*(-9*Cos[x^2] + Cos[3*x^2]) + 27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] - Sq
rt[6*Pi]*FresnelC[Sqrt[6/Pi]*x])/144
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.82

method	result
default	$-\frac{3x\cos(x^2)}{8}+\frac{3\,\mathrm{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{16}+\frac{x\cos(3x^2)}{24}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{\sqrt{\pi}}\right)}{144}$
risch	$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3i}\,x\right)}{96\sqrt{-3i}}+\frac{(-1)^{\frac{3}{4}}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,(-1)^{\frac{1}{4}}x\right)}{288}-\frac{3(-1)^{\frac{3}{4}}\sqrt{\pi}\,\mathrm{erf}\left((-1)^{\frac{1}{4}}x\right)}{32}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-i}\,x\right)}{32\sqrt{-i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^2)^3,x,method=_RETURNVERBOSE)
[Out] -3/8*x*cos(x^2)+3/16*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)+1/24*x*c
os(3*x^2)-1/144*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*
x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 97, normalized size = 1.37

$$\frac{1}{24}\,x\cos\left(3\,x^2\right)-\frac{3}{8}\,x\cos\left(x^2\right)+\frac{1}{1152}\,\sqrt{\pi}\left((2i-2)\,\sqrt{3}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{3i}\,x\right)-(2i+2)\,\sqrt{3}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{-3i}\,x\right)-(27i-27)\,\sqrt{2}\,\mathrm{erf}\left(\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\,x\right)-(27i+27)\,\sqrt{2}\,\mathrm{erf}\left(\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\,x\right)+(27i+27)\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{-i}\,x\right)-(27i-27)\,\sqrt{2}\,\mathrm{erf}\left((-1)^{\frac{1}{4}}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*x*cos(3*x^2) - 3/8*x*cos(x^2) + 1/1152*sqrt(pi)*((2*I - 2)*sqrt(3)*sqr
t(2)*erf(sqrt(3*I)*x) - (2*I + 2)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I
 - 27)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*
I - 1/2)*sqrt(2)*x) + (27*I + 27)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqr
t(2)*erf((-1)^(1/4)*x))
```

Fricas [A]
time = 0.37, size = 51, normalized size = 0.72

$$\frac{1}{6}\,x\cos\left(x^2\right)^3-\frac{1}{2}\,x\cos\left(x^2\right)-\frac{1}{144}\,\sqrt{6}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{6}\,x}{\sqrt{\pi}}\right)+\frac{3}{16}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x*cos(x^2)^3 - 1/2*x*cos(x^2) - 1/144*sqrt(6)*sqrt(pi)*fresnel_cos(sqrt
(6)*x/sqrt(pi)) + 3/16*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi))
```

Sympy [A]
time = 2.15, size = 116, normalized size = 1.63

$$-\frac{15x\cos\left(x^2\right)\Gamma\left(\frac{5}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)}+\frac{5x\cos\left(3x^2\right)\Gamma\left(\frac{5}{4}\right)}{96\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{2}\,\sqrt{\pi}\,C\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt{6}\,\sqrt{\pi}\,C\left(\frac{\sqrt{6}\,x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{576\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x**2)**3,x)
```

```
[Out] -15*x*cos(x**2)*gamma(5/4)/(32*gamma(9/4)) + 5*x*cos(3*x**2)*gamma(5/4)/(96
*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*gamma(5/4)/
(64*gamma(9/4)) - 5*sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt(pi))*gamma(5/4
)/(576*gamma(9/4))
```

Giac [C] Result contains complex when optimal does not.
time = 3.71, size = 97, normalized size = 1.37

$$\left(\frac{1}{576}i+\frac{1}{576}\right)\sqrt{6}\,\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\,x\right)-\left(\frac{1}{576}i-\frac{1}{576}\right)\sqrt{6}\,\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\,x\right)-\left(\frac{3}{64}i+\frac{3}{64}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\,x\right)+\left(\frac{3}{64}i-\frac{3}{64}\right)\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\,x\right)+\frac{1}{48}\,xe^{\left(3i\,x^2\right)}-\frac{3}{16}\,xe^{\left(i\,x^2\right)}-\frac{3}{16}\,xe^{\left(-i\,x^2\right)}+\frac{1}{48}\,xe^{\left(-3i\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] (1/576*I + 1/576)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) - (1/576*I
- 1/576)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) - (3/64*I + 3/64)*s
```

```
qrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) + (3/64*I - 3/64)*sqrt(2)*sqrt
(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) + 1/48*x*e^(3*I*x^2) - 3/16*x*e^(I*x^2)
- 3/16*x*e^(-I*x^2) + 1/48*x*e^(-3*I*x^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sin\left(x^2\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^2)^3,x)
```

```
[Out] int(x^2*sin(x^2)^3, x)
```

3.32 $\int x^4 \cos(x^2) \sin^2(x^2)\, dx$

Optimal. Leaf size=84

$$\frac{1}{4}x\cos\left(x^2\right)-\frac{1}{12}x\cos^3\left(x^2\right)-\frac{3}{16}\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,x\right)+\frac{1}{48}\sqrt{\frac{\pi}{6}}\,C\left(\sqrt{\frac{6}{\pi}}\,x\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)$$

```
[Out] 1/4*x*cos(x^2)-1/12*x*cos(x^2)^3+1/6*x^3*sin(x^2)^3+1/288*FresnelC(x*6^(1/2
)/Pi^(1/2))*6^(1/2)*Pi^(1/2)-3/32*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(
1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3524, 3484, 3466, 3433}

$$-\frac{3}{16}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,x\right)+\frac{1}{48}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,x\right)+\frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

```
[Out] (3*x*Cos[x^2])/16 - (x*Cos[3*x^2])/48 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x
])/16 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/48 + (x^3*Sin[x^2]^3)/6
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
```

```
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\cos\left(x^2\right)\sin^2\left(x^2\right)\,dx &= \frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{2}\int x^2\sin^3\left(x^2\right)\,dx\\ &= \frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{2}\int\left(\frac{3}{4}x^2\sin\left(x^2\right)-\frac{1}{4}x^2\sin\left(3x^2\right)\right)dx\\ &= \frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{1}{8}\int x^2\sin\left(3x^2\right)\,dx-\frac{3}{8}\int x^2\sin\left(x^2\right)\,dx\\ &= \frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{1}{48}\int\cos\left(3x^2\right)\,dx-\frac{3}{16}\int\cos\\ &= \frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)-\frac{3}{16}\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,x\right)+\frac{1}{48}\sqrt{\frac{\pi}{6}}\,C\left(\sqrt{\frac{6}{\pi}}\,x\right)+\end{aligned}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.89

$$\frac{1}{288}\left(-27\sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\,x\right)+\sqrt{6\pi}\,C\left(\sqrt{\frac{6}{\pi}}\,x\right)+6x\left(9\cos\left(x^2\right)-\cos\left(3x^2\right)+8x^2\sin^3\left(x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Cos[x^2]*Sin[x^2]^2,x]
[Out] (-27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] + Sqrt[6*Pi]*FresnelC[Sqrt[6/Pi]*x]
+ 6*x*(9*Cos[x^2] - Cos[3*x^2] + 8*x^2*Sin[x^2]^3))/288
```

Maple [A]
time = 0.07, size = 78, normalized size = 0.93

method	result
default	$\frac{x^3\sin(x^2)}{8}+\frac{3x\cos(x^2)}{16}-\frac{3\,\mathrm{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{32}-\frac{x^3\sin(3x^2)}{24}-\frac{x\cos(3x^2)}{48}+\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{FresnelC}\left(\right.}{288}$
risch	$-\frac{(-1)^{\frac{3}{4}}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,(-1)^{\frac{1}{4}}x\right)}{576}+\frac{3(-1)^{\frac{3}{4}}\sqrt{\pi}\,\mathrm{erf}\left((-1)^{\frac{1}{4}}x\right)}{64}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3i}\,x\right)}{192\sqrt{-3i}}-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-i}\,x\right)}{64\sqrt{-i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(x^2)*sin(x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^3*sin(x^2)+3/16*x*cos(x^2)-3/32*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*
Pi^(1/2)-1/24*x^3*sin(3*x^2)-1/48*x*cos(3*x^2)+1/288*2^(1/2)*Pi^(1/2)*3^(1/
2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 117, normalized size = 1.39

$$-\frac{1}{24}\,x^3\sin\left(3\,x^2\right)+\frac{1}{8}\,x^3\sin\left(x^2\right)-\frac{1}{48}\,x\cos\left(3\,x^2\right)+\frac{3}{16}\,x\cos\left(x^2\right)-\frac{1}{2304}\,\sqrt{\pi}\,\left((2i-2)\,\sqrt{3}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{3i}\,x\right)-(2i+2)\,\sqrt{3}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{-3i}\,x\right)-(27i-27)\,\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\,x\right)-(27i+27)\,\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\,x\right)+(27i+27)\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{-i}\,x\right)-(27i-27)\,\sqrt{2}\,\mathrm{erf}\left((-1)^{\frac{1}{4}}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*x^3*sin(3*x^2) + 1/8*x^3*sin(x^2) - 1/48*x*cos(3*x^2) + 3/16*x*cos(x^
2) - 1/2304*sqrt(pi)*((2*I - 2)*sqrt(3)*sqrt(2)*erf(sqrt(3*I)*x) - (2*I + 2
)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I - 27)*sqrt(2)*erf((1/2*I + 1/2)
*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*x) + (27*I + 27
)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqrt(2)*erf((-1)^(1/4)*x))
```

Fricas [A]
time = 0.40, size = 73, normalized size = 0.87

$$-\frac{1}{12}\,x\cos\left(x^2\right)^3+\frac{1}{4}\,x\cos\left(x^2\right)+\frac{1}{288}\,\sqrt{6}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{6}\,x}{\sqrt{\pi}}\right)-\frac{3}{32}\,\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)-\frac{1}{6}\,\left(x^3\cos\left(x^2\right)^2-x^3\right)\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*x*cos(x^2)^3 + 1/4*x*cos(x^2) + 1/288*sqrt(6)*sqrt(pi)*fresnel_cos(sq
rt(6)*x/sqrt(pi)) - 3/32*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi)) -
 1/6*(x^3*cos(x^2)^2 - x^3)*sin(x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(82) = 164$.
time = 2.20, size = 291, normalized size = 3.46

$$-\frac{9x^5\Gamma\left(-\frac{9}{4}\right)}{40\Gamma\left(-\frac{5}{4}\right)}+\frac{9x^3\sin\left(x^2\right)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)}-\frac{5x^3\sin\left(x^2\right)\Gamma\left(-\frac{5}{4}\right)}{16\Gamma\left(-\frac{1}{4}\right)}+\frac{3x^3\sin\left(3x^2\right)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)}+\frac{27x\cos\left(x^2\right)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)}-\frac{15x\cos\left(x^2\right)\Gamma\left(-\frac{5}{4}\right)}{32\Gamma\left(-\frac{1}{4}\right)}+\frac{3x\cos\left(3x^2\right)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)}+\frac{15\sqrt{2}\,\sqrt{\pi}\,C\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{5}{4}\right)}{64\Gamma\left(-\frac{1}{4}\right)}-\frac{27\sqrt{2}\,\sqrt{\pi}\,C\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{9}{4}\right)}{128\Gamma\left(-\frac{5}{4}\right)}-\frac{\sqrt{6}\,\sqrt{\pi}\,C\left(\frac{\sqrt{6}\,x}{\sqrt{\pi}}\right)\Gamma\left(-\frac{9}{4}\right)}{128\Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(x**2)*sin(x**2)**2,x)
```

```
[Out] -9*x**5*gamma(-9/4)/(40*gamma(-5/4)) + 9*x**3*sin(x**2)*gamma(-9/4)/(32*gam
ma(-5/4)) - 5*x**3*sin(x**2)*gamma(-5/4)/(16*gamma(-1/4)) + 3*x**3*sin(3*x*
*2)*gamma(-9/4)/(32*gamma(-5/4)) + 27*x*cos(x**2)*gamma(-9/4)/(64*gamma(-5/
4)) - 15*x*cos(x**2)*gamma(-5/4)/(32*gamma(-1/4)) + 3*x*cos(3*x**2)*gamma(-
9/4)/(64*gamma(-5/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*ga
mma(-5/4)/(64*gamma(-1/4)) - 27*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi
```

```
))*gamma(-9/4)/(128*gamma(-5/4)) - sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt
(pi))*gamma(-9/4)/(128*gamma(-5/4))
```

Giac [C] Result contains complex when optimal does not.
time = 3.08, size = 125, normalized size = 1.49

$$-\left(\frac{1}{1152}i+\frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\,x\right)+\left(\frac{1}{1152}i-\frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\,x\right)+\left(\frac{3}{128}i+\frac{3}{128}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\,x\right)-\left(\frac{3}{128}i-\frac{3}{128}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\,x\right)-\frac{1}{96}\left(-2i\,x^3+x\right)e^{\left(3i\,x^2\right)}-\frac{1}{32}\left(2i\,x^3-3\,x\right)e^{\left(i\,x^2\right)}-\frac{1}{32}\left(-2i\,x^3-3\,x\right)e^{\left(-i\,x^2\right)}-\frac{1}{96}\left(2i\,x^3+x\right)e^{\left(-3i\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="giac")
```

```
[Out] -(1/1152*I + 1/1152)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) + (1/115
2*I - 1/1152)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) + (3/128*I + 3
/128)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) - (3/128*I - 3/128)*sqr
t(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) - 1/96*(-2*I*x^3 + x)*e^(3*I*x^
2) - 1/32*(2*I*x^3 - 3*x)*e^(I*x^2) - 1/32*(-2*I*x^3 - 3*x)*e^(-I*x^2) - 1/
96*(2*I*x^3 + x)*e^(-3*I*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\cos\left(x^2\right)\,\sin\left(x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(x^2)*sin(x^2)^2,x)
```

```
[Out] int(x^4*cos(x^2)*sin(x^2)^2, x)
```

3.33 $\int x \sin^7(a+bx^2)\,dx$

Optimal. Leaf size=67

$$-\frac{\cos(a+bx^2)}{2b}+\frac{\cos^3(a+bx^2)}{2b}-\frac{3\cos^5(a+bx^2)}{10b}+\frac{\cos^7(a+bx^2)}{14b}$$

```
[Out] -1/2*cos(b*x^2+a)/b+1/2*cos(b*x^2+a)^3/b-3/10*cos(b*x^2+a)^5/b+1/14*cos(b*x
^2+a)^7/b
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3460, 2713}

$$\frac{\cos^7(a+bx^2)}{14b}-\frac{3\cos^5(a+bx^2)}{10b}+\frac{\cos^3(a+bx^2)}{2b}-\frac{\cos(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x^2]^7,x]
```

```
[Out] -1/2*Cos[a + b*x^2]/b + Cos[a + b*x^2]^3/(2*b) - (3*Cos[a + b*x^2]^5)/(10*b
) + Cos[a + b*x^2]^7/(14*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\sin^7(a+bx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sin^7(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\text{Subst}\left(\int (1-3x^2+3x^4-x^6)\,dx, x, \cos(a+bx^2)\right)}{2b}\\
&= -\frac{\cos(a+bx^2)}{2b}+\frac{\cos^3(a+bx^2)}{2b}-\frac{3\cos^5(a+bx^2)}{10b}+\frac{\cos^7(a+bx^2)}{14b}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 1.00

$$-\frac{35\cos(a+bx^2)}{128b}+\frac{7\cos(3(a+bx^2))}{128b}-\frac{7\cos(5(a+bx^2))}{640b}+\frac{\cos(7(a+bx^2))}{896b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x^2]^7,x]
```

```
[Out] (-35*Cos[a + b*x^2])/(128*b) + (7*Cos[3*(a + b*x^2)])/(128*b) - (7*Cos[5*(a
 + b*x^2)])/(640*b) + Cos[7*(a + b*x^2)]/(896*b)
```

Maple [A]
time = 0.05, size = 50, normalized size = 0.75

method	result	size
derivativedivides	$-\frac{\left(\frac{16}{5}+\sin^6(b\,x^2+a)+\frac{6\left(\sin^4(b\,x^2+a)\right)}{5}+\frac{8\left(\sin^2(b\,x^2+a)\right)}{5}\right)\cos(b\,x^2+a)}{14b}$	50
default	$-\frac{\left(\frac{16}{5}+\sin^6(b\,x^2+a)+\frac{6\left(\sin^4(b\,x^2+a)\right)}{5}+\frac{8\left(\sin^2(b\,x^2+a)\right)}{5}\right)\cos(b\,x^2+a)}{14b}$	50
risch	$-\frac{35\cos(b\,x^2+a)}{128b}+\frac{\cos(7b\,x^2+7a)}{896b}-\frac{7\cos(5b\,x^2+5a)}{640b}+\frac{7\cos(3b\,x^2+3a)}{128b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14/b*(16/5+sin(b*x^2+a)^6+6/5*sin(b*x^2+a)^4+8/5*sin(b*x^2+a)^2)*cos(b*x
^2+a)
```

Maxima [A]
time = 0.35, size = 55, normalized size = 0.82

$$\frac{5\cos(7\,bx^2+7\,a)-49\cos(5\,bx^2+5\,a)+245\cos(3\,bx^2+3\,a)-1225\cos(bx^2+a)}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/4480*(5*cos(7*b*x^2 + 7*a) - 49*cos(5*b*x^2 + 5*a) + 245*cos(3*b*x^2 + 3*
a) - 1225*cos(b*x^2 + a))/b
```

Fricas [A]
time = 0.36, size = 52, normalized size = 0.78

$$\frac{5\cos(bx^2+a)^7-21\cos(bx^2+a)^5+35\cos(bx^2+a)^3-35\cos(bx^2+a)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*c
os(b*x^2 + a))/b
```

Sympy [A]
time = 0.89, size = 95, normalized size = 1.42

$$\begin{cases} -\frac{\sin^6(a+bx^2)\cos(a+bx^2)}{2b} - \frac{\sin^4(a+bx^2)\cos^3(a+bx^2)}{b} - \frac{4\sin^2(a+bx^2)\cos^5(a+bx^2)}{5b} - \frac{8\cos^7(a+bx^2)}{35b} & \text{for } b \neq 0 \\ \frac{x^2\sin^7(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x**2+a)**7,x)
```

```
[Out] Piecewise((-sin(a + b*x**2)**6*cos(a + b*x**2)/(2*b) - sin(a + b*x**2)**4*c
os(a + b*x**2)**3/b - 4*sin(a + b*x**2)**2*cos(a + b*x**2)**5/(5*b) - 8*cos
(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sin(a)**7/2, True))
```

Giac [A]
time = 4.81, size = 52, normalized size = 0.78

$$\frac{5\cos\left(bx^2+a\right)^7 - 21\cos\left(bx^2+a\right)^5 + 35\cos\left(bx^2+a\right)^3 - 35\cos\left(bx^2+a\right)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*c
os(b*x^2 + a))/b
```

Mupad [B]
time = 4.99, size = 55, normalized size = 0.82

$$\frac{245\cos\left(3\,b\,x^2+3\,a\right) - 49\cos\left(5\,b\,x^2+5\,a\right) + 5\cos\left(7\,b\,x^2+7\,a\right) - 1225\cos\left(b\,x^2+a\right)}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x^2)^7,x)
```

```
[Out] (245*cos(3*a + 3*b*x^2) - 49*cos(5*a + 5*b*x^2) + 5*cos(7*a + 7*b*x^2) - 12
25*cos(a + b*x^2))/(4480*b)
```

3.34 $\int \frac{(1+\sin(x^2))^2}{x^3} \, dx$

Optimal. Leaf size=44

$$-\frac{3}{4x^2}+\frac{\cos(2x^2)}{4x^2}+\text{Ci}(x^2)-\frac{\sin(x^2)}{x^2}+\frac{\text{Si}(2x^2)}{2}$$

```
[Out] -3/4/x^2+Ci(x^2)+1/4*cos(2*x^2)/x^2+1/2*Si(2*x^2)-sin(x^2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3484, 3461, 3378, 3380, 3460, 3383}

$$\text{CosIntegral}(x^2)+\frac{\text{Si}(2x^2)}{2}-\frac{3}{4x^2}-\frac{\sin(x^2)}{x^2}+\frac{\cos(2x^2)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x^2])^2/x^3,x]
```

```
[Out] -3/(4*x^2) + Cos[2*x^2]/(4*x^2) + CosIntegral[x^2] - Sin[x^2]/x^2 + SinInte
gral[2*x^2]/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+\sin(x^2))^2}{x^3}\,dx &= \int \left(\frac{3}{2x^3}-\frac{\cos(2x^2)}{2x^3}+\frac{2\sin(x^2)}{x^3}\right)dx\\
&= -\frac{3}{4x^2}-\frac{1}{2}\int \frac{\cos(2x^2)}{x^3}\,dx+2\int \frac{\sin(x^2)}{x^3}\,dx\\
&= -\frac{3}{4x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{\cos(2x)}{x^2}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{\sin(x)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{3}{4x^2}+\frac{\cos(2x^2)}{4x^2}-\frac{\sin(x^2)}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{\cos(x)}{x}\,d\right.\\
&= -\frac{3}{4x^2}+\frac{\cos(2x^2)}{4x^2}+\text{Ci}(x^2)-\frac{\sin(x^2)}{x^2}+\frac{\text{Si}(2x^2)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.93

$$\frac{-3+\cos(2x^2)+4x^2\text{Ci}(x^2)-4\sin(x^2)+2x^2\text{Si}(2x^2)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x^2])^2/x^3,x]
[Out] (-3 + Cos[2*x^2] + 4*x^2*CosIntegral[x^2] - 4*Sin[x^2] + 2*x^2*SinIntegral[
2*x^2])/(4*x^2)
```

Maple [A]
time = 0.06, size = 39, normalized size = 0.89

method	result
default	$-\frac{3}{4x^2}+\text{cosineIntegral}\left(x^2\right)+\frac{\cos(2x^2)}{4x^2}+\frac{\text{sinIntegral}(2x^2)}{2}-\frac{\sin(x^2)}{x^2}$
risch	$\text{cosineIntegral}\left(x^2\right)-\frac{i\pi\,\text{csgn}(ix^2)\text{csgn}(x^2)}{2}+\frac{i\pi\,\text{csgn}(ix^2)}{2}-\frac{3}{4x^2}-\frac{\pi\,\text{csgn}(x^2)}{4}+\frac{\text{sinIntegral}(2x^2)}{2}-\frac{\sin(x^2)}{x^2}+\text{cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/x^2+Ci(x^2)+1/4*cos(2*x^2)/x^2+1/2*Si(2*x^2)-sin(x^2)/x^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 54, normalized size = 1.23

$$\frac{x^2(i\,\Gamma(-1,2i\,x^2)-i\,\Gamma(-1,-2i\,x^2))-1}{4\,x^2}-\frac{1}{2\,x^2}+\frac{1}{2}\,\Gamma\left(-1,i\,x^2\right)+\frac{1}{2}\,\Gamma\left(-1,-i\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(x^2*(I*gamma(-1, 2*I*x^2) - I*gamma(-1, -2*I*x^2)) - 1)/x^2 - 1/2/x^2
+ 1/2*gamma(-1, I*x^2) + 1/2*gamma(-1, -I*x^2)
```

Fricas [A]
time = 0.37, size = 47, normalized size = 1.07

$$\frac{x^2\,\text{Ci}\left(-x^2\right)+x^2\,\text{Ci}\left(x^2\right)+x^2\,\text{Si}\left(2\,x^2\right)+\cos\left(x^2\right)^2-2\,\sin\left(x^2\right)-2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*cos_integral(-x^2) + x^2*cos_integral(x^2) + x^2*sin_integral(2*x^
2) + cos(x^2)^2 - 2*sin(x^2) - 2)/x^2
```

Sympy [A]
time = 2.52, size = 51, normalized size = 1.16

$$-\log\left(x^2\right)+\frac{\log\left(x^4\right)}{2}+\text{Ci}\left(x^2\right)+\frac{\text{Si}\left(2x^2\right)}{2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{\cos\left(2x^2\right)}{4x^2}-\frac{3}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x**2))**2/x**3,x)
```

```
[Out] -log(x**2) + log(x**4)/2 + Ci(x**2) + Si(2*x**2)/2 - sin(x**2)/x**2 + cos(2
*x**2)/(4*x**2) - 3/(4*x**2)
```

Giac [A]
time = 6.05, size = 39, normalized size = 0.89

$$\frac{4\,x^2\,\mathrm{Ci}\left(x^2\right)+2\,x^2\,\mathrm{Si}\left(2\,x^2\right)+\cos\left(2\,x^2\right)-4\,\sin\left(x^2\right)-3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*x^2*cos_integral(x^2) + 2*x^2*sin_integral(2*x^2) + cos(2*x^2) - 4*s
in(x^2) - 3)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\mathrm{cosint}\left(x^2\right)+\frac{\mathrm{sinint}(2\,x^2)}{2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{{\cos\left(x^2\right)}^2}{2\,x^2}-\frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x^2) + 1)^2/x^3,x)
```

```
[Out] cosint(x^2) + sinint(2*x^2)/2 - sin(x^2)/x^2 + cos(x^2)^2/(2*x^2) - 1/x^2
```

3.35 $\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=362

$$-\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}+\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}-\frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}+\frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d^2}-$$

```
[Out] -1/2*I*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
+1/2*I*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
-x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2
)+x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/
2)-I*polylog(3,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2
)+I*polylog(3,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3460, 3404, 2296, 2221, 2611, 2320, 6724}

$$-\frac{i\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{i\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\sqrt{a^2-b^2}}-\frac{x^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{x^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\sqrt{a^2-b^2}}-\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d\sqrt{a^2-b^2}}+\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-1/2*I)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*d) + ((I/2)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*d) - (x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a -
 Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) + (x^2*PolyLog[2, (I*b*E^(I*(c +
d*x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) - (I*PolyLog[3, (I*b
*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3) + (I*Poly
Log[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\sin(c+dx)}\,dx, x, x^2\right)$$

$$= \text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)$$

$$= -\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}} + \frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-}\right.}{\sqrt{a^2-b^2}}$$

$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{i\text{Subst}\left(\int x\log\left(\right.\right.}{}$$

$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} - \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} - \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} - \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,d^2}$$

Mathematica [**A**]
time = 0.14, size = 289, normalized size = 0.80

$$\frac{-2dx^2\text{Li}_2\left(-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right) - i\left(d^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right) - d^2x^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right) + 2idx^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right) + 2\text{Li}_3\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right) - 2\text{Li}_3\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)\right)}{2\sqrt{a^2-b^2}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (-2*d*x^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - I
*(d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - d^2*x^4
*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*x^2*PolyL
og[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, (I*b*E^
(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])] - 2*PolyLog[3, (I*b*E^(I*(c + d*x^2
)))/(a + Sqrt[a^2 - b^2])]))/(2*Sqrt[a^2 - b^2]*d^3)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+b\sin(d\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*sin(d*x^2+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1435 vs. 2(300) = 600.
time = 0.51, size = 1435, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d
*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c)
- a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b
^2)/b^2) - b)/b + 1) - 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d
*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt
(-(a^2 - b^2)/b^2) - b)/b + 1) + 2*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog((
-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2
+ c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + b*c^2*sqrt(-(a^2 - b^2)/b^2)*log
(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2
*I*a) + b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x
^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - b*c^2*sqrt(-(a^2 - b^2)/b^2
)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^
2) + 2*I*a) - b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) - 2*I*b*
sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (b*d^2*x^4 - b*c^2)*
sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos
(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d^2*x
^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 +
 c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b
) - (b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(d*x^2 + c) -
a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2
```

```
)/b^2) - b)/b) + (b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos(
d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) + 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*c
os(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*
sqrt(-(a^2 - b^2)/b^2))/b) - 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*co
s(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2))/b) + 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*co
s(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2))/b) - 2*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*co
s(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2))/b))/((a^2 - b^2)*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + b\sin\left(c + dx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(x**5/(a + b*sin(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{a + b\,\sin\left(d\,x^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*sin(c + d*x^2)),x)
```

```
[Out] int(x^5/(a + b*sin(c + d*x^2)), x)
```

3.36 $\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=245

$$-\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}+\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d}-\frac{\mathrm{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d^2}+\frac{\mathrm{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d^2}$$

```
[Out] -1/2*I*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
+1/2*I*x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
-1/2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2
)+1/2*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/
2)
```

Rubi [A]
time = 0.34, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3460, 3404, 2296, 2221, 2317, 2438}

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\sqrt{a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d^2\sqrt{a^2-b^2}}-\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2d\sqrt{a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{2d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-1/2*I)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*d) + ((I/2)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt
[a^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^2)))/
(a + Sqrt[a^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{x^3}{a+b\sin(c+dx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^2\right)$$

$$= \text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)$$

$$= -\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}} + \frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{i\text{Subst}\left(\int\log\left(1\right.\right.}{}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{\text{Subst}\left(\int \frac{\log\left(1-\right.}{}\right.}{}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} + \frac{ix^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d} - \frac{\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right.}{2\sqrt{a^2-b^2}\,d^2}$$

Mathematica [A]
time = 0.04, size = 188, normalized size = 0.77

$$\frac{-idx^2\left(\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)\right)-\text{Li}_2\left(-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)+\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-I)*d*x^2*(Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - Log[
1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^
(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x^2)
))/(a + Sqrt[a^2 - b^2])])/(2*Sqrt[a^2 - b^2]*d^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b\sin(d\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^3/(a+b*sin(d*x^2+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(199) = 398.
time = 0.58, size = 1041, normalized size = 4.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + b*c*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*lo
g(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^
2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x
^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin(d
*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin
(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) + (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2
+ c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b) - (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*co
s(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^
2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/
b^2)*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b
*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2 - b^2)*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\sin\left(c + dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(x**3/(a + b*sin(c + d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + b\,\sin\left(d\,x^2 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*sin(c + d*x^2)),x)
```

```
[Out] int(x^3/(a + b*sin(c + d*x^2)), x)
```

3.37 $\int \frac{x}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$$

```
[Out] arctan((b+a*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/d/(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 2739, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx^2)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+b\sin(c+dx^2)}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^2\right)\\ &= \frac{\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{d}\\ &= -\frac{2\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sin[c + d*x^2]),x]
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Maple [A]
time = 0.06, size = 48, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$	48
default	$\frac{\arctan\left(\frac{2a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$	48

risch	$-\frac{\ln\left(e^{i(d x^2+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(e^{i(d x^2+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,d}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1/
2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8078 vs. $2(43) = 86$.
time = 28.31, size = 8078, normalized size = 168.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^4*cos(c)*sin(c) - 4*(a^2
*b^4 - b^6)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c)^
3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 4*(3*(a^3*b^3
 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^6)*c
os(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^3 +
4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 +
 b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6
)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 + 3*((a^3
*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c
))*sin(d*x^2 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5 + 2*(2*a^
5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)
^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3 + (2*a^5*b -
3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^2*b^4 - b^6)*s
in(c)^2)*cos(d*x^2 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*sin(c) - (a^3*b
^3 - a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + ((4*a^4*b^2 - 5*a^2*b^4 + b^6)*c
os(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x
^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^5*cos(c) - 4*a*b^4*cos(d*x^2 + 2*c)^4*cos
(c)*sin(c) + b^5*sin(d*x^2 + 2*c)^5*sin(c) + (b^5*cos(d*x^2 + 2*c)*cos(c) +
 4*a*b^4*cos(c)*sin(c))*sin(d*x^2 + 2*c)^4 + 2*((2*a^2*b^3 - b^5)*cos(c)^3
+ 3*(2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(b^5*cos(d*x^
2 + 2*c)^2*sin(c) + 3*(2*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (2*a^2*b^3 - b^5)
*sin(c)^3 + 2*(a*b^4*cos(c)^2 - a*b^4*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2
 + 2*c)^3 - 4*((4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + (4*a^3*b^2 - 3*a*b^4
)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 2*(b^5*cos(d*x^2 + 2*c)^3*cos(c) +
2*(4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + 2*(4*a^3*b^2 - 3*a*b^4)*cos(c)*si
```

```
n(c)^3 + 3*((2*a^2*b^3 - b^5)*cos(c)^3 - (2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)
*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + ((8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)
^5 + 2*(8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (8*a^4*b - 8*a^2*b^3
 + b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^4*sin(c)
+ (8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(8*a^4*b - 8*a^2*b^3 + b^
5)*cos(c)^2*sin(c)^3 + (8*a^4*b - 8*a^2*b^3 + b^5)*sin(c)^5 + 4*(a*b^4*cos(
c)^2 - a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 6*((2*a^2*b^3 - b^5)*cos(c)^2*s
in(c) - (2*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((4*a^3*b^2 - 3*
a*b^4)*cos(c)^4 - (4*a^3*b^2 - 3*a*b^4)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x
^2 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*
x^2 + 2*c)^5 + b^6*sin(d*x^2 + 2*c)^6 - 6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) +
 (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2
 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^
4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c
)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x
^2 + 2*c)^4 + 3*(b^6*cos(d*x^2 + 2*c)^2 - 2*a*b^5*cos(d*x^2 + 2*c)*sin(c) +
 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c
)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*
sin(c)^3)*cos(d*x^2 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(4*
a^3*b^3 - 3*a*b^5)*cos(c)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*s
in(c) + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 3*((8
*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(
c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^
2 + 3*(b^6*cos(d*x^2 + 2*c)^4 - 4*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^
4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^
2*sin(c)^2 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*
cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*(3*(4*a^3*b^3
 - 3*a*b^5)*cos(c)^2*sin(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2
*c))*sin(d*x^2 + 2*c)^2 - 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin
(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 2
0*a^3*b^3 + 5*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 6*(a*b^5*cos(d*x^2 + 2*c)
^4*cos(c) + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6
)*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*co
s(c)^3*sin(c)^2 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4
*a^3*b^3 - 3*a*b^5)*cos(c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos
(d*x^2 + 2*c)^2 - 4*((8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4
*b^2 - 8*a^2*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)
 - 2*(3*b^5*cos(c)*sin(d*x^2 + 2*c)^5 - 3*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)
*cos(c)^4*sin(c)^2 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 +
(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*sin(c)^6 + 3*(a...
```

Fricas [A]

time = 0.36, size = 208, normalized size = 4.33

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx^2+c)^2-2ab\sin(dx^2+c)-a^2-b^2+2(a\cos(dx^2+c)\sin(dx^2+c)+b\cos(dx^2+c))\sqrt{-a^2+b^2}}{b^2\cos(dx^2+c)^2-2ab\sin(dx^2+c)-a^2-b^2}\right)}{4(a^2-b^2)d}, -\frac{\arctan\left(-\frac{a\sin(dx^2+c)+b}{\sqrt{a^2-b^2}\cos(dx^2+c)}\right)}{2\sqrt{a^2-b^2}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^
2 + c) - a^2 - b^2 + 2*(a*cos(d*x^2 + c)*sin(d*x^2 + c) + b*cos(d*x^2 + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2
))/((a^2 - b^2)*d), -1/2*arctan(-(a*sin(d*x^2 + c) + b)/(sqrt(a^2 - b^2)*co
s(d*x^2 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(37) = 74$.
time = 5.72, size = 202, normalized size = 4.21

$$\begin{cases} \frac{\tilde{\infty}x^2}{\sin(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)\right)}{2bd} & \text{for } a=0 \\ \frac{x^2}{2(a+b\sin(c))} & \text{for } d=0 \\ \frac{\sqrt{b^2}}{b^2d\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)-bd\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{b^2d\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)+bd\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((zoo*x**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 +
 d*x**2/2))/(2*b*d), Eq(a, 0)), (x**2/(2*(a + b*sin(c))), Eq(d, 0)), (sqrt(
b**2)/(b**2*d*tan(c/2 + d*x**2/2) - b*d*sqrt(b**2)), Eq(a, -sqrt(b**2))), (
-sqrt(b**2)/(b**2*d*tan(c/2 + d*x**2/2) + b*d*sqrt(b**2)), Eq(a, sqrt(b**2)
)), (log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b**2)/a)/(2*d*sqrt(-a**2
+ b**2)) - log(tan(c/2 + d*x**2/2) + b/a + sqrt(-a**2 + b**2)/a)/(2*d*sqrt(
-a**2 + b**2)), True))
```

Giac [A]

time = 5.44, size = 63, normalized size = 1.31

$$\frac{\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2*
c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B]
time = 6.63, size = 128, normalized size = 2.67

$$-\frac{\ln\left(-x\,{\mathrm{e}}^{d\,x^2\,1\mathrm{i}}\,{\mathrm{e}}^{c\,1\mathrm{i}}\,2\mathrm{i}-\frac{2\,x\,\left(b\,1\mathrm{i}+a\,{\mathrm{e}}^{d\,x^2\,1\mathrm{i}}\,{\mathrm{e}}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)-\ln\left(-x\,{\mathrm{e}}^{d\,x^2\,1\mathrm{i}}\,{\mathrm{e}}^{c\,1\mathrm{i}}\,2\mathrm{i}+\frac{2\,x\,\left(b\,1\mathrm{i}+a\,{\mathrm{e}}^{d\,x^2\,1\mathrm{i}}\,{\mathrm{e}}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{2\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sin(c + d*x^2)),x)
```

```
[Out] -(log(- x*exp(d*x^2*1i)*exp(c*1i)*2i - (2*x*(b*1i + a*exp(d*x^2*1i)*exp(c*1
i)))/((a + b)^(1/2)*(b - a)^(1/2))) - log((2*x*(b*1i + a*exp(d*x^2*1i)*exp(
c*1i)))/((a + b)^(1/2)*(b - a)^(1/2)) - x*exp(d*x^2*1i)*exp(c*1i)*2i))/(2*d
*(a + b)^(1/2)*(b - a)^(1/2))
```

3.38 $\int \frac{1}{x(a+b\sin(c+dx^2))} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^2+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^2])),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)} \, dx = \int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)} \, dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^2])),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^2])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^2+c\right)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(1/x/(a+b*sin(d*x^2+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*sin(d*x^2 + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^{2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**2))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*sin(c + d*x^2))),x)
[Out] int(1/(x*(a + b*sin(c + d*x^2))), x)
```

3.39 $\int \frac{1}{x^3(a+b\sin(c+dx^2))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^2+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^2])),x]
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(d\,x^2+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^2+c)),x)
[Out] int(1/x^3/(a+b*sin(d*x^2+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*sin(d*x^2 + c) + a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \sin (c + dx^2))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**2))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \ (a + b \ \sin (d \, x^2 + c))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*sin(c + d*x^2))),x)
[Out] int(1/(x^3*(a + b*sin(c + d*x^2))), x)
```

3.40 $\int \frac{x^2}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{x^2}{a+b\sin\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sin(d*x^2+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/(a + b*Sin[c + d*x^2]),x]
[Out] Defer[Int][x^2/(a + b*Sin[c + d*x^2]), x]
```

Rubi steps

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^2]),x]
[Out] Integrate[x^2/(a + b*Sin[c + d*x^2]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\sin\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^2+c)),x)
[Out] int(x^2/(a+b*sin(d*x^2+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^2/(b*sin(d*x^2 + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \sin \left(c + dx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**2+c)),x)
[Out] Integral(x**2/(a + b*sin(c + d*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{a + b\,\sin \left(d\,x^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sin(c + d*x^2)),x)
[Out] int(x^2/(a + b*sin(c + d*x^2)), x)
```

3.41 $\int \frac{1}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{a+b\sin\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^2+c)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^2])^(-1),x]
[Out] Defer[Int][(a + b*Sin[c + d*x^2])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d*x^2])^(-1), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^2+c)),x)
[Out] int(1/(a+b*sin(d*x^2+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*sin(d*x^2 + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin\left(c + dx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(a + b*sin(c + d*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{a + b\,\sin\left(d\,x^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x^2)),x)
[Out] int(1/(a + b*sin(c + d*x^2)), x)
```

3.42 $\int \frac{1}{x^2(a+b\sin(c+dx^2))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^2])),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx = \int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(d\,x^2+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^2+c)),x)
[Out] int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*sin(d*x^2 + c) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \sin \left(c + dx^2\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**2+c)),x)
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**2))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\ \left(a + b\ \sin\left(d\,x^2 + c\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*sin(c + d*x^2))),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^2))), x)
```

3.43 $\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=663

$$\frac{ix^4}{2\left(a^2-b^2\right)d}-\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2}-\frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d}-\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2}+\frac{iax^4}{}$$

```
[Out] 1/2*I*x^4/(a^2-b^2)/d-x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a
^2-b^2)/d^2-1/2*I*a*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2
-b^2)^(3/2)/d-x^2*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/
d^2+1/2*I*a*x^4*ln(1-I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3
/2)/d+I*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3-a
*x^2*polylog(2,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^
2+I*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/d^3+a*x^2
*polylog(2,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-I*
a*polylog(3,I*b*exp(I*(d*x^2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3+I
*a*polylog(3,I*b*exp(I*(d*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^3+
1/2*b*x^4*cos(d*x^2+c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Rubi [A]
time = 0.84, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3460, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4615, 2317, 2438}

$$\frac{i\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\left(a^2-b^2\right)}+\frac{i\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\left(a^2-b^2\right)}-\frac{ia\text{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\left(a^2-b^2\right)^{3/2}}+\frac{ia\text{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\left(a^2-b^2\right)^{3/2}}-\frac{ax^2\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{ax^2\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)}-\frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)}-\frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d\left(a^2-b^2\right)^{3/2}}+\frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d\left(a^2-b^2\right)^{3/2}}+\frac{bx^4\cos\left(c+dx^2\right)}{2d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{ix^4}{2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((I/2)*x^4)/((a^2 - b^2)*d) - (x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqr
t[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*a*x^4*Log[1 - (I*b*E^(I*(c + d*x
^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (x^2*Log[1 - (I*b*E^(
I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + ((I/2)*a*x^4*Lo
g[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d)
 + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d^3) - (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])
/((a^2 - b^2)^(3/2)*d^2) + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
a^2 - b^2])])/((a^2 - b^2)*d^3) + (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (I*a*PolyLog[3, (I*b*E^(
I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) + (I*a*Poly
Log[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d
^3) + (b*x^4*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
```

```
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^5}{(a+b\sin(c+dx^2))^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{x^2}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2(a^2-b^2)} - \frac{b\text{Subst}\,\cdots}{\cdots}$$

$$= \frac{ix^4}{2(a^2-b^2)d} + \frac{bx^4\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c\cdots}}\cdots\right)}{a^2-b^2}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} + \frac{\cdots}{2(\cdots}$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \cdots$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \cdots$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \cdots$$

$$= \frac{ix^4}{2(a^2-b^2)d} - \frac{x^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \cdots$$

<u>Mathematica</u> [A]
time = 1.56, size = 513, normalized size = 0.77

$$\frac{id^2x^4 - 2dx^2\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right) - \frac{iad^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2dx^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right) + \frac{iad^2x^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \left(2i-\frac{2adx^2}{\sqrt{a^2-b^2}}\right)\text{Li}_2\left(-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right) + \left(2i+\frac{2adx^2}{\sqrt{a^2-b^2}}\right)\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right) - \frac{2ia\text{Li}_3\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2ia\text{Li}_3\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{bd^2x^4\cos(c+dx^2)}{a+b\sin(c+dx^2)}}{2(a^2-b^2)d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (I*d^2*x^4 - 2*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])
] - (I*a*d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/S
qrt[a^2 - b^2] - 2*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^
```

```
2])] + (I*a*d^2*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])
/Sqrt[a^2 - b^2] + (2*I - (2*a*d*x^2)/Sqrt[a^2 - b^2])*PolyLog[2, ((-I)*b*E
^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + (2*I + (2*a*d*x^2)/Sqrt[a^2 - b
^2])*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] - ((2*I)*a*P
olyLog[3, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] +
 ((2*I)*a*PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/Sqrt[a
^2 - b^2] + (b*d^2*x^4*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a^2 - b^
2)*d^3)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a +b \sin \left(d \,x^{2}+c \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*x^4*cos(d*x^2 + c) + ((a^2
*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 +
 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*
sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*
b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^
2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))*integrate(2*(2*a^2*d*x^5*co
s(d*x^2 + c)^2 + 2*a^2*d*x^5*sin(d*x^2 + c)^2 + a*b*d*x^5*sin(d*x^2 + c) -
2*a*b*x^3*cos(d*x^2 + c) - (a*b*d*x^5*sin(d*x^2 + c) + 2*a*b*x^3*cos(d*x^2
+ c))*cos(2*d*x^2 + 2*c) + (a*b*d*x^5*cos(d*x^2 + c) - 2*a*b*x^3*sin(d*x^2
+ c) - 2*b^2*x^3)*sin(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)
^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 +
 c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 -
a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2
 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(
2*d*x^2 + 2*c)), x) + (a*b*x^4*sin(d*x^2 + c) + b^2*x^4)*sin(2*d*x^2 + 2*c)
)/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 +
 c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 -
b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3
```

```
*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*s
in(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2469 vs. $2(565) = 1130$.
time = 0.60, size = 2469, normalized size = 3.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*b - b^3)*d^2*x^4*cos(d*x^2 + c) + 2*(a*b^2*sin(d*x^2 + c) + a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x^2 + c) + a*sin(d*x^2 +
c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2
*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos
(d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sq
rt(-(a^2 - b^2)/b^2))/b) + 2*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^
2)/b^2)*polylog(3, -(-I*a*cos(d*x^2 + c) + a*sin(d*x^2 + c) + (b*cos(d*x^2
+ c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 2*(a*b^2*sin(d*x^2
+ c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(-I*a*cos(d*x^2 + c) + a*s
in(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b
^2))/b) - 2*(-I*a^3 + I*a*b^2 + (-I*a^2*b + I*b^3)*sin(d*x^2 + c) + (-I*a*b
^2*d*x^2*sin(d*x^2 + c) - I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog((I*a
*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c)
)*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(-I*a^3 + I*a*b^2 + (-I*a^2*b + I*
b^3)*sin(d*x^2 + c) + (I*a*b^2*d*x^2*sin(d*x^2 + c) + I*a^2*b*d*x^2)*sqrt(-
(a^2 - b^2)/b^2))*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x
^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) - 2*(I*a^3
 - I*a*b^2 + (I*a^2*b - I*b^3)*sin(d*x^2 + c) + (I*a*b^2*d*x^2*sin(d*x^2 +
c) + I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog((-I*a*cos(d*x^2 + c) - a*
sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/
b^2) - b)/b + 1) - 2*(I*a^3 - I*a*b^2 + (I*a^2*b - I*b^3)*sin(d*x^2 + c) +
(-I*a*b^2*d*x^2*sin(d*x^2 + c) - I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dil
og((-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*
x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (2*(a^2*b - b^3)*c*sin(d*x^2
 + c) + 2*(a^3 - a*b^2)*c + (a*b^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a
^2 - b^2)/b^2))*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(
a^2 - b^2)/b^2) + 2*I*a) + (2*(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b
^2)*c + (a*b^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(
2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) + (2*(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b^2)*c - (a*b^2*c^2*s
in(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c)
+ 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + (2*(a^2*b -
b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b^2)*c - (a*b^2*c^2*sin(d*x^2 + c) + a^2
```

```
*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (2*(a^3 - a*b^2)*d*x^2 + 2*(a^3
 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) + (a
^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt
(-(a^2 - b^2)/b^2))*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*
x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (2*(a^3 - a
*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)
*sin(d*x^2 + c) - (a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))*log(-(I*a*cos(d*x^2 + c) - a*sin(d*
x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
 b)/b) - (2*(a^3 - a*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^
2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) + (a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d
^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))*log(-(-I*a*cos(
d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2) - b)/b) - (2*(a^3 - a*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c +
2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) - (a^2*b*d^2*x^4 -
 a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/
b^2))*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*
b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^4*b - 2*a^2*b^3 + b^5
)*d^3*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**2+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*sin(c + d*x^2))^2,x)
[Out] int(x^5/(a + b*sin(c + d*x^2))^2, x)
```

3.44 $\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=324

$$-\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d}+\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2\left(a^2-b^2\right)d^2}-\frac{a\text{Li}_2\left(\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d^2}$$

```
[Out] -1/2*ln(a+b*sin(d*x^2+c))/(a^2-b^2)/d^2-1/2*I*a*x^2*ln(1-I*b*exp(I*(d*x^2+c
))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+1/2*I*a*x^2*ln(1-I*b*exp(I*(d*x^2
+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-1/2*a*polylog(2,I*b*exp(I*(d*x^
2+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/2*a*polylog(2,I*b*exp(I*(d
*x^2+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/2*b*x^2*cos(d*x^2+c)/(a
^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Rubi [A]
time = 0.38, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3460, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\left(a^2-b^2\right)^{3/2}}+\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d^2\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2d^2\left(a^2-b^2\right)}-\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d\left(a^2-b^2\right)^{3/2}}+\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d\left(a^2-b^2\right)^{3/2}}+\frac{bx^2\cos\left(c+dx^2\right)}{2d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((-1/2*I)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*d) + ((I/2)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^2]]/(2*(a^2 -
b^2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(
2*(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
a^2 - b^2])])/(2*(a^2 - b^2)^(3/2)*d^2) + (b*x^2*Cos[c + d*x^2])/(2*(a^2 -
b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{x^3}{(a+b\sin(c+dx^2))^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2(a^2-b^2)} - \frac{b\text{Subs}}{}$$

$$= \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} + \frac{a\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2-b^2}$$

$$= -\frac{\log(a+b\sin(c+dx^2))}{2(a^2-b^2)d^2} + \frac{bx^2\cos(c+dx^2)}{2(a^2-b^2)d(a+b\sin(c+dx^2))} - \frac{(iab)\text{Subst}\Big(}{}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{\log(a+b\,\text{s}}{2(a^2-}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{\log(a+b\,\text{s}}{2(a^2-}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}d} - \frac{\log(a+b\,\text{s}}{2(a^2-}$$

Mathematica [A]
time = 0.64, size = 302, normalized size = 0.93

$$\frac{-\frac{iadx^2\log\left(1+\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{iadx^2\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{\log(a+b\sin(c+dx^2))}{a^2-b^2} - \frac{a\text{Li}_2\left(-\frac{ibe^{i(c+dx^2)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{bdx^2\cos(c+dx^2)}{(a^2-b^2)(a+b\sin(c+dx^2))}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (((-I)*a*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (I*a*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^2]]/(a^2 - b^2) - (a*Pol
yLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)^(3/
2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(a^2 - b
^2)^(3/2) + (b*d*x^2*Cos[c + d*x^2])/((a^2 - b^2)*(a + b*Sin[c + d*x^2])))/
(2*d^2)
```

Maple [**F**]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+b\sin \left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1509 vs. 2(274) = 548.
time = 0.70, size = 1509, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*b - b^3)*d*x^2*cos(d*x^2 + c) + (I*a*b^2*sin(d*x^2 + c) + I*a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (
b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(
d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqr
```

```
t(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)*sqrt(
-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*
x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*a*b^
2*sin(d*x^2 + c) + I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^2 +
c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x
^2 + c) + (b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b) + (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d
*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*x
^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^
2)*log(-(-I*a*cos(d*x^2 + c) - a*sin(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*s
in(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*x^2 + a^2*b*c + (a
*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos
(d*x^2 + c) - a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c)
+ (a*b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*
x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (a^
3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c) + (a*b^2*c*sin(d*x^2 + c) + a^2*b*
c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^
2 + c) - (a*b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*
b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c) - (a*b^2*c*sin(d*x^2 + c)
+ a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^
2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b - 2*a^2*b^3 + b^5)*d^
2*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b\sin(c + dx^2))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**3/(a + b*sin(c + d*x**2))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int(x^3/(a + b*sin(c + d*x^2))^2, x)
```

3.45 $\int \frac{x}{(a+b\sin(c+dx^2))^2}\,dx$

Optimal. Leaf size=91

$$\frac{a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}+\frac{b\cos(c+dx^2)}{2(a^2-b^2)\,d\,(a+b\sin(c+dx^2))}$$

```
[Out] a*arctan((b+a*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+1/2*
b*cos(d*x^2+c)/(a^2-b^2)/d/(a+b*sin(d*x^2+c))
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 2743, 12, 2739, 632, 210}

$$\frac{a\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx^2)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}+\frac{b\cos(c+dx^2)}{2d\,(a^2-b^2)\,(a+b\sin(c+dx^2))}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*d
) + (b*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b\sin(c+dx))^2}\,dx,x,x^2\right)\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{\text{Subst}\left(\int \frac{a}{a+b\sin(c+dx)}\,dx,x,x^2\right)}{2\left(a^2-b^2\right)}\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{a\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx,x,x^2\right)}{2\left(a^2-b^2\right)}\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{a\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)\right.}{\left(a^2-b^2\right)d}\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}-\frac{(2a)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\right.}{\left(a^2-b^2\right)d}\\
&= \frac{a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}+\frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}
\end{aligned}
$$

Mathematica [A]

time = 0.15, size = 91, normalized size = 1.00

$$\frac{\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b\cos(c+dx^2)}{a+b\sin(c+dx^2)}}{2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (b*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.14, size = 131, normalized size = 1.44

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{a^2-b^2}}{a\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{2d}$
default	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{a^2-b^2}}{a\left(\tan^2\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{2d}$
risch	$\frac{ib+a e^{i(dx^2+c)}}{(a^2-b^2)d\left(b e^{2i(dx^2+c)}-b+2ia e^{i(dx^2+c)}\right)}-\frac{a\ln\left(e^{i(dx^2+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}(a+b)(a-b)d}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2*(b^2/a/(a^2-b^2)*tan(1/2*d*x^2+1/2*c)+b/(a^2-b^2))/(a*tan(1/2*d*x^
2+1/2*c)^2+2*b*tan(1/2*d*x^2+1/2*c)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*
tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.41, size = 366, normalized size = 4.02

$$\left[\frac{\left(ab\sin\left(dx^2+c\right)+a^2\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2-2\,\left(a\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)+b\cos\left(dx^2+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2}\right)+2\,\left(a^2b-b^3\right)\cos\left(dx^2+c\right)}{4\,\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx^2+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)},-\frac{\left(ab\sin\left(dx^2+c\right)+a^2\right)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin\left(dx^2+c\right)+b}{\sqrt{a^2-b^2}\,\cos\left(dx^2+c\right)}\right)-\left(a^2b-b^3\right)\cos\left(dx^2+c\right)}{2\,\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx^2+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*sin(d*x^2 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d
*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2 - 2*(a*cos(d*x^2 + c)*sin(d*
x^2 + c) + b*cos(d*x^2 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*
b*sin(d*x^2 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^2 + c))/((a^4*b -
2*a^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/2*((a*
b*sin(d*x^2 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^2 + c) + b)/(sqr
t(a^2 - b^2)*cos(d*x^2 + c))) - (a^2*b - b^3)*cos(d*x^2 + c))/((a^4*b - 2*a
^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x/(a + b*sin(c + d*x**2))**2, x)
```

Giac [A]
time = 5.35, size = 144, normalized size = 1.58

$$\frac{\left(\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2d-b^2d\right)\sqrt{a^2-b^2}}+\frac{b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+ab}{\left(a^3d-ab^2d\right)\left(a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + (b^2*tan(1/
2*d*x^2 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^2 + 1/2*c)^2 + 2*
b*tan(1/2*d*x^2 + 1/2*c) + a))
```

Mupad [B]
time = 5.10, size = 178, normalized size = 1.96

$$\frac{\frac{b}{a^2-b^2}+\frac{b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{a\,(a^2-b^2)}}{d\,\left(a\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2+2\,b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+a\right)}+\frac{a\,\mathrm{atan}\left(\frac{(a^2-b^2)\,\left(\frac{a^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{a\,\left(2\,a^2\,b-2\,b^3\right)}{2\,(a+b)^{3/2}\,\left(a^2-b^2\right)\,(a-b)^{3/2}}\right)}{a}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] (b/(a^2 - b^2) + (b^2*tan(c/2 + (d*x^2)/2))/(a*(a^2 - b^2)))/(d*(a + a*tan(
c/2 + (d*x^2)/2)^2 + 2*b*tan(c/2 + (d*x^2)/2))) + (a*atan(((a^2 - b^2)*((a^
2*tan(c/2 + (d*x^2)/2))/((a + b)^(3/2)*(a - b)^(3/2)) + (a*(2*a^2*b - 2*b^3
))/(2*(a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/a))/(d*(a + b)^(3/2)*(a -
b)^(3/2))
```

3.46 $\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^2])^2),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2} dx = \int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2} dx$$

Mathematica [A]
time = 4.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^2])^2),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^2+c\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^2*cos(2*d*x^2 + 2*
c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c
)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2 - 2*(2*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*cos(d*x^2
) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2
*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2*d*x^2)*sin(2*c) + 2*
(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(d
*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b
^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*cos(
d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2
*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^2*cos(2*d*x^2)*sin(2*c)
 - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^2*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-2*(b^4*cos(2*c)*sin(2*d*x
^2) + b^4*cos(2*d*x^2)*sin(2*c) - 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) + 2*(
a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (a^3*b*d*x^2*sin(d*x^2 + c) - a^3*b*cos(
d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a^3*b - a*b^3 + (a*b^3*d*x^2*sin(2*c) + a
*b^3*cos(2*c))*cos(2*d*x^2) - 2*((a^4 - a^2*b^2)*d*x^2*cos(c) - (a^4 - a^2*
b^2)*sin(c))*cos(d*x^2) + (a*b^3*d*x^2*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x
^2) + 2*((a^4 - a^2*b^2)*d*x^2*sin(c) + (a^4 - a^2*b^2)*cos(c))*sin(d*x^2))
*cos(d*x^2 + c) - (a^3*b*d*x^2*cos(d*x^2 + c) + a^3*b*sin(d*x^2 + c) + a^2*
b^2)*sin(2*d*x^2 + 2*c) - ((a^3*b - a*b^3)*d*x^2 + (a*b^3*d*x^2*cos(2*c) -
a*b^3*sin(2*c))*cos(2*d*x^2) + 2*((a^4 - a^2*b^2)*d*x^2*sin(c) + (a^4 - a^2
*b^2)*cos(c))*cos(d*x^2) - (a*b^3*d*x^2*sin(2*c) + a*b^3*cos(2*c))*sin(2*d*
```

```
x^2) + 2*((a^4 - a^2*b^2)*d*x^2*cos(c) - (a^4 - a^2*b^2)*sin(c))*sin(d*x^2)
)*sin(d*x^2 + c))/(a^4*b^2*d*x^3*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^3*sin(2
*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 +
4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c
)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^3*sin(2*d*x
^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3*cos(c)*sin(d*x^2) + 4*((a^6 - 2
*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*
sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3*cos(d*x^2)*sin(c) + (a^4
*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (
a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^3*
cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c
)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x^3*cos(2*d*x^2)*co
s(2*c) - a^2*b^4*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^3*co
s(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^3*cos(d*x^2)*sin(c) + (a^4*b^2 -
a^2*b^4)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c
) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2) + 2*((a^3*b^3 - a*b
^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*sin(d*x^2) +
 (a^2*b^4 - b^6)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^3*cos(2*c)*s
in(2*d*x^2) + a^2*b^4*d*x^3*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x
^3*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^3*sin(d*x^2)*sin(c))*sin(2*d
*x^2 + 2*c)), x) + (a^3*b*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (a
*b^3*cos(2*c)*sin(2*d*x^2) + a*b^3*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^2
)*cos(d*x^2)*cos(c) + 2*(a^4 - a^2*b^2)*sin(d*x^2)*sin(c))*sin(d*x^2 + c))/
(a^4*b^2*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 +
(b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^
2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x
^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b -
 2*a^3*b^3 + a*b^5)*d*x^2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4
)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(
a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 +
 b^6)*d*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*c
os(2*c)*sin(c))*d*x^2*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3
*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x*cos(d*x^2 + c)^2 - 2*a*b*x*sin(d*x^2 + c) - (a^2 + b^2)*
x), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**2))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*sin(c + d*x^2))^2),x)
```

```
[Out] int(1/(x*(a + b*sin(c + d*x^2))^2), x)
```

3.47 $\int \frac{1}{x^3(a+b\sin(c+dx^2))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 5.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
[Out] int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^4*cos(2*d*x^2 + 2*
c)^2 + a^4*b^2*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^4*cos(c
)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
4*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^4 - 2*(2*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2
) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2
*b^4*d*x^4*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^4*sin(2*d*x^2)*sin(2*c) + 2*
(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(d
*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b
^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*cos(
d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*x^4*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^4*sin(2*c))*sin(2*d*x^2) - 2
*(a^2*b^4*d*x^4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^4*cos(2*d*x^2)*sin(2*c)
 - 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^4*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-2*(2*b^4*cos(2*c)*sin(2*d
*x^2) + 2*b^4*cos(2*d*x^2)*sin(2*c) - 4*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) +
 4*(a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (a^3*b*d*x^2*sin(d*x^2 + c) - 2*a^3*
b*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*a^3*b - 2*a*b^3 + (a*b^3*d*x^2*si
n(2*c) + 2*a*b^3*cos(2*c))*cos(2*d*x^2) - 2*((a^4 - a^2*b^2)*d*x^2*cos(c) -
 2*(a^4 - a^2*b^2)*sin(c))*cos(d*x^2) + (a*b^3*d*x^2*cos(2*c) - 2*a*b^3*sin
(2*c))*sin(2*d*x^2) + 2*((a^4 - a^2*b^2)*d*x^2*sin(c) + 2*(a^4 - a^2*b^2)*c
os(c))*sin(d*x^2))*cos(d*x^2 + c) - (a^3*b*d*x^2*cos(d*x^2 + c) + 2*a^3*b*s
in(d*x^2 + c) + 2*a^2*b^2)*sin(2*d*x^2 + 2*c) - ((a^3*b - a*b^3)*d*x^2 + (a
*b^3*d*x^2*cos(2*c) - 2*a*b^3*sin(2*c))*cos(2*d*x^2) + 2*((a^4 - a^2*b^2)*d
*x^2*sin(c) + 2*(a^4 - a^2*b^2)*cos(c))*cos(d*x^2) - (a*b^3*d*x^2*sin(2*c)
```

```
+ 2*a*b^3*cos(2*c))*sin(2*d*x^2) + 2*((a^4 - a^2*b^2)*d*x^2*cos(c) - 2*(a^4
 - a^2*b^2)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^2*d*x^5*cos(2*d*x^2
+ 2*c)^2 + a^4*b^2*d*x^5*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2
*c)^2)*d*x^5*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^
6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^5*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b
^6*sin(2*c)^2)*d*x^5*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^5*c
os(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b
^2 + a^2*b^4)*sin(c)^2)*d*x^5*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*
d*x^5*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^5 - 2*(2*((a^3*b^
3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^5*cos(d
*x^2) - (a^2*b^4 - b^6)*d*x^5*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(
c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^5*sin(d*x^2))*cos(2*d*x^2) - 2*
(a^2*b^4*d*x^5*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^5*sin(2*d*x^2)*sin(2*c)
+ 2*(a^5*b - a^3*b^3)*d*x^5*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^5*c
os(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^5)*cos(2*d*x^2 + 2*c) - 2*(2*((a
^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^5*
cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2
*c)*sin(c))*d*x^5*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^5*sin(2*c))*sin(2*d*x^2)
 - 2*(a^2*b^4*d*x^5*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^5*cos(2*d*x^2)*sin(
2*c) - 2*(a^5*b - a^3*b^3)*d*x^5*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*
x^5*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b*sin(d*x^2 + c) + a^
2*b^2)*sin(2*d*x^2 + 2*c) - (a*b^3*cos(2*c)*sin(2*d*x^2) + a*b^3*cos(2*d*x^
2)*sin(2*c) - 2*(a^4 - a^2*b^2)*cos(d*x^2)*cos(c) + 2*(a^4 - a^2*b^2)*sin(d
*x^2)*sin(c))*sin(d*x^2 + c))/(a^4*b^2*d*x^4*cos(2*d*x^2 + 2*c)^2 + a^4*b^2
*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*cos(2
*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^
2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x
^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^4*cos(c)*sin(d*x^2) +
 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(
c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^4*cos(d*x^2)*s
in(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^4 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*
sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2) - (a^2*b^4 -
 b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^3*cos(d*x^2 + c)^2 - 2*a*b*x^3*sin(d*x^2 + c) - (a^2 + b
^2)*x^3), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + b \sin \left(c + dx^2\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**2))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \left(a + b \sin \left(d\, x^2 + c\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*sin(c + d*x^2))^2),x)
```

```
[Out] int(1/(x^3*(a + b*sin(c + d*x^2))^2), x)
```

3.48 $\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

`[Out] Unintegrable(x^2/(a+b*sin(d*x^2+c))^2,x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[x^2/(a + b*Sin[c + d*x^2])^2,x]`

`[Out] Defer[Int][x^2/(a + b*Sin[c + d*x^2])^2, x]`

Rubi steps

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[x^2/(a + b*Sin[c + d*x^2])^2,x]`

`[Out] Integrate[x^2/(a + b*Sin[c + d*x^2])^2, x]`

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*x*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*x*cos(d*x^2 + c) + ((a^2*b^2
 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(
a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(
2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)
*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 +
c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c))*integrate((4*a^2*d*x^2*cos(d*x^
2 + c)^2 + 4*a^2*d*x^2*sin(d*x^2 + c)^2 + 2*a*b*d*x^2*sin(d*x^2 + c) - a*b*
cos(d*x^2 + c) - (2*a*b*d*x^2*sin(d*x^2 + c) + a*b*cos(d*x^2 + c))*cos(2*d*
x^2 + 2*c) + (2*a*b*d*x^2*cos(d*x^2 + c) - a*b*sin(d*x^2 + c) - b^2)*sin(2*
d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d
*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) +
 (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c
)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b
- a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b^4)*d)*cos(2*d*x^2 + 2*c)), x) + (a
*b*x*sin(d*x^2 + c) + b^2*x)*sin(2*d*x^2 + 2*c))/((a^2*b^2 - b^4)*d*cos(2*d
*x^2 + 2*c)^2 + 4*(a^4 - a^2*b^2)*d*cos(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*
cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^2*b^2 - b^4)*d*sin(2*d*x^2 + 2*c)^2
+ 4*(a^4 - a^2*b^2)*d*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*sin(d*x^2 + c)
 + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin(d*x^2 + c) + (a^2*b^2 - b
^4)*d)*cos(2*d*x^2 + 2*c))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-x^2/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + b\sin\left(c + dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**2/(a + b*sin(c + d*x**2))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int(x^2/(a + b*sin(c + d*x^2))^2, x)
```

3.49 $\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^2])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^2])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^(-2),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x^2])^(-2), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(1/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x*cos(2*d*x^2 + 2*c)
^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d
*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*
b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin(d*x^2)
 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*si
n(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x^2)*sin
(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(
2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4 - b^6)*
d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin
(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*d*x^2)*co
s(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x*cos(c)
*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^
4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3
*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)
*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*sin(d*x^2) + (a^2*b^4 -
b^6)*d*x*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*c)*sin(2*d*x^2) + a^
2*b^4*d*x*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*cos(c)
 + 2*(a^5*b - a^3*b^3)*d*x*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate
(-(b^4*cos(2*c)*sin(2*d*x^2) + b^4*cos(2*d*x^2)*sin(2*c) - 2*(a^3*b - a*b^3
)*cos(d*x^2)*cos(c) + 2*(a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (2*a^3*b*d*x^2*
sin(d*x^2 + c) - a^3*b*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a^3*b - a*b^3
+ (2*a*b^3*d*x^2*sin(2*c) + a*b^3*cos(2*c))*cos(2*d*x^2) - 2*(2*(a^4 - a^2*
b^2)*d*x^2*cos(c) - (a^4 - a^2*b^2)*sin(c))*cos(d*x^2) + (2*a*b^3*d*x^2*cos
(2*c) - a*b^3*sin(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*x^2*sin(c) +
(a^4 - a^2*b^2)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (2*a^3*b*d*x^2*cos(d*x
^2 + c) + a^3*b*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (2*(a^3*b -
a*b^3)*d*x^2 + (2*a*b^3*d*x^2*cos(2*c) - a*b^3*sin(2*c))*cos(2*d*x^2) + 2*(
2*(a^4 - a^2*b^2)*d*x^2*sin(c) + (a^4 - a^2*b^2)*cos(c))*cos(d*x^2) - (2*a*
b^3*d*x^2*sin(2*c) + a*b^3*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*
```

```
x^2*cos(c) - (a^4 - a^2*b^2)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^2*d
*x^2*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2
*c)^2 + b^6*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^
4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b
^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3
 + a*b^5)*d*x^2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2
 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*
a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^
2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*si
n(c))*d*x^2*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*
cos(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2
*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b -
 a^3*b^3)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2
+ 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*
c)*sin(c))*d*x^2*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b
^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^2*sin(2
*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^2*
cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^
5*b - a^3*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b*si
n(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (a*b^3*cos(2*c)*sin(2*d*x^2) +
 a*b^3*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^2)*cos(d*x^2)*cos(c) + 2*(a^4
 - a^2*b^2)*sin(d*x^2)*sin(c))*sin(d*x^2 + c))/(a^4*b^2*d*x*cos(2*d*x^2 + 2
*c)^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2
)*d*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a
^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*
c)^2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin(d*x
^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)
*sin(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x^2)*
sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*s
in(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4 - b^
6)*d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*
sin(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) -...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin\left(c + dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a + b\,\sin\left(d\,x^2 + c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int(1/(a + b*sin(c + d*x^2))^2, x)
```

3.50 $\int \frac{1}{x^2(a+b\sin(c+dx^2))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 4.85, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) - b^4*cos(2*c)*sin(2*d*x^2) - b^4*
cos(2*d*x^2)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) - 2*(a^3*b - a*
b^3)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c) - a*b^3*sin(2*d*x^2)*
sin(2*c) + a^3*b - a*b^3 + 2*(a^4 - a^2*b^2)*cos(c)*sin(d*x^2) + 2*(a^4 - a
^2*b^2)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + (a^4*b^2*d*x^3*cos(2*d*x^2 + 2*
c)^2 + a^4*b^2*d*x^3*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^
2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 -
2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*s
in(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3*cos(c
)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*sin(c)^2)*d*x^3*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
3*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2
) - (a^2*b^4 - b^6)*d*x^3*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) +
 (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2
*b^4*d*x^3*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*
(a^5*b - a^3*b^3)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^3*cos(d
*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b
^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^3*cos(
d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*x^3*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2
*(a^2*b^4*d*x^3*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^3*cos(2*d*x^2)*sin(2*c)
 - 2*(a^5*b - a^3*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^3*
sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-(3*b^4*cos(2*c)*sin(2*d*x
^2) + 3*b^4*cos(2*d*x^2)*sin(2*c) - 6*(a^3*b - a*b^3)*cos(d*x^2)*cos(c) + 6
*(a^3*b - a*b^3)*sin(d*x^2)*sin(c) + (2*a^3*b*d*x^2*sin(d*x^2 + c) - 3*a^3*
b*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (3*a^3*b - 3*a*b^3 + (2*a*b^3*d*x^2*
sin(2*c) + 3*a*b^3*cos(2*c))*cos(2*d*x^2) - 2*(2*(a^4 - a^2*b^2)*d*x^2*cos(
c) - 3*(a^4 - a^2*b^2)*sin(c))*cos(d*x^2) + (2*a*b^3*d*x^2*cos(2*c) - 3*a*b
^3*sin(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*x^2*sin(c) + 3*(a^4 - a^
2*b^2)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (2*a^3*b*d*x^2*cos(d*x^2 + c) +
 3*a^3*b*sin(d*x^2 + c) + 3*a^2*b^2)*sin(2*d*x^2 + 2*c) - (2*(a^3*b - a*b^3
)*d*x^2 + (2*a*b^3*d*x^2*cos(2*c) - 3*a*b^3*sin(2*c))*cos(2*d*x^2) + 2*(2*(
a^4 - a^2*b^2)*d*x^2*sin(c) + 3*(a^4 - a^2*b^2)*cos(c))*cos(d*x^2) - (2*a*b
```

```
^3*d*x^2*sin(2*c) + 3*a*b^3*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d
*x^2*cos(c) - 3*(a^4 - a^2*b^2)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^
2*d*x^4*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x^4*sin(2*d*x^2 + 2*c)^2 + (b^6*co
s(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2
*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 +
 (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*
b^3 + a*b^5)*d*x^4*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c
)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b -
 2*a^3*b^3 + a*b^5)*d*x^4*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d
*x^4 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)
*sin(c))*d*x^4*cos(d*x^2) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 -
a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2
))*cos(2*d*x^2) - 2*(a^2*b^4*d*x^4*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^4*si
n(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^5*
b - a^3*b^3)*d*x^4*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x
^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin
(2*c)*sin(c))*d*x^4*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^
3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^4*si
n(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x^4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x
^4*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*
(a^5*b - a^3*b^3)*d*x^4*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b
*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c) - (a*b^3*cos(2*c)*sin(2*d*x^2
) + a*b^3*cos(2*d*x^2)*sin(2*c) - 2*(a^4 - a^2*b^2)*cos(d*x^2)*cos(c) + 2*(
a^4 - a^2*b^2)*sin(d*x^2)*sin(c))*sin(d*x^2 + c))/(a^4*b^2*d*x^3*cos(2*d*x^
2 + 2*c)^2 + a^4*b^2*d*x^3*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin
(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (
a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*cos(d*x^2)^2 + (b^6*cos(2*c)^2 +
 b^6*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^3
*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4
*b^2 + a^2*b^4)*sin(c)^2)*d*x^3*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5
)*d*x^3*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^3 - 2*(2*((a^3*
b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^3*cos
(d*x^2) - (a^2*b^4 - b^6)*d*x^3*cos(2*c) - 2*((...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos(d*x^2 + c)^2 - 2*a*b*x^2*sin(d*x^2 + c) - (a^2 + b
^2)*x^2), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \sin \left(c + dx^2\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**2))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \left(a + b\ \sin \left(d\, x^2 + c\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*sin(c + d*x^2))^2),x)
```

```
[Out] int(1/(x^2*(a + b*sin(c + d*x^2))^2), x)
```

3.51 $\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p \, dx$

Optimal. Leaf size=23

$$\text{Int}\left((ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p ,x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p \, dx = \int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p \, dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a+b\sin \left(c+dx^2\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a+b\sin \left(d\,x^2+c\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**p,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="giac")
[Out] integrate((x*e)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {(e\,x)}^m\,{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^2))^p, x)
```

3.52 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 \, dx$

Optimal. Leaf size=444

$$\frac{a(2a^2+3b^2)\,(ex)^{1+m}}{2e(1+m)}+\frac{3ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\,(-idx^2)^{\frac{1}{2}(-1-m)}\,\Gamma\left(\frac{1+m}{2},-idx^2\right)}{16e}-\frac{3ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\,(i}{16}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*(e*x)^(1+m)/e/(1+m)+3/16*I*b*(4*a^2+b^2)*exp(I*c)*(e*x)
^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-I*d*x^2)/e-3/16*I*b*(4*a^2+
b^2)*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,I*d*x^2)/e/exp(I*c)
+3*2^(-7/2-1/2*m)*a*b^2*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMM
A(1/2+1/2*m,-2*I*d*x^2)/e+3*2^(-7/2-1/2*m)*a*b^2*(e*x)^(1+m)*(I*d*x^2)^(-1/
2-1/2*m)*GAMMA(1/2+1/2*m,2*I*d*x^2)/e/exp(2*I*c)-1/16*I*3^(-1/2-1/2*m)*b^3*
exp(3*I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-3*I*d*x^2)/
e+1/16*I*3^(-1/2-1/2*m)*b^3*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/
2*m,3*I*d*x^2)/e/exp(3*I*c)
```

Rubi [A]
time = 0.31, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3484, 6, 3471, 2250, 3470}

$$\frac{3ibe^{ic}(4a^2+b^2)\,(-idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{16e}-\frac{3ibe^{-ic}(4a^2+b^2)\,(idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)}{16e}+\frac{3ab^2e^{2ic}2^{-\frac{m}{2}-\frac{7}{2}}(-idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-2idx^2\right)}{e}+\frac{3ab^2e^{-2ic}2^{-\frac{m}{2}-\frac{7}{2}}(idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},2idx^2\right)}{e}-\frac{ib^3e^{3ic}3^{-\frac{m}{2}-\frac{1}{2}}(-idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-3idx^2\right)}{16e}+\frac{ib^3e^{-3ic}3^{-\frac{m}{2}-\frac{1}{2}}(idx^2)^{\frac{1}{2}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},3idx^2\right)}{16e}+\frac{a(2a^2+3b^2)\,(ex)^{m+1}}{2e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + (((3*I)/16)*b*(4*a^2 + b^
2)*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*
x^2])/e - (((3*I)/16)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c)) + (3*2^(-7/2 - m/2)*a*b^2*E^((2*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2])/e
 + (3*2^(-7/2 - m/2)*a*b^2*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 +
m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c)) - ((I/16)*3^(-1/2 - m/2)*b^3*E^((3*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2])/e
 + ((I/16)*3^(-1/2 - m/2)*b^3*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1
 + m)/2, (3*I)*d*x^2])/(e*E^((3*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx = \int \left(a^3(ex)^m+\frac{3}{2}ab^2(ex)^m-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)+3a^2b(ex)^m\,\mathrm{s}\right.$$
$$= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)+3a^2b(ex)^m\sin\right.$$
$$= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right)+\left(3a^2b+\frac{3b^3}{4}\right.\right.$$
$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)}-\frac{1}{2}\left(3ab^2\right)\int (ex)^m\cos\left(2c+2dx^2\right)\,dx-\frac{1}{4}b^3\int ($$
$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)}-\frac{1}{4}\left(3ab^2\right)\int e^{-2ic-2idx^2}(ex)^m\,dx-\frac{1}{4}\left(3ab^2\right)\int e$$
$$= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)}+\frac{3ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2}\right.}{16e}$$

Mathematica [A]
time = 8.33, size = 451, normalized size = 1.02

$$\frac{1}{16}ix(ex)^m\left(3b\left(4a^2+b^2\right)e^{ic}\left(-idx^2\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-idx^2\right)+\frac{-16ia^3dx^2-24iab^2dx^2+12ia^2be^{-ic}(1+m)\left(idx^2\right)^{\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right)+3ib^3e^{-ic}(1+m)\left(idx^2\right)^{\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right)+3\,2^{\frac{1}{2}-\frac{m}{2}}ab^2e^{2ic}(1+m)\left(-idx^2\right)^{\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-2idx^2\right)-3\,2^{\frac{1}{2}-\frac{m}{2}}ab^2e^{-2ic}(1+m)\left(idx^2\right)^{\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},2idx^2\right)-i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{3ic}(1+m)\left(-idx^2\right)^{\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},-3idx^2\right)-i3^{-\frac{1}{2}-\frac{m}{2}}b^3e^{-3ic}(1+m)\left(idx^2\right)^{\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1+m}{2},3idx^2\right)}{d(1+m)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

```
[Out] (I/16)*x*(e*x)^m*(3*b*(4*a^2 + b^2)*E^(I*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma
[(1 + m)/2, (-I)*d*x^2] + ((-16*I)*a^3*d*x^2 - (24*I)*a*b^2*d*x^2 + ((12*I)
*a^2*b*(1 + m)*(I*d*x^2)^(1/2 - m/2)*Gamma[(1 + m)/2, I*d*x^2])/E^(I*c) + (
(3*I)*b^3*(1 + m)*(I*d*x^2)^(1/2 - m/2)*Gamma[(1 + m)/2, I*d*x^2])/E^(I*c)
+ 3*2^(1/2 - m/2)*a*b^2*E^((2*I)*c)*(1 + m)*((-I)*d*x^2)^(1/2 - m/2)*Gamma[
(1 + m)/2, (-2*I)*d*x^2] - (3*2^(1/2 - m/2)*a*b^2*(1 + m)*(I*d*x^2)^(1/2 -
m/2)*Gamma[(1 + m)/2, (2*I)*d*x^2])/E^((2*I)*c) - I*3^(-1/2 - m/2)*b^3*E^((
3*I)*c)*(1 + m)*((-I)*d*x^2)^(1/2 - m/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2] - (
I*3^(-1/2 - m/2)*b^3*(1 + m)*(I*d*x^2)^(1/2 - m/2)*Gamma[(1 + m)/2, (3*I)*d
*x^2])/E^((3*I)*c))/(d*(1 + m)*x^2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(d\,x^2 + c\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="maxima")
```

```
[Out] (x*e)^(m + 1)*a^3*e^(-1)/(m + 1) + 1/8*(12*a*b^2*x*e^(m*log(x) + m) + 3*((4
*a^2*b + b^3)*m*sin(c) + (4*a^2*b + b^3)*sin(c))*e^m*integrate(x^m*cos(d*x^
2), x) + 3*(4*a^2*b + b^3 + (4*a^2*b + b^3)*m)*e^m*integrate(x^m*sin(d*x^2
+ c), x) + 3*((4*a^2*b + b^3)*m*cos(c) + (4*a^2*b + b^3)*cos(c))*e^m*integr
ate(x^m*sin(d*x^2), x) - 12*(a*b^2*m + a*b^2)*integrate(cos(2*d*x^2 + 2*c)*
e^(m*log(x) + m), x) - 2*(b^3*m + b^3)*integrate(e^(m*log(x) + m)*sin(3*d*x
^2 + 3*c), x))/(m + 1)
```

Fricas [A]
time = 0.14, size = 302, normalized size = 0.68

$$\frac{24\,(2\,a^3 + 3\,ab^2)(xe)^m\,dx + (b^3m + b^3)e^{\left(-\frac{1}{2}\,(m-1)\log\left(3i\,de^{(-2)}\right) - 3i\,c + 1\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, 3i\,dx^2\right) - 9i\,(ab^2m + ab^2)e^{\left(-\frac{1}{2}\,(m-1)\log\left(2i\,de^{(-2)}\right) - 2i\,c + 1\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, 2i\,dx^2\right) - 9\,(4\,a^2b + b^3 + (4\,a^2b + b^3)m)e^{\left(-\frac{1}{2}\,(m-1)\log\left(i\,de^{(-2)}\right) - i\,c + 1\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, i\,dx^2\right) - 9\,(4\,a^2b + b^3 + (4\,a^2b + b^3)m)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-i\,de^{(-2)}\right) + i\,c + 1\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, -i\,dx^2\right) + 9i\,(ab^2m + ab^2)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-2i\,de^{(-2)}\right) + 2i\,c + 1\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, -2i\,dx^2\right) + (b^3m + b^3)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-3i\,de^{(-2)}\right) + 3i\,c + 1\right)}\Gamma\left(\frac{1}{2}\,m + \frac{1}{2}, -3i\,dx^2\right)}{48\,(dm + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(24*(2*a^3 + 3*a*b^2)*(x*e)^m*d*x + (b^3*m + b^3)*e^(-1/2*(m - 1)*log(
3*I*d*e^(-2)) - 3*I*c + 1)*gamma(1/2*m + 1/2, 3*I*d*x^2) - 9*I*(a*b^2*m + a
*b^2)*e^(-1/2*(m - 1)*log(2*I*d*e^(-2)) - 2*I*c + 1)*gamma(1/2*m + 1/2, 2*I
*d*x^2) - 9*(4*a^2*b + b^3 + (4*a^2*b + b^3)*m)*e^(-1/2*(m - 1)*log(I*d*e^(
-2)) - I*c + 1)*gamma(1/2*m + 1/2, I*d*x^2) - 9*(4*a^2*b + b^3 + (4*a^2*b +
 b^3)*m)*e^(-1/2*(m - 1)*log(-I*d*e^(-2)) + I*c + 1)*gamma(1/2*m + 1/2, -I*
d*x^2) + 9*I*(a*b^2*m + a*b^2)*e^(-1/2*(m - 1)*log(-2*I*d*e^(-2)) + 2*I*c +
 1)*gamma(1/2*m + 1/2, -2*I*d*x^2) + (b^3*m + b^3)*e^(-1/2*(m - 1)*log(-3*I
*d*e^(-2)) + 3*I*c + 1)*gamma(1/2*m + 1/2, -3*I*d*x^2))/(d*m + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**3,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x^2 + c) + a)^3*(x*e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,x\right)}^m\,{\left(a+b\,\sin\left(d\,x^2+c\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2))^3,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^2))^3, x)
```

3.53 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=279

$$\frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)}+\frac{iabe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{2e}-\frac{iabe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx\right.}{2e}$$

```
[Out] 1/2*(2*a^2+b^2)*(e*x)^(1+m)/e/(1+m)+1/2*I*a*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^
2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-I*d*x^2)/e-1/2*I*a*b*(e*x)^(1+m)*(I*d*x^2)
^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,I*d*x^2)/e/exp(I*c)+2^(-7/2-1/2*m)*b^2*exp(2*
I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-2*I*d*x^2)/e+2^(-
7/2-1/2*m)*b^2*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,2*I*d*x^2
)/e/exp(2*I*c)
```

Rubi [A]
time = 0.15, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3484, 6, 3471, 2250, 3470}

$$\frac{iabe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{2e}-\frac{iabe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)}{2e}+\frac{b^2e^{2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-2idx^2\right)}{e}+\frac{b^2e^{-2ic}2^{-\frac{m}{2}-\frac{7}{2}}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},2idx^2\right)}{e}+\frac{(2a^2+b^2)(ex)^{m+1}}{2e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/2)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/2)*a*b*
(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
 + (2^(-7/2 - m/2)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, (-2*I)*d*x^2])/e + (2^(-7/2 - m/2)*b^2*(e*x)^(1 + m)*(I*d*
x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \int \left(a^2(ex)^m+\frac{1}{2}b^2(ex)^m-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^2\right)+2ab(ex)^m\sin\left(\right.\right.\\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^2\right)+2ab(ex)^m\sin\left(c+\right.\right.\\
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)}+(2ab)\int (ex)^m\sin\left(c+dx^2\right)\,dx-\frac{1}{2}b^2\int (ex)^m\,\mathrm{co}\\
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)}+(iab)\int e^{-ic-idx^2}(ex)^m\,dx-(iab)\int e^{ic+idx^2}(ex)^{n}\\
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)}+\frac{iabe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{2e}-
\end{aligned}$$

Mathematica [A]
time = 4.62, size = 551, normalized size = 1.97

$$\frac{2^{\frac{1}{2}(-7-m)}x(ex)^m\left(d^2x^4\right)^{\frac{1}{2}(-1-m)}\left(2^{\frac{7+m}{2}}a^2\left(d^2x^4\right)^{\frac{1+m}{2}}+2^{\frac{5+m}{2}}b^2\left(d^2x^4\right)^{\frac{1+m}{2}}+b^2\left(idx^2\right)^{\frac{1+m}{2}}\cos(2c)\Gamma\left(\frac{1+m}{2},-2idx^2\right)+b^2m\left(idx^2\right)^{\frac{1+m}{2}}\cos(2c)\Gamma\left(\frac{1+m}{2},-2idx^2\right)+b^2\left(-idx^2\right)^{\frac{1+m}{2}}\cos(2c)\Gamma\left(\frac{1+m}{2},2idx^2\right)+b^2m\left(-idx^2\right)^{\frac{1+m}{2}}\cos(2c)\Gamma\left(\frac{1+m}{2},2idx^2\right)-i2^{\frac{5+m}{2}}ab(1+m)\left(-idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right)(\cos(c)-i\sin(c))+i2^{\frac{5+m}{2}}ab(1+m)\left(idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-idx^2\right)(\cos(c)+i\sin(c))+ib^2\left(idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-2idx^2\right)\sin(2c)+ib^2m\left(idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-2idx^2\right)\sin(2c)-ib^2\left(-idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},2idx^2\right)\sin(2c)-ib^2m\left(-idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},2idx^2\right)\sin(2c)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (2^((-7 - m)/2)*x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(2^((7 + m)/2)*a^2*(d^2*x^
4)^((1 + m)/2) + 2^((5 + m)/2)*b^2*(d^2*x^4)^((1 + m)/2) + b^2*(I*d*x^2)^((
1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*m*(I*d*x^2)^((1 + m
)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*((-I)*d*x^2)^((1 + m)/2)
*Cos[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] + b^2*m*((-I)*d*x^2)^((1 + m)/2)*Co
s[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] - I*2^((5 + m)/2)*a*b*(1 + m)*((-I)*d*
```

```
x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*2^((5 +
m)/2)*a*b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c
] + I*Sin[c]) + I*b^2*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2]*
Sin[2*c] + I*b^2*m*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2]*Sin
[2*c] - I*b^2*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2]*Sin[2*
c] - I*b^2*m*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2]*Sin[2*c
]))/(1 + m)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(e x \right)^{m} \left(a +b \sin \left(d \,x^{2}+c \right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (x*e)^(m + 1)*a^2*e^(-1)/(m + 1) + 1/2*(b^2*x*e^(m*log(x) + m) - (b^2*m + b
^2)*integrate(cos(2*d*x^2 + 2*c)*e^(m*log(x) + m), x) + 4*(a*b*m + a*b)*int
egrate(e^(m*log(x) + m)*sin(d*x^2 + c), x))/(m + 1)
```

Fricas [A]
time = 0.16, size = 187, normalized size = 0.67

$$\frac{8\,(2\,a^2+b^2)(xe)^m\,dx - i\,(b^2m+b^2)e^{\left(-\frac{1}{2}\,(m-1)\log\left(2i\,de^{(-2)}\right)-2i\,c+1\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2i\,dx^2\right) - 8\,(abm+ab)e^{\left(-\frac{1}{2}\,(m-1)\log\left(i\,de^{(-2)}\right)-i\,c+1\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},i\,dx^2\right) - 8\,(abm+ab)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-i\,de^{(-2)}\right)+i\,c+1\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-i\,dx^2\right) + i\,(b^2m+b^2)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-2i\,de^{(-2)}\right)+2i\,c+1\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-2i\,dx^2\right)}{16\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*(2*a^2 + b^2)*(x*e)^m*d*x - I*(b^2*m + b^2)*e^(-1/2*(m - 1)*log(2*I
*d*e^(-2)) - 2*I*c + 1)*gamma(1/2*m + 1/2, 2*I*d*x^2) - 8*(a*b*m + a*b)*e^(
-1/2*(m - 1)*log(I*d*e^(-2)) - I*c + 1)*gamma(1/2*m + 1/2, I*d*x^2) - 8*(a*
b*m + a*b)*e^(-1/2*(m - 1)*log(-I*d*e^(-2)) + I*c + 1)*gamma(1/2*m + 1/2, -
I*d*x^2) + I*(b^2*m + b^2)*e^(-1/2*(m - 1)*log(-2*I*d*e^(-2)) + 2*I*c + 1)*
gamma(1/2*m + 1/2, -2*I*d*x^2))/(d*m + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^2*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,x)^m\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int((e*x)^m*(a + b*sin(c + d*x^2))^2, x)
```

3.54 $\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)\,dx$

Optimal. Leaf size=134

$$\frac{a(ex)^{1+m}}{e(1+m)}+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{4e}-\frac{ibe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},idx^2\right)}{4e}$$

```
[Out] a*(e*x)^(1+m)/e/(1+m)+1/4*I*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^2)^(-1/2-1/2*m)*
GAMMA(1/2+1/2*m,-I*d*x^2)/e-1/4*I*b*(e*x)^(1+m)*(I*d*x^2)^(-1/2-1/2*m)*GAMM
A(1/2+1/2*m,I*d*x^2)/e/exp(I*c)
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {14, 3470, 2250}

$$\frac{ibe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{4e}-\frac{ibe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)}{4e}+\frac{a(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/4)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)
^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/4)*b*(e*x)^(1 + m)*(I*d
*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int \left(a(ex)^m+b(ex)^m\sin\left(c+dx^2\right)\right)\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)}+b\int (ex)^m\sin\left(c+dx^2\right)\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)}+\frac{1}{2}(ib)\int e^{-ic-idx^2}(ex)^m\,dx-\frac{1}{2}(ib)\int e^{ic+idx^2}(ex)^m\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)}+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{4e}-\frac{ibe^{-ic}(ex)^{1\cdots}}{}\end{aligned}$$

Mathematica [A]
time = 1.05, size = 149, normalized size = 1.11

$$\frac{x(ex)^m\left(d^2x^4\right)^{\frac{1}{2}(-1-m)}\left(4a\left(d^2x^4\right)^{\frac{1+m}{2}}-ib(1+m)\left(-idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},idx^2\right)\left(\cos(c)-i\sin(c)\right)+ib(1+m)\left(idx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-idx^2\right)\left(\cos(c)+i\sin(c)\right)\right)}{4(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(4*a*(d^2*x^4)^((1 + m)/2) - I*b*(1 + m)*
((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*
b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c] + I*Si
n[c])))/(4*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m\left(a+b\sin\left(d\,x^2+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] (x*e)^(m + 1)*a*e^(-1)/(m + 1) + b*integrate(e^(m*log(x) + m)*sin(d*x^2 + c
), x)
```

Fricas [A]
time = 0.11, size = 93, normalized size = 0.69

$$\frac{4\,(xe)^{m}\,adx-(bm+b)e^{\left(-\frac{1}{2}\,(m-1)\log\left(i\,de^{(-2)}\right)-i\,c+1\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},i\,dx^{2}\right)-(bm+b)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-i\,de^{(-2)}\right)+i\,c+1\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-i\,dx^{2}\right)}{4\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(x*e)^m*a*d*x - (b*m + b)*e^(-1/2*(m - 1)*log(I*d*e^(-2)) - I*c + 1)
*gamma(1/2*m + 1/2, I*d*x^2) - (b*m + b)*e^(-1/2*(m - 1)*log(-I*d*e^(-2)) +
 I*c + 1)*gamma(1/2*m + 1/2, -I*d*x^2))/(d*m + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{m}\left(a+b\sin\left(c+dx^{2}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {(e\,x)}^m\,\left(a+b\,\sin\left(d\,x^2+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^2)),x)
```

```
[Out] int((e*x)^m*(a + b*sin(c + d*x^2)), x)
```

3.55 $\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(ex)^m}{a+b\sin\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^2]),x]
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^2]), x]
```

Rubi steps

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx = \int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]),x]
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{a+b\sin\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*sin(d*x^2+c)),x)
[Out] int((e*x)^m/(a+b*sin(d*x^2+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
[Out] integrate((x*e)^m/(b*sin(d*x^2 + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
[Out] integral((x*e)^m/(b*sin(d*x^2 + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{a + b\sin(c + dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**2+c)),x)
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="giac")
[Out] integrate((x*e)^m/(b*sin(d*x^2 + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{a + b\,\sin\left(d\,x^2 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a + b*sin(c + d*x^2)),x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^2)), x)
```

3.56 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

`[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^2+c))^2,x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]`

`[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]`

Rubi steps

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]`

`[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]`

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(a+b\sin\left(d\,x^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)

[Out] int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")

[Out] (a^3*b*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c)*e^(m*log(x) + m) - b^4*cos(2*c)*e^(m*log(x) + m)*sin(2*d*x^2) - b^4*cos(2*d*x^2)*e^(m*log(x) + m)*sin(2*c) + 2*(a^3*b - a*b^3)*cos(d*x^2)*cos(c)*e^(m*log(x) + m) - 2*(a^3*b - a*b^3)*e^(m*log(x) + m)*sin(d*x^2)*sin(c) - (a*b^3*cos(2*d*x^2)*cos(2*c)*e^(m*log(x) + m) - a*b^3*e^(m*log(x) + m)*sin(2*d*x^2)*sin(2*c) + 2*(a^4 - a^2*b^2)*cos(c)*e^(m*log(x) + m)*sin(d*x^2) + 2*(a^4 - a^2*b^2)*cos(d*x^2)*e^(m*log(x) + m)*sin(c) + (a^3*b - a*b^3)*e^(m*log(x) + m))*cos(d*x^2 + c) - (a^4*b^2*d*x*cos(2*d*x^2 + 2*c)^2 + a^4*b^2*d*x*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*cos(d*x^2)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*cos(d*x^2) - (a^2*b^4 - b^6)*d*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*sin(d*x^2))*cos(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x*sin(2*d*x^2)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x)*cos(2*d*x^2 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x*sin(d*x^2) + (a^2*b^4 - b^6)*d*x*sin(2*c))*sin(2*d*x^2) - 2*(a^2*b^4*d*x*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-((b^4*m*sin(2*c) - b^4*sin(2*c))*cos(2*d*x^2)*e^(m*log(x) + m) - 2*((a^3*b - a*b^3)*m*cos(c) - (a^3*b - a*b^3)*cos(c))*cos(d*x^2)*e^(m*log(x) + m) + (b^4*m*cos(2*c) - b^4*cos(2*c))*e^(m*log(x) + m)*sin(2*d*x^2) + 2*((a^3*b - a*b^3)*m*sin(c) - (a^3*b - a*b^3)*sin(c))*e^(m*log(x) + m)*sin(d*x^2) - (2*a^3*b*d*x^2*e^(m*log(x) + m)*sin(d*x^2 + c) + (a^3*b*m - a^3*b)*cos(d*x^2 + c)*e^(m*log(x) + m))*cos(2*d*x^2 + 2*c) - ((2*a*b^3*d*x^2*e^m*sin(2*c) - (a*b^3*m*cos(2*c) - a*b^3*cos(2*c))*e^m)*x^m*cos(2*d

```
*x^2) - 2*(2*(a^4 - a^2*b^2)*d*x^2*cos(c)*e^m + ((a^4 - a^2*b^2)*m*sin(c) -
 (a^4 - a^2*b^2)*sin(c))*e^m)*x^m*cos(d*x^2) + (2*a*b^3*d*x^2*cos(2*c)*e^m
+ (a*b^3*m*sin(2*c) - a*b^3*sin(2*c))*e^m)*x^m*sin(2*d*x^2) + 2*(2*(a^4 - a
^2*b^2)*d*x^2*e^m*sin(c) - ((a^4 - a^2*b^2)*m*cos(c) - (a^4 - a^2*b^2)*cos(
c))*e^m)*x^m*sin(d*x^2) + (a^3*b - a*b^3 - (a^3*b - a*b^3)*m)*e^(m*log(x) +
 m))*cos(d*x^2 + c) + (2*a^3*b*d*x^2*cos(d*x^2 + c)*e^(m*log(x) + m) - (a^3
*b*m - a^3*b)*e^(m*log(x) + m)*sin(d*x^2 + c) - (a^2*b^2*m - a^2*b^2)*e^(m*
log(x) + m))*sin(2*d*x^2 + 2*c) + (2*(a^3*b - a*b^3)*d*x^2*e^(m*log(x) + m)
 + (2*a*b^3*d*x^2*cos(2*c)*e^m + (a*b^3*m*sin(2*c) - a*b^3*sin(2*c))*e^m)*x
^m*cos(2*d*x^2) + 2*(2*(a^4 - a^2*b^2)*d*x^2*e^m*sin(c) - ((a^4 - a^2*b^2)*
m*cos(c) - (a^4 - a^2*b^2)*cos(c))*e^m)*x^m*cos(d*x^2) - (2*a*b^3*d*x^2*e^m
*sin(2*c) - (a*b^3*m*cos(2*c) - a*b^3*cos(2*c))*e^m)*x^m*sin(2*d*x^2) + 2*(
2*(a^4 - a^2*b^2)*d*x^2*cos(c)*e^m + ((a^4 - a^2*b^2)*m*sin(c) - (a^4 - a^2
*b^2)*sin(c))*e^m)*x^m*sin(d*x^2))*sin(d*x^2 + c))/(a^4*b^2*d*x^2*cos(2*d*x
^2 + 2*c)^2 + a^4*b^2*d*x^2*sin(2*d*x^2 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*si
n(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 +
(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (b^6*cos(2*c)^2
+ b^6*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^
2*cos(c)*sin(d*x^2) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^
4*b^2 + a^2*b^4)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^
5)*d*x^2*cos(d*x^2)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2 - 2*(2*((a^3
*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*co
s(d*x^2) - (a^2*b^4 - b^6)*d*x^2*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*c
os(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*d*x^2) -
 2*(a^2*b^4*d*x^2*cos(2*d*x^2)*cos(2*c) - a^2*b^4*d*x^2*sin(2*d*x^2)*sin(2*
c) + 2*(a^5*b - a^3*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^5*b - a^3*b^3)*d*x^
2*cos(d*x^2)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^2)*cos(2*d*x^2 + 2*c) - 2*(2*
((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x
^2*cos(d*x^2) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*co
s(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^2*b^4 - b^6)*d*x^2*sin(2*c))*sin(2*d*x
^2) - 2*(a^2*b^4*d*x^2*cos(2*c)*sin(2*d*x^2) + a^2*b^4*d*x^2*cos(2*d*x^2)*s
in(2*c) - 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^5*b - a^3*b^3)
*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^3*b*e^(m*log(x) + m)
*sin(d*x^2 + c) + a^2*b^2*e^(m*log(x) + m))*sin(2*d*x^2 + 2*c) - (a*b^3*cos
(2*c)*e^(m*log(x) + m)*sin(2*d*x^2) + a*b^3*cos...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(x*e)^m/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2)
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^2))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**2))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/(b*sin(d*x^2 + c) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{(a+b\,\sin(d\,x^2+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a + b*sin(c + d*x^2))^2,x)
```

```
[Out] int((e*x)^m/(a + b*sin(c + d*x^2))^2, x)
```

3.57 $\int x^5(a+b\sin(c+dx^3))\,dx$

Optimal. Leaf size=44

$$\frac{ax^6}{6}-\frac{bx^3\cos(c+dx^3)}{3d}+\frac{b\sin(c+dx^3)}{3d^2}$$

```
[Out] 1/6*a*x^6-1/3*b*x^3*cos(d*x^3+c)/d+1/3*b*sin(d*x^3+c)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3460, 3377, 2717}

$$\frac{ax^6}{6}+\frac{b\sin(c+dx^3)}{3d^2}-\frac{bx^3\cos(c+dx^3)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\sin\left(c+dx^3\right)\right)\,dx &= \int\left(ax^5+bx^5\sin\left(c+dx^3\right)\right)\,dx\\ &= \frac{ax^6}{6}+b\int x^5\sin\left(c+dx^3\right)\,dx\\ &= \frac{ax^6}{6}+\frac{1}{3}b\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^3\right)\\ &= \frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\text{Subst}\left(\int\cos(c+dx)\,dx,x,x^3\right)}{3d}\\ &= \frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 44, normalized size = 1.00

$$\frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^3]),x]
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Maple [**A**]
time = 0.04, size = 39, normalized size = 0.89

method	result	size
risch	$\frac{a\,x^6}{6}-\frac{b\,x^3\cos\left(d\,x^3+c\right)}{3d}+\frac{b\sin\left(d\,x^3+c\right)}{3d^2}$	39
norman	$\frac{\frac{a\,x^6}{6}+\frac{a\,x^6\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{6}+\frac{2b\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d^2}-\frac{b\,x^3}{3d}+\frac{b\,x^3\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d}}{1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
[Out] 1/6*a*x^6-1/3*b*x^3*cos(d*x^3+c)/d+1/3*b*sin(d*x^3+c)/d^2
```

Maxima [**A**]
time = 0.29, size = 37, normalized size = 0.84

$$\frac{1}{6}\,ax^6-\frac{\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)b}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 - 1/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*b/d^2
```

Fricas [A]
time = 0.36, size = 40, normalized size = 0.91

$$\frac{ad^2x^6 - 2\,bdx^3\cos\left(dx^3 + c\right) + 2\,b\sin\left(dx^3 + c\right)}{6\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^2*x^6 - 2*b*d*x^3*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c))/d^2
```

Sympy [A]
time = 0.39, size = 49, normalized size = 1.11

$$\begin{cases} \frac{ax^6}{6} - \frac{bx^3\cos\left(c+dx^3\right)}{3d} + \frac{b\sin\left(c+dx^3\right)}{3d^2} & \text{for}\ d \neq 0 \\ \frac{x^6\left(a+b\sin\left(c\right)\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((a*x**6/6 - b*x**3*cos(c + d*x**3)/(3*d) + b*sin(c + d*x**3)/(3*d
**2), Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Giac [A]
time = 4.90, size = 75, normalized size = 1.70

$$\frac{\left(dx^3 + c\right)^2a - 2\left(dx^3 + c\right)b\cos\left(dx^3 + c\right) + 2\,b\sin\left(dx^3 + c\right)}{6\,d^2} - \frac{\left(dx^3 + c\right)ac - bc\cos\left(dx^3 + c\right)}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 1/6*((d*x^3 + c)^2*a - 2*(d*x^3 + c)*b*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c))
/d^2 - 1/3*((d*x^3 + c)*a*c - b*c*cos(d*x^3 + c))/d^2
```

Mupad [B]
time = 0.17, size = 38, normalized size = 0.86

$$\frac{a\,x^6}{6} + \frac{\frac{b\,\sin\left(d\,x^3+c\right)}{3} - \frac{b\,d\,x^3\,\cos\left(d\,x^3+c\right)}{3}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*sin(c + d*x^3)),x)
```

```
[Out] (a*x^6)/6 + ((b*sin(c + d*x^3))/3 - (b*d*x^3*cos(c + d*x^3))/3)/d^2
```

3.58 $\int x^2(a+b\sin(c+dx^3))\,dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}-\frac{b\cos(c+dx^3)}{3d}$$

[Out] 1/3*a*x^3-1/3*b*cos(d*x^3+c)/d

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {14, 3460, 2718}

$$\frac{ax^3}{3}-\frac{b\cos(c+dx^3)}{3d}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*Sin[c + d*x^3]),x]

[Out] (a*x^3)/3 - (b*Cos[c + d*x^3])/(3*d)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\sin\left(c+dx^3\right)\right)\,dx &= \int \left(ax^2+bx^2\sin\left(c+dx^3\right)\right)\,dx \\
&= \frac{ax^3}{3}+b\int x^2\sin\left(c+dx^3\right)\,dx \\
&= \frac{ax^3}{3}+\frac{1}{3}b\text{Subst}\left(\int \sin(c+dx)\,dx,x,x^3\right) \\
&= \frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.64

$$\frac{ax^3}{3}-\frac{b\cos(c)\cos\left(dx^3\right)}{3d}+\frac{b\sin(c)\sin\left(dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^3]),x]
[Out] (a*x^3)/3 - (b*Cos[c]*Cos[d*x^3])/(3*d) + (b*Sin[c]*Sin[d*x^3])/(3*d)
```

Maple [A]
time = 0.02, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{a\,x^3}{3}-\frac{b\cos\left(d\,x^3+c\right)}{3d}$	22
derivativedivides	$\frac{\left(d\,x^3+c\right)a-b\cos\left(d\,x^3+c\right)}{3d}$	27
default	$\frac{\left(d\,x^3+c\right)a-b\cos\left(d\,x^3+c\right)}{3d}$	27
norman	$\frac{\frac{a\,x^3}{3}-\frac{2b}{3d}+\frac{a\,x^3\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3}}{1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
[Out] 1/3/d*((d*x^3+c)*a-b*cos(d*x^3+c))
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$\frac{1}{3}\,ax^3-\frac{b\cos\left(dx^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 - 1/3*b*cos(d*x^3 + c)/d
```

Fricas [A]
time = 0.37, size = 23, normalized size = 0.92

$$\frac{adx^3 - b\cos\left(dx^3 + c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d*x^3 - b*cos(d*x^3 + c))/d
```

Sympy [A]
time = 0.11, size = 31, normalized size = 1.24

$$\begin{cases} \frac{ax^3}{3} - \frac{b\cos\left(c+dx^3\right)}{3d} & \text{for}\ d \neq 0 \\ \frac{x^3(a+b\sin\left(c\right))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((a*x**3/3 - b*cos(c + d*x**3)/(3*d), Ne(d, 0)), (x**3*(a + b*sin(
c))/3, True))
```

Giac [A]
time = 3.85, size = 26, normalized size = 1.04

$$\frac{(dx^3 + c)a - b\cos\left(dx^3 + c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 1/3*((d*x^3 + c)*a - b*cos(d*x^3 + c))/d
```

Mupad [B]
time = 4.66, size = 21, normalized size = 0.84

$$\frac{a\,x^3}{3} - \frac{b\,\cos\left(d\,x^3 + c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*sin(c + d*x^3)),x)
```

```
[Out] (a*x^3)/3 - (b*cos(c + d*x^3))/(3*d)
```

3.59 $\int \frac{a+b\sin(c+dx^3)}{x}\,dx$

Optimal. Leaf size=31

$$a\log(x)+\frac{1}{3}b\text{Ci}(dx^3)\sin(c)+\frac{1}{3}b\cos(c)\text{Si}(dx^3)$$

```
[Out] a*ln(x)+1/3*b*cos(c)*Si(d*x^3)+1/3*b*Ci(d*x^3)*sin(c)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {14, 3458, 3457, 3456}

$$a\log(x)+\frac{1}{3}b\sin(c)\text{CosIntegral}(dx^3)+\frac{1}{3}b\cos(c)\text{Si}(dx^3)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x,x]
```

```
[Out] a*Log[x] + (b*CosIntegral[d*x^3]*Sin[c])/3 + (b*Cos[c]*SinIntegral[d*x^3])/
3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\int \frac{a+b\sin(c+dx^3)}{x}\,dx = \int \left(\frac{a}{x}+\frac{b\sin(c+dx^3)}{x}\right)dx$$
$$= a\log(x)+b\int \frac{\sin(c+dx^3)}{x}\,dx$$
$$= a\log(x)+(b\cos(c))\int \frac{\sin(dx^3)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx^3)}{x}\,dx$$
$$= a\log(x)+\frac{1}{3}b\text{Ci}\left(dx^3\right)\sin(c)+\frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.94

$$a\log(x)+\frac{1}{3}b\left(\text{Ci}\left(dx^3\right)\sin(c)+\cos(c)\text{Si}\left(dx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x,x]
[Out] a*Log[x] + (b*(CosIntegral[d*x^3]*Sin[c] + Cos[c]*SinIntegral[d*x^3]))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin(d\,x^3+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x,x)
[Out] int((a+b*sin(d*x^3+c))/x,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 50, normalized size = 1.61

$$-\frac{1}{6}\left(\left(i\,\text{Ei}\left(i\,dx^3\right)-i\,\text{Ei}\left(-i\,dx^3\right)\right)\cos(c)-\left(\text{Ei}\left(i\,dx^3\right)+\text{Ei}\left(-i\,dx^3\right)\right)\sin(c)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="maxima")
[Out] -1/6*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3)
)*sin(c))*b + a*log(x)
```

Fricas [A]
time = 0.39, size = 38, normalized size = 1.23

$$\frac{1}{3}\,b\cos\left(c\right)\mathrm{Si}\left(dx^{3}\right)+a\log\left(x\right)+\frac{1}{6}\left(b\,\mathrm{Ci}\left(dx^{3}\right)+b\,\mathrm{Ci}\left(-dx^{3}\right)\right)\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="fricas")
[Out] 1/3*b*cos(c)*sin_integral(d*x^3) + a*log(x) + 1/6*(b*cos_integral(d*x^3) +
b*cos_integral(-d*x^3))*sin(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+dx^{3}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x,x)
[Out] Integral((a + b*sin(c + d*x**3))/x, x)
```

Giac [A]
time = 3.69, size = 32, normalized size = 1.03

$$\frac{1}{3}\,b\,\mathrm{Ci}\left(dx^{3}\right)\sin\left(c\right)+\frac{1}{3}\,b\cos\left(c\right)\mathrm{Si}\left(dx^{3}\right)+\frac{1}{3}\,a\log\left(dx^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="giac")
[Out] 1/3*b*cos_integral(d*x^3)*sin(c) + 1/3*b*cos(c)*sin_integral(d*x^3) + 1/3*a
*log(d*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$a\,\ln\left(x\right)+\frac{b\,\sin\left(c\right)\,\mathrm{cosint}\left(d\,x^{3}\right)}{3}+\frac{b\,\cos\left(c\right)\,\mathrm{sinint}\left(d\,x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))/x,x)
[Out] a*log(x) + (b*sin(c)*cosint(d*x^3))/3 + (b*cos(c)*sinint(d*x^3))/3
```

3.60 $\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx$

Optimal. Leaf size=53

$$-\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{Ci}\left(dx^3\right)-\frac{b\sin\left(c+dx^3\right)}{3x^3}-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)$$

```
[Out] -1/3*a/x^3+1/3*b*d*Ci(d*x^3)*cos(c)-1/3*b*d*Si(d*x^3)*sin(c)-1/3*b*sin(d*x^
3+c)/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3460, 3378, 3384, 3380, 3383}

$$-\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{CosIntegral}\left(dx^3\right)-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)-\frac{b\sin\left(c+dx^3\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*d*Cos[c]*CosIntegral[d*x^3])/3 - (b*Sin[c + d*x^3])/(3*x^3)
 - (b*d*Sin[c]*SinIntegral[d*x^3])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b\sin\left(c+dx^3\right)}{x^4}\right)\,dx \\
&= -\frac{a}{3x^3}+b\int \frac{\sin\left(c+dx^3\right)}{x^4}\,dx \\
&= -\frac{a}{3x^3}+\frac{1}{3}b\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^3\right) \\
&= -\frac{a}{3x^3}-\frac{b\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,x^3\right) \\
&= -\frac{a}{3x^3}-\frac{b\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}(bd\cos(c))\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,x^3\right)-\frac{1}{3}(bd\sin(c) \\
&= -\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{Ci}\left(dx^3\right)-\frac{b\sin\left(c+dx^3\right)}{3x^3}-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 0.91

$$-\frac{a-bdx^3\cos(c)\text{Ci}(dx^3)+b\sin\left(c+dx^3\right)+bdx^3\sin(c)\text{Si}(dx^3)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^4,x]
```

```
[Out] -1/3*(a - b*d*x^3*Cos[c]*CosIntegral[d*x^3] + b*Sin[c + d*x^3] + b*d*x^3*Si
n[c]*SinIntegral[d*x^3])/x^3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^4,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^4,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 57, normalized size = 1.08

$$\frac{1}{6}\left(\left(\Gamma\left(-1, i\,dx^{3}\right)+\Gamma\left(-1, -i\,dx^{3}\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1, i\,dx^{3}\right)-i\,\Gamma\left(-1, -i\,dx^{3}\right)\right)\sin\left(c\right)\right)bd-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*x
^3) - I*gamma(-1, -I*d*x^3))*sin(c))*b*d - 1/3*a/x^3
```

Fricas [A]
time = 0.36, size = 65, normalized size = 1.23

$$-\frac{2\,bdx^{3}\sin\left(c\right)\mathrm{Si}\left(dx^{3}\right)-\left(bdx^{3}\,\mathrm{Ci}\left(dx^{3}\right)+bdx^{3}\,\mathrm{Ci}\left(-dx^{3}\right)\right)\cos\left(c\right)+2\,b\sin\left(dx^{3}+c\right)+2\,a}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*d*x^3*sin(c)*sin_integral(d*x^3) - (b*d*x^3*cos_integral(d*x^3) +
 b*d*x^3*cos_integral(-d*x^3))*cos(c) + 2*b*sin(d*x^3 + c) + 2*a)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(45) = 90$.
time = 7.44, size = 99, normalized size = 1.87

$$\frac{(dx^3+c)bd^2\cos(c)\,\mathrm{Ci}(dx^3)-bcd^2\cos(c)\,\mathrm{Ci}(dx^3)-(dx^3+c)bd^2\sin(c)\,\mathrm{Si}(dx^3)+bcd^2\sin(c)\,\mathrm{Si}(dx^3)-bd^2\sin(dx^3+c)-ad^2}{3\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="giac")
```

```
[Out] 1/3*((d*x^3 + c)*b*d^2*cos(c)*cos_integral(d*x^3) - b*c*d^2*cos(c)*cos_inte
gral(d*x^3) - (d*x^3 + c)*b*d^2*sin(c)*sin_integral(d*x^3) + b*c*d^2*sin(c)
*sin_integral(d*x^3) - b*d^2*sin(d*x^3 + c) - a*d^2)/(d^2*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))/x^4,x)
```

```
[Out] int((a + b*sin(c + d*x^3))/x^4, x)
```

3.61 $\int x^4(a+b\sin(c+dx^3))\,dx$

Optimal. Leaf size=112

$$\frac{ax^5}{5}-\frac{bx^2\cos(c+dx^3)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\,(-idx^3)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\,(idx^3)^{2/3}}$$

```
[Out] 1/5*a*x^5-1/3*b*x^2*cos(d*x^3+c)/d-1/9*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/d
/(-I*d*x^3)^(2/3)-1/9*b*x^2*GAMMA(2/3,I*d*x^3)/d/exp(I*c)/(I*d*x^3)^(2/3)
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3466, 3471, 2250}

$$-\frac{be^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{9d\,(-idx^3)^{2/3}}-\frac{be^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{9d\,(idx^3)^{2/3}}+\frac{ax^5}{5}-\frac{bx^2\cos(c+dx^3)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^5)/5 - (b*x^2*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x^2*Gamma[2/3, (-I)*d
*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (b*x^2*Gamma[2/3, I*d*x^3])/(9*d*E^(I*c)*
(I*d*x^3)^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx &= \int\left(ax^4+bx^4\sin\left(c+dx^3\right)\right)\,dx\\
&= \frac{ax^5}{5}+b\int x^4\sin\left(c+dx^3\right)\,dx\\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}+\frac{(2b)\int x\cos\left(c+dx^3\right)\,dx}{3d}\\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}+\frac{b\int e^{-ic-idx^3}x\,dx}{3d}+\frac{b\int e^{ic+idx^3}x\,dx}{3d}\\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 124, normalized size = 1.11

$$\frac{dx^8\left(3(d^2x^6)^{2/3}\left(3adx^3-5b\cos\left(c+dx^3\right)\right)-5b(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c))-5b(idx^3)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c))\right)}{45\left(d^2x^6\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (d*x^8*(3*(d^2*x^6)^(2/3)*(3*a*d*x^3 - 5*b*Cos[c + d*x^3]) - 5*b*((-I)*d*x^
3)^(2/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 5*b*(I*d*x^3)^(2/3)*Gamm
a[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(45*(d^2*x^6)^(5/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4\left(a+b\sin\left(d\,x^3+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^4*(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.31, size = 109, normalized size = 0.97

$$\frac{1}{5}\,ax^5-\frac{\left(6\,dx^3\cos\left(dx^3+c\right)-\left(dx^3\right)^{\frac{1}{3}}\left(\left(\left(i\,\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i\,dx^3\right)+\left(-i\,\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)\right)b}{18\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 - 1/18*(6*d*x^3*cos(d*x^3 + c) - (d*x^3)^(1/3)*(((I*sqrt(3) - 1)*
gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*cos(c) + ((sqr
t(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*sin(c))
)*b/(d^2*x)
```

Fricas [A]
time = 0.10, size = 70, normalized size = 0.62

$$\frac{9\,ad^2x^5-15\,bdx^2\cos\left(dx^3+c\right)+5i\,b(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)-5i\,b(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)}{45\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/45*(9*a*d^2*x^5 - 15*b*d*x^2*cos(d*x^3 + c) + 5*I*b*(I*d)^(1/3)*e^(-I*c)*
gamma(2/3, I*d*x^3) - 5*I*b*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -I*d*x^3))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a+b\sin\left(c+dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a + b \sin \left(d x^3 + c\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*sin(c + d*x^3)),x)
[Out] int(x^4*(a + b*sin(c + d*x^3)), x)
```

3.62 $\int x(a+b\sin(c+dx^3))\,dx$

Optimal. Leaf size=91

$$\frac{ax^2}{2}+\frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}$$

```
[Out] 1/2*a*x^2+1/6*I*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)-1/6*I*b
*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 3470, 2250}

$$\frac{ibe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}+\frac{ax^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^2)/2 + ((I/6)*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3)
 - ((I/6)*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b \sin \left(c+d x^3\right)\right) \, dx &= \int \left(a x+b x \sin \left(c+d x^3\right)\right) \, dx \\
&= \frac{a x^2}{2}+b \int x \sin \left(c+d x^3\right) \, dx \\
&= \frac{a x^2}{2}+\frac{1}{2} (i b) \int e^{-i c-i d x^3} x \, dx-\frac{1}{2} (i b) \int e^{i c+i d x^3} x \, dx \\
&= \frac{a x^2}{2}+\frac{i b e^{i c} x^2 \Gamma \left(\frac{2}{3},-i d x^3\right)}{6 \left(-i d x^3\right)^{2/3}}-\frac{i b e^{-i c} x^2 \Gamma \left(\frac{2}{3},i d x^3\right)}{6 \left(i d x^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.09, size = 108, normalized size = 1.19

$$\frac{x^2 \left(3 a \left(d^2 x^6\right)^{2/3}+b \left(-i d x^3\right)^{2/3} \Gamma \left(\frac{2}{3},i d x^3\right) (-i \cos (c)-\sin (c))+i b \left(i d x^3\right)^{2/3} \Gamma \left(\frac{2}{3},-i d x^3\right) (\cos (c)+i \sin (c))\right)}{6 \left(d^2 x^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (x^2*(3*a*(d^2*x^6)^(2/3) + b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*((-I)*
Cos[c] - Sin[c]) + I*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*S
in[c])))/(6*(d^2*x^6)^(2/3))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x \left(a +b \sin \left(d \,x^{3}+c \right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x*(a+b*sin(d*x^3+c)),x)
```

Maxima [**A**]
time = 0.33, size = 93, normalized size = 1.02

$$\frac{1}{2} \, a x^{2} - \frac{\left(d x^{3}\right)^{\frac{1}{3}} {\left({\left({\left(\sqrt{3} + i\right)} \Gamma\left(\frac{2}{3}, i \, d x^{3}\right) + {\left(\sqrt{3} - i\right)} \Gamma\left(\frac{2}{3}, -i \, d x^{3}\right)\right)} \cos\left(c\right) - {\left({\left(i \, \sqrt{3} - 1\right)} \Gamma\left(\frac{2}{3}, i \, d x^{3}\right) + {\left(-i \, \sqrt{3} - 1\right)} \Gamma\left(\frac{2}{3}, -i \, d x^{3}\right)\right)} \sin\left(c\right)\right)} b}{12 \, d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

[Out] 1/2*a*x^2 - 1/12*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*sin(c))*b/(d*x)

Fricas [**A**]
time = 0.13, size = 53, normalized size = 0.58

$$\frac{3\,adx^2 - b(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3}, i\,dx^3\right) - b(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3}, -i\,dx^3\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="fricas")

[Out] 1/6*(3*a*d*x^2 - b*(I*d)^(1/3)*e^(-I*c)*gamma(2/3, I*d*x^3) - b*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -I*d*x^3))/d

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*sin(d*x**3+c)),x)

[Out] Integral(x*(a + b*sin(c + d*x**3)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="giac")

[Out] integrate((b*sin(d*x^3 + c) + a)*x, x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a + b\,\sin\left(d\,x^3 + c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*sin(c + d*x^3)),x)

[Out] int(x*(a + b*sin(c + d*x^3)), x)

3.63 $\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx$

Optimal. Leaf size=101

$$-\frac{a}{x}-\frac{bde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{bde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{b\sin\left(c+dx^3\right)}{x}$$

```
[Out] -a/x-1/2*b*d*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)-1/2*b*d*x^2*
GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)-b*sin(d*x^3+c)/x
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {14, 3468, 3471, 2250}

$$-\frac{be^{ic}dx^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}dx^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^2,x]
```

```
[Out] -(a/x) - (b*d*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(2/3)) -
(b*d*x^2*Gamma[2/3, I*d*x^3])/(2*E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^
3])/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+dx^3\right)}{x^2}\,dx &= \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^3\right)}{x^2}\right)dx \\
&= -\frac{a}{x}+b\int \frac{\sin\left(c+dx^3\right)}{x^2}\,dx \\
&= -\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}+(3bd)\int x\cos\left(c+dx^3\right)\,dx \\
&= -\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}+\frac{1}{2}(3bd)\int e^{-ic-idx^3}x\,dx+\frac{1}{2}(3bd)\int e^{ic+idx^3}x\,dx \\
&= -\frac{a}{x}-\frac{bde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{bde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{b\sin\left(c+dx^3\right)}{x}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 120, normalized size = 1.19

$$
\frac{-ib(-idx^3)^{5/3}\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c))+ib(idx^3)^{5/3}\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c))-2(d^2x^6)^{2/3}\left(a+b\sin\left(c+dx^3\right)\right)}{2x\left(d^2x^6\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^2,x]
[Out] ((-I)*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(I
*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(2/3
)*(a + b*Sin[c + d*x^3]))/(2*x*(d^2*x^6)^(2/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{a+b\sin\left(d\,x^3+c\right)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^2,x)
[Out] int((a+b*sin(d*x^3+c))/x^2,x)
```

Maxima [A]
time = 0.32, size = 89, normalized size = 0.88

$$-\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)b}{12\,x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="maxima")
```

```
[Out] -1/12*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(-1/3, I*d*x^3) + (-I*sqrt(3) -
1)*gamma(-1/3, -I*d*x^3))*cos(c) + ((sqrt(3) + I)*gamma(-1/3, I*d*x^3) + (s
qrt(3) - I)*gamma(-1/3, -I*d*x^3))*sin(c))*b/x - a/x
```

Fricas [A]
time = 0.11, size = 62, normalized size = 0.61

$$\frac{i\,b(i\,d)^{\frac{1}{3}}\,xe^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)-i\,b(-i\,d)^{\frac{1}{3}}\,xe^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)-2\,b\sin\left(dx^3+c\right)-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*b*(I*d)^(1/3)*x*e^(-I*c)*gamma(2/3, I*d*x^3) - I*b*(-I*d)^(1/3)*x*e^
(I*c)*gamma(2/3, -I*d*x^3) - 2*b*sin(d*x^3 + c) - 2*a)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+dx^3\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\sin\left(d\,x^3+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))/x^2,x)
```

```
[Out] int((a + b*sin(c + d*x^3))/x^2, x)
```

3.64 $\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$

Optimal. Leaf size=130

$$-\frac{a}{4x^4}-\frac{3bd\cos(c+dx^3)}{4x}-\frac{3ibd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{b\sin(c+dx^3)}{4x^4}$$

```
[Out] -1/4*a/x^4-3/4*b*d*cos(d*x^3+c)/x-3/8*I*b*d^2*exp(I*c)*x^2*GAMMA(2/3,-I*d*x
^3)/(-I*d*x^3)^(2/3)+3/8*I*b*d^2*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^
(2/3)-1/4*b*sin(d*x^3+c)/x^4
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {14, 3468, 3469, 3470, 2250}

$$-\frac{3ibe^{ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibe^{-ic}d^2x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{a}{4x^4}-\frac{3bd\cos(c+dx^3)}{4x}-\frac{b\sin(c+dx^3)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*d*Cos[c + d*x^3])/(4*x) - (((3*I)/8)*b*d^2*E^(I*c)*x^2*Ga
mma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) + (((3*I)/8)*b*d^2*x^2*Gamma[2/3,
I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^3])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+dx^3\right)}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^3\right)}{x^5}\right)\,dx \\
&= -\frac{a}{4x^4}+b\int \frac{\sin\left(c+dx^3\right)}{x^5}\,dx \\
&= -\frac{a}{4x^4}-\frac{b\sin\left(c+dx^3\right)}{4x^4}+\frac{1}{4}(3bd)\int \frac{\cos\left(c+dx^3\right)}{x^2}\,dx \\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{1}{4}\left(9bd^2\right)\int x\sin\left(c+dx^3\right)\,dx \\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{1}{8}\left(9ibd^2\right)\int e^{-ic-idx^3}x\,dx+\frac{1}{8}(9ib \\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{3ibd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 143, normalized size = 1.10

$$
\frac{3bd^2x^6\left(idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)\left(-i\cos(c)+\sin(c)\right)+3bd^2x^6\left(-idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)\left(i\cos(c)+\sin(c)\right)-2\left(d^2x^6\right)^{2/3}\left(a+3bdx^3\cos\left(c+dx^3\right)+b\sin\left(c+dx^3\right)\right)}{8x^4\left(d^2x^6\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^5,x]
```

```
[Out] (3*b*d^2*x^6*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
+ 3*b*d^2*x^6*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(I*Cos[c] + Sin[c]) -
2*(d^2*x^6)^(2/3)*(a + 3*b*d*x^3*Cos[c + d*x^3] + b*Sin[c + d*x^3]))/(8*x^4
*(d^2*x^6)^(2/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(d\,x^{3}+c\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^5,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^5,x)
```

Maxima [A]
time = 0.32, size = 91, normalized size = 0.70

$$\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3},i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3},i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)bd}{12\,x}-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(-4/3, I*d*x^3) + (sqrt(3) - I)*gam
ma(-4/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(-4/3, I*d*x^3) + (-I*sq
rt(3) - 1)*gamma(-4/3, -I*d*x^3))*sin(c))*b*d/x - 1/4*a/x^4
```

Fricas [A]
time = 0.13, size = 83, normalized size = 0.64

$$\frac{3\,b(i\,d)^{\frac{1}{3}}\,dx^4e^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)+3\,b(-i\,d)^{\frac{1}{3}}\,dx^4e^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)-6\,bdx^3\cos\left(dx^3+c\right)-2\,b\sin\left(dx^3+c\right)-2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*(I*d)^(1/3)*d*x^4*e^(-I*c)*gamma(2/3, I*d*x^3) + 3*b*(-I*d)^(1/3)*
d*x^4*e^(I*c)*gamma(2/3, -I*d*x^3) - 6*b*d*x^3*cos(d*x^3 + c) - 2*b*sin(d*x
^3 + c) - 2*a)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+dx^{3}\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\sin\left(d\,x^3 + c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))/x^5,x)
[Out] int((a + b*sin(c + d*x^3))/x^5, x)
```

3.65 $\int x^3(a + b\sin(c + dx^3))\,dx$

Optimal. Leaf size=106

$$\frac{ax^4}{4} - \frac{bx\cos(c + dx^3)}{3d} - \frac{be^{ic}x\Gamma\left(\frac{1}{3}, -idx^3\right)}{18d\sqrt[3]{-idx^3}} - \frac{be^{-ic}x\Gamma\left(\frac{1}{3}, idx^3\right)}{18d\sqrt[3]{idx^3}}$$

```
[Out] 1/4*a*x^4-1/3*b*x*cos(d*x^3+c)/d-1/18*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/d/(-
I*d*x^3)^(1/3)-1/18*b*x*GAMMA(1/3,I*d*x^3)/d/exp(I*c)/(I*d*x^3)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {14, 3466, 3437, 2239}

$$-\frac{be^{ic}x\text{Gamma}\left(\frac{1}{3}, -idx^3\right)}{18d\sqrt[3]{-idx^3}} - \frac{be^{-ic}x\text{Gamma}\left(\frac{1}{3}, idx^3\right)}{18d\sqrt[3]{idx^3}} + \frac{ax^4}{4} - \frac{bx\cos(c + dx^3)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^4)/4 - (b*x*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3
])/(18*d*((-I)*d*x^3)^(1/3)) - (b*x*Gamma[1/3, I*d*x^3])/(18*d*E^(I*c)*(I*d
*x^3)^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\sin\left(c+dx^3\right)\right)\,dx&=\int\left(ax^3+bx^3\sin\left(c+dx^3\right)\right)\,dx\\&=\frac{ax^4}{4}+b\int x^3\sin\left(c+dx^3\right)\,dx\\&=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}+\frac{b\int\cos\left(c+dx^3\right)\,dx}{3d}\\&=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}+\frac{b\int e^{-ic-idx^3}\,dx}{6d}+\frac{b\int e^{ic+idx^3}\,dx}{6d}\\&=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 124, normalized size = 1.17

$$\frac{dx^7\left(3\sqrt[3]{d^2x^6}\left(3adx^3-4b\cos\left(c+dx^3\right)\right)-2b\sqrt[3]{-idx^3}\,\Gamma\left(\frac{1}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)-2b\sqrt[3]{idx^3}\,\Gamma\left(\frac{1}{3},-idx^3\right)\left(\cos(c)+i\sin(c)\right)\right)}{36\left(d^2x^6\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (d*x^7*(3*(d^2*x^6)^(1/3)*(3*a*d*x^3 - 4*b*Cos[c + d*x^3]) - 2*b*((-I)*d*x^
3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 2*b*(I*d*x^3)^(1/3)*Gamm
a[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(36*(d^2*x^6)^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\sin\left(d\,x^3+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^3*(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.30, size = 110, normalized size = 1.04

$$\frac{1}{4}\,ax^4-\frac{\left(12\left(dx^3\right)^{\frac{1}{3}}x\cos\left(dx^3+c\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)x\right)b}{36\left(dx^3\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 - 1/36*(12*(d*x^3)^(1/3)*x*cos(d*x^3 + c) + (((sqrt(3) - I)*gamma
(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -I*d*x^3))*cos(c) + ((-I*sqrt(3)
- 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*d*x^3))*sin(c))*x)
*b/((d*x^3)^(1/3)*d)
```

Fricas [A]
time = 0.11, size = 68, normalized size = 0.64

$$\frac{9\,ad^2x^4 - 12\,bdx\cos\left(dx^3+c\right) + 2i\,b(i\,d)^{\frac{2}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{1}{3}, i\,dx^3\right) - 2i\,b(-i\,d)^{\frac{2}{3}}\,e^{(i\,c)}\Gamma\left(\frac{1}{3}, -i\,dx^3\right)}{36\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/36*(9*a*d^2*x^4 - 12*b*d*x*cos(d*x^3 + c) + 2*I*b*(I*d)^(2/3)*e^(-I*c)*ga
mma(1/3, I*d*x^3) - 2*I*b*(-I*d)^(2/3)*e^(I*c)*gamma(1/3, -I*d*x^3))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x**3*(a + b*sin(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*sin(c + d*x^3)),x)
```

```
[Out] int(x^3*(a + b*sin(c + d*x^3)), x)
```

3.66 $\int (a + b\sin(c + dx^3))\, dx$

Optimal. Leaf size=82

$$ax + \frac{ibe^{ic}x\Gamma\left(\frac{1}{3}, -idx^3\right)}{6\sqrt[3]{-idx^3}} - \frac{ibe^{-ic}x\Gamma\left(\frac{1}{3}, idx^3\right)}{6\sqrt[3]{idx^3}}$$

```
[Out] a*x+1/6*I*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/6*I*b*x*GAMMA
(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3436, 2239}

$$\frac{ibe^{ic}x\text{Gamma}\left(\frac{1}{3}, -idx^3\right)}{6\sqrt[3]{-idx^3}} - \frac{ibe^{-ic}x\text{Gamma}\left(\frac{1}{3}, idx^3\right)}{6\sqrt[3]{idx^3}} + ax$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x^3],x]
```

```
[Out] a*x + ((I/6)*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - ((I/6
)*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int (a + b\sin(c + dx^3))\, dx &= ax + b\int \sin(c + dx^3)\, dx \\
&= ax + \frac{1}{2}(ib)\int e^{-ic-idx^3}\, dx - \frac{1}{2}(ib)\int e^{ic+idx^3}\, dx \\
&= ax + \frac{ibe^{ic}x\Gamma\left(\frac{1}{3}, -idx^3\right)}{6\sqrt[3]{-idx^3}} - \frac{ibe^{-ic}x\Gamma\left(\frac{1}{3}, idx^3\right)}{6\sqrt[3]{idx^3}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 138, normalized size = 1.68

$$ax-\frac{1}{2}ib\cos(c)\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}+\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)+\frac{1}{2}b\left(-\frac{x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)\sin(c)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x^3],x]
```

```
[Out] a*x - (I/2)*b*Cos[c]*(-1/3*(x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) +
(x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3))) + (b*(-1/3*(x*Gamma[1/3, (-I)*
d*x^3])/((-I)*d*x^3)^(1/3) - (x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3)))*S
in[c])/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int a+b\sin\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(d*x^3+c),x)
```

```
[Out] int(a+b*sin(d*x^3+c),x)
```

Maxima [A]
time = 0.31, size = 85, normalized size = 1.04

$$\frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos(c)-\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\sin(c)\right)bx}{12\,\left(dx^3\right)^{\frac{1}{3}}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/12*(((-I*sqrt(3) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I
*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(
1/3, -I*d*x^3))*sin(c))*b*x/(d*x^3)^(1/3) + a*x
```

Fricas [A]
time = 0.10, size = 49, normalized size = 0.60

$$-\frac{b(i\,d)^{\frac{2}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)+b(-i\,d)^{\frac{2}{3}}\,e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-6\,adx}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(b*(I*d)^(2/3)*e^(-I*c)*gamma(1/3, I*d*x^3) + b*(-I*d)^(2/3)*e^(I*c)*g
amma(1/3, -I*d*x^3) - 6*a*d*x)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x**3+c),x)
[Out] Integral(a + b*sin(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="giac")
[Out] integrate(b*sin(d*x^3 + c) + a, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int a + b\,\sin\left(d\,x^3 + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(c + d*x^3),x)
[Out] int(a + b*sin(c + d*x^3), x)
```

3.67 $\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{a}{2x^2}-\frac{bde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{bde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{2x^2}$$

```
[Out] -1/2*a/x^2-1/4*b*d*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/4*b*d*
x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)-1/2*b*sin(d*x^3+c)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {14, 3468, 3437, 2239}

$$-\frac{be^{ic}dx\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{be^{-ic}dx\text{Gamma}\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*d*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/(4*((-I)*d*x^3)^(1/3))
- (b*d*x*Gamma[1/3, I*d*x^3])/(4*E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^
3])/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b\sin(c+dx^3)}{x^3}\right)dx \\
&= -\frac{a}{2x^2}+b\int \frac{\sin(c+dx^3)}{x^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{b\sin(c+dx^3)}{2x^2}+\frac{1}{2}(3bd)\int \cos\left(c+dx^3\right)\,dx \\
&= -\frac{a}{2x^2}-\frac{b\sin(c+dx^3)}{2x^2}+\frac{1}{4}(3bd)\int e^{-ic-idx^3}\,dx+\frac{1}{4}(3bd)\int e^{ic+idx^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{bde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{bde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin(c+dx^3)}{2x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 120, normalized size = 1.19

$$\frac{-ib(-idx^3)^{4/3}\Gamma\left(\frac{1}{3},idx^3\right)(\cos(c)-i\sin(c))+ib(idx^3)^{4/3}\Gamma\left(\frac{1}{3},-idx^3\right)(\cos(c)+i\sin(c))-2\sqrt[3]{d^2x^6}\left(a+b\sin\left(c+dx^3\right)\right)}{4x^2\sqrt[3]{d^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^3,x]
[Out] ((-I)*b*((-I)*d*x^3)^(4/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(I
*d*x^3)^(4/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(1/3
)*(a + b*Sin[c + d*x^3]))/(4*x^2*(d^2*x^6)^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(d\,x^3+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^3,x)
[Out] int((a+b*sin(d*x^3+c))/x^3,x)
```

Maxima [A]
time = 0.31, size = 90, normalized size = 0.89

$$\frac{(dx^3)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos(c)-\left(\left(i\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(-i\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\sin(c)\right)b}{12\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-2/3, I*d*x^3) + (sqrt(3) + I)*gam
ma(-2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) + 1)*gamma(-2/3, I*d*x^3) + (-I*sq
rt(3) + 1)*gamma(-2/3, -I*d*x^3))*sin(c))*b/x^2 - 1/2*a/x^2
```

Fricas [A]
time = 0.11, size = 66, normalized size = 0.65

$$\frac{i\,b(i\,d)^{\frac{2}{3}}\,x^2e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)-i\,b(-i\,d)^{\frac{2}{3}}\,x^2e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-2\,b\sin\left(dx^3+c\right)-2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(I*b*(I*d)^(2/3)*x^2*e^(-I*c)*gamma(1/3, I*d*x^3) - I*b*(-I*d)^(2/3)*x^
2*e^(I*c)*gamma(1/3, -I*d*x^3) - 2*b*sin(d*x^3 + c) - 2*a)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\sin\left(d\,x^3+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^3))/x^3, x)
```

3.68 $\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$

Optimal. Leaf size=126

$$-\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{3ibd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{5x^5}$$

```
[Out] -1/5*a/x^5-3/10*b*d*cos(d*x^3+c)/x^2-3/20*I*b*d^2*exp(I*c)*x*GAMMA(1/3,-I*d
*x^3)/(-I*d*x^3)^(1/3)+3/20*I*b*d^2*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)
^(1/3)-1/5*b*sin(d*x^3+c)/x^5
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {14, 3468, 3469, 3436, 2239}

$$-\frac{3ibe^{ic}d^2x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibe^{-ic}d^2x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{a}{5x^5}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*d*Cos[c + d*x^3])/(10*x^2) - (((3*I)/20)*b*d^2*E^(I*c)*x*
Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (((3*I)/20)*b*d^2*x*Gamma[1/3,
 I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^3])/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+dx^3\right)}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b\sin\left(c+dx^3\right)}{x^6}\right)\,dx \\
&= -\frac{a}{5x^5}+b\int \frac{\sin\left(c+dx^3\right)}{x^6}\,dx \\
&= -\frac{a}{5x^5}-\frac{b\sin\left(c+dx^3\right)}{5x^5}+\frac{1}{5}(3bd)\int \frac{\cos\left(c+dx^3\right)}{x^3}\,dx \\
&= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{1}{10}\left(9bd^2\right)\int \sin\left(c+dx^3\right)\,dx \\
&= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{1}{20}\left(9ibd^2\right)\int e^{-ic-idx^3}\,dx+\frac{1}{20}(9ib \\
&= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{3ibd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin
\end{aligned}
$$

Mathematica [A]
time = 0.31, size = 146, normalized size = 1.16

$$
\frac{3bd^2x^6\sqrt[3]{idx^3}\,\Gamma\left(\frac{1}{3},-idx^3\right)\left(-i\cos(c)+\sin(c)\right)+3bd^2x^6\sqrt[3]{-idx^3}\,\Gamma\left(\frac{1}{3},idx^3\right)\left(i\cos(c)+\sin(c)\right)-2\sqrt[3]{d^2x^6}\left(2a+3bdx^3\cos\left(c+dx^3\right)+2b\sin\left(c+dx^3\right)\right)}{20x^5\sqrt[3]{d^2x^6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^6,x]
```

```
[Out] (3*b*d^2*x^6*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
+ 3*b*d^2*x^6*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(I*Cos[c] + Sin[c]) -
2*(d^2*x^6)^(1/3)*(2*a + 3*b*d*x^3*Cos[c + d*x^3] + 2*b*Sin[c + d*x^3]))/(2
0*x^5*(d^2*x^6)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a + b\sin\left(d\,x^{3} + c\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^6,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^6,x)
```

Maxima [A]
time = 0.32, size = 91, normalized size = 0.72

$$-\frac{\left(dx^{3}\right)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3}, i\,dx^{3}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3}, -i\,dx^{3}\right)\right)\cos\left(c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{5}{3}, i\,dx^{3}\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{5}{3}, -i\,dx^{3}\right)\right)\sin\left(c\right)\right)bd}{12\,x^{2}}-\frac{a}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="maxima")
```

```
[Out] -1/12*(d*x^3)^(2/3)*(((-I*sqrt(3) - 1)*gamma(-5/3, I*d*x^3) + (I*sqrt(3) -
1)*gamma(-5/3, -I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(-5/3, I*d*x^3) + (s
qrt(3) + I)*gamma(-5/3, -I*d*x^3))*sin(c))*b*d/x^2 - 1/5*a/x^5
```

Fricas [A]
time = 0.11, size = 83, normalized size = 0.66

$$\frac{3\,b\left(i\,d\right)^{\frac{2}{3}}\,dx^{5}e^{\left(-i\,c\right)}\Gamma\left(\frac{1}{3}, i\,dx^{3}\right)+3\,b\left(-i\,d\right)^{\frac{2}{3}}\,dx^{5}e^{\left(i\,c\right)}\Gamma\left(\frac{1}{3}, -i\,dx^{3}\right)-6\,bdx^{3}\cos\left(dx^{3}+c\right)-4\,b\sin\left(dx^{3}+c\right)-4\,a}{20\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(3*b*(I*d)^(2/3)*d*x^5*e^(-I*c)*gamma(1/3, I*d*x^3) + 3*b*(-I*d)^(2/3)
*d*x^5*e^(I*c)*gamma(1/3, -I*d*x^3) - 6*b*d*x^3*cos(d*x^3 + c) - 4*b*sin(d*
x^3 + c) - 4*a)/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sin\left(c + dx^{3}\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**6,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(d\,x^3+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))/x^6,x)
[Out] int((a + b*sin(c + d*x^3))/x^6, x)
```

3.69 $\int x^5(a + b\sin(c + dx^3))^2\, dx$

Optimal. Leaf size=107

$$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2abx^3\cos(c+dx^3)}{3d}+\frac{2ab\sin(c+dx^3)}{3d^2}-\frac{b^2x^3\cos(c+dx^3)\sin(c+dx^3)}{6d}+\frac{b^2\sin^2(c+dx^3)}{12d^2}$$

```
[Out] 1/6*a^2*x^6+1/12*b^2*x^6-2/3*a*b*x^3*cos(d*x^3+c)/d+2/3*a*b*sin(d*x^3+c)/d^
2-1/6*b^2*x^3*cos(d*x^3+c)*sin(d*x^3+c)/d+1/12*b^2*sin(d*x^3+c)^2/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3460, 3398, 3377, 2717, 3391, 30}

$$\frac{a^2x^6}{6}+\frac{2ab\sin(c+dx^3)}{3d^2}-\frac{2abx^3\cos(c+dx^3)}{3d}+\frac{b^2\sin^2(c+dx^3)}{12d^2}-\frac{b^2x^3\sin(c+dx^3)\cos(c+dx^3)}{6d}+\frac{b^2x^6}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (a^2*x^6)/6 + (b^2*x^6)/12 - (2*a*b*x^3*Cos[c + d*x^3])/(3*d) + (2*a*b*Sin[
c + d*x^3])/(3*d^2) - (b^2*x^3*Cos[c + d*x^3]*Sin[c + d*x^3])/(6*d) + (b^2*
Sin[c + d*x^3]^2)/(12*d^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \frac{1}{3}\text{Subst}\left(\int x(a+b\sin(c+dx))^2\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(a^2x+2abx\sin(c+dx)+b^2x\sin^2(c+dx)\right)\,dx,x,x^3\right)\\
&= \frac{a^2x^6}{6}+\frac{1}{3}(2ab)\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^3\right)+\frac{1}{3}b^2\text{Subst}\left(\int x\sin^2(c+\right.\\
&= \frac{a^2x^6}{6}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^3\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}+\frac{b^2\sin^2\left(c+\right.}{12d^2}\\
&= \frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{b^2x^3\cos\left(c+dx^3\right)}{6d}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 92, normalized size = 0.86

$$\frac{4a^2d^2x^6+2b^2d^2x^6-16abdx^3\cos\left(c+dx^3\right)-b^2\cos\left(2\left(c+dx^3\right)\right)+16ab\sin\left(c+dx^3\right)-2b^2dx^3\sin\left(2\left(c+dx^3\right)\right)}{24d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (4*a^2*d^2*x^6 + 2*b^2*d^2*x^6 - 16*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c
 + d*x^3)] + 16*a*b*Sin[c + d*x^3] - 2*b^2*d*x^3*Sin[2*(c + d*x^3)])/(24*d^
2)
```

Maple [A]
time = 0.22, size = 137, normalized size = 1.28

method	result
risch	$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2ab\,x^3\cos\left(d\,x^3+c\right)}{3d}+\frac{2ab\sin\left(d\,x^3+c\right)}{3d^2}-\frac{b^2\cos\left(2d\,x^3+2c\right)}{24d^2}-\frac{b^2x^3\sin\left(2d\,x^3+2c\right)}{12d}$
default	$\frac{a^2x^6}{6}+\frac{b^2x^6}{6}-\frac{b^2\left(\frac{x^6}{6}+\frac{\frac{1}{6d^2}+\frac{x^3\tan\left(d\,x^3+c\right)}{3d}}{1+\tan^2\left(d\,x^3+c\right)}\right)}{2}-\frac{-\frac{8ab\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d^2}+\frac{4ab\,x^3}{3d}-\frac{4ab\,x^3\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d}}{2\left(1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}$
norman	$\frac{\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^6\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{6}+\frac{b^2}{12}\right)x^6\left(\tan^4\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)+\frac{b^2\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)}{3d^2}-\frac{b^2x^3\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+\frac{b^2x^3\left(\tan \cdots\right.}{\cdots}\cdots}{\cdots\left(1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x^6+1/6*b^2*x^6-1/2*b^2*(1/6*x^6+(1/6/d^2+1/3*x^3/d*tan(d*x^3+c))/(
1+tan(d*x^3+c)^2))-1/2*(-8/3/d^2*a*b*tan(1/2*d*x^3+1/2*c)+4/3/d*a*b*x^3-4/3
/d*a*b*x^3*tan(1/2*d*x^3+1/2*c)^2)/(1+tan(1/2*d*x^3+1/2*c)^2)
```

Maxima [A]
time = 0.29, size = 87, normalized size = 0.81

$$\frac{1}{6}\,a^2x^6-\frac{2\,(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right))ab}{3\,d^2}+\frac{(2\,d^2x^6-2\,dx^3\sin\left(2\,dx^3+2\,c\right)-\cos\left(2\,dx^3+2\,c\right))b^2}{24\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 - 2/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*a*b/d^2 + 1/24*(2
*d^2*x^6 - 2*d*x^3*sin(2*d*x^3 + 2*c) - cos(2*d*x^3 + 2*c))*b^2/d^2
```

Fricas [A]
time = 0.36, size = 84, normalized size = 0.79

$$\frac{(2\,a^2+b^2)d^2x^6-8\,abdx^3\cos\left(dx^3+c\right)-b^2\cos\left(dx^3+c\right)^2-2\,(b^2dx^3\cos\left(dx^3+c\right)-4\,ab)\sin\left(dx^3+c\right)}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*((2*a^2 + b^2)*d^2*x^6 - 8*a*b*d*x^3*cos(d*x^3 + c) - b^2*cos(d*x^3 +
c)^2 - 2*(b^2*d*x^3*cos(d*x^3 + c) - 4*a*b)*sin(d*x^3 + c))/d^2
```

Sympy [A]
time = 0.61, size = 143, normalized size = 1.34

$$\begin{cases} \frac{a^2x^6}{6} - \frac{2abx^3\cos(c+dx^3)}{3d} + \frac{2ab\sin(c+dx^3)}{3d^2} + \frac{b^2x^6\sin^2(c+dx^3)}{12} + \frac{b^2x^6\cos^2(c+dx^3)}{12} - \frac{b^2x^3\sin(c+dx^3)\cos(c+dx^3)}{6d} + \frac{b^2\sin^2(c+dx^3)}{12d^2} & \text{for } d \neq 0 \\ \frac{x^6(a+b\sin(c))^2}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 - 2*a*b*x**3*cos(c + d*x**3)/(3*d) + 2*a*b*sin(c + d
*x**3)/(3*d**2) + b**2*x**6*sin(c + d*x**3)**2/12 + b**2*x**6*cos(c + d*x**
3)**2/12 - b**2*x**3*sin(c + d*x**3)*cos(c + d*x**3)/(6*d) + b**2*sin(c + d
*x**3)**2/(12*d**2), Ne(d, 0)), (x**6*(a + b*sin(c))**2/6, True))
```

Giac [A]
time = 7.24, size = 165, normalized size = 1.54

$$\frac{4\,(dx^3+c)^2a^2 + 2\,(dx^3+c)^2b^2 - 16\,(dx^3+c)ab\cos(dx^3+c) - 2\,(dx^3+c)b^2\sin(2\,dx^3+2\,c) - b^2\cos(2\,dx^3+2\,c) + 16\,ab\sin(dx^3+c)}{24\,d^2} - \frac{4\,(dx^3+c)a^2c + (2\,dx^3+2\,c-\sin(2\,dx^3+2\,c))b^2c - 8\,abc\cos(dx^3+c)}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(4*(d*x^3 + c)^2*a^2 + 2*(d*x^3 + c)^2*b^2 - 16*(d*x^3 + c)*a*b*cos(d*
x^3 + c) - 2*(d*x^3 + c)*b^2*sin(2*d*x^3 + 2*c) - b^2*cos(2*d*x^3 + 2*c) +
16*a*b*sin(d*x^3 + c))/d^2 - 1/12*(4*(d*x^3 + c)*a^2*c + (2*d*x^3 + 2*c - s
in(2*d*x^3 + 2*c))*b^2*c - 8*a*b*c*cos(d*x^3 + c))/d^2
```

Mupad [B]
time = 0.27, size = 95, normalized size = 0.89

$$-\frac{b^2\cos\left(d\,x^3+c\right)^2 - 2\,a^2\,d^2\,x^6 - b^2\,d^2\,x^6 - 8\,a\,b\,\sin\left(d\,x^3+c\right) + 8\,a\,b\,d\,x^3\,\cos\left(d\,x^3+c\right) + 2\,b^2\,d\,x^3\,\cos\left(d\,x^3+c\right)\,\sin\left(d\,x^3+c\right)}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] -(b^2*cos(c + d*x^3)^2 - 2*a^2*d^2*x^6 - b^2*d^2*x^6 - 8*a*b*sin(c + d*x^3)
 + 8*a*b*d*x^3*cos(c + d*x^3) + 2*b^2*d*x^3*cos(c + d*x^3)*sin(c + d*x^3))/
(12*d^2)
```

3.70 $\int x^2(a+b\sin(c+dx^3))^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{2ab\cos\left(c+dx^3\right)}{3d}-\frac{b^2\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}$$

```
[Out] 1/6*(2*a^2+b^2)*x^3-2/3*a*b*cos(d*x^3+c)/d-1/6*b^2*cos(d*x^3+c)*sin(d*x^3+c
)/d
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {3460, 2723}

$$\frac{1}{6}x^3\left(2a^2+b^2\right)-\frac{2ab\cos\left(c+dx^3\right)}{3d}-\frac{b^2\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (2*a*b*Cos[c + d*x^3])/(3*d) - (b^2*Cos[c + d*x^3]*
Sin[c + d*x^3])/(6*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx&=\frac{1}{3}\text{Subst}\left(\int(a+b\sin(c+dx))^2\,dx,x,x^3\right)\\&=\frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{2ab\cos\left(c+dx^3\right)}{3d}-\frac{b^2\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 52, normalized size = 0.87

$$-\frac{-2(2a^2+b^2)\,(c+dx^3)+8ab\cos\left(c+dx^3\right)+b^2\sin\left(2(c+dx^3)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] -1/12*(-2*(2*a^2 + b^2)*(c + d*x^3) + 8*a*b*Cos[c + d*x^3] + b^2*Sin[2*(c +
 d*x^3)])/d
```

Maple [A]
time = 0.06, size = 62, normalized size = 1.03

method	result
risch	$\frac{x^3a^2}{3}+\frac{x^3b^2}{6}-\frac{2ab\cos\left(d\,x^3+c\right)}{3d}-\frac{b^2\sin\left(2d\,x^3+2c\right)}{12d}$
derivativedivides	$\frac{b^2\left(-\frac{\cos\left(d\,x^3+c\right)\sin\left(d\,x^3+c\right)}{2}+\frac{d\,x^3}{2}+\frac{c}{2}\right)-2ab\cos\left(d\,x^3+c\right)+a^2\left(d\,x^3+c\right)}{3d}$
default	$\frac{b^2\left(-\frac{\cos\left(d\,x^3+c\right)\sin\left(d\,x^3+c\right)}{2}+\frac{d\,x^3}{2}+\frac{c}{2}\right)-2ab\cos\left(d\,x^3+c\right)+a^2\left(d\,x^3+c\right)}{3d}$
norman	$\frac{\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^3+\left(\frac{a^2}{3}+\frac{b^2}{6}\right)x^3\left(\tan^4\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)+\left(\frac{2a^2}{3}+\frac{b^2}{3}\right)x^3\left(\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)-\frac{4ab}{3d}-\frac{b^2\tan\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3d}+\frac{b^2\left(\tan^3\left(\frac{d\,x^3}{2}+\cdots\right)\right.}{3d}\cdots}{\left(1+\tan^2\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(b^2*(-1/2*cos(d*x^3+c)*sin(d*x^3+c)+1/2*d*x^3+1/2*c)-2*a*b*cos(d*x^3
+c)+a^2*(d*x^3+c))
```

Maxima [A]
time = 0.35, size = 52, normalized size = 0.87

$$\frac{1}{3}\,a^2x^3+\frac{\left(2\,dx^3-\sin\left(2\,dx^3+2\,c\right)\right)b^2}{12\,d}-\frac{2\,ab\cos\left(dx^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/12*(2*d*x^3 - sin(2*d*x^3 + 2*c))*b^2/d - 2/3*a*b*cos(d*x^3
 + c)/d
```

Fricas [A]
time = 0.39, size = 53, normalized size = 0.88

$$\frac{(2\,a^2+b^2)dx^3-b^2\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-4\,ab\cos\left(dx^3+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2 + b^2)*d*x^3 - b^2*cos(d*x^3 + c)*sin(d*x^3 + c) - 4*a*b*cos(d*
x^3 + c))/d
```

Sympy [A]
time = 0.18, size = 99, normalized size = 1.65

$$\begin{cases}\frac{a^2x^3}{3}-\frac{2ab\cos\left(c+dx^3\right)}{3d}+\frac{b^2x^3\sin^2\left(c+dx^3\right)}{6}+\frac{b^2x^3\cos^2\left(c+dx^3\right)}{6}-\frac{b^2\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d} & \text{for } d\neq 0\\ \frac{x^3(a+b\sin\left(c\right))^2}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Piecewise((a**2*x**3/3 - 2*a*b*cos(c + d*x**3)/(3*d) + b**2*x**3*sin(c + d*
x**3)**2/6 + b**2*x**3*cos(c + d*x**3)**2/6 - b**2*sin(c + d*x**3)*cos(c +
d*x**3)/(6*d), Ne(d, 0)), (x**3*(a + b*sin(c))**2/3, True))
```

Giac [A]
time = 4.20, size = 57, normalized size = 0.95

$$\frac{4\,(dx^3+c)a^2+(2\,dx^3+2\,c-\sin\left(2\,dx^3+2\,c\right))b^2-8\,ab\cos\left(dx^3+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(4*(d*x^3 + c)*a^2 + (2*d*x^3 + 2*c - sin(2*d*x^3 + 2*c))*b^2 - 8*a*b*
cos(d*x^3 + c))/d
```

Mupad [B]
time = 4.72, size = 51, normalized size = 0.85

$$\frac{a^2\,x^3}{3}+\frac{b^2\,x^3}{6}-\frac{b^2\,\sin\left(2\,d\,x^3+2\,c\right)}{12\,d}-\frac{2\,a\,b\,\cos\left(d\,x^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3)/6 - (b^2*sin(2*c + 2*d*x^3))/(12*d) - (2*a*b*cos(c
+ d*x^3))/(3*d)
```

3.71 $\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{6}b^2\cos(2c)\text{Ci}(2dx^3)+\frac{1}{2}(2a^2+b^2)\log(x)+\frac{2}{3}ab\text{Ci}(dx^3)\sin(c)+\frac{2}{3}ab\cos(c)\text{Si}(dx^3)+\frac{1}{6}b^2\sin(2c)\text{Si}(2dx^3)$$

```
[Out] -1/6*b^2*Ci(2*d*x^3)*cos(2*c)+1/2*(2*a^2+b^2)*ln(x)+2/3*a*b*cos(c)*Si(d*x^3
)+2/3*a*b*Ci(d*x^3)*sin(c)+1/6*b^2*Si(2*d*x^3)*sin(2*c)
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3484, 6, 3459, 3457, 3456, 3458}

$$\frac{1}{2}(2a^2+b^2)\log(x)+\frac{2}{3}ab\sin(c)\text{CosIntegral}(dx^3)+\frac{2}{3}ab\cos(c)\text{Si}(dx^3)-\frac{1}{6}b^2\cos(2c)\text{CosIntegral}(2dx^3)+\frac{1}{6}b^2\sin(2c)\text{Si}(2dx^3)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x,x]
```

```
[Out] -1/6*(b^2*Cos[2*c]*CosIntegral[2*d*x^3]) + ((2*a^2 + b^2)*Log[x])/2 + (2*a*
b*CosIntegral[d*x^3]*Sin[c])/3 + (2*a*b*Cos[c]*SinIntegral[d*x^3])/3 + (b^2
*Sin[2*c]*SinIntegral[2*d*x^3])/6
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x}\,dx &= \int \left(\frac{a^2}{x}+\frac{b^2}{2x}-\frac{b^2\cos(2c+2dx^3)}{2x}+\frac{2ab\sin(c+dx^3)}{x}\right)dx\\ &= \int \left(\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\cos(2c+2dx^3)}{2x}+\frac{2ab\sin(c+dx^3)}{x}\right)dx\\ &= \frac{1}{2}(2a^2+b^2)\log(x)+(2ab)\int \frac{\sin(c+dx^3)}{x}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x}\,dx\\ &= \frac{1}{2}(2a^2+b^2)\log(x)+(2ab\cos(c))\int \frac{\sin(dx^3)}{x}\,dx-\frac{1}{2}(b^2\cos(2c))\int \frac{\cos(2dx^3}{x}\\ &= -\frac{1}{6}b^2\cos(2c)\text{Ci}(2dx^3)+\frac{1}{2}(2a^2+b^2)\log(x)+\frac{2}{3}ab\text{Ci}(dx^3)\sin(c)+\frac{2}{3}ab\cos(c\end{aligned}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.89

$$\frac{1}{2}(2a^2+b^2)\log(x)-\frac{1}{6}b(b\cos(2c)\text{Ci}(2dx^3)-4a\text{Ci}(dx^3)\sin(c)-4a\cos(c)\text{Si}(dx^3)-b\sin(2c)\text{Si}(2dx^3))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x,x]
```

```
[Out] ((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^3] - 4*a*CosInt
egral[d*x^3]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^3] - b*Sin[2*c]*SinIntegra
l[2*d*x^3]))/6
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.43, size = 108, normalized size = 1.35

$$-\frac{1}{3}\left(\left(i\operatorname{Ei}\left(i\,dx^3\right)-i\operatorname{Ei}\left(-i\,dx^3\right)\right)\cos\left(c\right)-\left(\operatorname{Ei}\left(i\,dx^3\right)+\operatorname{Ei}\left(-i\,dx^3\right)\right)\sin\left(c\right)\right)ab-\frac{1}{12}\left(\left(\operatorname{Ei}\left(2i\,dx^3\right)+\operatorname{Ei}\left(-2i\,dx^3\right)\right)\cos\left(2\,c\right)-\left(-i\operatorname{Ei}\left(2i\,dx^3\right)+i\operatorname{Ei}\left(-2i\,dx^3\right)\right)\sin\left(2\,c\right)-6\,\log\left(x\right)\right)b^2+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3)
)*sin(c))*a*b - 1/12*((Ei(2*I*d*x^3) + Ei(-2*I*d*x^3))*cos(2*c) - (-I*Ei(2*
I*d*x^3) + I*Ei(-2*I*d*x^3))*sin(2*c) - 6*log(x))*b^2 + a^2*log(x)
```

Fricas [A]
time = 0.37, size = 95, normalized size = 1.19

$$\frac{1}{6}\,b^2\sin\left(2\,c\right)\operatorname{Si}\left(2\,dx^3\right)+\frac{2}{3}\,ab\cos\left(c\right)\operatorname{Si}\left(dx^3\right)-\frac{1}{12}\left(b^2\operatorname{Ci}\left(2\,dx^3\right)+b^2\operatorname{Ci}\left(-2\,dx^3\right)\right)\cos\left(2\,c\right)+\frac{1}{2}\left(2\,a^2+b^2\right)\log\left(x\right)+\frac{1}{3}\left(ab\operatorname{Ci}\left(dx^3\right)+ab\operatorname{Ci}\left(-dx^3\right)\right)\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*sin(2*c)*sin_integral(2*d*x^3) + 2/3*a*b*cos(c)*sin_integral(d*x^3)
 - 1/12*(b^2*cos_integral(2*d*x^3) + b^2*cos_integral(-2*d*x^3))*cos(2*c) +
 1/2*(2*a^2 + b^2)*log(x) + 1/3*(a*b*cos_integral(d*x^3) + a*b*cos_integral
(-d*x^3))*sin(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x, x)
```

Giac [A]
time = 4.16, size = 79, normalized size = 0.99

$$-\frac{1}{6}\,b^2\cos\left(2\,c\right)\operatorname{Ci}\left(2\,dx^3\right)+\frac{2}{3}\,ab\operatorname{Ci}\left(dx^3\right)\sin\left(c\right)+\frac{2}{3}\,ab\cos\left(c\right)\operatorname{Si}\left(dx^3\right)-\frac{1}{6}\,b^2\sin\left(2\,c\right)\operatorname{Si}\left(-2\,dx^3\right)+\frac{1}{3}\,a^2\log\left(dx^3\right)+\frac{1}{6}\,b^2\log\left(dx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="giac")
```

```
[Out] -1/6*b^2*cos(2*c)*cos_integral(2*d*x^3) + 2/3*a*b*cos_integral(d*x^3)*sin(c
) + 2/3*a*b*cos(c)*sin_integral(d*x^3) - 1/6*b^2*sin(2*c)*sin_integral(-2*d
*x^3) + 1/3*a^2*log(d*x^3) + 1/6*b^2*log(d*x^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2/x,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x, x)
```

3.72 $\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$

Optimal. Leaf size=122

$$-\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2(c+dx^3)\right)}{6x^3}+\frac{2}{3}abd\cos(c)\mathrm{Ci}\left(dx^3\right)+\frac{1}{3}b^2d\mathrm{Ci}\left(2dx^3\right)\sin(2c)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}-\frac{2}{3}abd\sin(c)\mathrm{S}$$

```
[Out] 1/6*(-2*a^2-b^2)/x^3+2/3*a*b*d*Ci(d*x^3)*cos(c)+1/6*b^2*cos(2*d*x^3+2*c)/x^
3+1/3*b^2*d*cos(2*c)*Si(2*d*x^3)-2/3*a*b*d*Si(d*x^3)*sin(c)+1/3*b^2*d*Ci(2*
d*x^3)*sin(2*c)-2/3*a*b*sin(d*x^3+c)/x^3
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3484, 6, 3461, 3378, 3384, 3380, 3383, 3460}

$$-\frac{2a^2+b^2}{6x^3}+\frac{2}{3}abd\cos(c)\mathrm{CosIntegral}\left(dx^3\right)-\frac{2}{3}abd\sin(c)\mathrm{Si}\left(dx^3\right)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}b^2d\sin(2c)\mathrm{CosIntegral}\left(2dx^3\right)+\frac{1}{3}b^2d\cos(2c)\mathrm{Si}\left(2dx^3\right)+\frac{b^2\cos\left(2(c+dx^3)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

```
[Out] -1/6*(2*a^2 + b^2)/x^3 + (b^2*Cos[2*(c + d*x^3)])/(6*x^3) + (2*a*b*d*Cos[c]
*CosIntegral[d*x^3])/3 + (b^2*d*CosIntegral[2*d*x^3]*Sin[2*c])/3 - (2*a*b*S
in[c + d*x^3])/(3*x^3) - (2*a*b*d*Sin[c]*SinIntegral[d*x^3])/3 + (b^2*d*Cos
[2*c]*SinIntegral[2*d*x^3])/3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx &= \int \left(\frac{a^2}{x^4}+\frac{b^2}{2x^4}-\frac{b^2\cos(2c+2dx^3)}{2x^4}+\frac{2ab\sin(c+dx^3)}{x^4}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^4}-\frac{b^2\cos(2c+2dx^3)}{2x^4}+\frac{2ab\sin(c+dx^3)}{x^4}\right)dx \\
&= -\frac{2a^2+b^2}{6x^3}+(2ab)\int \frac{\sin(c+dx^3)}{x^4}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x^4}\,dx \\
&= -\frac{2a^2+b^2}{6x^3}+\frac{1}{3}(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^3\right)-\frac{1}{6}b^2\text{Subst}\left(\int \frac{\cos(2c+
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos(2(c+dx^3))}{6x^3}-\frac{2ab\sin(c+dx^3)}{3x^3}+\frac{1}{3}(2abd)\text{Subst}\left(\int \frac{\cos(c
\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos(2(c+dx^3))}{6x^3}-\frac{2ab\sin(c+dx^3)}{3x^3}+\frac{1}{3}(2abd\cos(c))\text{Subst}\Bigg(\int
\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos(2(c+dx^3))}{6x^3}+\frac{2}{3}abd\cos(c)\text{Ci}(dx^3)+\frac{1}{3}b^2d\text{Ci}(2dx^3)\sin(2c
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 116, normalized size = 0.95

$$
\frac{-2a^2-b^2+b^2\cos(2(c+dx^3))+4abdx^3\cos(c)\text{Ci}(dx^3)+2b^2dx^3\text{Ci}(2dx^3)\sin(2c)-4ab\sin(c+dx^3)-4abdx^3\sin(c)\text{Si}(dx^3)+2b^2dx^3\cos(2c)\text{Si}(2dx^3)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^3)] + 4*a*b*d*x^3*Cos[c]*CosIntegral[d*x
^3] + 2*b^2*d*x^3*CosIntegral[2*d*x^3]*Sin[2*c] - 4*a*b*Sin[c + d*x^3] - 4*
a*b*d*x^3*Sin[c]*SinIntegral[d*x^3] + 2*b^2*d*x^3*Cos[2*c]*SinIntegral[2*d*
x^3])/(6*x^3)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$
\int \frac{(a+b\sin(d\,x^3+c))^2}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^4,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^4,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.41, size = 124, normalized size = 1.02

$$
\frac{1}{3}\left(\left(\Gamma(-1,i\,dx^3)+\Gamma(-1,-i\,dx^3)\right)\cos(c)-\left(i\,\Gamma(-1,i\,dx^3)-i\,\Gamma(-1,-i\,dx^3)\right)\sin(c)\right)abd+\frac{(((i\,\Gamma(-1,2i\,dx^3)-i\,\Gamma(-1,-2i\,dx^3))\cos(2\,c)+(\Gamma(-1,2i\,dx^3)+\Gamma(-1,-2i\,dx^3))\sin(2\,c))dx^3-1)b^2}{6\,x^3}-\frac{a^2}{3\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*x
^3) - I*gamma(-1, -I*d*x^3))*sin(c))*a*b*d + 1/6*(((I*gamma(-1, 2*I*d*x^3)
- I*gamma(-1, -2*I*d*x^3))*cos(2*c) + (gamma(-1, 2*I*d*x^3) + gamma(-1, -2*
I*d*x^3))*sin(2*c))*d*x^3 - 1)*b^2/x^3 - 1/3*a^2/x^3
```

Fricas [A]
time = 0.41, size = 147, normalized size = 1.20

$$\frac{2\,b^2dx^3\cos\left(2\,c\right)\operatorname{Si}\left(2\,dx^3\right)-4\,abdx^3\sin\left(c\right)\operatorname{Si}\left(dx^3\right)+2\,b^2\cos\left(dx^3+c\right)^2-4\,ab\sin\left(dx^3+c\right)-2\,a^2-2\,b^2+2\left(abdx^3\operatorname{Ci}\left(dx^3\right)+abdx^3\operatorname{Ci}\left(-dx^3\right)\right)\cos\left(c\right)+\left(b^2dx^3\operatorname{Ci}\left(2\,dx^3\right)+b^2dx^3\operatorname{Ci}\left(-2\,dx^3\right)\right)\sin\left(2\,c\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d*x^3*cos(2*c)*sin_integral(2*d*x^3) - 4*a*b*d*x^3*sin(c)*sin_in
tegral(d*x^3) + 2*b^2*cos(d*x^3 + c)^2 - 4*a*b*sin(d*x^3 + c) - 2*a^2 - 2*b
^2 + 2*(a*b*d*x^3*cos_integral(d*x^3) + a*b*d*x^3*cos_integral(-d*x^3))*cos
(c) + (b^2*d*x^3*cos_integral(2*d*x^3) + b^2*d*x^3*cos_integral(-2*d*x^3))*
sin(2*c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(109) = 218.
time = 5.30, size = 226, normalized size = 1.85

$$\frac{4\left(dx^3+c\right)abd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-4\,abcd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)+2\left(dx^3+c\right)b^2d^2\operatorname{Ci}\left(2\,dx^3\right)\sin\left(2\,c\right)-2\,b^2cd^2\operatorname{Ci}\left(2\,dx^3\right)\sin\left(2\,c\right)-4\left(dx^3+c\right)abd^2\sin\left(c\right)\operatorname{Si}\left(dx^3\right)+4\,abcd^2\sin\left(c\right)\operatorname{Si}\left(dx^3\right)-2\left(dx^3+c\right)b^2d^2\cos\left(2\,c\right)\operatorname{Si}\left(-2\,dx^3\right)+2\,b^2cd^2\cos\left(2\,c\right)\operatorname{Si}\left(-2\,dx^3\right)+b^2d^2\cos\left(2\,dx^3+2\,c\right)-4\,abd^2\sin\left(dx^3+c\right)-2\,a^2d^2-b^2d^2}{6\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(4*(d*x^3 + c)*a*b*d^2*cos(c)*cos_integral(d*x^3) - 4*a*b*c*d^2*cos(c)*
cos_integral(d*x^3) + 2*(d*x^3 + c)*b^2*d^2*cos_integral(2*d*x^3)*sin(2*c)
- 2*b^2*c*d^2*cos_integral(2*d*x^3)*sin(2*c) - 4*(d*x^3 + c)*a*b*d^2*sin(c)
*sin_integral(d*x^3) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^3) - 2*(d*x^3 +
```

```
c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^3) + 2*b^2*c*d^2*cos(2*c)*sin_integ
ral(-2*d*x^3) + b^2*d^2*cos(2*d*x^3 + 2*c) - 4*a*b*d^2*sin(d*x^3 + c) - 2*a
^2*d^2 - b^2*d^2)/(d^2*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2/x^4,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x^4, x)
```

3.73 $\int x^4(a+b\sin(c+dx^3))^2\,dx$

Optimal. Leaf size=249

$$\frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}+\frac{ib^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}-$$

```
[Out] 1/10*(2*a^2+b^2)*x^5-2/3*a*b*x^2*cos(d*x^3+c)/d-2/9*a*b*exp(I*c)*x^2*GAMMA(
2/3,-I*d*x^3)/d/(-I*d*x^3)^(2/3)-2/9*a*b*x^2*GAMMA(2/3,I*d*x^3)/d/exp(I*c)/
(I*d*x^3)^(2/3)+1/72*I*b^2*exp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/d/(
-I*d*x^3)^(2/3)-1/72*I*b^2*x^2*GAMMA(2/3,2*I*d*x^3)*2^(1/3)/d/exp(2*I*c)/(I
*d*x^3)^(2/3)-1/12*b^2*x^2*sin(2*d*x^3+2*c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.389$, Rules used = {3484, 6, 3467, 3470, 2250, 3466, 3471}

$$-\frac{2abe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}+\frac{ib^2e^{2ic}x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}x^2\text{Gamma}\left(\frac{2}{3},2idx^3\right)}{36\ 2^{2/3}d\left(idx^3\right)^{2/3}}+\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^5)/10 - (2*a*b*x^2*Cos[c + d*x^3])/(3*d) - (2*a*b*E^(I*c)*
x^2*Gamma[2/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (2*a*b*x^2*Gamma[2/3
, I*d*x^3])/(9*d*E^(I*c)*(I*d*x^3)^(2/3)) + ((I/36)*b^2*E^((2*I)*c)*x^2*Gam
ma[2/3, (-2*I)*d*x^3])/(2^(2/3)*d*((-I)*d*x^3)^(2/3)) - ((I/36)*b^2*x^2*Gam
ma[2/3, (2*I)*d*x^3])/(2^(2/3)*d*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (b^2*x^2*Si
n[2*c + 2*d*x^3])/(12*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int\left(a^2x^4+\frac{b^2x^4}{2}-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+2abx^4\sin\left(c+dx^3\right)\right)dx\\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x^4-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+2abx^4\sin\left(c+dx^3\right)\right)dx\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5+(2ab)\int x^4\sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int x^4\cos\left(2c+2dx^3\right)\,
\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}+\frac{(4ab)\int x\,\mathrm{co}
\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}+\frac{(2ab)\int e^{-i}}{3d}
\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2}{9d\,(id}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 339, normalized size = 1.36

$$\frac{dx^8\left(72a^2dx^3(d^2x^6)^{2/3}+36b^2dx^3(d^2x^6)^{2/3}-240ab(d^2x^6)^{2/3}\cos\left(c+dx^3\right)+5i\sqrt[3]{2}\,b^2(idx^3)^{2/3}\cos(2c)\Gamma\left(\frac{2}{3},-2idx^3\right)-5i\sqrt[3]{2}\,b^2(-idx^3)^{2/3}\cos(2c)\Gamma\left(\frac{2}{3},2idx^3\right)-80ab(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c))-80ab(idx^3)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c))-5\sqrt[3]{2}\,b^2(idx^3)^{2/3}\Gamma\left(\frac{2}{3},-2idx^3\right)\sin(2c)-5\sqrt[3]{2}\,b^2(-idx^3)^{2/3}\Gamma\left(\frac{2}{3},2idx^3\right)\sin(2c)-30b^2(d^2x^6)^{2/3}\sin\left(2(c+dx^3)\right)\right)}{360\left(d^2x^6\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (d*x^8*(72*a^2*d*x^3*(d^2*x^6)^(2/3) + 36*b^2*d*x^3*(d^2*x^6)^(2/3) - 240*a
*b*(d^2*x^6)^(2/3)*Cos[c + d*x^3] + (5*I)*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Cos[2
*c]*Gamma[2/3, (-2*I)*d*x^3] - (5*I)*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Cos[2*c
]*Gamma[2/3, (2*I)*d*x^3] - 80*a*b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(
Cos[c] - I*Sin[c]) - 80*a*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c]
+ I*Sin[c]) - 5*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*
c] - 5*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 30
*b^2*(d^2*x^6)^(2/3)*Sin[2*(c + d*x^3)]))/(360*(d^2*x^6)^(5/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int x^{4}\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.36, size = 234, normalized size = 0.94

$$\frac{1}{5}\,a^2x^5-\frac{\left(6\,dx^3\cos\left(dx^3+c\right)-\left(dx^3\right)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)\right)ab}{9\,d^2x}+\frac{\left(72\,d^2x^6-60\,dx^3\sin\left(2\,dx^3+2\,c\right)-5\cdot2^{\frac{1}{3}}\left(dx^3\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},2i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)+\left(\left(-i\sqrt{3}+1\right)\Gamma\left(\frac{2}{3},2i\,dx^3\right)+\left(i\sqrt{3}+1\right)\Gamma\left(\frac{2}{3},-2i\,dx^3\right)\right)\sin\left(2\,c\right)\right)\right)b^2}{720\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - 1/9*(6*d*x^3*cos(d*x^3 + c) - (d*x^3)^(1/3)*(((I*sqrt(3) - 1)
*gamma(2/3, I*d*x^3) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*cos(c) + ((sq
rt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt(3) - I)*gamma(2/3, -I*d*x^3))*sin(c)
))*a*b/(d^2*x) + 1/720*(72*d^2*x^6 - 60*d*x^3*sin(2*d*x^3 + 2*c) - 5*2^(1/3
)*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, 2*I*d*x^3) + (sqrt(3) - I)*gamma
(2/3, -2*I*d*x^3))*cos(2*c) + ((-I*sqrt(3) + 1)*gamma(2/3, 2*I*d*x^3) + (I*
sqrt(3) + 1)*gamma(2/3, -2*I*d*x^3))*sin(2*c)))*b^2/(d^2*x)
```

Fricas [A]
time = 0.14, size = 150, normalized size = 0.60

$$\frac{36\left(2\,a^2+b^2\right)d^2x^5-60\,b^2dx^2\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-240\,abdx^2\cos\left(dx^3+c\right)-5\,b^2(2i\,d)^{\frac{1}{3}}\,e^{(-2i\,c)}\Gamma\left(\frac{2}{3},2i\,dx^3\right)+80i\,ab(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)-80i\,ab(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)-5\,b^2(-2i\,d)^{\frac{1}{3}}\,e^{(2i\,c)}\Gamma\left(\frac{2}{3},-2i\,dx^3\right)}{360\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/360*(36*(2*a^2 + b^2)*d^2*x^5 - 60*b^2*d*x^2*cos(d*x^3 + c)*sin(d*x^3 + c
) - 240*a*b*d*x^2*cos(d*x^3 + c) - 5*b^2*(2*I*d)^(1/3)*e^(-2*I*c)*gamma(2/3
, 2*I*d*x^3) + 80*I*a*b*(I*d)^(1/3)*e^(-I*c)*gamma(2/3, I*d*x^3) - 80*I*a*b
*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -I*d*x^3) - 5*b^2*(-2*I*d)^(1/3)*e^(2*I*c)
*gamma(2/3, -2*I*d*x^3))/d^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \sin \left(c + dx^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**3))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \, \left(a + b \, \sin \left(d \, x^3 + c\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x^4*(a + b*sin(c + d*x^3))^2, x)
```

3.74 $\int x(a+b\sin(c+dx^3))^2\,dx$

Optimal. Leaf size=193

$$\frac{1}{4}\left(2a^2+b^2\right)x^2+\frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}}+\frac{b^2e^{-2ic}x^2\Gamma\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}$$

```
[Out] 1/4*(2*a^2+b^2)*x^2+1/3*I*a*b*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(
2/3)-1/3*I*a*b*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)+1/24*b^2*exp
(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(2/3)+1/24*b^2*x^2*GAM
MA(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3)
```

Rubi [A]
time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3484, 6, 3471, 2250, 3470}

$$\frac{iabe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}}+\frac{b^2e^{-2ic}x^2\text{Gamma}\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}+\frac{1}{4}x^2\left(2a^2+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^2)/4 + ((I/3)*a*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I
)*d*x^3)^(2/3) - ((I/3)*a*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/
3)) + (b^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(12*2^(2/3)*((-I)*d*x^
3)^(2/3)) + (b^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(12*2^(2/3)*E^((2*I)*c)*(I*d*
x^3)^(2/3))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int\left(a^2x+\frac{b^2x}{2}-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+2abx\sin\left(c+dx^3\right)\right)dx\\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+2abx\sin\left(c+dx^3\right)\right)dx\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+(2ab)\int x\sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int x\cos\left(2c+2dx^3\right)\,dx\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+(iab)\int e^{-ic-idx^3}x\,dx-(iab)\int e^{ic+idx^3}x\,dx-\frac{1}{4}b^2\int e^{-2ic-}\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+\frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},\right.}{12\ 2^{2/3}\left(-\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 283, normalized size = 1.47

$$
\frac{x^2\left(12a^2\left(d^2x^6\right)^{2/3}+6b^2\left(d^2x^6\right)^{2/3}+\sqrt[3]{2}\,b^2\left(idx^3\right)^{2/3}\cos(2c)\Gamma\left(\frac{2}{3},-2idx^3\right)+\sqrt[3]{2}\,b^2\left(-idx^3\right)^{2/3}\cos(2c)\Gamma\left(\frac{2}{3},2idx^3\right)-8iab\left(-idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)+8iab\left(idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},-idx^3\right)\left(\cos(c)+i\sin(c)\right)+i\sqrt[3]{2}\,b^2\left(idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},-2idx^3\right)\sin(2c)-i\sqrt[3]{2}\,b^2\left(-idx^3\right)^{2/3}\Gamma\left(\frac{2}{3},2idx^3\right)\sin(2c)\right)}{24\left(d^2x^6\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (x^2*(12*a^2*(d^2*x^6)^(2/3) + 6*b^2*(d^2*x^6)^(2/3) + 2^(1/3)*b^2*(I*d*x^3
)^(2/3)*Cos[2*c]*Gamma[2/3, (-2*I)*d*x^3] + 2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*
Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] - (8*I)*a*b*((-I)*d*x^3)^(2/3)*Gamma[2/3,
I*d*x^3]*(Cos[c] - I*Sin[c]) + (8*I)*a*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*
x^3]*(Cos[c] + I*Sin[c]) + I*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Gamma[2/3, (-2*I)*
d*x^3]*Sin[2*c] - I*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Gamma[2/3, (2*I)*d*x^3]*
Sin[2*c]))/(24*(d^2*x^6)^(2/3))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int x \left(a + b \sin \left(d\,x^{3}+c\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.35, size = 199, normalized size = 1.03

$$\frac{1}{2}\,a^{2}x^{2} - \frac{(dx^{3})^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3}, i\,dx^{3}\right) + \left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3}, -i\,dx^{3}\right)\right)\cos\left(c\right) - \left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3}, i\,dx^{3}\right) + \left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3}, -i\,dx^{3}\right)\right)\sin\left(c\right)\right)ab}{6\,dx} + \frac{\left(12\,dx^{3} - 2^{\frac{1}{3}}(dx^{3})^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3}, 2i\,dx^{3}\right) + \left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3}, -2i\,dx^{3}\right)\right)\cos\left(2\,c\right) + \left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3}, 2i\,dx^{3}\right) + \left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3}, -2i\,dx^{3}\right)\right)\sin\left(2\,c\right)\right)\right)b^{2}}{48\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 - 1/6*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*d*x^3) + (sqrt
(3) - I)*gamma(2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(2/3, I*d*x^3
) + (-I*sqrt(3) - 1)*gamma(2/3, -I*d*x^3))*sin(c))*a*b/(d*x) + 1/48*(12*d*x
^3 - 2^(1/3)*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(2/3, 2*I*d*x^3) + (-I*sq
rt(3) - 1)*gamma(2/3, -2*I*d*x^3))*cos(2*c) + ((sqrt(3) + I)*gamma(2/3, 2*I
*d*x^3) + (sqrt(3) - I)*gamma(2/3, -2*I*d*x^3))*sin(2*c)))*b^2/(d*x)
```

Fricas [A]
time = 0.14, size = 107, normalized size = 0.55

$$\frac{-i\,b^{2}(2i\,d)^{\frac{1}{3}}\,e^{(-2i\,c)}\Gamma\left(\frac{2}{3}, 2i\,dx^{3}\right) - 8\,ab(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3}, i\,dx^{3}\right) - 8\,ab(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3}, -i\,dx^{3}\right) + i\,b^{2}(-2i\,d)^{\frac{1}{3}}\,e^{(2i\,c)}\Gamma\left(\frac{2}{3}, -2i\,dx^{3}\right) + 6\,(2\,a^{2}+b^{2})dx^{2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*b^2*(2*I*d)^(1/3)*e^(-2*I*c)*gamma(2/3, 2*I*d*x^3) - 8*a*b*(I*d)^(
1/3)*e^(-I*c)*gamma(2/3, I*d*x^3) - 8*a*b*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -
I*d*x^3) + I*b^2*(-2*I*d)^(1/3)*e^(2*I*c)*gamma(2/3, -2*I*d*x^3) + 6*(2*a^2
 + b^2)*d*x^2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \sin \left(c + dx^{3}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x*(a + b*sin(c + d*x**3))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x*(a + b*sin(c + d*x^3))^2, x)
```

3.75 $\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$

Optimal. Leaf size=231

$$\frac{-2a^2-b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{abde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{(-idx^3)^{2/3}}-\frac{abde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{(idx^3)^{2/3}}+\frac{ib^2de^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{2\,2^{2/3}(-idx^3)^{2/3}}-\frac{ib^2d}{2}$$

```
[Out] 1/2*(-2*a^2-b^2)/x+1/2*b^2*cos(2*d*x^3+2*c)/x-a*b*d*exp(I*c)*x^2*GAMMA(2/3,
-I*d*x^3)/(-I*d*x^3)^(2/3)-a*b*d*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^
(2/3)+1/4*I*b^2*d*exp(2*I*c)*x^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(
2/3)-1/4*I*b^2*d*x^2*GAMMA(2/3,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3
)-2*a*b*sin(d*x^3+c)/x
```

Rubi [A]
time = 0.12, antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3484, 6, 3469, 3470, 2250, 3468, 3471}

$$-\frac{abe^{ic}dx^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{(-idx^3)^{2/3}}-\frac{abe^{-ic}dx^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{(idx^3)^{2/3}}+\frac{ib^2e^{2ic}dx^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{2\,2^{2/3}(-idx^3)^{2/3}}-\frac{ib^2e^{-2ic}dx^2\text{Gamma}\left(\frac{2}{3},2idx^3\right)}{2\,2^{2/3}(idx^3)^{2/3}}-\frac{2a^2+b^2}{2x}-\frac{2ab\sin(c+dx^3)}{x}+\frac{b^2\cos(2c+2dx^3)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

```
[Out] -1/2*(2*a^2 + b^2)/x + (b^2*Cos[2*c + 2*d*x^3])/(2*x) - (a*b*d*E^(I*c)*x^2*
Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) - (a*b*d*x^2*Gamma[2/3, I*d*x^3]
)/(E^(I*c)*(I*d*x^3)^(2/3)) + ((I/2)*b^2*d*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I
)*d*x^3])/(2^(2/3)*((-I)*d*x^3)^(2/3)) - ((I/2)*b^2*d*x^2*Gamma[2/3, (2*I)*
d*x^3])/(2^(2/3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (2*a*b*Sin[c + d*x^3])/x
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx &= \int \left(\frac{a^2}{x^2}+\frac{b^2}{2x^2}-\frac{b^2\cos(2c+2dx^3)}{2x^2}+\frac{2ab\sin(c+dx^3)}{x^2}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^2}-\frac{b^2\cos(2c+2dx^3)}{2x^2}+\frac{2ab\sin(c+dx^3)}{x^2}\right)dx \\
&= -\frac{2a^2+b^2}{2x}+(2ab)\int \frac{\sin(c+dx^3)}{x^2}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x^2}\,dx \\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{2ab\sin(c+dx^3)}{x}+(6abd)\int x\cos(c+dx^3) \\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{2ab\sin(c+dx^3)}{x}+(3abd)\int e^{-ic-idx^3}x\,dx+ \\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos(2c+2dx^3)}{2x}-\frac{abde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{(-idx^3)^{2/3}}-\frac{abde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{(idx^3)^{2/3}}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 332, normalized size = 1.44

$$\frac{-4a^2(d^2x^6)^{2/3} - 2b^2(d^2x^6)^{2/3} + 2b^2(d^2x^6)^{2/3}\cos(2(c+dx^3)) + \sqrt[3]{2}\,b^2(idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{2}{3},-2idx^3\right) + \sqrt[3]{2}\,b^2(-idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{2}{3},2idx^3\right) - 4iab(-idx^3)^{5/3}\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c)) + 4iab(idx^3)^{5/3}\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c)) + i\sqrt[3]{2}\,b^2(idx^3)^{5/3}\Gamma\left(\frac{2}{3},-2idx^3\right)\sin(2c) - i\sqrt[3]{2}\,b^2(-idx^3)^{5/3}\Gamma\left(\frac{2}{3},2idx^3\right)\sin(2c) - 8ab(d^2x^6)^{2/3}\sin(c+dx^3)}{4x(d^2x^6)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

```
[Out] (-4*a^2*(d^2*x^6)^(2/3) - 2*b^2*(d^2*x^6)^(2/3) + 2*b^2*(d^2*x^6)^(2/3)*Cos
[2*(c + d*x^3)] + 2^(1/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (-2*I)*d*
x^3] + 2^(1/3)*b^2*((-I)*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] - (4
*I)*a*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (4*I)*
a*b*(I*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*2^(1/3)*
b^2*(I*d*x^3)^(5/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*c] - I*2^(1/3)*b^2*((-I)
*d*x^3)^(5/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 8*a*b*(d^2*x^6)^(2/3)*Sin[
c + d*x^3])/(4*x*(d^2*x^6)^(2/3))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^2,x)
```

Maxima [A]
time = 0.36, size = 187, normalized size = 0.81

$$-\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\cos(c)+\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\sin(c)\right)ab}{6\,x}+\frac{\left(2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},2i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{1}{3},-2i\,dx^3\right)\right)\cos(2\,c)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},2i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},-2i\,dx^3\right)\right)\sin(2\,c)\right)-12\right)b^2}{24\,x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(d*x^3)^(1/3)*(((I*sqrt(3) - 1)*gamma(-1/3, I*d*x^3) + (-I*sqrt(3) - 1
)*gamma(-1/3, -I*d*x^3))*cos(c) + ((sqrt(3) + I)*gamma(-1/3, I*d*x^3) + (sq
rt(3) - I)*gamma(-1/3, -I*d*x^3))*sin(c))*a*b/x + 1/24*(2^(1/3)*(d*x^3)^(1/
3)*(((sqrt(3) + I)*gamma(-1/3, 2*I*d*x^3) + (sqrt(3) - I)*gamma(-1/3, -2*I*
d*x^3))*cos(2*c) - ((I*sqrt(3) - 1)*gamma(-1/3, 2*I*d*x^3) + (-I*sqrt(3) -
1)*gamma(-1/3, -2*I*d*x^3))*sin(2*c)) - 12)*b^2/x - a^2/x
```

Fricas [A]
time = 0.13, size = 131, normalized size = 0.57

$$-\frac{b^2(2i\,d)^{\frac{1}{3}}\,xe^{(-2i\,c)}\Gamma\left(\frac{2}{3},2i\,dx^3\right)-4i\,ab(i\,d)^{\frac{1}{3}}\,xe^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)+4i\,ab(-i\,d)^{\frac{1}{3}}\,xe^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)+b^2(-2i\,d)^{\frac{1}{3}}\,xe^{(2i\,c)}\Gamma\left(\frac{2}{3},-2i\,dx^3\right)-4\,b^2\cos\left(dx^3+c\right)^2+8\,ab\sin\left(dx^3+c\right)+4\,a^2+4\,b^2}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*(2*I*d)^(1/3)*x*e^(-2*I*c)*gamma(2/3, 2*I*d*x^3) - 4*I*a*b*(I*d)^
(1/3)*x*e^(-I*c)*gamma(2/3, I*d*x^3) + 4*I*a*b*(-I*d)^(1/3)*x*e^(I*c)*gamma
(2/3, -I*d*x^3) + b^2*(-2*I*d)^(1/3)*x*e^(2*I*c)*gamma(2/3, -2*I*d*x^3) - 4
*b^2*cos(d*x^3 + c)^2 + 8*a*b*sin(d*x^3 + c) + 4*a^2 + 4*b^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(c + dx^3))^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\sin(d\,x^3 + c))^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2/x^2,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x^2, x)
```

3.76 $\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx$

Optimal. Leaf size=285

$$\frac{-2a^2-b^2}{8x^4}-\frac{3abd\cos\left(c+dx^3\right)}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}-\frac{3iabd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iabd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}}-\frac{3b^2\cdots}{\cdots}$$

```
[Out] 1/8*(-2*a^2-b^2)/x^4-3/2*a*b*d*cos(d*x^3+c)/x+1/8*b^2*cos(2*d*x^3+2*c)/x^4-
3/4*I*a*b*d^2*exp(I*c)*x^2*GAMMA(2/3,-I*d*x^3)/(-I*d*x^3)^(2/3)+3/4*I*a*b*d
^2*x^2*GAMMA(2/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(2/3)-3/8*b^2*d^2*exp(2*I*c)*x
^2*GAMMA(2/3,-2*I*d*x^3)*2^(1/3)/(-I*d*x^3)^(2/3)-3/8*b^2*d^2*x^2*GAMMA(2/3
,2*I*d*x^3)*2^(1/3)/exp(2*I*c)/(I*d*x^3)^(2/3)-1/2*a*b*sin(d*x^3+c)/x^4-3/4
*b^2*d*sin(2*d*x^3+2*c)/x
```

Rubi [A]
time = 0.16, antiderivative size = 283, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3484, 6, 3469, 3468, 3471, 2250, 3470}

$$-\frac{3iabe^{ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iabe^{-ic}d^2x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}}-\frac{3b^2e^{2ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{4\ 2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{3b^2e^{-2ic}d^2x^2\text{Gamma}\left(\frac{2}{3},2idx^3\right)}{4\ 2^{2/3}\left(idx^3\right)^{2/3}}-\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos\left(c+dx^3\right)}{2x}-\frac{ab\sin\left(c+dx^3\right)}{2x^4}-\frac{3b^2d\sin\left(2c+2dx^3\right)}{4x}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2)/x^4 - (3*a*b*d*Cos[c + d*x^3])/(2*x) + (b^2*Cos[2*c + 2*
d*x^3])/(8*x^4) - (((3*I)/4)*a*b*d^2*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((
-I)*d*x^3)^(2/3) + (((3*I)/4)*a*b*d^2*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*
d*x^3)^(2/3)) - (3*b^2*d^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(4*2^(
2/3)*((-I)*d*x^3)^(2/3)) - (3*b^2*d^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(4*2^(2/
3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (a*b*Sin[c + d*x^3])/(2*x^4) - (3*b^2*d*S
in[2*c + 2*d*x^3])/(4*x)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx = \int \left(\frac{a^2}{x^5}+\frac{b^2}{2x^5}-\frac{b^2\cos(2c+2dx^3)}{2x^5}+\frac{2ab\sin(c+dx^3)}{x^5}\right)dx$$

$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x^5}-\frac{b^2\cos(2c+2dx^3)}{2x^5}+\frac{2ab\sin(c+dx^3)}{x^5}\right)dx$$

$$= -\frac{2a^2+b^2}{8x^4}+(2ab)\int \frac{\sin(c+dx^3)}{x^5}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x^5}\,dx$$

$$= -\frac{2a^2+b^2}{8x^4}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{ab\sin(c+dx^3)}{2x^4}+\frac{1}{2}(3abd)\int \frac{\cos(c+dx^3)}{x^2}$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos(c+dx^3)}{2x}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{ab\sin(c+dx^3)}{2x^4}-3b$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos(c+dx^3)}{2x}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{ab\sin(c+dx^3)}{2x^4}-3b$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos(c+dx^3)}{2x}+\frac{b^2\cos(2c+2dx^3)}{8x^4}-\frac{3iabd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx\right.}{4(-idx^3)^{2/3}}$$

Mathematica [**A**]
time = 1.57, size = 292, normalized size = 1.02

$$-\frac{2a^2+b^2+12abdx^3\cos(c+dx^3)-b^2\cos(2(c+dx^3))-3\sqrt[3]{2}b^2(idx^3)^{4/3}\cos(2c)\Gamma\left(\frac{2}{3},2idx^3\right)+6iab(idx^3)^{4/3}\Gamma\left(\frac{2}{3},idx^3\right)(\cos(c)-i\sin(c))+6iab(idx^3)^{2/3}\sqrt[3]{d^2x^6}\Gamma\left(\frac{2}{3},-idx^3\right)(\cos(c)+i\sin(c))-3\sqrt[3]{2}b^2(-idx^3)^{4/3}\Gamma\left(\frac{2}{3},-2idx^3\right)(\cos(2c)+i\sin(2c))+3i\sqrt[3]{2}b^2(idx^3)^{4/3}\Gamma\left(\frac{2}{3},2idx^3\right)\sin(2c)+4ab\sin(c+dx^3)+6b^2dx^3\sin(2(c+dx^3))}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

```
[Out] -1/8*(2*a^2 + b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c + d*x^3)] -
3*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] + (6*I)*a*b*
(I*d*x^3)^(4/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I*d*x^
3)^(2/3)*(d^2*x^6)^(1/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 3*2^(
1/3)*b^2*((-I)*d*x^3)^(4/3)*Gamma[2/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]
) + (3*I)*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] + 4*
a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/x^4
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^5,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^5,x)
```

Maxima [A]
time = 0.35, size = 194, normalized size = 0.68

$$\frac{(dx^3)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3}, i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3}, -i\,dx^3\right)\right)\cos(c)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3}, i\,dx^3\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{4}{3}, -i\,dx^3\right)\right)\sin(c)\right)abd}{6\,x}-\frac{\left(2\cdot 2^{\frac{1}{3}}(dx^3)^{\frac{1}{3}}\left(\left(\left(-i\sqrt{3}+1\right)\Gamma\left(-\frac{4}{3}, 2i\,dx^3\right)+\left(i\sqrt{3}+1\right)\Gamma\left(-\frac{4}{3}, -2i\,dx^3\right)\right)\cos(2\,c)-\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{4}{3}, 2i\,dx^3\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{4}{3}, -2i\,dx^3\right)\right)\sin(2\,c)\right)dx^3+3\right)b^2}{24\,x^4}-\frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^3)^(1/3)*(((sqrt(3) + I)*gamma(-4/3, I*d*x^3) + (sqrt(3) - I)*gamm
a(-4/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) - 1)*gamma(-4/3, I*d*x^3) + (-I*sqr
t(3) - 1)*gamma(-4/3, -I*d*x^3))*sin(c))*a*b*d/x - 1/24*(2*2^(1/3)*(d*x^3)^
(1/3)*(((-I*sqrt(3) + 1)*gamma(-4/3, 2*I*d*x^3) + (I*sqrt(3) + 1)*gamma(-4/
3, -2*I*d*x^3))*cos(2*c) - ((sqrt(3) + I)*gamma(-4/3, 2*I*d*x^3) + (sqrt(3)
 - I)*gamma(-4/3, -2*I*d*x^3))*sin(2*c))*d*x^3 + 3)*b^2/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time = 0.13, size = 180, normalized size = 0.63

$$\frac{3i\,b^2(2i\,d)^{\frac{1}{3}}\,dx^4e^{(-2i\,c)}\Gamma\left(\frac{2}{3}, 2i\,dx^3\right)+6\,ab(i\,d)^{\frac{1}{3}}\,dx^4e^{(-i\,c)}\Gamma\left(\frac{2}{3}, i\,dx^3\right)+6\,ab(-i\,d)^{\frac{1}{3}}\,dx^4e^{(i\,c)}\Gamma\left(\frac{2}{3}, -i\,dx^3\right)-3i\,b^2(-2i\,d)^{\frac{1}{3}}\,dx^4e^{(2i\,c)}\Gamma\left(\frac{2}{3}, -2i\,dx^3\right)-12\,abdx^3\cos\left(dx^3+c\right)+2\,b^2\cos\left(dx^3+c\right)^2-2\,a^2-2\,b^2-4\left(3\,b^2dx^3\cos\left(dx^3+c\right)+ab\right)\sin\left(dx^3+c\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*b^2*(2*I*d)^(1/3)*d*x^4*e^(-2*I*c)*gamma(2/3, 2*I*d*x^3) + 6*a*b*(
I*d)^(1/3)*d*x^4*e^(-I*c)*gamma(2/3, I*d*x^3) + 6*a*b*(-I*d)^(1/3)*d*x^4*e^
(I*c)*gamma(2/3, -I*d*x^3) - 3*I*b^2*(-2*I*d)^(1/3)*d*x^4*e^(2*I*c)*gamma(2
/3, -2*I*d*x^3) - 12*a*b*d*x^3*cos(d*x^3 + c) + 2*b^2*cos(d*x^3 + c)^2 - 2*
a^2 - 2*b^2 - 4*(3*b^2*d*x^3*cos(d*x^3 + c) + a*b)*sin(d*x^3 + c))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sin\left(c + dx^3\right)\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2/x^5,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x^5, x)
```

3.77 $\int x^3(a+b\sin(c+dx^3))^2\,dx$

Optimal. Leaf size=237

$$\frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}\,d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}}{72\sqrt[3]{2}}$$

```
[Out] 1/8*(2*a^2+b^2)*x^4-2/3*a*b*x*cos(d*x^3+c)/d-1/9*a*b*exp(I*c)*x*GAMMA(1/3,-
I*d*x^3)/d/(-I*d*x^3)^(1/3)-1/9*a*b*x*GAMMA(1/3,I*d*x^3)/d/exp(I*c)/(I*d*x^
3)^(1/3)+1/144*I*b^2*exp(2*I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/d/(-I*d*x^3
)^(1/3)-1/144*I*b^2*x*GAMMA(1/3,2*I*d*x^3)*2^(2/3)/d/exp(2*I*c)/(I*d*x^3)^(
1/3)-1/12*b^2*x*sin(2*d*x^3+2*c)/d
```

Rubi [A]

time = 0.10, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3484, 6, 3467, 3436, 2239, 3466, 3437}

$$-\frac{abe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}\,d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}\,d\sqrt[3]{idx^3}}+\frac{1}{8}x^4\left(2a^2+b^2\right)-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^4)/8 - (2*a*b*x*Cos[c + d*x^3])/(3*d) - (a*b*E^(I*c)*x*Gam
ma[1/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(1/3)) - (a*b*x*Gamma[1/3, I*d*x^3])
/(9*d*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/72)*b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I
)*d*x^3])/(2^(1/3)*d*((-I)*d*x^3)^(1/3)) - ((I/72)*b^2*x*Gamma[1/3, (2*I)*d
*x^3])/(2^(1/3)*d*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (b^2*x*Sin[2*c + 2*d*x^3])
/(12*d)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int\left(a^2x^3+\frac{b^2x^3}{2}-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+2abx^3\sin\left(c+dx^3\right)\right)\,dx\\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x^3-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+2abx^3\sin\left(c+dx^3\right)\right)\,dx\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4+(2ab)\int x^3\sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int x^3\cos\left(2c+2dx^3\right)\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}+\frac{(2ab)\int\cos\left(c\right.}{3d}\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}+\frac{(ab)\int e^{-ic-id}}{3d}\\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},\right.}{9d\sqrt[3]{idx^3}}
\end{aligned}
$$

Mathematica [A]

time = 0.35, size = 339, normalized size = 1.43

$$\frac{dx^7\left(36a^2dx^3\sqrt[3]{d^2x^6}+18b^2dx^3\sqrt[3]{d^2x^6}-96ab\sqrt[3]{d^2x^6}\cos\left(c+dx^3\right)+i2^{2/3}b^2\sqrt[3]{idx^3}\cos(2c)\Gamma\left(\frac{1}{3},-2idx^3\right)-i2^{2/3}b^2\sqrt[3]{-idx^3}\cos(2c)\Gamma\left(\frac{1}{3},2idx^3\right)-16ab\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},idx^3\right)(\cos(c)-i\sin(c))-16ab\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3},-idx^3\right)(\cos(c)+i\sin(c))-2^{2/3}b^2\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3},-2idx^3\right)\sin(2c)-2^{2/3}b^2\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},2idx^3\right)\sin(2c)-12b^2\sqrt[3]{d^2x^6}\sin\left(2\left(c+dx^3\right)\right)\right)}{144\left(d^2x^6\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (d*x^7*(36*a^2*d*x^3*(d^2*x^6)^(1/3) + 18*b^2*d*x^3*(d^2*x^6)^(1/3) - 96*a*
b*(d^2*x^6)^(1/3)*Cos[c + d*x^3] + I*2^(2/3)*b^2*(I*d*x^3)^(1/3)*Cos[2*c]*G
amma[1/3, (-2*I)*d*x^3] - I*2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Cos[2*c]*Gamma[1
/3, (2*I)*d*x^3] - 16*a*b*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] -
I*Sin[c]) - 16*a*b*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c
]) - 2^(2/3)*b^2*(I*d*x^3)^(1/3)*Gamma[1/3, (-2*I)*d*x^3]*Sin[2*c] - 2^(2/3
)*b^2*((-I)*d*x^3)^(1/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] - 12*b^2*(d^2*x^6
)^(1/3)*Sin[2*(c + d*x^3)]))/(144*(d^2*x^6)^(4/3))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\sin\left(d\,x^3+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.34, size = 240, normalized size = 1.01

$$\frac{1}{4}\,a^2x^4-\frac{2^{\frac{2}{3}}\left(\left(\left(\left(i\,\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(-i\,\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)+\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\sin\left(2\,c\right)\right)x-6\cdot 2^{\frac{1}{3}}\left(3\,dx^4-2\,x\sin\left(2\,dx^3+2\,c\right)\right)\left(dx^3\right)^{\frac{1}{3}}\right)b^2}{288\,\left(dx^3\right)^{\frac{1}{3}}\,d}-\frac{\left(12\,\left(dx^3\right)^{\frac{1}{3}}\,x\cos\left(dx^3+c\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(c\right)+\left(\left(-i\,\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(i\,\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)x\right)ab}{18\,\left(dx^3\right)^{\frac{1}{3}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 - 1/288*2^(2/3)*((((I*sqrt(3) + 1)*gamma(1/3, 2*I*d*x^3) + (-I*
sqrt(3) + 1)*gamma(1/3, -2*I*d*x^3))*cos(2*c) + ((sqrt(3) - I)*gamma(1/3, 2
*I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -2*I*d*x^3))*sin(2*c))*x - 6*2^(1/3)*(
3*d*x^4 - 2*x*sin(2*d*x^3 + 2*c))*(d*x^3)^(1/3))*b^2/((d*x^3)^(1/3)*d) - 1/
18*(12*(d*x^3)^(1/3)*x*cos(d*x^3 + c) + (((sqrt(3) - I)*gamma(1/3, I*d*x^3)
 + (sqrt(3) + I)*gamma(1/3, -I*d*x^3))*cos(c) + ((-I*sqrt(3) - 1)*gamma(1/3
, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*d*x^3))*sin(c))*x)*a*b/((d*x^3)^
(1/3)*d)
```

Fricas [A]
time = 0.12, size = 146, normalized size = 0.62

$$\frac{18\,(2\,a^2+b^2)d^2x^4-24\,b^2dx\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-96\,abdx\cos\left(dx^3+c\right)-b^2(2i\,d)^{\frac{2}{3}}\,e^{(-2i\,c)}\Gamma\left(\frac{1}{3},2i\,dx^3\right)+16i\,ab(i\,d)^{\frac{2}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)-16i\,ab(-i\,d)^{\frac{2}{3}}\,e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-b^2(-2i\,d)^{\frac{2}{3}}\,e^{(2i\,c)}\Gamma\left(\frac{1}{3},-2i\,dx^3\right)}{144\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(18*(2*a^2 + b^2)*d^2*x^4 - 24*b^2*d*x*cos(d*x^3 + c)*sin(d*x^3 + c)
- 96*a*b*d*x*cos(d*x^3 + c) - b^2*(2*I*d)^(2/3)*e^(-2*I*c)*gamma(1/3, 2*I*d
*x^3) + 16*I*a*b*(I*d)^(2/3)*e^(-I*c)*gamma(1/3, I*d*x^3) - 16*I*a*b*(-I*d)
^(2/3)*e^(I*c)*gamma(1/3, -I*d*x^3) - b^2*(-2*I*d)^(2/3)*e^(2*I*c)*gamma(1/
3, -2*I*d*x^3))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \sin \left(c + dx^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x**3*(a + b*sin(c + d*x**3))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \, \left(a + b \, \sin \left(d \, x^3 + c\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x^3*(a + b*sin(c + d*x^3))^2, x)
```

3.78 $\int (a+b\sin(c+dx^3))^2\, dx$

Optimal. Leaf size=183

$$\frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\Gamma\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}$$

```
[Out] 1/2*(2*a^2+b^2)*x+1/3*I*a*b*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)
-1/3*I*a*b*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)+1/24*b^2*exp(2*I*c
)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*x^3)^(1/3)+1/24*b^2*x*GAMMA(1/3,2*I
*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3438, 3437, 2239, 3436}

$$\frac{iabe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}+\frac{1}{2}x\left(2a^2+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + ((I/3)*a*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*
x^3)^(1/3) - ((I/3)*a*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) +
(b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(12*2^(1/3)*((-I)*d*x^3)^(1/3)
) + (b^2*x*Gamma[1/3, (2*I)*d*x^3])/(12*2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)
)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2dx^3\right)+2ab\sin\left(c+dx^3\right)\right)\,dx\\ &= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int \cos\left(2c+2dx^3\right)\,dx\\ &= \frac{1}{2}\left(2a^2+b^2\right)x+(iab)\int e^{-ic-idx^3}\,dx-(iab)\int e^{ic+idx^3}\,dx-\frac{1}{4}b^2\int e^{-2ic-2idx}\\ &= \frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2\right.}{12\sqrt[3]{2}\,\sqrt[3]{-id}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 281, normalized size = 1.54

$$\frac{x\left(24a^2\sqrt[3]{d^2x^6}+12b^2\sqrt[3]{d^2x^6}+2^{2/3}b^2\sqrt[3]{idx^3}\cos(2c)\Gamma\left(\frac{1}{3},-2idx^3\right)+2^{2/3}b^2\sqrt[3]{-idx^3}\cos(2c)\Gamma\left(\frac{1}{3},2idx^3\right)-8iab\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},idx^3\right)(\cos(c)-i\sin(c))+8iab\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3},-idx^3\right)(\cos(c)+i\sin(c))+i2^{2/3}b^2\sqrt[3]{idx^3}\Gamma\left(\frac{1}{3},-2idx^3\right)\sin(2c)-i2^{2/3}b^2\sqrt[3]{-idx^3}\Gamma\left(\frac{1}{3},2idx^3\right)\sin(2c)\right)}{24\sqrt[3]{d^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (x*(24*a^2*(d^2*x^6)^(1/3) + 12*b^2*(d^2*x^6)^(1/3) + 2^(2/3)*b^2*(I*d*x^3)
^(1/3)*Cos[2*c]*Gamma[1/3, (-2*I)*d*x^3] + 2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*C
os[2*c]*Gamma[1/3, (2*I)*d*x^3] - (8*I)*a*b*((-I)*d*x^3)^(1/3)*Gamma[1/3, I
*d*x^3]*(Cos[c] - I*Sin[c]) + (8*I)*a*b*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x
^3]*(Cos[c] + I*Sin[c]) + I*2^(2/3)*b^2*(I*d*x^3)^(1/3)*Gamma[1/3, (-2*I)*d
*x^3]*Sin[2*c] - I*2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Gamma[1/3, (2*I)*d*x^3]*S
in[2*c]))/(24*(d^2*x^6)^(1/3))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(d\,x^3+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.38, size = 192, normalized size = 1.05

$$\frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)abx}{6\,\left(dx^3\right)^{\frac{1}{3}}}+\frac{2^{\frac{2}{3}}\left(\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},2i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-2i\,dx^3\right)\right)\sin\left(2\,c\right)\right)x+12\cdot2^{\frac{1}{3}}\left(dx^3\right)^{\frac{1}{3}}x\right)b^2}{48\,\left(dx^3\right)^{\frac{1}{3}}}+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(((-I*sqrt(3) - 1)*gamma(1/3, I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -I*
d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(1/3, I*d*x^3) + (sqrt(3) + I)*gamma(1
/3, -I*d*x^3))*sin(c))*a*b*x/(d*x^3)^(1/3) + 1/48*2^(2/3)*((((sqrt(3) - I)*
gamma(1/3, 2*I*d*x^3) + (sqrt(3) + I)*gamma(1/3, -2*I*d*x^3))*cos(2*c) + ((
-I*sqrt(3) - 1)*gamma(1/3, 2*I*d*x^3) + (I*sqrt(3) - 1)*gamma(1/3, -2*I*d*x
^3))*sin(2*c))*x + 12*2^(1/3)*(d*x^3)^(1/3)*x)*b^2/(d*x^3)^(1/3) + a^2*x
```

Fricas [A]
time = 0.12, size = 105, normalized size = 0.57

$$\frac{-i\,b^2(2i\,d)^{\frac{2}{3}}\,e^{(-2i\,c)}\Gamma\left(\frac{1}{3},2i\,dx^3\right)-8\,ab(i\,d)^{\frac{2}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)-8\,ab(-i\,d)^{\frac{2}{3}}\,e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)+i\,b^2(-2i\,d)^{\frac{2}{3}}\,e^{(2i\,c)}\Gamma\left(\frac{1}{3},-2i\,dx^3\right)+12\,(2\,a^2+b^2)dx}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*b^2*(2*I*d)^(2/3)*e^(-2*I*c)*gamma(1/3, 2*I*d*x^3) - 8*a*b*(I*d)^(
2/3)*e^(-I*c)*gamma(1/3, I*d*x^3) - 8*a*b*(-I*d)^(2/3)*e^(I*c)*gamma(1/3, -
I*d*x^3) + I*b^2*(-2*I*d)^(2/3)*e^(2*I*c)*gamma(1/3, -2*I*d*x^3) + 12*(2*a^
2 + b^2)*d*x)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\sin\left(d\,x^3+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2,x)
[Out] int((a + b*sin(c + d*x^3))^2, x)
```

3.79 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=227

$$\frac{-2a^2-b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{abde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}+\frac{ib^2de^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2de^{-2ic}}{4\sqrt[3]{2}}$$

```
[Out] 1/4*(-2*a^2-b^2)/x^2+1/4*b^2*cos(2*d*x^3+2*c)/x^2-1/2*a*b*d*exp(I*c)*x*GAMM
A(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)-1/2*a*b*d*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I
*d*x^3)^(1/3)+1/8*I*b^2*d*exp(2*I*c)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*
x^3)^(1/3)-1/8*I*b^2*d*x*GAMMA(1/3,2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^
(1/3)-a*b*sin(d*x^3+c)/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3484, 6, 3469, 3436, 2239, 3468, 3437}

$$-\frac{abe^{ic}dx\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abe^{-ic}dx\text{Gamma}\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}dx\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}dx\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{idx^3}}-\frac{2a^2+b^2}{4x^2}-\frac{ab\sin\left(c+dx^3\right)}{x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

```
[Out] -1/4*(2*a^2 + b^2)/x^2 + (b^2*Cos[2*c + 2*d*x^3])/(4*x^2) - (a*b*d*E^(I*c)*
x*Gamma[1/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(1/3)) - (a*b*d*x*Gamma[1/3, I*d*
x^3])/(2*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/4)*b^2*d*E^((2*I)*c)*x*Gamma[1/3, (
-2*I)*d*x^3])/(2^(1/3)*((-I)*d*x^3)^(1/3)) - ((I/4)*b^2*d*x*Gamma[1/3, (2*I
)*d*x^3])/(2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (a*b*Sin[c + d*x^3])/x^2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{b^2}{2x^3}-\frac{b^2\cos(2c+2dx^3)}{2x^3}+\frac{2ab\sin(c+dx^3)}{x^3}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^3}-\frac{b^2\cos(2c+2dx^3)}{2x^3}+\frac{2ab\sin(c+dx^3)}{x^3}\right)dx\\
&= -\frac{2a^2+b^2}{4x^2}+(2ab)\int \frac{\sin(c+dx^3)}{x^3}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x^3}\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2c+2dx^3)}{4x^2}-\frac{ab\sin(c+dx^3)}{x^2}+(3abd)\int \cos\left(c+dx^3\right)\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2c+2dx^3)}{4x^2}-\frac{ab\sin(c+dx^3)}{x^2}+\frac{1}{2}(3abd)\int e^{-ic-idx^3}\,dx\\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos(2c+2dx^3)}{4x^2}-\frac{abde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 332, normalized size = 1.46

$$\frac{-4a^2\sqrt[3]{d^2x^6}-2b^2\sqrt[3]{d^2x^6}+2b^2\sqrt[3]{d^2x^6}\cos\left(2(c+dx^3)\right)+2^{2/3}b^2(idx^3)^{4/3}\cos(2c)\Gamma\left(\frac{1}{3},-2idx^3\right)+2^{2/3}b^2(-idx^3)^{4/3}\cos(2c)\Gamma\left(\frac{1}{3},2idx^3\right)-4iab(-idx^3)^{4/3}\Gamma\left(\frac{1}{3},idx^3\right)(\cos(c)-i\sin(c))+4iab(idx^3)^{4/3}\Gamma\left(\frac{1}{3},-idx^3\right)(\cos(c)+i\sin(c))+i2^{2/3}b^2(idx^3)^{4/3}\Gamma\left(\frac{1}{3},-2idx^3\right)\sin(2c)-i2^{2/3}b^2(-idx^3)^{4/3}\Gamma\left(\frac{1}{3},2idx^3\right)\sin(2c)-8ab\sqrt[3]{d^2x^6}\sin\left(c+dx^3\right)}{8x^2\sqrt[3]{d^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

```
[Out] (-4*a^2*(d^2*x^6)^(1/3) - 2*b^2*(d^2*x^6)^(1/3) + 2*b^2*(d^2*x^6)^(1/3)*Cos
[2*(c + d*x^3)] + 2^(2/3)*b^2*(I*d*x^3)^(4/3)*Cos[2*c]*Gamma[1/3, (-2*I)*d*
x^3] + 2^(2/3)*b^2*((-I)*d*x^3)^(4/3)*Cos[2*c]*Gamma[1/3, (2*I)*d*x^3] - (4
*I)*a*b*((-I)*d*x^3)^(4/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (4*I)*
a*b*(I*d*x^3)^(4/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*2^(2/3)*
b^2*(I*d*x^3)^(4/3)*Gamma[1/3, (-2*I)*d*x^3]*Sin[2*c] - I*2^(2/3)*b^2*((-I)
*d*x^3)^(4/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] - 8*a*b*(d^2*x^6)^(1/3)*Sin[
c + d*x^3])/(8*x^2*(d^2*x^6)^(1/3))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(d\,x^{3}+c\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^3,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^3,x)
```

Maxima [A]
time = 0.35, size = 188, normalized size = 0.83

$$\frac{(dx^3)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos\left(c\right)-\left(\left(i\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3},i\,dx^3\right)+\left(-i\sqrt{3}+1\right)\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\sin\left(c\right)\right)ab}{6\,x^2}-\frac{\left(2^{\frac{2}{3}}(dx^3)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},2i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},-2i\,dx^3\right)\right)\cos\left(2\,c\right)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{2}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{2}{3},-2i\,dx^3\right)\right)\sin\left(2\,c\right)\right)+6\right)b^2}{24\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^3)^(2/3)*(((sqrt(3) - I)*gamma(-2/3, I*d*x^3) + (sqrt(3) + I)*gamm
a(-2/3, -I*d*x^3))*cos(c) - ((I*sqrt(3) + 1)*gamma(-2/3, I*d*x^3) + (-I*sqr
t(3) + 1)*gamma(-2/3, -I*d*x^3))*sin(c))*a*b/x^2 - 1/24*(2^(2/3)*(d*x^3)^(2
/3)*(((-I*sqrt(3) - 1)*gamma(-2/3, 2*I*d*x^3) + (I*sqrt(3) - 1)*gamma(-2/3,
 -2*I*d*x^3))*cos(2*c) - ((sqrt(3) - I)*gamma(-2/3, 2*I*d*x^3) + (sqrt(3) +
 I)*gamma(-2/3, -2*I*d*x^3))*sin(2*c)) + 6)*b^2/x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.12, size = 139, normalized size = 0.61

$$-\frac{b^2(2i\,d)^{\frac{2}{3}}\,x^2e^{(-2i\,c)}\Gamma\left(\frac{1}{3},2i\,dx^3\right)-4i\,ab(i\,d)^{\frac{2}{3}}\,x^2e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)+4i\,ab(-i\,d)^{\frac{2}{3}}\,x^2e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)+b^2(-2i\,d)^{\frac{2}{3}}\,x^2e^{(2i\,c)}\Gamma\left(\frac{1}{3},-2i\,dx^3\right)-4\,b^2\cos\left(dx^3+c\right)^2+8\,ab\sin\left(dx^3+c\right)+4\,a^2+4\,b^2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*(2*I*d)^(2/3)*x^2*e^(-2*I*c)*gamma(1/3, 2*I*d*x^3) - 4*I*a*b*(I*d
)^(2/3)*x^2*e^(-I*c)*gamma(1/3, I*d*x^3) + 4*I*a*b*(-I*d)^(2/3)*x^2*e^(I*c)
*gamma(1/3, -I*d*x^3) + b^2*(-2*I*d)^(2/3)*x^2*e^(2*I*c)*gamma(1/3, -2*I*d*
x^3) - 4*b^2*cos(d*x^3 + c)^2 + 8*a*b*sin(d*x^3 + c) + 4*a^2 + 4*b^2)/x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(c + dx^3))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\sin(d\,x^3 + c))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2/x^3,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x^3, x)
```

3.80 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=277

$$\frac{-2a^2-b^2}{10x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{3iabd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}}-\frac{3b^2d^2e^{2}\ldots}{10\ldots}$$

```
[Out] 1/10*(-2*a^2-b^2)/x^5-3/5*a*b*d*cos(d*x^3+c)/x^2+1/10*b^2*cos(2*d*x^3+2*c)/
x^5-3/10*I*a*b*d^2*exp(I*c)*x*GAMMA(1/3,-I*d*x^3)/(-I*d*x^3)^(1/3)+3/10*I*a
*b*d^2*x*GAMMA(1/3,I*d*x^3)/exp(I*c)/(I*d*x^3)^(1/3)-3/20*b^2*d^2*exp(2*I*c
)*x*GAMMA(1/3,-2*I*d*x^3)*2^(2/3)/(-I*d*x^3)^(1/3)-3/20*b^2*d^2*x*GAMMA(1/3
,2*I*d*x^3)*2^(2/3)/exp(2*I*c)/(I*d*x^3)^(1/3)-2/5*a*b*sin(d*x^3+c)/x^5-3/1
0*b^2*d*sin(2*d*x^3+2*c)/x^2
```

Rubi [A]

time = 0.11, antiderivative size = 275, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3484, 6, 3469, 3468, 3437, 2239, 3436}

$$-\frac{3iabe^{ic}d^2x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabe^{-ic}d^2x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}}-\frac{3b^2e^{2ic}d^2x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{3b^2e^{-2ic}d^2x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{idx^3}}-\frac{2a^2+b^2}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{3b^2d\sin\left(2c+2dx^3\right)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

```
[Out] -1/10*(2*a^2 + b^2)/x^5 - (3*a*b*d*Cos[c + d*x^3])/(5*x^2) + (b^2*Cos[2*c +
 2*d*x^3])/(10*x^5) - (((3*I)/10)*a*b*d^2*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])
/((-I)*d*x^3)^(1/3) + (((3*I)/10)*a*b*d^2*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(
I*d*x^3)^(1/3)) - (3*b^2*d^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(10*2^
(1/3)*((-I)*d*x^3)^(1/3)) - (3*b^2*d^2*x*Gamma[1/3, (2*I)*d*x^3])/(10*2^(1/
3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (2*a*b*Sin[c + d*x^3])/(5*x^5) - (3*b^2*d
*Sin[2*c + 2*d*x^3])/(10*x^2)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
```

```
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sin(c+dx^3))^2}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{b^2}{2x^6}-\frac{b^2\cos(2c+2dx^3)}{2x^6}+\frac{2ab\sin(c+dx^3)}{x^6}\right)dx \\ &= \int \left(\frac{a^2+\frac{b^2}{2}}{x^6}-\frac{b^2\cos(2c+2dx^3)}{2x^6}+\frac{2ab\sin(c+dx^3)}{x^6}\right)dx \\ &= -\frac{2a^2+b^2}{10x^5}+(2ab)\int \frac{\sin(c+dx^3)}{x^6}\,dx-\frac{1}{2}b^2\int \frac{\cos(2c+2dx^3)}{x^6}\,dx \\ &= -\frac{2a^2+b^2}{10x^5}+\frac{b^2\cos(2c+2dx^3)}{10x^5}-\frac{2ab\sin(c+dx^3)}{5x^5}+\frac{1}{5}(6abd)\int \frac{\cos(c+dx^3)}{x^3} \\ &= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos(c+dx^3)}{5x^2}+\frac{b^2\cos(2c+2dx^3)}{10x^5}-\frac{2ab\sin(c+dx^3)}{5x^5}-\frac{3b^2}{} \\ &= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos(c+dx^3)}{5x^2}+\frac{b^2\cos(2c+2dx^3)}{10x^5}-\frac{2ab\sin(c+dx^3)}{5x^5}-\frac{3b^2}{} \\ &= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos(c+dx^3)}{5x^2}+\frac{b^2\cos(2c+2dx^3)}{10x^5}-\frac{3iabd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}\end{aligned}$$

<u>Mathematica</u> [A]
time = 1.58, size = 294, normalized size = 1.06

$$-\frac{4a^2+2b^2+12abdx^3\cos(c+dx^3)-2b^2\cos(2(c+dx^3))-3\,2^{2/3}b^2(idx^3)^{5/3}\cos(2c)\Gamma\left(\frac{1}{3},2idx^3\right)+6iab(idx^3)^{5/3}\Gamma\left(\frac{1}{3},idx^3\right)(\cos(c)-i\sin(c))+6iab\sqrt[3]{idx^3}(d^2x^6)^{2/3}\Gamma\left(\frac{1}{3},-idx^3\right)(\cos(c)+i\sin(c))-3\,2^{2/3}b^2(-idx^3)^{5/3}\Gamma\left(\frac{1}{3},-2idx^3\right)(\cos(2c)+i\sin(2c))+3i2^{2/3}b^2(idx^3)^{5/3}\Gamma\left(\frac{1}{3},2idx^3\right)\sin(2c)+8ab\sin(c+dx^3)+6b^2dx^3\sin(2(c+dx^3))}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

```
[Out] -1/20*(4*a^2 + 2*b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - 2*b^2*Cos[2*(c + d*x^3
)] - 3*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[1/3, (2*I)*d*x^3] + (6*I)
*a*b*(I*d*x^3)^(5/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I
*d*x^3)^(1/3)*(d^2*x^6)^(2/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) -
3*2^(2/3)*b^2*((-I)*d*x^3)^(5/3)*Gamma[1/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin
[2*c]) + (3*I)*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c]
 + 8*a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/x^5
```

<u>Maple</u> [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(d\,x^3+c))^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^6,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^6,x)
```

Maxima [A]
time = 0.36, size = 193, normalized size = 0.70

$$-\frac{(dx^3)^{\frac{2}{3}}\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},-i\,dx^3\right)\right)\cos(c)-\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{5}{3},i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{5}{3},-i\,dx^3\right)\right)\sin(c)\right)abd}{6\,x^2}-\frac{\left(5\cdot 2^{\frac{2}{3}}(dx^3)^{\frac{2}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(-\frac{5}{3},2i\,dx^3\right)+\left(\sqrt{3}+i\right)\Gamma\left(-\frac{5}{3},-2i\,dx^3\right)\right)\cos(2\,c)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},2i\,dx^3\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{5}{3},-2i\,dx^3\right)\right)\sin(2\,c)\right)dx^3+6\right)b^2}{60\,x^5}-\frac{a^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*(d*x^3)^(2/3)*(((-I*sqrt(3) - 1)*gamma(-5/3, I*d*x^3) + (I*sqrt(3) - 1
)*gamma(-5/3, -I*d*x^3))*cos(c) - ((sqrt(3) - I)*gamma(-5/3, I*d*x^3) + (sq
rt(3) + I)*gamma(-5/3, -I*d*x^3))*sin(c))*a*b*d/x^2 - 1/60*(5*2^(2/3)*(d*x^
3)^(2/3)*(((sqrt(3) - I)*gamma(-5/3, 2*I*d*x^3) + (sqrt(3) + I)*gamma(-5/3,
 -2*I*d*x^3))*cos(2*c) + ((-I*sqrt(3) - 1)*gamma(-5/3, 2*I*d*x^3) + (I*sqrt
(3) - 1)*gamma(-5/3, -2*I*d*x^3))*sin(2*c))*d*x^3 + 6)*b^2/x^5 - 1/5*a^2/x^
5
```

Fricas [A]
time = 0.13, size = 181, normalized size = 0.65

$$\frac{3i\,b^2(2i\,d)^{\frac{2}{3}}\,dx^5e^{(-2i\,c)}\Gamma\left(\frac{1}{3},2i\,dx^3\right)+6\,ab(i\,d)^{\frac{2}{3}}\,dx^5e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)+6\,ab(-i\,d)^{\frac{2}{3}}\,dx^5e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-3i\,b^2(-2i\,d)^{\frac{2}{3}}\,dx^5e^{(2i\,c)}\Gamma\left(\frac{1}{3},-2i\,dx^3\right)-12\,abdx^3\cos\left(dx^3+c\right)+4\,b^2\cos\left(dx^3+c\right)^2-4\,a^2-4\,b^2-4\left(3\,b^2dx^3\cos\left(dx^3+c\right)+2\,ab\right)\sin\left(dx^3+c\right)}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(3*I*b^2*(2*I*d)^(2/3)*d*x^5*e^(-2*I*c)*gamma(1/3, 2*I*d*x^3) + 6*a*b*
(I*d)^(2/3)*d*x^5*e^(-I*c)*gamma(1/3, I*d*x^3) + 6*a*b*(-I*d)^(2/3)*d*x^5*e
^(I*c)*gamma(1/3, -I*d*x^3) - 3*I*b^2*(-2*I*d)^(2/3)*d*x^5*e^(2*I*c)*gamma(
1/3, -2*I*d*x^3) - 12*a*b*d*x^3*cos(d*x^3 + c) + 4*b^2*cos(d*x^3 + c)^2 - 4
*a^2 - 4*b^2 - 4*(3*b^2*d*x^3*cos(d*x^3 + c) + 2*a*b)*sin(d*x^3 + c))/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**6,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \sin (d x^3 + c))^2}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x^3))^2/x^6,x)
```

```
[Out] int((a + b*sin(c + d*x^3))^2/x^6, x)
```

3.81 $\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=245

$$-\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d}-\frac{\mathrm{Li}_2\left(\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d^2}+\frac{\mathrm{Li}_2\left(\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d^2}$$

```
[Out] -1/3*I*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
+1/3*I*x^3*ln(1-I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)
-1/3*polylog(2,I*b*exp(I*(d*x^3+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2
)+1/3*polylog(2,I*b*exp(I*(d*x^3+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/
2)
```

Rubi [A]
time = 0.32, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3460, 3404, 2296, 2221, 2317, 2438}

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\sqrt{a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^3)}}{\sqrt{a^2-b^2}+a}\right)}{3d^2\sqrt{a^2-b^2}}-\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{\sqrt{a^2-b^2}+a}\right)}{3d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] ((-1/3*I)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(Sqrt
[a^2 - b^2]*d) + ((I/3)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 -
 b^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt
[a^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^3)))/
(a + Sqrt[a^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{x^5}{a+b\sin(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^3\right)$$

$$= -\frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}} + \frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c}}{2a+2\sqrt{a^2-}}\right.}{3\sqrt{a^2-}}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d} + \frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d} + \frac{i\text{Subst}\left(\int\log\left(1\right.\right.}{}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d} + \frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d} + \frac{\text{Subst}\left(\int \frac{\log\left(1-\right.}{}\right.}{}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d} + \frac{ix^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d} - \frac{\text{Li}_2\left(\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right.}{3\sqrt{a^2-b^2}\,d^2}$$

Mathematica [A]
time = 0.10, size = 188, normalized size = 0.77

$$\frac{-idx^3\left(\log\left(1+\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)-\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)\right)-\text{Li}_2\left(-\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)+\text{Li}_2\left(\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] ((-I)*d*x^3*(Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] - Log[
1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^
(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x^3)
))/(a + Sqrt[a^2 - b^2])])/(3*Sqrt[a^2 - b^2]*d^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+b\sin(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^3+c)),x)
[Out] int(x^5/(a+b*sin(d*x^3+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(199) = 398.
time = 0.56, size = 1041, normalized size = 4.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] -1/6*(b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3 +
 c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + b*c*sqrt(-(a^2 - b^2)/b^2)*log(
2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*
I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - b*c*sqrt(-(a^2 - b^2)/b^2)*lo
g(-2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) -
 2*I*a) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x^
3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b
)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x
^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b + 1) + I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin(d
*x^3 + c) + (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)
- b)/b + 1) - I*b*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin
(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2
) - b)/b + 1) + (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3
+ c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^
2 - b^2)/b^2) - b)/b) - (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*co
s(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*s
qrt(-(a^2 - b^2)/b^2) - b)/b) + (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(
-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) - I*b*sin(d*x^
3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/
b^2)*log(-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b
*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**3+c)),x)
[Out] Integral(x**5/(a + b*sin(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{a+b\,\sin\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*sin(c + d*x^3)),x)
[Out] int(x^5/(a + b*sin(c + d*x^3)), x)
```

3.82 $\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=51

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d}$$

```
[Out] 2/3*arctan((b+a*tan(1/2*d*x^3+1/2*c))/(a^2-b^2)^(1/2))/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3460, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx^3)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{a+b\sin(c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^3\right) \\
&= \frac{2\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^3)\right)\right)}{3d} \\
&= -\frac{4\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx^3)\right)\right)}{3d} \\
&= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^3]),x]
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.96

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$	49
default	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$	49

risch	$-\frac{\ln\left(e^{i(d x^3+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{3\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(e^{i(d x^3+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{3\sqrt{-a^2+b^2}\,d}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^3+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^(
1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8078 vs. $2(44) = 88$.
time = 28.81, size = 8078, normalized size = 158.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^4*cos(c)*sin(c) - 4*(a^2
*b^4 - b^6)*cos(c)*sin(d*x^3 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c)^
3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 4*(3*(a^3*b^3
 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^6)*c
os(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^3 +
4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 +
 b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6
)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 + 3*((a^3
*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c
))*sin(d*x^3 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5 + 2*(2*a^
5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)
^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3 + (2*a^5*b -
3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^2*b^4 - b^6)*s
in(c)^2)*cos(d*x^3 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*sin(c) - (a^3*b
^3 - a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + ((4*a^4*b^2 - 5*a^2*b^4 + b^6)*c
os(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c))*sin(d*x
^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^5*cos(c) - 4*a*b^4*cos(d*x^3 + 2*c)^4*cos
(c)*sin(c) + b^5*sin(d*x^3 + 2*c)^5*sin(c) + (b^5*cos(d*x^3 + 2*c)*cos(c) +
 4*a*b^4*cos(c)*sin(c))*sin(d*x^3 + 2*c)^4 + 2*((2*a^2*b^3 - b^5)*cos(c)^3
+ 3*(2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 + 2*(b^5*cos(d*x^
3 + 2*c)^2*sin(c) + 3*(2*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (2*a^2*b^3 - b^5)
*sin(c)^3 + 2*(a*b^4*cos(c)^2 - a*b^4*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3
 + 2*c)^3 - 4*((4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + (4*a^3*b^2 - 3*a*b^4
)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + 2*(b^5*cos(d*x^3 + 2*c)^3*cos(c) +
2*(4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + 2*(4*a^3*b^2 - 3*a*b^4)*cos(c)*si
```

n(c)^3 + 3*((2*a^2*b^3 - b^5)*cos(c)^3 - (2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)
*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 + ((8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)
^5 + 2*(8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (8*a^4*b - 8*a^2*b^3
 + b^5)*cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^4*sin(c)
+ (8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(8*a^4*b - 8*a^2*b^3 + b^
5)*cos(c)^2*sin(c)^3 + (8*a^4*b - 8*a^2*b^3 + b^5)*sin(c)^5 + 4*(a*b^4*cos(
c)^2 - a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 6*((2*a^2*b^3 - b^5)*cos(c)^2*s
in(c) - (2*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + 4*((4*a^3*b^2 - 3*
a*b^4)*cos(c)^4 - (4*a^3*b^2 - 3*a*b^4)*sin(c)^4)*cos(d*x^3 + 2*c))*sin(d*x
^3 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*
x^3 + 2*c)^5 + b^6*sin(d*x^3 + 2*c)^6 - 6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) +
 (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2
 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^
4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c
)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x
^3 + 2*c)^4 + 3*(b^6*cos(d*x^3 + 2*c)^2 - 2*a*b^5*cos(d*x^3 + 2*c)*sin(c) +
 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c
)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*
sin(c)^3)*cos(d*x^3 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(4*
a^3*b^3 - 3*a*b^5)*cos(c)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*s
in(c) + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 3*((8
*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(
c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^
2 + 3*(b^6*cos(d*x^3 + 2*c)^4 - 4*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^
4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^
2*sin(c)^2 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*
cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*(3*(4*a^3*b^3
 - 3*a*b^5)*cos(c)^2*sin(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2
*c))*sin(d*x^3 + 2*c)^2 - 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin
(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 2
0*a^3*b^3 + 5*a*b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 6*(a*b^5*cos(d*x^3 + 2*c)
^4*cos(c) + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6
)*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*co
s(c)^3*sin(c)^2 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4
*a^3*b^3 - 3*a*b^5)*cos(c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos
(d*x^3 + 2*c)^2 - 4*((8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4
*b^2 - 8*a^2*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)
 - 2*(3*b^5*cos(c)*sin(d*x^3 + 2*c)^5 - 3*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)
*cos(c)^4*sin(c)^2 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 +
(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*sin(c)^6 + 3*(a...

Fricas [A]

time = 0.38, size = 208, normalized size = 4.08

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2+2(a\cos(dx^3+c)\sin(dx^3+c)+b\cos(dx^3+c))\sqrt{-a^2+b^2}}{b^2\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2}\right)}{6(a^2-b^2)d}, -\frac{\arctan\left(-\frac{a\sin(dx^3+c)+b}{\sqrt{a^2-b^2}\cos(dx^3+c)}\right)}{3\sqrt{a^2-b^2}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^
3 + c) - a^2 - b^2 + 2*(a*cos(d*x^3 + c)*sin(d*x^3 + c) + b*cos(d*x^3 + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2
))/((a^2 - b^2)*d), -1/3*arctan(-(a*sin(d*x^3 + c) + b)/(sqrt(a^2 - b^2)*co
s(d*x^3 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(41) = 82$.
time = 7.88, size = 212, normalized size = 4.16

$$\begin{cases} \frac{\tilde{\infty}x^3}{\sin(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)\right)}{3bd} & \text{for } a=0 \\ \frac{x^3}{3(a+b\sin(c))} & \text{for } d=0 \\ \frac{2\sqrt{b^2}}{3b^2d\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)-3bd\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{3b^2d\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)+3bd\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{3d\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{3d\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((zoo*x**3/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 +
 d*x**3/2))/(3*b*d), Eq(a, 0)), (x**3/(3*(a + b*sin(c))), Eq(d, 0)), (2*sqr
t(b**2)/(3*b**2*d*tan(c/2 + d*x**3/2) - 3*b*d*sqrt(b**2)), Eq(a, -sqrt(b**2
))), (-2*sqrt(b**2)/(3*b**2*d*tan(c/2 + d*x**3/2) + 3*b*d*sqrt(b**2)), Eq(a
, sqrt(b**2))), (log(tan(c/2 + d*x**3/2) + b/a - sqrt(-a**2 + b**2)/a)/(3*d
*sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**3/2) + b/a + sqrt(-a**2 + b**2)/a
)/(3*d*sqrt(-a**2 + b**2)), True))
```

Giac [A]

time = 4.85, size = 64, normalized size = 1.25

$$\frac{2 \left(\pi \left\lfloor \frac{dx^3+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{3\sqrt{a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B]
time = 6.52, size = 136, normalized size = 2.67

$$-\frac{\ln\left(-x^2\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}-\frac{2\,x^2\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)-\ln\left(-x^2\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}+\frac{2\,x^2\,\left(b\,1\mathrm{i}+a\,\mathrm{e}^{d\,x^3\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{3\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sin(c + d*x^3)),x)
```

```
[Out] -(log(- x^2*exp(d*x^3*1i)*exp(c*1i)*2i - (2*x^2*(b*1i + a*exp(d*x^3*1i)*exp
(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2))) - log((2*x^2*(b*1i + a*exp(d*x^3*1i
)*exp(c*1i)))/((a + b)^(1/2)*(b - a)^(1/2)) - x^2*exp(d*x^3*1i)*exp(c*1i)*2
i))/(3*d*(a + b)^(1/2)*(b - a)^(1/2))
```

3.83 $\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^3])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*sin(d*x^3 + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + b\sin\left(c + dx^{3}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x*(a + b*sin(c + d*x**3))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x*(a + b*sin(c + d*x^3))), x)
```

3.84 $\int \frac{1}{x^4(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^4\,(a+b\sin(c+dx^3))},x\right)$$

```
[Out] Unintegrable(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\,(a+b\sin(c+dx^3))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(a + b*Sin[c + d*x^3])),x]
[Out] Defer[Int][1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x^4\,(a+b\sin(c+dx^3))}\,dx = \int \frac{1}{x^4\,(a+b\sin(c+dx^3))}\,dx$$

Mathematica [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\,(a+b\sin(c+dx^3))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\,(a+b\sin(d\,x^3+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^4*sin(d*x^3 + c) + a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \sin(c + dx^3))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x**4*(a + b*sin(c + d*x**3))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^4\, (a + b\, \sin(d\, x^3 + c))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x^4*(a + b*sin(c + d*x^3))), x)
```

3.85 $\int \frac{x}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{x}{a+b\sin\left(c+dx^3\right)},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(a + b*Sin[c + d*x^3]),x]
[Out] Defer[Int][x/(a + b*Sin[c + d*x^3]), x]
```

Rubi steps

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx = \int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(a + b*Sin[c + d*x^3]),x]
[Out] Integrate[x/(a + b*Sin[c + d*x^3]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\sin\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^3+c)),x)
[Out] int(x/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(x/(b*sin(d*x^3 + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\sin\left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**3+c)),x)
[Out] Integral(x/(a + b*sin(c + d*x**3)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{a + b\,\sin\left(d\,x^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sin(c + d*x^3)),x)
```

```
[Out] int(x/(a + b*sin(c + d*x^3)), x)
```

3.86 $\int \frac{1}{x^2(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^3])),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx = \int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(d\,x^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*sin(d*x^3 + c) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \sin \left(c + dx^3\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**3))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\, \left(a + b\, \sin \left(d\, x^3 + c\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^3))), x)
```

3.87 $\int \frac{1}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{a+b\sin\left(c+dx^3\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^3])^(-1),x]
[Out] Defer[Int][(a + b*Sin[c + d*x^3])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx = \int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d*x^3])^(-1), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(1/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*sin(d*x^3 + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin\left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x**3)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{a + b\,\sin\left(d\,x^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x^3)),x)
```

```
[Out] int(1/(a + b*sin(c + d*x^3)), x)
```

3.88 $\int \frac{1}{x^3(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^3])),x]
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx = \int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(d\,x^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^3+c)),x)
[Out] int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*sin(d*x^3 + c) + a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + b \sin \left(c + dx^3\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**3+c)),x)
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**3))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*sin(c + d*x^3))),x)
[Out] int(1/(x^3*(a + b*sin(c + d*x^3))), x)
```

3.89 $\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=324

$$-\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}+\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3\left(a^2-b^2\right)d^2}-\frac{a\mathrm{Li}_2\left(\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right.}{3\left(a^2-b^2\right)^{3/2}d^2}$$

```
[Out] -1/3*ln(a+b*sin(d*x^3+c))/(a^2-b^2)/d^2-1/3*I*a*x^3*ln(1-I*b*exp(I*(d*x^3+c
))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+1/3*I*a*x^3*ln(1-I*b*exp(I*(d*x^3
+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-1/3*a*polylog(2,I*b*exp(I*(d*x^
3+c))/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/3*a*polylog(2,I*b*exp(I*(d
*x^3+c))/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+1/3*b*x^3*cos(d*x^3+c)/(a
^2-b^2)/d/(a+b*sin(d*x^3+c))
```

Rubi [A]
time = 0.40, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3460, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\left(a^2-b^2\right)^{3/2}}+\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d^2\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3d^2\left(a^2-b^2\right)}-\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{bx^3\cos\left(c+dx^3\right)}{3d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((-1/3*I)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/((a
^2 - b^2)^(3/2)*d) + ((I/3)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt
[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^3]]/(3*(a^2 -
b^2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(
3*(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[
a^2 - b^2])])/(3*(a^2 - b^2)^(3/2)*d^2) + (b*x^3*Cos[c + d*x^3])/(3*(a^2 -
b^2)*d*(a + b*Sin[c + d*x^3]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{x^5}{(a+b\sin(c+dx^3))^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+b\sin(c+dx))^2}\,dx, x, x^3\right)$$

$$= \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} + \frac{a\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^3\right)}{3(a^2-b^2)} - \frac{b\text{Subst}\,\cdots}{\cdots}$$

$$= \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} + \frac{(2a)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^3\cdots\right.}{3(a^2-b^2)}$$

$$= -\frac{\log(a+b\sin(c+dx^3))}{3(a^2-b^2)d^2} + \frac{bx^3\cos(c+dx^3)}{3(a^2-b^2)d(a+b\sin(c+dx^3))} - \frac{(2iab)\text{Subst}\Big(\cdots}{\cdots}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin\cdots}{3(a^2-\cdots}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin\cdots}{3(a^2-\cdots}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a-\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{3(a^2-b^2)^{3/2}d} - \frac{\log(a+b\sin\cdots}{3(a^2-\cdots}$$

Mathematica [A]
time = 0.65, size = 302, normalized size = 0.93

$$\frac{-\frac{iadx^3\log\left(1+\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{iadx^3\log\left(1-\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{\log(a+b\sin(c+dx^3))}{a^2-b^2} - \frac{a\text{Li}_2\left(-\frac{ibe^{i(c+dx^3)}}{-a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\text{Li}_2\left(\frac{ibe^{i(c+dx^3)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{bdx^3\cos(c+dx^3)}{(a^2-b^2)(a+b\sin(c+dx^3))}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (((-I)*a*d*x^3*Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (I*a*d*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 -
 b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^3]]/(a^2 - b^2) - (a*Pol
yLog[2, ((-I)*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)^(3/
2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])])/(a^2 - b
^2)^(3/2) + (b*d*x^3*Cos[c + d*x^3])/((a^2 - b^2)*(a + b*Sin[c + d*x^3])))/
(3*d^2)
```

Maple [**F**]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1509 vs. 2(274) = 548.
time = 0.61, size = 1509, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2*b - b^3)*d*x^3*cos(d*x^3 + c) + (I*a*b^2*sin(d*x^3 + c) + I*a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (
b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) +
 (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((I*a*cos(
d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqr
```

```
t(-(a^2 - b^2)/b^2) - b)/b + 1) + (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)*sqrt(
-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*
x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b + 1) + (I*a*b^
2*sin(d*x^3 + c) + I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog((-I*a*cos(d*x^3 +
c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2
- b^2)/b^2) - b)/b + 1) - (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*
sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3 + c) - a*sin(d*x
^3 + c) + (b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) -
b)/b) + (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sq
rt(-(a^2 - b^2)/b^2)*log(-(I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d
*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) - (a^2*b*d*x
^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^
2)*log(-(-I*a*cos(d*x^3 + c) - a*sin(d*x^3 + c) + (b*cos(d*x^3 + c) - I*b*s
in(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) - b)/b) + (a^2*b*d*x^3 + a^2*b*c + (a
*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(-(-I*a*cos
(d*x^3 + c) - a*sin(d*x^3 + c) - (b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sq
rt(-(a^2 - b^2)/b^2) - b)/b) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c)
+ (a*b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*
x^3 + c) + 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - (a^
3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c) + (a*b^2*c*sin(d*x^3 + c) + a^2*b*
c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) +
2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^
3 + c) - (a*b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*
b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*
a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c) - (a*b^2*c*sin(d*x^3 + c)
+ a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^
3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b - 2*a^2*b^3 + b^5)*d^
2*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a + b\,\sin\left(d\,x^3 + c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*sin(c + d*x^3))^2,x)
[Out] int(x^5/(a + b*sin(c + d*x^3))^2, x)
```

3.90 $\int \frac{x^2}{(a+b\sin(c+dx^3))^2}\,dx$

Optimal. Leaf size=94

$$\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx^3)\right)}{\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}+\frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}$$

```
[Out] 2/3*a*arctan((b+a*tan(1/2*d*x^3+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+
1/3*b*cos(d*x^3+c)/(a^2-b^2)/d/(a+b*sin(d*x^3+c))
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3460, 2743, 12, 2739, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx^3)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^3\right)}{3d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*(a^2 - b^2)^(3/
2)*d) + (b*Cos[c + d*x^3])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x^3]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+b\sin(c+dx))^2}\,dx,x,x^3\right)\\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}+\frac{\text{Subst}\left(\int \frac{a}{a+b\sin(c+dx)}\,dx,x,x^3\right)}{3\left(a^2-b^2\right)}\\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}+\frac{a\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx,x,x^3\right)}{3\left(a^2-b^2\right)}\\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}+\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}\left(c+d\right.\right.\right.}{3\left(a^2-b^2\right)d}\\
&= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}-\frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\,\text{ta}\right.}{3\left(a^2-b^2\right)d}\\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}+\frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}
\end{aligned}
$$

Mathematica [A]

time = 0.14, size = 91, normalized size = 0.97

$$\frac{\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b\cos(c+dx^3)}{a+b\sin(c+dx^3)}}{3(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (b*Cos[c + d*x^3])/(a + b*Sin[c + d*x^3]))/(3*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.13, size = 131, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{a^2-b^2}}{a\left(\tan^2\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{3d}$
default	$\frac{\frac{\frac{2b^2\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{a^2-b^2}}{a\left(\tan^2\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{3d}$
risch	$\frac{\frac{2ib}{3}+\frac{2a\,e^{i(dx^3+c)}}{3}}{(a^2-b^2)d\left(b\,e^{2i(dx^3+c)}-b+2ia\,e^{i(dx^3+c)}\right)}-\frac{a\ln\left(e^{i(dx^3+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{3\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{a\ln\left(e^{i(dx^3+c)}+\frac{ia\sqrt{\ }}{}\right.}{3\sqrt{-a^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^3+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*(b^2/a/(a^2-b^2)*tan(1/2*d*x^3+1/2*c)+b/(a^2-b^2))/(a*tan(1/2*d*x^
3+1/2*c)^2+2*b*tan(1/2*d*x^3+1/2*c)+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*
tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.40, size = 366, normalized size = 3.89

$$\left[\frac{\left(ab\sin\left(dx^3+c\right)+a^2\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{(2a^2-b^2)\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2-2\,(a\cos(dx^3+c)\sin(dx^3+c)+b\cos(dx^3+c))\sqrt{-a^2+b^2}}{b^2\cos(dx^3+c)^2-2ab\sin(dx^3+c)-a^2-b^2}\right)+2\,(a^2b-b^3)\cos\left(dx^3+c\right)}{6\left((a^4b-2\,a^2b^3+b^5)d\sin\left(dx^3+c\right)+(a^5-2\,a^3b^2+ab^4)d\right)},-\frac{\left(ab\sin\left(dx^3+c\right)+a^2\right)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(dx^3+c)+b}{\sqrt{a^2-b^2}\cos(dx^3+c)}\right)-(a^2b-b^3)\cos\left(dx^3+c\right)}{3\left((a^4b-2\,a^2b^3+b^5)d\sin\left(dx^3+c\right)+(a^5-2\,a^3b^2+ab^4)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*sin(d*x^3 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d
*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2 - 2*(a*cos(d*x^3 + c)*sin(d*
x^3 + c) + b*cos(d*x^3 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*
b*sin(d*x^3 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^3 + c))/((a^4*b -
2*a^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/3*((a*
b*sin(d*x^3 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^3 + c) + b)/(sqr
t(a^2 - b^2)*cos(d*x^3 + c))) - (a^2*b - b^3)*cos(d*x^3 + c))/((a^4*b - 2*a
^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.22, size = 146, normalized size = 1.55

$$\frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{3\,(a^2d-b^2d)\sqrt{a^2-b^2}}+\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+ab\right)}{3\,(a^3d-ab^2d)\left(a\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx^3+\frac{1}{2}\,c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + 2/3*(b^
2*tan(1/2*d*x^3 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^3 + 1/2*c
)^2 + 2*b*tan(1/2*d*x^3 + 1/2*c) + a))
```

Mupad [B]
time = 4.95, size = 186, normalized size = 1.98

$$\frac{\frac{2\,b}{a^2-b^2}+\frac{2\,b^2\,\mathrm{tan}\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{a\,\left(a^2-b^2\right)}}{d\,\left(3\,a\,{\mathrm{tan}\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}^2+6\,b\,\mathrm{tan}\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)+3\,a\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{3\,\left(a^2-b^2\right)\,\left(\frac{2\,a^2\,\mathrm{tan}\left(\frac{d\,x^3}{2}+\frac{c}{2}\right)}{3\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a\,\left(3\,a^2\,b-3\,b^3\right)}{9\,{\left(a+b\right)}^{3/2}\,\left(a^2-b^2\right)\,{\left(a-b\right)}^{3/2}}\right)}{2\,a}\right)}{3\,d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] ((2*b)/(a^2 - b^2) + (2*b^2*tan(c/2 + (d*x^3)/2))/(a*(a^2 - b^2)))/(d*(3*a
+ 3*a*tan(c/2 + (d*x^3)/2)^2 + 6*b*tan(c/2 + (d*x^3)/2))) + (2*a*atan((3*(a
^2 - b^2)*((2*a^2*tan(c/2 + (d*x^3)/2))/(3*(a + b)^(3/2)*(a - b)^(3/2)) + (
2*a*(3*a^2*b - 3*b^3))/(9*(a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/(2*a))
)/(3*d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.91 $\int \frac{1}{x(a+b\sin(c+dx^3))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

`[Out] Unintegrable(1/x/(a+b*sin(d*x^3+c))^2,x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/(x*(a + b*Sin[c + d*x^3])^2),x]`

`[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^3])^2), x]`

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 6.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/(x*(a + b*Sin[c + d*x^3])^2),x]`

`[Out] Integrate[1/(x*(a + b*Sin[c + d*x^3])^2), x]`

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) + 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^3 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^3*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^3*sin(2*c))*sin
(2*d*x^3))*integrate(-2*(b^4*cos(2*c)*sin(2*d*x^3) + b^4*cos(2*d*x^3)*sin(2
*c) - 2*(a^3*b - a*b^3)*cos(d*x^3)*cos(c) + 2*(a^3*b - a*b^3)*sin(d*x^3)*si
n(c) + (a^3*b*d*x^3*sin(d*x^3 + c) - a^3*b*cos(d*x^3 + c))*cos(2*d*x^3 + 2*
c) + (a^3*b - a*b^3 + (a*b^3*d*x^3*sin(2*c) + a*b^3*cos(2*c))*cos(2*d*x^3)
- 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - (a^4 - a^2*b^2)*sin(c))*cos(d*x^3) + (a
*b^3*d*x^3*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x^3) + 2*((a^4 - a^2*b^2)*d*x
^3*sin(c) + (a^4 - a^2*b^2)*cos(c))*sin(d*x^3))*cos(d*x^3 + c) - (a^3*b*d*x
^3*cos(d*x^3 + c) + a^3*b*sin(d*x^3 + c) + a^2*b^2)*sin(2*d*x^3 + 2*c) - ((
a^3*b - a*b^3)*d*x^3 + (a*b^3*d*x^3*cos(2*c) - a*b^3*sin(2*c))*cos(2*d*x^3)
 + 2*((a^4 - a^2*b^2)*d*x^3*sin(c) + (a^4 - a^2*b^2)*cos(c))*cos(d*x^3) - (
a*b^3*d*x^3*sin(2*c) + a*b^3*cos(2*c))*sin(2*d*x^3) + 2*((a^4 - a^2*b^2)*d*
x^3*cos(c) - (a^4 - a^2*b^2)*sin(c))*sin(d*x^3))*sin(d*x^3 + c))/(a^4*b^2*d
*x^4*cos(2*d*x^3 + 2*c)^2 + a^4*b^2*d*x^4*sin(2*d*x^3 + 2*c)^2 + (b^6*cos(2
*c)^2 + b^6*sin(2*c)^2)*d*x^4*cos(2*d*x^3)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^
4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^3)^2 + (b
^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^4*sin(2*d*x^3)^2 + 4*(a^5*b - 2*a^3*b^3
 + a*b^5)*d*x^4*cos(c)*sin(d*x^3) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2
 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^3)^2 + 4*(a^5*b - 2*
a^3*b^3 + a*b^5)*d*x^4*cos(d*x^3)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^
```

```
4 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*si
n(c))*d*x^4*cos(d*x^3) - (a^2*b^4 - b^6)*d*x^4*cos(2*c) - 2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^3))*
cos(2*d*x^3) - 2*(a^2*b^4*d*x^4*cos(2*d*x^3)*cos(2*c) - a^2*b^4*d*x^4*sin(2
*d*x^3)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^3) + 2*(a^5*b -
 a^3*b^3)*d*x^4*cos(d*x^3)*sin(c) + (a^4*b^2 - a^2*b^4)*d*x^4)*cos(2*d*x^3
+ 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*
c)*sin(c))*d*x^4*cos(d*x^3) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b
^3 - a*b^5)*cos(2*c)*sin(c))*d*x^4*sin(d*x^3) + (a^2*b^4 - b^6)*d*x^4*sin(2
*c))*sin(2*d*x^3) - 2*(a^2*b^4*d*x^4*cos(2*c)*sin(2*d*x^3) + a^2*b^4*d*x^4*
cos(2*d*x^3)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^3)*cos(c) + 2*(a^
5*b - a^3*b^3)*d*x^4*sin(d*x^3)*sin(c))*sin(2*d*x^3 + 2*c)), x) + 2*(2*a^2*
cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(2*d*x^3)*sin(2*c) -
 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a
^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^
2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c
)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*
d*x^3*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin
(c)^2)*d*x^3*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^
2*b^2 - b^4)*d*x^3 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3
)*cos(2*c)*sin(c))*d*x^3*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((
a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin
(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*
b^3)*sin(2*c)*sin(c))*d*x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
 - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^
3*sin(2*c))*sin(2*d*x^3))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x*cos(d*x^3 + c)^2 - 2*a*b*x*sin(d*x^3 + c) - (a^2 + b^2)*
x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + b\sin\left(c + dx^3\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**3))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,{\left(a+b\,\sin\left(d\,x^3+c\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x*(a + b*sin(c + d*x^3))^2), x)
```

3.92 $\int \frac{1}{x^4(a+b\sin(c+dx^3))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
[Out] Defer[Int][1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 8.08, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) + 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^6*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^6*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^6*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^6 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^6*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^6*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^6*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin
(2*d*x^3))*integrate(-2*(2*b^4*cos(2*c)*sin(2*d*x^3) + 2*b^4*cos(2*d*x^3)*s
in(2*c) - 4*(a^3*b - a*b^3)*cos(d*x^3)*cos(c) + 4*(a^3*b - a*b^3)*sin(d*x^3
)*sin(c) + (a^3*b*d*x^3*sin(d*x^3 + c) - 2*a^3*b*cos(d*x^3 + c))*cos(2*d*x^
3 + 2*c) + (2*a^3*b - 2*a*b^3 + (a*b^3*d*x^3*sin(2*c) + 2*a*b^3*cos(2*c))*c
os(2*d*x^3) - 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - 2*(a^4 - a^2*b^2)*sin(c))*c
os(d*x^3) + (a*b^3*d*x^3*cos(2*c) - 2*a*b^3*sin(2*c))*sin(2*d*x^3) + 2*((a^
4 - a^2*b^2)*d*x^3*sin(c) + 2*(a^4 - a^2*b^2)*cos(c))*sin(d*x^3))*cos(d*x^3
 + c) - (a^3*b*d*x^3*cos(d*x^3 + c) + 2*a^3*b*sin(d*x^3 + c) + 2*a^2*b^2)*s
in(2*d*x^3 + 2*c) - ((a^3*b - a*b^3)*d*x^3 + (a*b^3*d*x^3*cos(2*c) - 2*a*b^
3*sin(2*c))*cos(2*d*x^3) + 2*((a^4 - a^2*b^2)*d*x^3*sin(c) + 2*(a^4 - a^2*b
^2)*cos(c))*cos(d*x^3) - (a*b^3*d*x^3*sin(2*c) + 2*a*b^3*cos(2*c))*sin(2*d*
x^3) + 2*((a^4 - a^2*b^2)*d*x^3*cos(c) - 2*(a^4 - a^2*b^2)*sin(c))*sin(d*x^
3))*sin(d*x^3 + c))/(a^4*b^2*d*x^7*cos(2*d*x^3 + 2*c)^2 + a^4*b^2*d*x^7*sin
(2*d*x^3 + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^7*cos(2*d*x^3)^2
+ 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin
(c)^2)*d*x^7*cos(d*x^3)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*d*x^7*sin(2*d
*x^3)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^7*cos(c)*sin(d*x^3) + 4*((a^6 -
 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*d*x^
7*sin(d*x^3)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^7*cos(d*x^3)*sin(c) + (a
```

```
^4*b^2 - 2*a^2*b^4 + b^6)*d*x^7 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) -
 (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^7*cos(d*x^3) - (a^2*b^4 - b^6)*d*x^
7*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2
*c)*sin(c))*d*x^7*sin(d*x^3))*cos(2*d*x^3) - 2*(a^2*b^4*d*x^7*cos(2*d*x^3)*
cos(2*c) - a^2*b^4*d*x^7*sin(2*d*x^3)*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*x^7*
cos(c)*sin(d*x^3) + 2*(a^5*b - a^3*b^3)*d*x^7*cos(d*x^3)*sin(c) + (a^4*b^2
- a^2*b^4)*d*x^7)*cos(2*d*x^3 + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos
(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^7*cos(d*x^3) + 2*((a^3*b^3 - a
*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^7*sin(d*x^3)
 + (a^2*b^4 - b^6)*d*x^7*sin(2*c))*sin(2*d*x^3) - 2*(a^2*b^4*d*x^7*cos(2*c)
*sin(2*d*x^3) + a^2*b^4*d*x^7*cos(2*d*x^3)*sin(2*c) - 2*(a^5*b - a^3*b^3)*d
*x^7*cos(d*x^3)*cos(c) + 2*(a^5*b - a^3*b^3)*d*x^7*sin(d*x^3)*sin(c))*sin(2
*d*x^3 + 2*c)), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3)
 + a*b*cos(2*d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((
a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^6*cos(2*d*x^3)^
2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*cos(d*x^3
)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^6*sin(2
*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*
cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)
*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^6 + 2*(2*((a^3*b - a*b^3)*co
s(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^6*cos(d*x^3) - (a^2*b^
2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b
^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)
*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*cos(d*x^3) + 2*((
a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^6*sin
(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin(2*d*x^3))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^4*cos(d*x^3 + c)^2 - 2*a*b*x^4*sin(d*x^3 + c) - (a^2 + b
^2)*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + b \sin \left(c + dx^3\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x**4*(a + b*sin(c + d*x**3))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^4\,{\left(a+b\,\sin\left(d\,x^3+c\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x^4*(a + b*sin(c + d*x^3))^2), x)
```

3.93 $\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(a + b*Sin[c + d*x^3])^2,x]
[Out] Defer[Int][x/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 4.52, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(a + b*Sin[c + d*x^3])^2,x]
[Out] Integrate[x/(a + b*Sin[c + d*x^3])^2, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-x/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b\sin\left(c + dx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x/(a + b*sin(c + d*x**3))**2, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int(x/(a + b*sin(c + d*x^3))^2, x)
```

3.94 $\int \frac{1}{x^2(a+b\sin(c+dx^3))^2} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 7.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos(d*x^3 + c)^2 - 2*a*b*x^2*sin(d*x^3 + c) - (a^2 + b
^2)*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \sin \left(c + dx^3\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**3))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*sin(c + d*x^3))^2),x)
[Out] int(1/(x^2*(a + b*sin(c + d*x^3))^2), x)
```

3.95 $\int \frac{1}{(a+b\sin(c+dx^3))^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^3])^(-2),x]
[Out] Defer[Int][(a + b*Sin[c + d*x^3])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 5.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d*x^3])^(-2), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d
*x^3) + 2*b^2*cos(2*d*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) - (2*a*b - (3
*a*b*d*x^3*sin(2*c) + 2*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c)
- 2*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 2*a*b*sin(2*c))*sin(2*
d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c) +
 (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 2*a*b*sin(2*c))*cos(2*d*x^3) + 2*(3
*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 2*a*
b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 2*a^2*sin(c))*sin(d*x^3)
)*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2
)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(
c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*si
n(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(c)*sin(d*x^3)
+ 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*sin(d*x^3)^
2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^3 + 2*(
2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3
*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*
cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*x^3))*cos(2*d*x^3) +
2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*
x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2
```

```
*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^3*sin(2*c))*sin(2*d*x^3)
), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(2*
d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4
)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 -
 a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b
^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4
*(a^3*b - a*b^3)*d*x^2*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a
^4 - a^2*b^2)*sin(c)^2)*d*x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*
x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
 - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^
2*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*
sin(c))*d*x^2*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos
(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3
)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^
2*b^2 - b^4)*d*x^2*sin(2*c))*sin(2*d*x^3))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sin \left(c + dx^{3}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**3+c))**2,x)
[Out] Integral((a + b*sin(c + d*x**3))**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+b\,\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x^3))^2,x)
[Out] int(1/(a + b*sin(c + d*x^3))^2, x)
```

3.96 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 7.74, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sin\left(d\,x^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^5*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(10*a*b*cos(d*x^3)*cos(c) + 5*b^2*cos(2*c)*sin(2*
d*x^3) + 5*b^2*cos(2*d*x^3)*sin(2*c) - 10*a*b*sin(d*x^3)*sin(c) - (5*a*b -
(3*a*b*d*x^3*sin(2*c) + 5*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c
) - 5*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 5*a*b*sin(2*c))*sin(
2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c)
 + (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 5*a*b*sin(2*c))*cos(2*d*x^3) + 2*
(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 5*
a*b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 5*a^2*sin(c))*sin(d*x^
3))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)
^2)*d*x^6*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*si
n(c)^2)*d*x^6*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*
sin(2*c)^2)*d*x^6*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*sin(d*x^3
) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*sin(d*x^3
)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^6 + 2
*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x
^6*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c
)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*d*x^3)
+ 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*
d*x^6*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
```

```
(2*c)*sin(c))*d*x^6*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin(2*d*x^
3)), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(
2*d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b
^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4
 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2
*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 +
 4*(a^3*b - a*b^3)*d*x^5*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 +
(a^4 - a^2*b^2)*sin(c)^2)*d*x^5*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(
d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*
c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*
x^5*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c
)*sin(c))*d*x^5*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*c
os(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b
^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (
a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin(2*d*x^3))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^3*cos(d*x^3 + c)^2 - 2*a*b*x^3*sin(d*x^3 + c) - (a^2 + b
^2)*x^3), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + b \sin \left(c + dx^3\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**3))**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3\,{\left(a+b\,\sin\left(d\,x^3+c\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*sin(c + d*x^3))^2),x)
```

```
[Out] int(1/(x^3*(a + b*sin(c + d*x^3))^2), x)
```

3.97 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left((ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p,x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx = \int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a+b\sin\left(d\,x^3+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**p,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="giac")
[Out] integrate((x*e)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e\,x\right)}^m\,{\left(a+b\,\sin\left(d\,x^3+c\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^3))^p, x)
```

3.98 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\,dx$

Optimal. Leaf size=442

$$\frac{a(2a^2+3b^2)\,(ex)^{1+m}}{2e(1+m)}+\frac{ib(4a^2+b^2)\,e^{ic}(ex)^{1+m}\,(-idx^3)^{\frac{1}{3}(-1-m)}\,\Gamma\left(\frac{1+m}{3},-idx^3\right)}{8e}-\frac{ib(4a^2+b^2)\,e^{-ic}(ex)^{1+m}\,(idx}{8e}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*(e*x)^(1+m)/e/(1+m)+1/8*I*b*(4*a^2+b^2)*exp(I*c)*(e*x)^
(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/8*I*b*(4*a^2+b^
2)*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c)+2
^(-7/3-1/3*m)*a*b^2*exp(2*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/
3+1/3*m,-2*I*d*x^3)/e+2^(-7/3-1/3*m)*a*b^2*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*
m)*GAMMA(1/3+1/3*m,2*I*d*x^3)/e/exp(2*I*c)-1/8*I*3^(-4/3-1/3*m)*b^3*exp(3*I
*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-3*I*d*x^3)/e+1/8*I
*3^(-4/3-1/3*m)*b^3*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,3*I*
d*x^3)/e/exp(3*I*c)
```

Rubi [A]

time = 0.28, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3484, 6, 3471, 2250, 3470}

$$\frac{ibe^{ic}(4a^2+b^2)\,(-idx^3)^{\frac{1}{3}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{8e}-\frac{ibe^{-ic}(4a^2+b^2)\,(idx^3)^{\frac{1}{3}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{8e}+\frac{ab^2e^{2ic}2^{-\frac{m}{3}-\frac{7}{3}}(-idx^3)^{\frac{1}{3}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-2idx^3\right)}{e}+\frac{ab^2e^{-2ic}2^{-\frac{m}{3}-\frac{7}{3}}(idx^3)^{\frac{1}{3}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},2idx^3\right)}{e}-\frac{ib^3e^{3ic}3^{-\frac{m}{3}-\frac{4}{3}}(-idx^3)^{\frac{1}{3}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-3idx^3\right)}{8e}+\frac{ib^3e^{-3ic}3^{-\frac{m}{3}-\frac{4}{3}}(idx^3)^{\frac{1}{3}(-m-1)}\,(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},3idx^3\right)}{8e}+\frac{a(2a^2+3b^2)\,(ex)^{m+1}}{2e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/8)*b*(4*a^2 + b^2)*E^
(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])
/e - ((I/8)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 +
 m)/3, I*d*x^3])/(e*E^(I*c)) + (2^(-7/3 - m/3)*a*b^2*E^((2*I)*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3])/e + (2^(-7/3
- m/3)*a*b^2*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*
x^3])/(e*E^((2*I)*c)) - ((I/8)*3^(-4/3 - m/3)*b^3*E^((3*I)*c)*(e*x)^(1 + m)
*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3])/e + ((I/8)*3^(-4
/3 - m/3)*b^3*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (3*I)*d
*x^3])/(e*E^((3*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3\, dx &= \int \left(a^3(ex)^m+\frac{3}{2}ab^2(ex)^m-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right)+3a^2b(ex)^m \mathrm{s}\right.\\
&= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right)+3a^2b(ex)^m \sin\right.\\
&= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m-\frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right)+\left(3a^2b+\frac{3b^3}{4}\right.\right.\\
&= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)}-\frac{1}{2}\left(3ab^2\right)\int (ex)^m\cos\left(2c+2dx^3\right)\, dx-\frac{1}{4}b^3\int (\\
&= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)}-\frac{1}{4}\left(3ab^2\right)\int e^{-2ic-2idx^3}(ex)^m\, dx-\frac{1}{4}\left(3ab^2\right)\int e\\
&= \frac{a(2a^2+3b^2)(ex)^{1+m}}{2e(1+m)}+\frac{ib(4a^2+b^2)e^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3}\right.}{8e}
\end{aligned}$$

Mathematica [A]
time = 10.86, size = 451, normalized size = 1.02

$$\frac{1}{24}ix(ex)^m\left(3b\left(4a^2+b^2\right)e^{ic}\left(-idx^3\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-idx^3\right)+\frac{-24ia^3dx^3-36iab^2dx^3+12ia^2be^{-ic}(1+m)\left(idx^3\right)^{\frac{2}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)+3ib^3e^{-ic}(1+m)\left(idx^3\right)^{\frac{2}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)+3\ 2^{\frac{2}{3}-\frac{m}{3}}ab^2e^{2ic}(1+m)\left(-idx^3\right)^{\frac{2}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-2idx^3\right)-3\ 2^{\frac{2}{3}-\frac{m}{3}}ab^2e^{-2ic}(1+m)\left(idx^3\right)^{\frac{2}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},2idx^3\right)-i3^{-\frac{1}{3}-\frac{m}{3}}b^3e^{3ic}(1+m)\left(-idx^3\right)^{\frac{2}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},-3idx^3\right)-i3^{-\frac{1}{3}-\frac{m}{3}}b^3e^{-3ic}(1+m)\left(idx^3\right)^{\frac{2}{3}-\frac{m}{3}}\Gamma\left(\frac{1+m}{3},3idx^3\right)}{d(1+m)x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

```
[Out] (I/24)*x*(e*x)^m*(3*b*(4*a^2 + b^2)*E^(I*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma
[(1 + m)/3, (-I)*d*x^3] + ((-24*I)*a^3*d*x^3 - (36*I)*a*b^2*d*x^3 + ((12*I)
*a^2*b*(1 + m)*(I*d*x^3)^(2/3 - m/3)*Gamma[(1 + m)/3, I*d*x^3])/E^(I*c) + (
(3*I)*b^3*(1 + m)*(I*d*x^3)^(2/3 - m/3)*Gamma[(1 + m)/3, I*d*x^3])/E^(I*c)
+ 3*2^(2/3 - m/3)*a*b^2*E^((2*I)*c)*(1 + m)*((-I)*d*x^3)^(2/3 - m/3)*Gamma[
(1 + m)/3, (-2*I)*d*x^3] - (3*2^(2/3 - m/3)*a*b^2*(1 + m)*(I*d*x^3)^(2/3 -
m/3)*Gamma[(1 + m)/3, (2*I)*d*x^3])/E^((2*I)*c) - I*3^(-1/3 - m/3)*b^3*E^((
3*I)*c)*(1 + m)*((-I)*d*x^3)^(2/3 - m/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3] - (
I*3^(-1/3 - m/3)*b^3*(1 + m)*(I*d*x^3)^(2/3 - m/3)*Gamma[(1 + m)/3, (3*I)*d
*x^3])/E^((3*I)*c))/(d*(1 + m)*x^3))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(e x \right)^{m} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="maxima")
```

```
[Out] (x*e)^(m + 1)*a^3*e^(-1)/(m + 1) + 1/8*(12*a*b^2*x*e^(m*log(x) + m) + 3*((4
*a^2*b + b^3)*m*sin(c) + (4*a^2*b + b^3)*sin(c))*e^m*integrate(x^m*cos(d*x^
3), x) + 3*(4*a^2*b + b^3 + (4*a^2*b + b^3)*m)*e^m*integrate(x^m*sin(d*x^3
+ c), x) + 3*((4*a^2*b + b^3)*m*cos(c) + (4*a^2*b + b^3)*cos(c))*e^m*integr
ate(x^m*sin(d*x^3), x) - 12*(a*b^2*m + a*b^2)*integrate(cos(2*d*x^3 + 2*c)*
e^(m*log(x) + m), x) - 2*(b^3*m + b^3)*integrate(e^(m*log(x) + m)*sin(3*d*x
^3 + 3*c), x))/(m + 1)
```

Fricas [A]
time = 0.13, size = 302, normalized size = 0.68

$$\frac{36 \, {\left(2 \, a^{3} + 3 \, a b^{2}\right)} \left(x e\right)^{m} d x + {\left(b^{3} m + b^{3}\right)} e^{\left(-\frac{1}{3} \, {\left(m - 2\right)} \log\left(3 i \, d e^{\left(-3\right)}\right) - 3 i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, 3 i \, d x^{3}\right) - 9 i \, {\left(a b^{2} m + a b^{2}\right)} e^{\left(-\frac{1}{3} \, {\left(m - 2\right)} \log\left(2 i \, d e^{\left(-3\right)}\right) - 2 i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, 2 i \, d x^{3}\right) - 9 \, {\left(4 \, a^{2} b + b^{3} + {\left(4 \, a^{2} b + b^{3}\right)} m\right)} e^{\left(-\frac{1}{3} \, {\left(m - 2\right)} \log\left(i \, d e^{\left(-3\right)}\right) - i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, i \, d x^{3}\right) - 9 \, {\left(4 \, a^{2} b + b^{3} + {\left(4 \, a^{2} b + b^{3}\right)} m\right)} e^{\left(-\frac{1}{3} \, {\left(m - 2\right)} \log\left(-i \, d e^{\left(-3\right)}\right) + i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, -i \, d x^{3}\right) + 9 i \, {\left(a b^{2} m + a b^{2}\right)} e^{\left(-\frac{1}{3} \, {\left(m - 2\right)} \log\left(-2 i \, d e^{\left(-3\right)}\right) + 2 i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, -2 i \, d x^{3}\right) + {\left(b^{3} m + b^{3}\right)} e^{\left(-\frac{1}{3} \, {\left(m - 2\right)} \log\left(-3 i \, d e^{\left(-3\right)}\right) + 3 i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, -3 i \, d x^{3}\right)}{72 \, {\left(d m + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*(2*a^3 + 3*a*b^2)*(x*e)^m*d*x + (b^3*m + b^3)*e^(-1/3*(m - 2)*log(
3*I*d*e^(-3)) - 3*I*c + 2)*gamma(1/3*m + 1/3, 3*I*d*x^3) - 9*I*(a*b^2*m + a
*b^2)*e^(-1/3*(m - 2)*log(2*I*d*e^(-3)) - 2*I*c + 2)*gamma(1/3*m + 1/3, 2*I
*d*x^3) - 9*(4*a^2*b + b^3 + (4*a^2*b + b^3)*m)*e^(-1/3*(m - 2)*log(I*d*e^(
-3)) - I*c + 2)*gamma(1/3*m + 1/3, I*d*x^3) - 9*(4*a^2*b + b^3 + (4*a^2*b +
 b^3)*m)*e^(-1/3*(m - 2)*log(-I*d*e^(-3)) + I*c + 2)*gamma(1/3*m + 1/3, -I*
d*x^3) + 9*I*(a*b^2*m + a*b^2)*e^(-1/3*(m - 2)*log(-2*I*d*e^(-3)) + 2*I*c +
 2)*gamma(1/3*m + 1/3, -2*I*d*x^3) + (b^3*m + b^3)*e^(-1/3*(m - 2)*log(-3*I
*d*e^(-3)) + 3*I*c + 2)*gamma(1/3*m + 1/3, -3*I*d*x^3))/(d*m + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**3,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x^3 + c) + a)^3*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,x\right)}^m\,{\left(a+b\,\sin\left(d\,x^3+c\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3))^3,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^3))^3, x)
```

3.99 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$

Optimal. Leaf size=285

$$\frac{(2a^2+b^2)(ex)^{1+m}}{2e(1+m)}+\frac{iabe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{3e}-\frac{iabe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{3e}$$

```
[Out] 1/2*(2*a^2+b^2)*(e*x)^(1+m)/e/(1+m)+1/3*I*a*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^
3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/3*I*a*b*(e*x)^(1+m)*(I*d*x^3)
^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,I*d*x^3)/e/exp(I*c)+1/3*2^(-7/3-1/3*m)*b^2*ex
p(2*I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-2*I*d*x^3)/e+
1/3*2^(-7/3-1/3*m)*b^2*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,2
*I*d*x^3)/e/exp(2*I*c)
```

Rubi [A]

time = 0.16, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3484, 6, 3471, 2250, 3470}

$$\frac{iabe^{ic}(-idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{3e}-\frac{iabe^{-ic}(idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{3e}+\frac{b^2e^{2ic}2^{-\frac{m}{3}-\frac{7}{3}}(-idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-2idx^3\right)}{3e}+\frac{b^2e^{-2ic}2^{-\frac{m}{3}-\frac{7}{3}}(idx^3)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},2idx^3\right)}{3e}+\frac{(2a^2+b^2)(ex)^{m+1}}{2e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/3)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/3)*a*b*
(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
 + (2^(-7/3 - m/3)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*
Gamma[(1 + m)/3, (-2*I)*d*x^3])/(3*e) + (2^(-7/3 - m/3)*b^2*(e*x)^(1 + m)*(
I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3])/(3*e*E^((2*I)*c))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int \left(a^2(ex)^m+\frac{1}{2}b^2(ex)^m-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right)+2ab(ex)^m\sin\left(\right.\right. \\
&= \int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m-\frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right)+2ab(ex)^m\sin\left(c+\right.\right. \\
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)}+(2ab)\int (ex)^m\sin\left(c+dx^3\right)\,dx-\frac{1}{2}b^2\int (ex)^m\,\mathrm{co} \\
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)}+(iab)\int e^{-ic-idx^3}(ex)^m\,dx-(iab)\int e^{ic+idx^3}(ex)^{m} \\
&= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)}+\frac{iabe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{3e}-
\end{aligned}$$

Mathematica [A]
time = 5.22, size = 556, normalized size = 1.95

$$\frac{2^{\frac{1}{3}(-7-m)}x(ex)^m\left(d^2x^6\right)^{\frac{1}{3}(-1-m)}\left(3\,2^{\frac{7+m}{3}}a^2\left(d^2x^6\right)^{\frac{1+m}{3}}+3\,2^{\frac{4+m}{3}}b^2\left(d^2x^6\right)^{\frac{1+m}{3}}+b^2\left(idx^3\right)^{\frac{1+m}{3}}\cos(2c)\Gamma\left(\frac{1+m}{3},-2idx^3\right)+b^2m\left(idx^3\right)^{\frac{1+m}{3}}\cos(2c)\Gamma\left(\frac{1+m}{3},-2idx^3\right)+b^2\left(-idx^3\right)^{\frac{1+m}{3}}\cos(2c)\Gamma\left(\frac{1+m}{3},2idx^3\right)+b^2m\left(-idx^3\right)^{\frac{1+m}{3}}\cos(2c)\Gamma\left(\frac{1+m}{3},2idx^3\right)-i2^{\frac{7+m}{3}}ab(1+m)\left(-idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)\left(\cos(c)-i\sin(c)\right)+i2^{\frac{7+m}{3}}ab(1+m)\left(idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},-idx^3\right)\left(\cos(c)+i\sin(c)\right)+ib^2\left(idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},-2idx^3\right)\sin(2c)+ib^2m\left(idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},-2idx^3\right)\sin(2c)-ib^2\left(-idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},2idx^3\right)\sin(2c)-ib^2m\left(-idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},2idx^3\right)\sin(2c)\right)}{3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (2^((-7 - m)/3)*x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(3*2^((7 + m)/3)*a^2*(d^2*
x^6)^((1 + m)/3) + 3*2^((4 + m)/3)*b^2*(d^2*x^6)^((1 + m)/3) + b^2*(I*d*x^3
)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*m*(I*d*x^3)^((1
 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*((-I)*d*x^3)^((1 + m
)/3)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] + b^2*m*((-I)*d*x^3)^((1 + m)/3
)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] - I*2^((7 + m)/3)*a*b*(1 + m)*((-I
```

```
)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*2^((
7 + m)/3)*a*b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(C
os[c] + I*Sin[c]) + I*b^2*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x
^3]*Sin[2*c] + I*b^2*m*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3]
*Sin[2*c] - I*b^2*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3]*Si
n[2*c] - I*b^2*m*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3]*Sin
[2*c]))/(3*(1 + m))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(e x\right)^{m} \left(a +b \sin \left(d \,x^{3}+c \right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] (x*e)^(m + 1)*a^2*e^(-1)/(m + 1) + 1/2*(b^2*x*e^(m*log(x) + m) - (b^2*m + b
^2)*integrate(cos(2*d*x^3 + 2*c)*e^(m*log(x) + m), x) + 4*(a*b*m + a*b)*int
egrate(e^(m*log(x) + m)*sin(d*x^3 + c), x))/(m + 1)
```

Fricas [A]
time = 0.13, size = 187, normalized size = 0.66

$$\frac{12 \,{\left(2 \, a^{2} + b^{2}\right)} \left(x e\right)^{m} d x - i \,{\left(b^{2} m + b^{2}\right)} e^{\left(-\frac{1}{3} \,{\left(m - 2\right)} \log\left(2 i \, d e^{\left(-3\right)}\right) - 2 i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, 2 i \, d x^{3}\right) - 8 \,{\left(a b m + a b\right)} e^{\left(-\frac{1}{3} \,{\left(m - 2\right)} \log\left(i \, d e^{\left(-3\right)}\right) - i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, i \, d x^{3}\right) - 8 \,{\left(a b m + a b\right)} e^{\left(-\frac{1}{3} \,{\left(m - 2\right)} \log\left(-i \, d e^{\left(-3\right)}\right) + i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, -i \, d x^{3}\right) + i \,{\left(b^{2} m + b^{2}\right)} e^{\left(-\frac{1}{3} \,{\left(m - 2\right)} \log\left(-2 i \, d e^{\left(-3\right)}\right) + 2 i \, c + 2\right)} \Gamma\left(\frac{1}{3} \, m + \frac{1}{3}, -2 i \, d x^{3}\right)}{24 \,{\left(d m + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(2*a^2 + b^2)*(x*e)^m*d*x - I*(b^2*m + b^2)*e^(-1/3*(m - 2)*log(2*
I*d*e^(-3)) - 2*I*c + 2)*gamma(1/3*m + 1/3, 2*I*d*x^3) - 8*(a*b*m + a*b)*e^
(-1/3*(m - 2)*log(I*d*e^(-3)) - I*c + 2)*gamma(1/3*m + 1/3, I*d*x^3) - 8*(a
*b*m + a*b)*e^(-1/3*(m - 2)*log(-I*d*e^(-3)) + I*c + 2)*gamma(1/3*m + 1/3,
-I*d*x^3) + I*(b^2*m + b^2)*e^(-1/3*(m - 2)*log(-2*I*d*e^(-3)) + 2*I*c + 2)
*gamma(1/3*m + 1/3, -2*I*d*x^3))/(d*m + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*(x*e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,x)^m \left(a + b\,\sin\left(d\,x^3 + c\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3))^2,x)
```

```
[Out] int((e*x)^m*(a + b*sin(c + d*x^3))^2, x)
```

3.100 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal. Leaf size=134

$$\frac{a(ex)^{1+m}}{e(1+m)}+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},idx^3\right)}{6e}$$

```
[Out] a*(e*x)^(1+m)/e/(1+m)+1/6*I*b*exp(I*c)*(e*x)^(1+m)*(-I*d*x^3)^(-1/3-1/3*m)*
GAMMA(1/3+1/3*m,-I*d*x^3)/e-1/6*I*b*(e*x)^(1+m)*(I*d*x^3)^(-1/3-1/3*m)*GAMM
A(1/3+1/3*m,I*d*x^3)/e/exp(I*c)
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {14, 3470, 2250}

$$\frac{ibe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{6e}+\frac{a(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/6)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)
^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/6)*b*(e*x)^(1 + m)*(I*d
*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx &= \int \left(a(ex)^m+b(ex)^m\sin\left(c+dx^3\right)\right)\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)}+b\int (ex)^m\sin\left(c+dx^3\right)\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)}+\frac{1}{2}(ib)\int e^{-ic-idx^3}(ex)^m\,dx-\frac{1}{2}(ib)\int e^{ic+idx^3}(ex)^m\,dx\\ &= \frac{a(ex)^{1+m}}{e(1+m)}+\frac{ibe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}(ex)^{1\cdots}}{}\end{aligned}$$

Mathematica [A]
time = 1.01, size = 149, normalized size = 1.11

$$\frac{x(ex)^m\left(d^2x^6\right)^{\frac{1}{3}(-1-m)}\left(6a\left(d^2x^6\right)^{\frac{1+m}{3}}-ib(1+m)\left(-idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},idx^3\right)(\cos(c)-i\sin(c))+ib(1+m)\left(idx^3\right)^{\frac{1+m}{3}}\Gamma\left(\frac{1+m}{3},-idx^3\right)(\cos(c)+i\sin(c))\right)}{6(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(6*a*(d^2*x^6)^((1 + m)/3) - I*b*(1 + m)*
((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*
b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(Cos[c] + I*Si
n[c])))/(6*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m\left(a+b\sin\left(d\,x^3+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] (x*e)^(m + 1)*a*e^(-1)/(m + 1) + b*integrate(e^(m*log(x) + m)*sin(d*x^3 + c
), x)
```

Fricas [A]
time = 0.11, size = 93, normalized size = 0.69

$$\frac{6\,(xe)^{m}\,adx-(bm+b)e^{\left(-\frac{1}{3}\,(m-2)\log\left(i\,de^{(-3)}\right)-i\,c+2\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},i\,dx^{3}\right)-(bm+b)e^{\left(-\frac{1}{3}\,(m-2)\log\left(-i\,de^{(-3)}\right)+i\,c+2\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},-i\,dx^{3}\right)}{6\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(x*e)^m*a*d*x - (b*m + b)*e^(-1/3*(m - 2)*log(I*d*e^(-3)) - I*c + 2)
*gamma(1/3*m + 1/3, I*d*x^3) - (b*m + b)*e^(-1/3*(m - 2)*log(-I*d*e^(-3)) +
 I*c + 2)*gamma(1/3*m + 1/3, -I*d*x^3))/(d*m + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{m}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {(e\,x)}^m\,\left(a+b\,\sin\left(d\,x^3+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^3)),x)
```

```
[Out] int((e*x)^m*(a + b*sin(c + d*x^3)), x)
```

3.101 $\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(ex)^m}{a+b\sin\left(c+dx^3\right)},x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Rubi steps

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx = \int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{a+b\sin\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*sin(d*x^3+c)),x)
[Out] int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
[Out] integrate((x*e)^m/(b*sin(d*x^3 + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
[Out] integral((x*e)^m/(b*sin(d*x^3 + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{a + b\sin(c + dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**3+c)),x)
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**3)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="giac")
[Out] integrate((x*e)^m/(b*sin(d*x^3 + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{a + b\,\sin\left(d\,x^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a + b*sin(c + d*x^3)),x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^3)), x)
```

3.102 $\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{(ex)^m}{(a+b\sin(c+dx^3))^2},x\right)$$

```
[Out] Unintegrable((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi steps

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx = \int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+b\sin(c+dx^3))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+b\sin(d\,x^3+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c)*e^(m*log(x) + m) + 2*b^2*cos(2*c)*e^(m*log(x)
+ m)*sin(2*d*x^3) + 2*b^2*cos(2*d*x^3)*e^(m*log(x) + m)*sin(2*c) - 4*a*b*e^
(m*log(x) + m)*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c)*e^(m*log(x)
 + m) - 2*a^2*cos(c)*e^(m*log(x) + m)*sin(d*x^3) - a*b*e^(m*log(x) + m)*sin
(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*e^(m*log(x) + m)*sin(c) - a*b*e^(m*lo
g(x) + m))*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4
)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a
^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b
^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(c)
*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2
*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)
*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*s
in(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b - a*b^
3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^3))*cos
(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c
)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b -
a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*sin(2*c))*
sin(2*d*x^3))*integrate(2/3*((b^2*m*sin(2*c) - 2*b^2*sin(2*c))*cos(2*d*x^3)
*e^(m*log(x) + m) + 2*(a*b*m*cos(c) - 2*a*b*cos(c))*cos(d*x^3)*e^(m*log(x)
+ m) + (b^2*m*cos(2*c) - 2*b^2*cos(2*c))*e^(m*log(x) + m)*sin(2*d*x^3) - 2*
(a*b*m*sin(c) - 2*a*b*sin(c))*e^(m*log(x) + m)*sin(d*x^3) - ((3*a*b*d*x^3*e
^m*sin(2*c) - (a*b*m*cos(2*c) - 2*a*b*cos(2*c))*e^m)*x^m*cos(2*d*x^3) + 2*(
3*a^2*d*x^3*cos(c)*e^m + (a^2*m*sin(c) - 2*a^2*sin(c))*e^m)*x^m*cos(d*x^3)
+ (3*a*b*d*x^3*cos(2*c)*e^m + (a*b*m*sin(2*c) - 2*a*b*sin(2*c))*e^m)*x^m*si
n(2*d*x^3) - 2*(3*a^2*d*x^3*e^m*sin(c) - (a^2*m*cos(c) - 2*a^2*cos(c))*e^m)
*x^m*sin(d*x^3) + (a*b*m - 2*a*b)*e^(m*log(x) + m))*cos(d*x^3 + c) - (3*a*b
*d*x^3*e^(m*log(x) + m) - (3*a*b*d*x^3*cos(2*c)*e^m + (a*b*m*sin(2*c) - 2*a
*b*sin(2*c))*e^m)*x^m*cos(2*d*x^3) + 2*(3*a^2*d*x^3*e^m*sin(c) - (a^2*m*cos
(c) - 2*a^2*cos(c))*e^m)*x^m*cos(d*x^3) + (3*a*b*d*x^3*e^m*sin(2*c) - (a*b*
m*cos(2*c) - 2*a*b*cos(2*c))*e^m)*x^m*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c)*
e^m + (a^2*m*sin(c) - 2*a^2*sin(c))*e^m)*x^m*sin(d*x^3))*sin(d*x^3 + c))/((
(a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*cos(2*d*x^3)
```

```
^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*cos(d*x^
3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^3*sin(
2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)
*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*sin(d*x^3)^2 + 4*(a^3*b - a*b^3
)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^3 + 2*(2*((a^3*b - a*b^3)*c
os(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3*cos(d*x^3) - (a^2*b
^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*
b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3
)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*cos(d*x^3) + 2*(
(a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3*si
n(d*x^3) + (a^2*b^2 - b^4)*d*x^3*sin(2*c))*sin(2*d*x^3)), x) + 2*(2*a^2*cos
(d*x^3)*cos(c)*e^(m*log(x) + m) + a*b*cos(2*c)*e^(m*log(x) + m)*sin(2*d*x^3
) + a*b*cos(2*d*x^3)*e^(m*log(x) + m)*sin(2*c) - 2*a^2*e^(m*log(x) + m)*sin
(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b
^4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 -
 a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2
*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(
c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x
^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 - b^
4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)
*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b - a*
b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^3))*c
os(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2
*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b
- a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*sin(2*c)
)*sin(2*d*x^3))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
[Out] integral(-(x*e)^m/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2)
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a + b\sin{(c + dx^3)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**3))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
[Out] integrate((x*e)^m/(b*sin(d*x^3 + c) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{(a+b\,\sin\left(d\,x^3+c\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a + b*sin(c + d*x^3))^2,x)
[Out] int((e*x)^m/(a + b*sin(c + d*x^3))^2, x)
```

3.103 $\int x^2 \sin\left(a + \frac{b}{x}\right) \, dx$

Optimal. Leaf size=78

$$\frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)+\frac{1}{6}b^3\cos(a)\mathrm{Ci}\left(\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\mathrm{Si}\left(\frac{b}{x}\right)$$

```
[Out] 1/6*b^3*Ci(b/x)*cos(a)+1/6*b*x^2*cos(a+b/x)-1/6*b^3*Si(b/x)*sin(a)-1/6*b^2*
x*sin(a+b/x)+1/3*x^3*sin(a+b/x)
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\frac{1}{6}b^3\cos(a)\mathrm{CosIntegral}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\mathrm{Si}\left(\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b/x],x]
```

```
[Out] (b*x^2*Cos[a + b/x])/6 + (b^3*Cos[a]*CosIntegral[b/x])/6 - (b^2*x*Sin[a + b
/x])/6 + (x^3*Sin[a + b/x])/3 - (b^3*Sin[a]*SinIntegral[b/x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin\left(a+\frac{b}{x}\right) dx &= -\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) - \frac{1}{3}b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}bx^2 \cos\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) + \frac{1}{6}b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}bx^2 \cos\left(a+\frac{b}{x}\right) - \frac{1}{6}b^2x \sin\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) + \frac{1}{6}b^3\text{Subst}\left(\int \frac{\cos(a}{x}\right.\\
&= \frac{1}{6}bx^2 \cos\left(a+\frac{b}{x}\right) - \frac{1}{6}b^2x \sin\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) + \frac{1}{6}\left(b^3\cos(a)\right)\text{Subst}\left(\right.\\
&= \frac{1}{6}bx^2 \cos\left(a+\frac{b}{x}\right) + \frac{1}{6}b^3\cos(a)\text{Ci}\left(\frac{b}{x}\right) - \frac{1}{6}b^2x \sin\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sin\left(a+\frac{b}{x}\right) -
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 0.90

$$\frac{1}{6}\left(b^3\cos(a)\text{Ci}\left(\frac{b}{x}\right) + x\left(bx\cos\left(a+\frac{b}{x}\right) - b^2\sin\left(a+\frac{b}{x}\right) + 2x^2\sin\left(a+\frac{b}{x}\right)\right) - b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b/x],x]
```

```
[Out] (b^3*Cos[a]*CosIntegral[b/x] + x*(b*x*Cos[a + b/x] - b^2*Sin[a + b/x] + 2*x
^2*Sin[a + b/x]) - b^3*Sin[a]*SinIntegral[b/x])/6
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.94

method	result

derivativedivides	$-b^3\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^3}{3b^3}-\frac{\cos\left(a+\frac{b}{x}\right)x^2}{6b^2}+\frac{\sin\left(a+\frac{b}{x}\right)x}{6b}+\frac{\text{sinIntegral}\left(\frac{b}{x}\right)\sin(a)}{6}-\frac{\text{cosineIntegral}\left(\frac{b}{x}\right)\cos(a)}{6}\right)$
default	$-b^3\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^3}{3b^3}-\frac{\cos\left(a+\frac{b}{x}\right)x^2}{6b^2}+\frac{\sin\left(a+\frac{b}{x}\right)x}{6b}+\frac{\text{sinIntegral}\left(\frac{b}{x}\right)\sin(a)}{6}-\frac{\text{cosineIntegral}\left(\frac{b}{x}\right)\cos(a)}{6}\right)$
risch	$\frac{i\pi\,\text{csgn}\left(\frac{b}{x}\right)e^{-ia}b^3}{12}-\frac{i\,\text{sinIntegral}\left(\frac{b}{x}\right)e^{-ia}b^3}{6}-\frac{\text{expIntegral}\left(1,-\frac{ib}{x}\right)e^{-ia}b^3}{12}-\frac{b^3\,\text{expIntegral}\left(1,-\frac{ib}{x}\right)e^{ia}}{12}+\frac{x^2b\cos}{}$
meijerg	$-\frac{b^3\sqrt{\pi}\,\cos(a)\left(-\frac{8x^2}{\sqrt{\pi}\,b^2}-\frac{4\left(2\gamma-\frac{11}{3}-2\ln(x)+2\ln(b)\right)}{3\sqrt{\pi}}+\frac{8x^2\left(-\frac{55b^2}{2x^2}+45\right)}{45\sqrt{\pi}\,b^2}+\frac{8\gamma}{3\sqrt{\pi}}+\frac{8\ln(2)}{3\sqrt{\pi}}+\frac{8\ln\left(\frac{b}{2x}\right)}{3\sqrt{\pi}}-\frac{8x^2\cos\left(\frac{b}{x}\right)}{3\sqrt{\pi}\,b^2}-\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b/x),x,method=_RETURNVERBOSE)
[Out] -b^3*(-1/3*sin(a+b/x)/b^3*x^3-1/6*cos(a+b/x)/b^2*x^2+1/6*sin(a+b/x)/b*x+1/6
*Si(b/x)*sin(a)-1/6*Ci(b/x)*cos(a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 86, normalized size = 1.10

$$\frac{1}{12}\left(\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b^3+\frac{1}{6}\,bx^2\cos\left(\frac{ax+b}{x}\right)-\frac{1}{6}\left(b^2x-2\,x^3\right)\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="maxima")
[Out] 1/12*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) + (I*Ei(I*b/x) - I*Ei(-I*b/x))*sin(a)
)*b^3 + 1/6*b*x^2*cos((a*x + b)/x) - 1/6*(b^2*x - 2*x^3)*sin((a*x + b)/x)
```

Fricas [A]
time = 0.38, size = 79, normalized size = 1.01

$$-\frac{1}{6}\,b^3\sin\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)+\frac{1}{6}\,bx^2\cos\left(\frac{ax+b}{x}\right)+\frac{1}{12}\left(b^3\,\mathrm{Ci}\left(\frac{b}{x}\right)+b^3\,\mathrm{Ci}\left(-\frac{b}{x}\right)\right)\cos\left(a\right)-\frac{1}{6}\left(b^2x-2\,x^3\right)\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="fricas")
[Out] -1/6*b^3*sin(a)*sin_integral(b/x) + 1/6*b*x^2*cos((a*x + b)/x) + 1/12*(b^3*
cos_integral(b/x) + b^3*cos_integral(-b/x))*cos(a) - 1/6*(b^2*x - 2*x^3)*si
n((a*x + b)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b/x),x)
```

```
[Out] Integral(x**2*sin(a + b/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(68) = 136$.

time = 6.68, size = 400, normalized size = 5.13

$$\frac{a^3b^4\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)+a^3b^4\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)-\frac{3(ax+b)a^2b^4\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)}{x}-\frac{3(ax+b)a^2b^4\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x}+\frac{3(ax+b)^2ab^4\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)}{x^2}+a^2b^4\sin\left(\frac{ax+b}{x}\right)+\frac{3(ax+b)^2ab^4\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x^2}+ab^4\cos\left(\frac{ax+b}{x}\right)-\frac{(ax+b)^3b^4\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)}{x^3}-\frac{2(ax+b)ab^4\sin\left(\frac{ax+b}{x}\right)}{x}-\frac{(ax+b)^3b^4\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x^3}-\frac{(ax+b)b^4\cos\left(\frac{ax+b}{x}\right)}{x}-2b^4\sin\left(\frac{ax+b}{x}\right)+\frac{(ax+b)^2b^4\sin\left(\frac{ax+b}{x}\right)}{x^2}}{6\left(a^3-\frac{3(ax+b)a^2}{x}+\frac{3(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="giac")
```

```
[Out] 1/6*(a^3*b^4*cos(a)*cos_integral(-a + (a*x + b)/x) + a^3*b^4*sin(a)*sin_int
egral(a - (a*x + b)/x) - 3*(a*x + b)*a^2*b^4*cos(a)*cos_integral(-a + (a*x
+ b)/x)/x - 3*(a*x + b)*a^2*b^4*sin(a)*sin_integral(a - (a*x + b)/x)/x + 3*
(a*x + b)^2*a*b^4*cos(a)*cos_integral(-a + (a*x + b)/x)/x^2 + a^2*b^4*sin((
a*x + b)/x) + 3*(a*x + b)^2*a*b^4*sin(a)*sin_integral(a - (a*x + b)/x)/x^2
+ a*b^4*cos((a*x + b)/x) - (a*x + b)^3*b^4*cos(a)*cos_integral(-a + (a*x +
b)/x)/x^3 - 2*(a*x + b)*a*b^4*sin((a*x + b)/x)/x - (a*x + b)^3*b^4*sin(a)*s
in_integral(a - (a*x + b)/x)/x^3 - (a*x + b)*b^4*cos((a*x + b)/x)/x - 2*b^4
*sin((a*x + b)/x) + (a*x + b)^2*b^4*sin((a*x + b)/x)/x^2)/((a^3 - 3*(a*x +
b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sin\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b/x),x)
```

```
[Out] int(x^2*sin(a + b/x), x)
```

3.104 $\int x \sin\left(a + \frac{b}{x}\right) \, dx$

Optimal. Leaf size=60

$$\frac{1}{2} b x \cos\left(a + \frac{b}{x}\right) + \frac{1}{2} b^2 \text{Ci}\left(\frac{b}{x}\right) \sin(a) + \frac{1}{2} x^2 \sin\left(a + \frac{b}{x}\right) + \frac{1}{2} b^2 \cos(a) \text{Si}\left(\frac{b}{x}\right)$$

```
[Out] 1/2*b*x*cos(a+b/x)+1/2*b^2*cos(a)*Si(b/x)+1/2*b^2*Ci(b/x)*sin(a)+1/2*x^2*si
n(a+b/x)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\frac{1}{2} b^2 \sin(a) \text{CosIntegral}\left(\frac{b}{x}\right) + \frac{1}{2} b^2 \cos(a) \text{Si}\left(\frac{b}{x}\right) + \frac{1}{2} x^2 \sin\left(a + \frac{b}{x}\right) + \frac{1}{2} b x \cos\left(a + \frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b/x],x]
```

```
[Out] (b*x*Cos[a + b/x])/2 + (b^2*CosIntegral[b/x]*Sin[a])/2 + (x^2*Sin[a + b/x])
/2 + (b^2*Cos[a]*SinIntegral[b/x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\sin\left(a+\frac{b}{x}\right)\,dx &= -\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}\left(b^2\cos(a)\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{x}\right)+\\
&= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\text{Ci}\left(\frac{b}{x}\right)\sin(a)+\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.87

$$\frac{1}{2}\left(b^2\text{Ci}\left(\frac{b}{x}\right)\sin(a)+x\left(b\cos\left(a+\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)\right)+b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b/x],x]
[Out] (b^2*CosIntegral[b/x]*Sin[a] + x*(b*Cos[a + b/x] + x*Sin[a + b/x]) + b^2*Co
s[a]*SinIntegral[b/x])/2
```

Maple [A]
time = 0.06, size = 57, normalized size = 0.95

method	result
derivativedivides	$-b^2\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^2}{2b^2}-\frac{\cos\left(a+\frac{b}{x}\right)x}{2b}-\frac{\cos(a)\,\text{sinIntegral}\left(\frac{b}{x}\right)}{2}-\frac{\text{cosineIntegral}\left(\frac{b}{x}\right)\sin(a)}{2}\right)$

default	$-b^2\left(-\frac{\sin\left(a+\frac{b}{x}\right)x^2}{2b^2}-\frac{\cos\left(a+\frac{b}{x}\right)x}{2b}-\frac{\cos(a)\,\mathrm{sinIntegral}\left(\frac{b}{x}\right)}{2}-\frac{\mathrm{cosineIntegral}\left(\frac{b}{x}\right)\sin(a)}{2}\right)$
risch	$-\frac{\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b^2}{4}+\frac{\mathrm{sinIntegral}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b^2}{2}-\frac{i\,\mathrm{expIntegral}\left(1,-\frac{ib}{x}\right)\mathrm{e}^{-ia}b^2}{4}+\frac{ib^2\,\mathrm{expIntegral}\left(1,-\frac{ib}{x}\right)\mathrm{e}^{ia}}{4}+\frac{bx\,\mathrm{co}}{}$
meijerg	$-\frac{b^2\sqrt{\pi}\,\cos(a)\left(-\frac{4x\cos\left(\frac{b}{x}\right)}{b\sqrt{\pi}}-\frac{4x^2\sin\left(\frac{b}{x}\right)}{b^2\sqrt{\pi}}-\frac{4\,\mathrm{sinIntegral}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{8}-\frac{b^2\sqrt{\pi}\,\sin(a)\left(-\frac{4x^2}{\sqrt{\pi}\,b^2}-\frac{2\left(2\gamma-3-2\ln(x)+\ln\left(b^2\right)\right.}{\sqrt{\pi}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*(-1/2*sin(a+b/x)/b^2*x^2-1/2*cos(a+b/x)/b*x-1/2*cos(a)*Si(b/x)-1/2*Ci(
b/x)*sin(a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 76, normalized size = 1.27

$$\frac{1}{4}\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b^2+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/4*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a)
)*b^2 + 1/2*b*x*cos((a*x + b)/x) + 1/2*x^2*sin((a*x + b)/x)
```

Fricas [A]
time = 0.37, size = 69, normalized size = 1.15

$$\frac{1}{2}\,b^2\cos\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)+\frac{1}{4}\left(b^2\,\mathrm{Ci}\left(\frac{b}{x}\right)+b^2\,\mathrm{Ci}\left(-\frac{b}{x}\right)\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/2*b^2*cos(a)*sin_integral(b/x) + 1/2*b*x*cos((a*x + b)/x) + 1/2*x^2*sin((
a*x + b)/x) + 1/4*(b^2*cos_integral(b/x) + b^2*cos_integral(-b/x))*sin(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x)
```

```
[Out] Integral(x*sin(a + b/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(52) = 104$.
time = 5.25, size = 251, normalized size = 4.18

$$\frac{a^2b^3\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin\left(a\right)-a^2b^3\cos\left(a\right)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)-\frac{2\,(ax+b)ab^3\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)}{x}+\frac{2\,(ax+b)ab^3\cos(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x}-ab^3\cos\left(\frac{ax+b}{x}\right)+\frac{(ax+b)^2b^3\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)}{x^2}-\frac{(ax+b)^2b^3\cos(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x^2}+\frac{(ax+b)b^3\cos\left(\frac{ax+b}{x}\right)}{x}+b^3\sin\left(\frac{ax+b}{x}\right)}{2\left(a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*b^3*cos_integral(-a + (a*x + b)/x)*sin(a) - a^2*b^3*cos(a)*sin_int
egral(a - (a*x + b)/x) - 2*(a*x + b)*a*b^3*cos_integral(-a + (a*x + b)/x)*s
in(a)/x + 2*(a*x + b)*a*b^3*cos(a)*sin_integral(a - (a*x + b)/x)/x - a*b^3*
cos((a*x + b)/x) + (a*x + b)^2*b^3*cos_integral(-a + (a*x + b)/x)*sin(a)/x^
2 - (a*x + b)^2*b^3*cos(a)*sin_integral(a - (a*x + b)/x)/x^2 + (a*x + b)*b^
3*cos((a*x + b)/x)/x + b^3*sin((a*x + b)/x))/((a^2 - 2*(a*x + b)*a/x + (a*x
 + b)^2/x^2)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sin\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b/x),x)
```

```
[Out] int(x*sin(a + b/x), x)
```

3.105 $\int \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=32

$$-b\cos(a)\text{Ci}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)+b\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[Out] -b*Ci(b/x)*cos(a)+b*Si(b/x)*sin(a)+x*sin(a+b/x)
```

Rubi [A]
time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3442, 3378, 3384, 3380, 3383}

$$-b\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)+b\sin(a)\text{Si}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x],x]
```

```
[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{x}\right)\,dx = -\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= x\sin\left(a+\frac{b}{x}\right) - b\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= x\sin\left(a+\frac{b}{x}\right) - (b\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{x}\right) + (b\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\right.$$

$$= -b\cos(a)\text{Ci}\left(\frac{b}{x}\right) + x\sin\left(a+\frac{b}{x}\right) + b\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$-b\cos(a)\text{Ci}\left(\frac{b}{x}\right) + x\sin\left(a+\frac{b}{x}\right) + b\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x],x]
[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Maple [A]
time = 0.05, size = 38, normalized size = 1.19

method	result
derivativedivides	$-b\left(-\frac{\sin\left(a+\frac{b}{x}\right)x}{b} - \text{sinIntegral}\left(\frac{b}{x}\right)\sin(a) + \text{cosineIntegral}\left(\frac{b}{x}\right)\cos(a)\right)$
default	$-b\left(-\frac{\sin\left(a+\frac{b}{x}\right)x}{b} - \text{sinIntegral}\left(\frac{b}{x}\right)\sin(a) + \text{cosineIntegral}\left(\frac{b}{x}\right)\cos(a)\right)$
risch	$\frac{b\,\text{expIntegral}\left(1,-\frac{ib}{x}\right)e^{ia}}{2} - \frac{i\pi\,\text{csgn}\left(\frac{b}{x}\right)e^{-ia}b}{2} + i\,\text{sinIntegral}\left(\frac{b}{x}\right)e^{-ia}b + \frac{\text{expIntegral}\left(1,-\frac{ib}{x}\right)e^{-ia}b}{2} + \sin$

meijerg	$-\frac{\sqrt{\pi}\,\cos(a)b\left(\frac{4\gamma-4-4\ln(x)+4\ln(b)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}-\frac{4x\sin\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b}+\frac{4\,\mathrm{cosineIntegral}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{4}-\frac{\sqrt{\pi}\,\mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -b*(-sin(a+b/x)/b*x-Si(b/x)*sin(a)+Ci(b/x)*cos(a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 58, normalized size = 1.81

$$-\frac{1}{2}\left(\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)\right)b+x\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) - (-I*Ei(I*b/x) + I*Ei(-I*b/x))*sin(a
))*b + x*sin((a*x + b)/x)
```

Fricas [A]
time = 0.38, size = 45, normalized size = 1.41

$$b\sin\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)-\frac{1}{2}\left(b\,\mathrm{Ci}\left(\frac{b}{x}\right)+b\,\mathrm{Ci}\left(-\frac{b}{x}\right)\right)\cos\left(a\right)+x\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x, algorithm="fricas")
```

```
[Out] b*sin(a)*sin_integral(b/x) - 1/2*(b*cos_integral(b/x) + b*cos_integral(-b/x
))*cos(a) + x*sin((a*x + b)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x)
```

```
[Out] Integral(sin(a + b/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(32) = 64.
time = 7.82, size = 132, normalized size = 4.12

$$-\frac{ab^2\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)+ab^2\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)-\frac{(ax+b)b^2\cos(a)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)}{x}-\frac{(ax+b)b^2\sin(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x}+b^2\sin\left(\frac{ax+b}{x}\right)}{\left(a-\frac{ax+b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x, algorithm="giac")
```

```
[Out] -(a*b^2*cos(a)*cos_integral(-a + (a*x + b)/x) + a*b^2*sin(a)*sin_integral(a
 - (a*x + b)/x) - (a*x + b)*b^2*cos(a)*cos_integral(-a + (a*x + b)/x)/x - (
a*x + b)*b^2*sin(a)*sin_integral(a - (a*x + b)/x)/x + b^2*sin((a*x + b)/x))
/((a - (a*x + b)/x)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sin\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x),x)
```

```
[Out] int(sin(a + b/x), x)
```

3.106 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal. Leaf size=21

$$-\text{Ci}\left(\frac{b}{x}\right)\sin(a)-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

`[Out] -cos(a)*Si(b/x)-Ci(b/x)*sin(a)`

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3458, 3457, 3456}

$$\sin(a)\left(-\text{CosIntegral}\left(\frac{b}{x}\right)\right)-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

`[In] Int[Sin[a + b/x]/x,x]`

`[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]`

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx&=\cos(a)\int \frac{\sin\left(\frac{b}{x}\right)}{x}\,dx+\sin(a)\int \frac{\cos\left(\frac{b}{x}\right)}{x}\,dx\\&=-\text{Ci}\left(\frac{b}{x}\right)\sin(a)-\cos(a)\text{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 21, normalized size = 1.00

$$-\mathrm{Ci}\left(\frac{b}{x}\right)\sin(a)-\cos(a)\mathrm{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x,x]
```

```
[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Maple [**A**]
time = 0.04, size = 22, normalized size = 1.05

method	result
derivativedivides	$-\cos\left(a\right)\mathrm{sinIntegral}\left(\frac{b}{x}\right)-\mathrm{cosineIntegral}\left(\frac{b}{x}\right)\sin\left(a\right)$
default	$-\cos\left(a\right)\mathrm{sinIntegral}\left(\frac{b}{x}\right)-\mathrm{cosineIntegral}\left(\frac{b}{x}\right)\sin\left(a\right)$
risch	$-\frac{i\,\mathrm{e}^{ia}\,\mathrm{expIntegral}\left(1,-\frac{ib}{x}\right)}{2}+\frac{\mathrm{e}^{-ia}\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)}{2}-\mathrm{e}^{-ia}\,\mathrm{sinIntegral}\left(\frac{b}{x}\right)+\frac{i\,\mathrm{expIntegral}\left(1,-\frac{ib}{x}\right)\mathrm{e}^{-ia}}{2}$
meijerg	$-\cos\left(a\right)\mathrm{sinIntegral}\left(\frac{b}{x}\right)-\frac{\sqrt{\pi}\,\sin(a)\left(\frac{2\gamma-2\ln(x)+\ln\left(b^{2}\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{cosineIntegral}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -cos(a)*Si(b/x)-Ci(b/x)*sin(a)
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.33, size = 43, normalized size = 2.05

$$\frac{1}{2}\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)-\frac{1}{2}\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*cos(a) - 1/2*(Ei(I*b/x) + Ei(-I*b/x))*sin(
a)
```

Fricas [**A**]
time = 0.36, size = 29, normalized size = 1.38

$$-\frac{1}{2}\left(\mathrm{Ci}\left(\frac{b}{x}\right)+\mathrm{Ci}\left(-\frac{b}{x}\right)\right)\sin\left(a\right)-\cos\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(cos_integral(b/x) + cos_integral(-b/x))*sin(a) - cos(a)*sin_integral(
b/x)
```

Sympy [A]
time = 0.61, size = 17, normalized size = 0.81

$$- \sin{\left(a \right)} \operatorname{Ci}{\left(\frac{b}{x} \right)} - \cos{\left(a \right)} \operatorname{Si}{\left(\frac{b}{x} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x)
```

```
[Out] -sin(a)*Ci(b/x) - cos(a)*Si(b/x)
```

Giac [A]
time = 6.51, size = 42, normalized size = 2.00

$$-\frac{b\,\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin\left(a\right)-b\cos\left(a\right)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x, algorithm="giac")
```

```
[Out] -(b*cos_integral(-a + (a*x + b)/x)*sin(a) - b*cos(a)*sin_integral(a - (a*x
+ b)/x))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$-\sin\left(a\right)\ \mathrm{cosint}\left(\frac{b}{x}\right)-\cos\left(a\right)\ \mathrm{sinint}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)/x,x)
```

```
[Out] - sin(a)*cosint(b/x) - cos(a)*sinint(b/x)
```

3.107 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=12

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] cos(a+b/x)/b
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3460, 2718}

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^2,x]
```

```
[Out] Cos[a + b/x]/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^2,x]
```

```
[Out] Cos[a + b/x]/b
```

Maple [A]
time = 0.02, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$	13
default	$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$	13
risch	$\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$	15
norman	$\frac{2}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}$	23
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}-\frac{\sin(a)\sin\left(\frac{b}{x}\right)}{b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] cos(a+b/x)/b
```

Maxima [A]
time = 0.28, size = 12, normalized size = 1.00

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] cos(a + b/x)/b
```

Fricas [A]
time = 0.39, size = 14, normalized size = 1.17

$$\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] cos((a*x + b)/x)/b
```

Sympy [A]
time = 0.33, size = 14, normalized size = 1.17

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**2,x)
```

```
[Out] Piecewise((cos(a + b/x)/b, Ne(b, 0)), (-sin(a)/x, True))
```

Giac [A]
time = 3.49, size = 14, normalized size = 1.17

$$\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="giac")
```

```
[Out] cos((a*x + b)/x)/b
```

Mupad [B]
time = 4.53, size = 12, normalized size = 1.00

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)/x^2,x)
```

```
[Out] cos(a + b/x)/b
```

3.108 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3} dx$

Optimal. Leaf size=29

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

```
[Out] cos(a+b/x)/b/x-sin(a+b/x)/b^2
```

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 3377, 2717}

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^3,x]
```

```
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^3,x]
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Maple [A]
time = 0.04, size = 42, normalized size = 1.45

method	result	size
risch	$\frac{\cos\left(\frac{ax+b}{x}\right)}{bx} - \frac{\sin\left(\frac{ax+b}{x}\right)}{b^2}$	34
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)}{b^2}$	42
default	$-\frac{\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)}{b^2}$	42
norman	$\frac{\frac{x}{b}-\frac{2x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^2}$	66
meijerg	$-\frac{2\sqrt{\pi}\,\cos(a)\left(-\frac{b\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}+\frac{\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}}\right)}{b^2} - \frac{2\sqrt{\pi}\,\sin(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/b^2*(sin(a+b/x)-(a+b/x)*cos(a+b/x)+a*cos(a+b/x))
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 50, normalized size = 1.72

$$-\frac{\left(i\,\Gamma\left(2,\frac{ib}{x}\right)-i\,\Gamma\left(2,-\frac{ib}{x}\right)\right)\cos(a)+\left(\Gamma\left(2,\frac{ib}{x}\right)+\Gamma\left(2,-\frac{ib}{x}\right)\right)\sin(a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((I*gamma(2, I*b/x) - I*gamma(2, -I*b/x))*cos(a) + (gamma(2, I*b/x) +
gamma(2, -I*b/x))*sin(a))/b^2
```

Fricas [A]
time = 0.35, size = 33, normalized size = 1.14

$$\frac{b\cos\left(\frac{ax+b}{x}\right) - x\sin\left(\frac{ax+b}{x}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] (b*cos((a*x + b)/x) - x*sin((a*x + b)/x))/(b^2*x)
```

Sympy [A]
time = 0.51, size = 29, normalized size = 1.00

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**3,x)
```

```
[Out] Piecewise((cos(a + b/x)/(b*x) - sin(a + b/x)/b**2, Ne(b, 0)), (-sin(a)/(2*x
**2), True))
```

Giac [A]
time = 2.67, size = 48, normalized size = 1.66

$$-\frac{a\cos\left(\frac{ax+b}{x}\right) - \frac{(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x} + \sin\left(\frac{ax+b}{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -(a*cos((a*x + b)/x) - (a*x + b)*cos((a*x + b)/x)/x + sin((a*x + b)/x))/b^2
```

Mupad [B]
time = 4.54, size = 29, normalized size = 1.00

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b\,x} - \frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)/x^3,x)
```

```
[Out] cos(a + b/x)/(b*x) - sin(a + b/x)/b^2
```

3.109 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4} dx$

Optimal. Leaf size=45

$$-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}$$

```
[Out] -2*cos(a+b/x)/b^3+cos(a+b/x)/b/x^2-2*sin(a+b/x)/b^2/x
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 3377, 2718}

$$-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^4,x]
```

```
[Out] (-2*Cos[a + b/x])/b^3 + Cos[a + b/x]/(b*x^2) - (2*Sin[a + b/x])/(b^2*x)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x} + \frac{2\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}$$
$$= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} + \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.84

$$\frac{(b^2-2x^2)\cos\left(a+\frac{b}{x}\right) - 2bx\sin\left(a+\frac{b}{x}\right)}{b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^4,x]
[Out] ((b^2 - 2*x^2)*Cos[a + b/x] - 2*b*x*Sin[a + b/x])/(b^3*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(45) = 90$.
time = 0.04, size = 95, normalized size = 2.11

method	result
risch	$\frac{(b^2-2x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2} - \frac{2\sin\left(\frac{ax+b}{x}\right)}{b^2x}$
norman	$\frac{\frac{x}{b}+\frac{4x^3\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^3}-\frac{4x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^3}$
derivativedivides	$-\frac{-a^2\cos\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{b^3}$
default	$-\frac{-a^2\cos\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{b^3}$
meijerg	$-\frac{4\sqrt{\pi}\cos(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{b^2}{2x^2}+1\right)\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^3} - \frac{4\sqrt{\pi}\sin(a)\sqrt{b^2}\left(\frac{(b^2)^{\frac{3}{2}}\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x\,b^2}-\frac{(b^2)^{\frac{3}{2}}\left(-\frac{3b^2}{2x^2}+\right.}{6\sqrt{\pi}}\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-a^2*cos(a+b/x)-2*a*(sin(a+b/x)-(a+b/x)*cos(a+b/x))-(a+b/x)^2*cos(a
+b/x)+2*cos(a+b/x)+2*(a+b/x)*sin(a+b/x))
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.34, size = 51, normalized size = 1.13

$$-\frac{\left(\Gamma\left(3, \frac{ib}{x}\right) + \Gamma\left(3, -\frac{ib}{x}\right)\right)\cos\left(a\right) - \left(i\,\Gamma\left(3, \frac{ib}{x}\right) - i\,\Gamma\left(3, -\frac{ib}{x}\right)\right)\sin\left(a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*((gamma(3, I*b/x) + gamma(3, -I*b/x))*cos(a) - (I*gamma(3, I*b/x) - I*
gamma(3, -I*b/x))*sin(a))/b^3
```

Fricas [A]
time = 0.34, size = 44, normalized size = 0.98

$$-\frac{2\,bx\sin\left(\frac{ax+b}{x}\right) - \left(b^2 - 2\,x^2\right)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] -(2*b*x*sin((a*x + b)/x) - (b^2 - 2*x^2)*cos((a*x + b)/x))/(b^3*x^2)
```

Sympy [A]
time = 0.77, size = 46, normalized size = 1.02

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x} - \frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} & \text{for}\: b \neq 0 \\ -\frac{\sin\left(a\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**4,x)
```

```
[Out] Piecewise((cos(a + b/x)/(b*x**2) - 2*sin(a + b/x)/(b**2*x) - 2*cos(a + b/x)
/b**3, Ne(b, 0)), (-sin(a)/(3*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(45) = 90.
time = 3.64, size = 106, normalized size = 2.36

$$\frac{a^2\cos\left(\frac{ax+b}{x}\right) - \frac{2\,(ax+b)a\cos\left(\frac{ax+b}{x}\right)}{x} + 2\,a\sin\left(\frac{ax+b}{x}\right) + \frac{(ax+b)^2\cos\left(\frac{ax+b}{x}\right)}{x^2} - \frac{2\,(ax+b)\sin\left(\frac{ax+b}{x}\right)}{x} - 2\,\cos\left(\frac{ax+b}{x}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] (a^2*cos((a*x + b)/x) - 2*(a*x + b)*a*cos((a*x + b)/x)/x + 2*a*sin((a*x + b
)/x) + (a*x + b)^2*cos((a*x + b)/x)/x^2 - 2*(a*x + b)*sin((a*x + b)/x)/x -
2*cos((a*x + b)/x))/b^3
```

Mupad [B]
time = 4.63, size = 46, normalized size = 1.02

$$\frac{b^2\,\cos\left(a+\frac{b}{x}\right)-2\,b\,x\,\sin\left(a+\frac{b}{x}\right)}{b^3\,x^2}-\frac{2\,\cos\left(a+\frac{b}{x}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)/x^4,x)
```

```
[Out] (b^2*cos(a + b/x) - 2*b*x*sin(a + b/x))/(b^3*x^2) - (2*cos(a + b/x))/b^3
```

3.110 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5} dx$

Optimal. Leaf size=61

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}$$

```
[Out] cos(a+b/x)/b/x^3-6*cos(a+b/x)/b^3/x+6*sin(a+b/x)/b^4-3*sin(a+b/x)/b^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 3377, 2717}

$$\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^5,x]
```

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx &= -\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b^3}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 1.00

$$
\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^5,x]
```

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(61) = 122.
time = 0.06, size = 165, normalized size = 2.70

method	result
risch	$\frac{(b^2-6x^2)\cos\left(\frac{ax+b}{x}\right)}{b^3x^3} - \frac{3(b^2-2x^2)\sin\left(\frac{ax+b}{x}\right)}{x^2b^4}$
norman	$\frac{\frac{x}{b}-\frac{6x^3}{b^3}+\frac{12x^4\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^4}+\frac{6x^3\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^3}-\frac{6x^2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^2}-\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^4}$
meijerg	$-\frac{8\sqrt{\pi}\,\cos(a)\left(\frac{b\left(-\frac{5b^2}{2x^2}+15\right)\cos\left(\frac{b}{x}\right)}{20\sqrt{\pi}\,x}-\frac{\left(-\frac{15b^2}{2x^2}+15\right)\sin\left(\frac{b}{x}\right)}{20\sqrt{\pi}}\right)}{b^4} - \frac{8\sqrt{\pi}\,\sin(a)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3b^2}{2x^2}+3\right)\cos\left(\frac{b}{x}\right)}{4\sqrt{\pi}}-\frac{b\left(-\frac{b}{2x}\right.}{4}\right.}{b^4}$
derivativedivides	$-\frac{a^3\cos\left(a+\frac{b}{x}\right)+3a^2\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-3a\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\right)}{b^4}$
default	$-\frac{a^3\cos\left(a+\frac{b}{x}\right)+3a^2\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-3a\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\right)}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(a^3*cos(a+b/x)+3*a^2*(sin(a+b/x)-(a+b/x)*cos(a+b/x))-3*a*(-(a+b/x)^
2*cos(a+b/x)+2*cos(a+b/x)+2*(a+b/x)*sin(a+b/x))-(a+b/x)^3*cos(a+b/x)+3*(a+b
/x)^2*sin(a+b/x)-6*sin(a+b/x)+6*(a+b/x)*cos(a+b/x))
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 50, normalized size = 0.82

$$\frac{\left(i\,\Gamma\left(4,\frac{i\,b}{x}\right)-i\,\Gamma\left(4,-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(\Gamma\left(4,\frac{i\,b}{x}\right)+\Gamma\left(4,-\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((I*gamma(4, I*b/x) - I*gamma(4, -I*b/x))*cos(a) + (gamma(4, I*b/x) + g
amma(4, -I*b/x))*sin(a))/b^4
```

Fricas [A]
time = 0.36, size = 52, normalized size = 0.85

$$\frac{\left(b^3-6\,bx^2\right)\cos\left(\frac{ax+b}{x}\right)-3\,\left(b^2x-2\,x^3\right)\sin\left(\frac{ax+b}{x}\right)}{b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] ((b^3 - 6*b*x^2)*cos((a*x + b)/x) - 3*(b^2*x - 2*x^3)*sin((a*x + b)/x))/(b^
4*x^3)
```

Sympy [A]
time = 1.04, size = 61, normalized size = 1.00

$$\begin{cases}\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} & \text{for}\ b\neq 0\\ -\frac{\sin\left(a\right)}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**5,x)
```

```
[Out] Piecewise((cos(a + b/x)/(b*x**3) - 3*sin(a + b/x)/(b**2*x**2) - 6*cos(a + b
/x)/(b**3*x) + 6*sin(a + b/x)/b**4, Ne(b, 0)), (-sin(a)/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(61) = 122.
time = 4.06, size = 191, normalized size = 3.13

$$-\frac{a^3\cos\left(\frac{ax+b}{x}\right)-\frac{3\,(ax+b)a^2\cos\left(\frac{ax+b}{x}\right)}{x}+3\,a^2\sin\left(\frac{ax+b}{x}\right)-6\,a\cos\left(\frac{ax+b}{x}\right)+\frac{3\,(ax+b)^2a\cos\left(\frac{ax+b}{x}\right)}{x^2}-\frac{6\,(ax+b)a\sin\left(\frac{ax+b}{x}\right)}{x}-\frac{(ax+b)^3\cos\left(\frac{ax+b}{x}\right)}{x^3}+\frac{6\,(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x}+\frac{3\,(ax+b)^2\sin\left(\frac{ax+b}{x}\right)}{x^2}-6\,\sin\left(\frac{ax+b}{x}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="giac")
```

```
[Out] -(a^3*cos((a*x + b)/x) - 3*(a*x + b)*a^2*cos((a*x + b)/x)/x + 3*a^2*sin((a*
x + b)/x) - 6*a*cos((a*x + b)/x) + 3*(a*x + b)^2*a*cos((a*x + b)/x)/x^2 - 6
*(a*x + b)*a*sin((a*x + b)/x)/x - (a*x + b)^3*cos((a*x + b)/x)/x^3 + 6*(a*x
 + b)*cos((a*x + b)/x)/x + 3*(a*x + b)^2*sin((a*x + b)/x)/x^2 - 6*sin((a*x
+ b)/x))/b^4
```

Mupad [B]
time = 4.77, size = 64, normalized size = 1.05

$$\frac{6\,\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\,b\,x^2\,\cos\left(a+\frac{b}{x}\right)-b^3\,\cos\left(a+\frac{b}{x}\right)+3\,b^2\,x\,\sin\left(a+\frac{b}{x}\right)}{b^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)/x^5,x)
```

```
[Out] (6*sin(a + b/x))/b^4 - (6*b*x^2*cos(a + b/x) - b^3*cos(a + b/x) + 3*b^2*x*s
in(a + b/x))/(b^4*x^3)
```

3.111 $\int x^2 \sin^2\left(a+\frac{b}{x}\right) \, dx$

Optimal. Leaf size=97

$$\frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{2}{3}b^3\text{Ci}\left(\frac{2b}{x}\right)\sin(2a)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{2}{3}b^3\cos$$

```
[Out] 1/6*x^3+1/3*b^2*x*cos(2*a+2*b/x)-1/6*x^3*cos(2*a+2*b/x)+2/3*b^3*cos(2*a)*Si
(2*b/x)+2/3*b^3*Ci(2*b/x)*sin(2*a)+1/6*b*x^2*sin(2*a+2*b/x)
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3506, 3461, 3378, 3384, 3380, 3383}

$$\frac{2}{3}b^3\sin(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{2}{3}b^3\cos(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b/x]^2,x]
```

```
[Out] x^3/6 + (b^2*x*Cos[2*(a + b/x)])/3 - (x^3*Cos[2*(a + b/x)])/6 + (2*b^3*CosI
ntegral[(2*b)/x]*Sin[2*a])/3 + (b*x^2*Sin[2*(a + b/x)])/6 + (2*b^3*Cos[2*a]
*SinIntegral[(2*b)/x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx &= \int \left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+\frac{2b}{x}\right)\right)\,dx\\
&= \frac{x^3}{6}-\frac{1}{2}\int x^2\cos\left(2a+\frac{2b}{x}\right)\,dx\\
&= \frac{x^3}{6}+\frac{1}{2}\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^4}\,dx,x,\frac{1}{x}\right)\\
&= \frac{x^3}{6}-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{3}b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= \frac{x^3}{6}-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{3}b^2\text{Subst}\left(\int \frac{\cos(2a+2b}{x^2}\right.\\
&= \frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\\
&= \frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\\
&= \frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{2}{3}b^3\text{Ci}\left(\frac{2b}{x}\right)\sin(2a)+\frac{1}{6}ba
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.89

$$\frac{1}{6}\left(4b^3\text{Ci}\left(\frac{2b}{x}\right)\sin(2a)+x\left(x^2+2b^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-x^2\cos\left(2\left(a+\frac{b}{x}\right)\right)+bx\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)+4b^3\cos(2a)\text{Si}\left(\frac{2b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b/x]^2,x]
```

```
[Out] (4*b^3*CosIntegral[(2*b)/x]*Sin[2*a] + x*(x^2 + 2*b^2*Cos[2*(a + b/x)] - x^
2*Cos[2*(a + b/x)] + b*x*Sin[2*(a + b/x)]) + 4*b^3*Cos[2*a]*SinIntegral[(2*
b)/x])/6
```

Maple [A]
time = 0.10, size = 96, normalized size = 0.99

method	result
derivativedivides	$-b^3\left(-\frac{x^3}{6b^3}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^3}{6b^3}-\frac{\sin\left(2a+\frac{2b}{x}\right)x^2}{6b^2}-\frac{\cos\left(2a+\frac{2b}{x}\right)x}{3b}-\frac{2\,\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\cos(2a)}{3}-\frac{2\,\mathrm{cosineInte}}{}\right.$
default	$-b^3\left(-\frac{x^3}{6b^3}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^3}{6b^3}-\frac{\sin\left(2a+\frac{2b}{x}\right)x^2}{6b^2}-\frac{\cos\left(2a+\frac{2b}{x}\right)x}{3b}-\frac{2\,\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\cos(2a)}{3}-\frac{2\,\mathrm{cosineInte}}{}\right.$
risch	$-\frac{\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}b^3}{3}+\frac{2\,\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}b^3}{3}-\frac{i\,\mathrm{expIntegral}\left(1,-\frac{2ib}{x}\right)\mathrm{e}^{-2ia}b^3}{3}+\frac{ib^3\,\mathrm{expIntegral}\left(1,-\frac{2ib}{x}\right)\mathrm{e}^{2i}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*(-1/6/b^3*x^3+1/6*cos(2*a+2*b/x)/b^3*x^3-1/6*sin(2*a+2*b/x)/b^2*x^2-1/
3*cos(2*a+2*b/x)/b*x-2/3*Si(2*b/x)*cos(2*a)-2/3*Ci(2*b/x)*sin(2*a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 99, normalized size = 1.02

$$-\frac{1}{3}\left(\left(i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos(2\,a)-\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin(2\,a)\right)b^3+\frac{1}{6}\,bx^2\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{6}\,x^3+\frac{1}{6}\,(2\,b^2x-x^3)\cos\left(\frac{2\,(ax+b)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*((I*Ei(2*I*b/x) - I*Ei(-2*I*b/x))*cos(2*a) - (Ei(2*I*b/x) + Ei(-2*I*b/
x))*sin(2*a))*b^3 + 1/6*b*x^2*sin(2*(a*x + b)/x) + 1/6*x^3 + 1/6*(2*b^2*x -
 x^3)*cos(2*(a*x + b)/x)
```

Fricas [A]
time = 0.39, size = 109, normalized size = 1.12

$$\frac{1}{3}\,bx^2\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+\frac{2}{3}\,b^3\cos(2\,a)\,\mathrm{Si}\left(\frac{2\,b}{x}\right)-\frac{1}{3}\,b^2x+\frac{1}{3}\,x^3+\frac{1}{3}\,(2\,b^2x-x^3)\cos\left(\frac{ax+b}{x}\right)^2+\frac{1}{3}\left(b^3\,\mathrm{Ci}\left(\frac{2\,b}{x}\right)+b^3\,\mathrm{Ci}\left(-\frac{2\,b}{x}\right)\right)\sin(2\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^2*cos((a*x + b)/x)*sin((a*x + b)/x) + 2/3*b^3*cos(2*a)*sin_integral
(2*b/x) - 1/3*b^2*x + 1/3*x^3 + 1/3*(2*b^2*x - x^3)*cos((a*x + b)/x)^2 + 1/
3*(b^3*cos_integral(2*b/x) + b^3*cos_integral(-2*b/x))*sin(2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b/x)**2,x)
```

```
[Out] Integral(x**2*sin(a + b/x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(88) = 176.
time = 4.93, size = 442, normalized size = 4.56

$$\frac{4\,a^3b^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin\left(2\,a\right)-4\,a^3b^4\cos\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2b^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x}+\frac{12\,(ax+b)a^2b^4\cos(2\,a)\,\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)}{x}-2\,a^2b^4\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{12\,(ax+b)^2ab^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x^2}-\frac{12\,(ax+b)^2ab^4\cos(2\,a)\,\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)}{x^2}+\frac{4\,(ax+b)ab^4\cos\left(\frac{2\,(ax+b)}{x}\right)}{x}-\frac{4\,(ax+b)^3b^4\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x^3}+ab^4\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{4\,(ax+b)^3b^4\cos(2\,a)\,\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)}{x^3}+b^4\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{2\,(ax+b)^2b^4\cos\left(\frac{2\,(ax+b)}{x}\right)}{x^2}-\frac{(ax+b)b^4\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}-b^4}{6\left(a^3-\frac{3\,(ax+b)a^2}{x}+\frac{3\,(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] 1/6*(4*a^3*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a) - 4*a^3*b^4*cos(
2*a)*sin_integral(2*a - 2*(a*x + b)/x) - 12*(a*x + b)*a^2*b^4*cos_integral(
-2*a + 2*(a*x + b)/x)*sin(2*a)/x + 12*(a*x + b)*a^2*b^4*cos(2*a)*sin_integr
al(2*a - 2*(a*x + b)/x)/x - 2*a^2*b^4*cos(2*(a*x + b)/x) + 12*(a*x + b)^2*a
*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a)/x^2 - 12*(a*x + b)^2*a*b^4
*cos(2*a)*sin_integral(2*a - 2*(a*x + b)/x)/x^2 + 4*(a*x + b)*a*b^4*cos(2*(
a*x + b)/x)/x - 4*(a*x + b)^3*b^4*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*
a)/x^3 + a*b^4*sin(2*(a*x + b)/x) + 4*(a*x + b)^3*b^4*cos(2*a)*sin_integral
(2*a - 2*(a*x + b)/x)/x^3 + b^4*cos(2*(a*x + b)/x) - 2*(a*x + b)^2*b^4*cos(
2*(a*x + b)/x)/x^2 - (a*x + b)*b^4*sin(2*(a*x + b)/x)/x - b^4)/((a^3 - 3*(a
*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\sin\left(a+\frac{b}{x}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b/x)^2,x)
```

```
[Out] int(x^2*sin(a + b/x)^2, x)
```

3.112 $\int x \sin^2\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=65

$$-b^2\cos(2a)\mathrm{Ci}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)+b^2\sin(2a)\mathrm{Si}\left(\frac{2b}{x}\right)$$

```
[Out] -b^2*Ci(2*b/x)*cos(2*a)+b^2*Si(2*b/x)*sin(2*a)+1/2*x^2*sin(a+b/x)^2+1/2*b*x
*sin(2*a+2*b/x)
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3474, 4669, 3454, 3442, 3378, 3384, 3380, 3383}

$$b^2(-\cos(2a))\mathrm{CosIntegral}\left(\frac{2b}{x}\right)+b^2\sin(2a)\mathrm{Si}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b/x]^2,x]
```

```
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x^2*Sin[a + b/x]^2)/2 + (b*x*Sin[2*
(a + b/x)])/2 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3454

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3474

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int x\sin^2\left(a+\frac{b}{x}\right)\,dx &= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+b\int\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\,dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}b\int\sin\left(2\left(a+\frac{b}{x}\right)\right)\,dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}b\int\sin\left(2a+\frac{2b}{x}\right)\,dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)-b^2\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)-\left(b^2\cos(2a)\right)\text{Subst}\left(\int\frac{\cos(2bx)}{x}\,dx,x,\right.\\
&= -b^2\cos(2a)\text{Ci}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 65, normalized size = 1.00

$$-b^2\cos(2a)\mathrm{Ci}\left(\frac{2b}{x}\right)+\frac{1}{4}x\left(x-x\cos\left(2\left(a+\frac{b}{x}\right)\right)+2b\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)+b^2\sin(2a)\mathrm{Si}\left(\frac{2b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b/x]^2,x]
```

```
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x*(x - x*Cos[2*(a + b/x)] + 2*b*Sin
[2*(a + b/x)]))/4 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Maple [A]
time = 0.08, size = 76, normalized size = 1.17

method	result
derivativedivides	$-b^2\left(-\frac{x^2}{4b^2}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^2}{4b^2}-\frac{\sin\left(2a+\frac{2b}{x}\right)x}{2b}-\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\sin(2a)+\mathrm{cosineIntegral}\left(\frac{2b}{x}\right)\right.$
default	$-b^2\left(-\frac{x^2}{4b^2}+\frac{\cos\left(2a+\frac{2b}{x}\right)x^2}{4b^2}-\frac{\sin\left(2a+\frac{2b}{x}\right)x}{2b}-\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\sin(2a)+\mathrm{cosineIntegral}\left(\frac{2b}{x}\right)\right.$
risch	$-\frac{i\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}b^2}{2}+i\,\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}b^2+\frac{\mathrm{expIntegral}\left(1,-\frac{2ib}{x}\right)\mathrm{e}^{-2ia}b^2}{2}+\frac{b^2\,\mathrm{expIntegral}\left(1,-\frac{2i}{x}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*(-1/4/b^2*x^2+1/4*cos(2*a+2*b/x)/b^2*x^2-1/2*sin(2*a+2*b/x)/b*x-Si(2*b
/x)*sin(2*a)+Ci(2*b/x)*cos(2*a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 87, normalized size = 1.34

$$-\frac{1}{2}\left(\left(\mathrm{Ei}\left(\frac{2ib}{x}\right)+\mathrm{Ei}\left(-\frac{2ib}{x}\right)\right)\cos(2a)+\left(i\,\mathrm{Ei}\left(\frac{2ib}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2ib}{x}\right)\right)\sin(2a)\right)b^2-\frac{1}{4}x^2\cos\left(\frac{2(ax+b)}{x}\right)+\frac{1}{2}bx\sin\left(\frac{2(ax+b)}{x}\right)+\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + (I*Ei(2*I*b/x) - I*Ei(-2*I*b/
x))*sin(2*a))*b^2 - 1/4*x^2*cos(2*(a*x + b)/x) + 1/2*b*x*sin(2*(a*x + b)/x)
 + 1/4*x^2
```

Fricas [A]
time = 0.35, size = 90, normalized size = 1.38

$$-\frac{1}{2}x^2\cos\left(\frac{ax+b}{x}\right)^2+bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+b^2\sin(2a)\,\mathrm{Si}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2-\frac{1}{2}\left(b^2\,\mathrm{Ci}\left(\frac{2b}{x}\right)+b^2\,\mathrm{Ci}\left(-\frac{2b}{x}\right)\right)\cos(2a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos((a*x + b)/x)^2 + b*x*cos((a*x + b)/x)*sin((a*x + b)/x) + b^2*s
in(2*a)*sin_integral(2*b/x) + 1/2*x^2 - 1/2*(b^2*cos_integral(2*b/x) + b^2*
cos_integral(-2*b/x))*cos(2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^2\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)**2,x)
```

```
[Out] Integral(x*sin(a + b/x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(62) = 124.
time = 4.92, size = 283, normalized size = 4.35

$$-\frac{4\,a^2b^3\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)+4\,a^2b^3\sin\left(2\,a\right)\operatorname{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{8\,(ax+b)ab^3\cos(2\,a)\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)}{x}-\frac{8\,(ax+b)ab^3\sin(2\,a)\operatorname{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)}{x}+\frac{4\,(ax+b)^2b^3\cos(2\,a)\operatorname{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)}{x^2}+2\,ab^3\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{4\,(ax+b)^2b^3\sin(2\,a)\operatorname{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)}{x^2}+b^3\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{2\,(ax+b)b^3\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}-b^3}{4\left(a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*b^3*cos(2*a)*cos_integral(-2*a + 2*(a*x + b)/x) + 4*a^2*b^3*sin
(2*a)*sin_integral(2*a - 2*(a*x + b)/x) - 8*(a*x + b)*a*b^3*cos(2*a)*cos_in
tegral(-2*a + 2*(a*x + b)/x)/x - 8*(a*x + b)*a*b^3*sin(2*a)*sin_integral(2*
a - 2*(a*x + b)/x)/x + 4*(a*x + b)^2*b^3*cos(2*a)*cos_integral(-2*a + 2*(a*
x + b)/x)/x^2 + 2*a*b^3*sin(2*(a*x + b)/x) + 4*(a*x + b)^2*b^3*sin(2*a)*sin
_integral(2*a - 2*(a*x + b)/x)/x^2 + b^3*cos(2*(a*x + b)/x) - 2*(a*x + b)*b
^3*sin(2*(a*x + b)/x)/x - b^3)/((a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2)*b
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,{\sin\left(a+\frac{b}{x}\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b/x)^2,x)
```

```
[Out] int(x*sin(a + b/x)^2, x)
```

3.113 $\int \sin^2\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=41

$$-b\mathrm{Ci}\left(\frac{2b}{x}\right)\sin(2a)+x\sin^2\left(a+\frac{b}{x}\right)-b\cos(2a)\mathrm{Si}\left(\frac{2b}{x}\right)$$

```
[Out] -b*cos(2*a)*Si(2*b/x)-b*Ci(2*b/x)*sin(2*a)+x*sin(a+b/x)^2
```

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3442, 3394, 12, 3384, 3380, 3383}

$$-b\sin(2a)\mathrm{CosIntegral}\left(\frac{2b}{x}\right)-b\cos(2a)\mathrm{Si}\left(\frac{2b}{x}\right)+x\sin^2\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2,x]
```

```
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \sin^2\left(a+\frac{b}{x}\right)\,dx &= -\text{Subst}\left(\int \frac{\sin^2(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= x\sin^2\left(a+\frac{b}{x}\right)-(2b)\text{Subst}\left(\int \frac{\sin(2a+2bx)}{2x}\,dx, x, \frac{1}{x}\right)\\ &= x\sin^2\left(a+\frac{b}{x}\right)-b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\,dx, x, \frac{1}{x}\right)\\ &= x\sin^2\left(a+\frac{b}{x}\right)-(b\cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, \frac{1}{x}\right)-(b\sin(2a))\text{Subst}\left(\int \cos(\right.\\ &= -b\text{Ci}\left(\frac{2b}{x}\right)\sin(2a)+x\sin^2\left(a+\frac{b}{x}\right)-b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 1.00

$$-b\text{Ci}\left(\frac{2b}{x}\right)\sin(2a)+x\sin^2\left(a+\frac{b}{x}\right)-b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2,x]
```

```
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Maple [A]
time = 0.06, size = 52, normalized size = 1.27

method	result
derivativedivides	$-b\left(-\frac{x}{2b}+\frac{\cos\left(2a+\frac{2b}{x}\right)x}{2b}+\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\cos\left(2a\right)+\mathrm{cosineIntegral}\left(\frac{2b}{x}\right)\sin\left(2a\right)\right)$
default	$-b\left(-\frac{x}{2b}+\frac{\cos\left(2a+\frac{2b}{x}\right)x}{2b}+\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\cos\left(2a\right)+\mathrm{cosineIntegral}\left(\frac{2b}{x}\right)\sin\left(2a\right)\right)$
risch	$\frac{\pi\,\mathrm{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-2ia}b}{2}-\mathrm{sinIntegral}\left(\frac{2b}{x}\right)\mathrm{e}^{-2ia}b+\frac{i\,\mathrm{expIntegral}\left(1,-\frac{2ib}{x}\right)\mathrm{e}^{-2ia}b}{2}-\frac{ib\,\mathrm{expIntegral}\left(1,-\frac{2ib}{x}\right)\mathrm{e}^{2ia}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*(-1/2*x/b+1/2*cos(2*a+2*b/x)/b*x+Si(2*b/x)*cos(2*a)+Ci(2*b/x)*sin(2*a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 66, normalized size = 1.61

$$-\frac{1}{2}\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)+\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b-\frac{1}{2}\,x\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((-I*Ei(2*I*b/x) + I*Ei(-2*I*b/x))*cos(2*a) + (Ei(2*I*b/x) + Ei(-2*I*b
/x))*sin(2*a))*b - 1/2*x*cos(2*(a*x + b)/x) + 1/2*x
```

Fricas [A]
time = 0.36, size = 56, normalized size = 1.37

$$-x\cos\left(\frac{ax+b}{x}\right)^{2}-b\cos\left(2\,a\right)\mathrm{Si}\left(\frac{2\,b}{x}\right)-\frac{1}{2}\left(b\,\mathrm{Ci}\left(\frac{2\,b}{x}\right)+b\,\mathrm{Ci}\left(-\frac{2\,b}{x}\right)\right)\sin\left(2\,a\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] -x*cos((a*x + b)/x)^2 - b*cos(2*a)*sin_integral(2*b/x) - 1/2*(b*cos_integra
l(2*b/x) + b*cos_integral(-2*b/x))*sin(2*a) + x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin^{2}\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2,x)
```

```
[Out] Integral(sin(a + b/x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(41) = 82.
time = 5.27, size = 153, normalized size = 3.73

$$-\frac{2\,ab^{2}\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin\left(2\,a\right)-2\,ab^{2}\cos\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-\frac{2\,(ax+b)b^{2}\,\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)\sin(2\,a)}{x}+\frac{2\,(ax+b)b^{2}\cos(2\,a)\,\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)}{x}-b^{2}\cos\left(\frac{2\,(ax+b)}{x}\right)+b^{2}}{2\left(a-\frac{ax+b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^2*cos_integral(-2*a + 2*(a*x + b)/x)*sin(2*a) - 2*a*b^2*cos(2*a
)*sin_integral(2*a - 2*(a*x + b)/x) - 2*(a*x + b)*b^2*cos_integral(-2*a + 2
*(a*x + b)/x)*sin(2*a)/x + 2*(a*x + b)*b^2*cos(2*a)*sin_integral(2*a - 2*(a
*x + b)/x)/x - b^2*cos(2*(a*x + b)/x) + b^2)/((a - (a*x + b)/x)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(a+\frac{b}{x}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)^2,x)
```

```
[Out] int(sin(a + b/x)^2, x)
```

3.114 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x} dx$

Optimal. Leaf size=37

$$\frac{1}{2}\cos(2a)\text{Ci}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)$$

```
[Out] 1/2*Ci(2*b/x)*cos(2*a)+1/2*ln(x)-1/2*Si(2*b/x)*sin(2*a)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3506, 3459, 3457, 3456}

$$\frac{1}{2}\cos(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[(2*b)/x])/2 + Log[x]/2 - (Sin[2*a]*SinIntegral[(2*b)/
x])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx &= \int \left(\frac{1}{2x} - \frac{\cos\left(2a+\frac{2b}{x}\right)}{2x}\right) dx \\ &= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos\left(2a+\frac{2b}{x}\right)}{x}\,dx \\ &= \frac{\log(x)}{2} - \frac{1}{2}\cos(2a)\int \frac{\cos\left(\frac{2b}{x}\right)}{x}\,dx + \frac{1}{2}\sin(2a)\int \frac{\sin\left(\frac{2b}{x}\right)}{x}\,dx \\ &= \frac{1}{2}\cos(2a)\text{Ci}\left(\frac{2b}{x}\right) + \frac{\log(x)}{2} - \frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.86

$$\frac{1}{2}\left(\cos(2a)\text{Ci}\left(\frac{2b}{x}\right) + \log(x) - \sin(2a)\text{Si}\left(\frac{2b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x,x]
[Out] (Cos[2*a]*CosIntegral[(2*b)/x] + Log[x] - Sin[2*a]*SinIntegral[(2*b)/x])/2
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.97

method	result
derivativedivides	$-\frac{\ln\left(\frac{b}{x}\right)}{2} - \frac{\text{sinIntegral}\left(\frac{2b}{x}\right)\sin(2a)}{2} + \frac{\text{cosineIntegral}\left(\frac{2b}{x}\right)\cos(2a)}{2}$
default	$-\frac{\ln\left(\frac{b}{x}\right)}{2} - \frac{\text{sinIntegral}\left(\frac{2b}{x}\right)\sin(2a)}{2} + \frac{\text{cosineIntegral}\left(\frac{2b}{x}\right)\cos(2a)}{2}$
risch	$\frac{i e^{-2ia}\pi\,\text{csgn}\left(\frac{b}{x}\right)}{4} - \frac{i e^{-2ia}\,\text{sinIntegral}\left(\frac{2b}{x}\right)}{2} - \frac{e^{-2ia}\,\text{expIntegral}\left(1, -\frac{2ib}{x}\right)}{4} - \frac{e^{2ia}\,\text{expIntegral}\left(1, -\frac{2ib}{x}\right)}{4} + \frac{\ln(x)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/2*ln(b/x)-1/2*Si(2*b/x)*sin(2*a)+1/2*Ci(2*b/x)*cos(2*a)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 51, normalized size = 1.38

$$\frac{1}{4}\left(\text{Ei}\left(\frac{2i\,b}{x}\right) + \text{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right) + \frac{1}{4}\left(i\,\text{Ei}\left(\frac{2i\,b}{x}\right) - i\,\text{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right) + \frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*(Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + 1/4*(I*Ei(2*I*b/x) - I*Ei(-2*I*
b/x))*sin(2*a) + 1/2*log(x)
```

Fricas [A]
time = 0.36, size = 39, normalized size = 1.05

$$\frac{1}{4}\left(\mathrm{Ci}\left(\frac{2\,b}{x}\right)+\mathrm{Ci}\left(-\frac{2\,b}{x}\right)\right)\cos\left(2\,a\right)-\frac{1}{2}\,\sin\left(2\,a\right)\mathrm{Si}\left(\frac{2\,b}{x}\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*(cos_integral(2*b/x) + cos_integral(-2*b/x))*cos(2*a) - 1/2*sin(2*a)*si
n_integral(2*b/x) + 1/2*log(x)
```

Sympy [A]
time = 1.35, size = 31, normalized size = 0.84

$$\frac{\log\left(x\right)}{2}-\frac{\sin\left(2a\right)\mathrm{Si}\left(\frac{2b}{x}\right)}{2}+\frac{\cos\left(2a\right)\mathrm{Ci}\left(\frac{2b}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x,x)
```

```
[Out] log(x)/2 - sin(2*a)*Si(2*b/x)/2 + cos(2*a)*Ci(2*b/x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.
time = 4.77, size = 65, normalized size = 1.76

$$\frac{b\cos\left(2\,a\right)\mathrm{Ci}\left(-2\,a+\frac{2\,(ax+b)}{x}\right)+b\sin\left(2\,a\right)\mathrm{Si}\left(2\,a-\frac{2\,(ax+b)}{x}\right)-b\log\left(-a+\frac{ax+b}{x}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*cos(2*a)*cos_integral(-2*a + 2*(a*x + b)/x) + b*sin(2*a)*sin_integra
l(2*a - 2*(a*x + b)/x) - b*log(-a + (a*x + b)/x))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sin\left(a+\frac{b}{x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)^2/x,x)
```

```
[Out] int(sin(a + b/x)^2/x, x)
```

3.115 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{2x}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}$$

```
[Out] -1/2/x+1/2*cos(a+b/x)*sin(a+b/x)/b
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3460, 2715, 8}

$$\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}-\frac{1}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^2,x]
```

```
[Out] -1/2*1/x + (Cos[a + b/x]*Sin[a + b/x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\text{Subst}\left(\int \sin^2(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b} - \frac{1}{2}\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2x} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.03

$$-\frac{a+\frac{b}{x}}{2b} + \frac{\sin\left(2\left(a+\frac{b}{x}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^2,x]
```

```
[Out] -1/2*(a + b/x)/b + Sin[2*(a + b/x)]/(4*b)
```

Maple [A]
time = 0.04, size = 34, normalized size = 1.10

method	result	size
risch	$-\frac{1}{2x}+\frac{\sin\left(\frac{2ax+2b}{x}\right)}{4b}$	23
derivativedivides	$-\frac{-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}}{b}$	34
default	$-\frac{-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}}{b}$	34
norman	$\frac{-\frac{1}{2}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2}-\frac{x\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2 x}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.81

$$\frac{x\sin\left(\frac{2\,(ax+b)}{x}\right)-2\,b}{4\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x*sin(2*(a*x + b)/x) - 2*b)/(b*x)
```

Fricas [A]
time = 0.37, size = 34, normalized size = 1.10

$$\frac{x\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-b}{2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos((a*x + b)/x)*sin((a*x + b)/x) - b)/(b*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(20) = 40$.
time = 1.25, size = 262, normalized size = 8.45

$$\begin{cases} -\frac{b\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} - \frac{2b\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} - \frac{b}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} - \frac{2x\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} + \frac{2x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx} & \text{for}\ b \neq 0 \\ -\frac{\sin^2(a)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**2,x)
```

```
[Out] Piecewise((-b*tan(a/2 + b/(2*x))**4/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*ta
n(a/2 + b/(2*x))**2 + 2*b*x) - 2*b*tan(a/2 + b/(2*x))**2/(2*b*x*tan(a/2 + b
/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - b/(2*b*x*tan(a/2 + b/(2
*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - 2*x*tan(a/2 + b/(2*x))**3/
(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) + 2*x*t
an(a/2 + b/(2*x))/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**
2 + 2*b*x), Ne(b, 0)), (-sin(a)**2/x, True))
```

Giac [A]
time = 6.41, size = 29, normalized size = 0.94

$$-\frac{\frac{2\,(ax+b)}{x}-\sin\left(\frac{2\,(ax+b)}{x}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(a*x + b)/x - sin(2*(a*x + b)/x))/b
```

Mupad [B]
time = 4.57, size = 22, normalized size = 0.71

$$\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)^2/x^2,x)
```

```
[Out] sin(2*a + (2*b)/x)/(4*b) - 1/(2*x)
```

3.116 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4x^2}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}$$

```
[Out] -1/4/x^2+1/2*cos(a+b/x)*sin(a+b/x)/b/x-1/4*sin(a+b/x)^2/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3460, 3391, 30}

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}-\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^3,x]
```

```
[Out] -1/4*1/x^2 + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x) - Sin[a + b/x]^2/(4*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\text{Subst}\left(\int x\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2} - \frac{1}{2}\text{Subst}\left(\int x\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{4x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.84

$$\frac{x^2\cos\left(2\left(a+\frac{b}{x}\right)\right) - 2b\left(b - x\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{8b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^3,x]
[Out] (x^2*Cos[2*(a + b/x)] - 2*b*(b - x*Sin[2*(a + b/x)]))/(8*b^2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(45) = 90$.
time = 0.05, size = 97, normalized size = 1.90

method	result	size
risch	$-\frac{1}{4x^2}+\frac{\cos\left(\frac{2ax+2b}{x}\right)}{8b^2}+\frac{\sin\left(\frac{2ax+2b}{x}\right)}{4bx}$	42
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}-a\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)}{b^2}$	97
default	$-\frac{\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}-a\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)}{b^2}$	97
norman	$\frac{-\frac{1}{4}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{x^2\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^2}-\frac{\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2}-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{4}-\frac{x\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2x^2}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/b^2*((a+b/x)*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/4*(a+b/x)^2+1/
4*sin(a+b/x)^2-a*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x))
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 68, normalized size = 1.33

$$\frac{\left(\left(\Gamma\left(2, \frac{2i\,b}{x}\right) + \Gamma\left(2, -\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right) - \left(i\,\Gamma\left(2, \frac{2i\,b}{x}\right) - i\,\Gamma\left(2, -\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^2 - 4\,b^2}{16\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(((gamma(2, 2*I*b/x) + gamma(2, -2*I*b/x))*cos(2*a) - (I*gamma(2, 2*I*
b/x) - I*gamma(2, -2*I*b/x))*sin(2*a))*x^2 - 4*b^2)/(b^2*x^2)
```

Fricas [A]
time = 0.36, size = 60, normalized size = 1.18

$$\frac{2\,x^2\cos\left(\frac{ax+b}{x}\right)^2+4\,bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-2\,b^2-x^2}{8\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^2*cos((a*x + b)/x)^2 + 4*b*x*cos((a*x + b)/x)*sin((a*x + b)/x) - 2
*b^2 - x^2)/(b^2*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(37) = 74.
time = 1.61, size = 391, normalized size = 7.67

$$\begin{cases} -\frac{b^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}-\frac{2b^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}-\frac{b^2}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}-\frac{4bx\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}+\frac{4bx\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2}-\frac{4x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2} & \text{for } b \neq 0 \\ -\frac{\sin^2(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**3,x)
```

```
[Out] Piecewise((-b**2*tan(a/2 + b/(2*x))**4/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 +
 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 2*b**2*tan(a/2 + b/(2*x
))**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**
2 + 4*b**2*x**2) - b**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*ta
n(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*b*x*tan(a/2 + b/(2*x))**3/(4*b**2*x*
*2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2)
 + 4*b*x*tan(a/2 + b/(2*x))/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**
2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*x**2*tan(a/2 + b/(2*x))**2/(4*b*
*2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*
x**2), Ne(b, 0)), (-sin(a)**2/(2*x**2), True))
```

Giac [A]
time = 6.72, size = 77, normalized size = 1.51

$$-\frac{2\,a\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{4\,(ax+b)a}{x}-\frac{2\,(ax+b)\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{2\,(ax+b)^2}{x^2}-\cos\left(\frac{2\,(ax+b)}{x}\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*a*sin(2*(a*x + b)/x) - 4*(a*x + b)*a/x - 2*(a*x + b)*sin(2*(a*x + b
)/x)/x + 2*(a*x + b)^2/x^2 - cos(2*(a*x + b)/x))/b^2
```

Mupad [B]
time = 4.62, size = 41, normalized size = 0.80

$$\frac{\cos\left(2\,a+\frac{2\,b}{x}\right)}{8\,b^2}-\frac{1}{4\,x^2}+\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)^2/x^3,x)
```

```
[Out] cos(2*a + (2*b)/x)/(8*b^2) - 1/(4*x^2) + sin(2*a + (2*b)/x)/(4*b*x)
```

3.117 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{6x^3}+\frac{1}{4b^2x}-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}$$

```
[Out] -1/6/x^3+1/4/b^2/x-1/4*cos(a+b/x)*sin(a+b/x)/b^3+1/2*cos(a+b/x)*sin(a+b/x)/
b/x^2-1/2*sin(a+b/x)^2/b^2/x
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3460, 3392, 30, 2715, 8}

$$-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}+\frac{1}{4b^2x}-\frac{1}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^4,x]
```

```
[Out] -1/6*1/x^3 + 1/(4*b^2*x) - (Cos[a + b/x]*Sin[a + b/x])/(4*b^3) + (Cos[a + b
/x]*Sin[a + b/x])/(2*b*x^2) - Sin[a + b/x]^2/(2*b^2*x)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x} - \frac{1}{2}\text{Subst}\left(\int x^2\,dx, x, \frac{1}{x}\right) + \frac{\text{Subst}\left(\int \sin^2(a+\right.}{2b^2}\\
&= -\frac{1}{6x^3} - \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x} + \frac{\text{Subst}\left(\int\right.}{4}\\
&= -\frac{1}{6x^3} + \frac{1}{4b^2x} - \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.62

$$\frac{-4b^3 + 6bx^2\cos\left(2\left(a+\frac{b}{x}\right)\right) - 3(-2b^2x + x^3)\sin\left(2\left(a+\frac{b}{x}\right)\right)}{24b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^4,x]
[Out] (-4*b^3 + 6*b*x^2*Cos[2*(a + b/x)] - 3*(-2*b^2*x + x^3)*Sin[2*(a + b/x)])/(
24*b^3*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(77) = 154.
time = 0.06, size = 197, normalized size = 2.26

method	result
risch	$-\frac{1}{6x^3} + \frac{\cos\left(\frac{2ax+2b}{x}\right)}{4b^2x} + \frac{(2b^2-x^2)\sin\left(\frac{2ax+2b}{x}\right)}{8b^3x^2}$

norman	$\frac{-\frac{1}{6}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{3}-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{6}+\frac{x^2}{4b^2}-\frac{x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2b^3}+\frac{x^3\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2b^3}-\frac{3x^2\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2b^2}+\frac{x^2}{}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2x^3}$
derivativedivides	$-\frac{a^2\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-2a\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)+\left(a+\right.}{b^3}$
default	$-\frac{a^2\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-2a\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)+\left(a+\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(a^2*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-2*a*((a+b/x)*(-1/2*c
os(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/4*(a+b/x)^2+1/4*sin(a+b/x)^2)+(a+b/x)
^2*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/2*(a+b/x)*cos(a+b/x)^2+1/4*
cos(a+b/x)*sin(a+b/x)+1/4*b/x+1/4*a-1/3*(a+b/x)^3)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 69, normalized size = 0.79

$$\frac{3\left(\left(-i\,\Gamma\left(3,\frac{2i\,b}{x}\right)+i\,\Gamma\left(3,-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(\Gamma\left(3,\frac{2i\,b}{x}\right)+\Gamma\left(3,-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^3-16\,b^3}{96\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*((-I*gamma(3, 2*I*b/x) + I*gamma(3, -2*I*b/x))*cos(2*a) - (gamma(3,
 2*I*b/x) + gamma(3, -2*I*b/x))*sin(2*a))*x^3 - 16*b^3)/(b^3*x^3)
```

Fricas [A]
time = 0.37, size = 72, normalized size = 0.83

$$\frac{6\,bx^2\cos\left(\frac{ax+b}{x}\right)^2-2\,b^3-3\,bx^2+3\left(2\,b^2x-x^3\right)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{12\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*x^2*cos((a*x + b)/x)^2 - 2*b^3 - 3*b*x^2 + 3*(2*b^2*x - x^3)*cos(
(a*x + b)/x)*sin((a*x + b)/x))/(b^3*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(68) = 136.
time = 2.25, size = 654, normalized size = 7.52

$$\begin{cases} -\frac{2b^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}-\frac{4b^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}-\frac{2b^3}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}-\frac{12b^2x\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}+\frac{12b^2x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}+\frac{3bx^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}-\frac{18bx^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}+\frac{3bx^2}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}+\frac{6x^3\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3}-\frac{6x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{12b^3x^3\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+24b^3x^3\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+12b^3x^3} & \text{for}\: b \neq 0 \\ -\frac{\sin^2(a)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**4,x)
```

```
[Out] Piecewise((-2*b**3*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**
4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 4*b**3*tan(a/2 + b
/(2*x))**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(
2*x))**2 + 12*b**3*x**3) - 2*b**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 12*b**2*x*tan(a/2 + b/(2*
x))**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x)
)**2 + 12*b**3*x**3) + 12*b**2*x*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 +
 b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 3*b*x**
2*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*
tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 18*b*x**2*tan(a/2 + b/(2*x))**2/(12
*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*
b**3*x**3) + 3*b*x**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*ta
n(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 6*x**3*tan(a/2 + b/(2*x))**3/(12*b**3
*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*
x**3) - 6*x**3*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3), Ne(b, 0)), (-sin(a)**2/(3*
x**3), True))
```

Giac [A]
time = 3.55, size = 153, normalized size = 1.76

$$\frac{6\,a^2\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2}{x}-6\,a\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{12\,(ax+b)^2a}{x^2}+\frac{6\,(ax+b)\cos\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{6\,(ax+b)^2\sin\left(\frac{2\,(ax+b)}{x}\right)}{x^2}-\frac{4\,(ax+b)^3}{x^3}-3\,\sin\left(\frac{2\,(ax+b)}{x}\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(6*a^2*sin(2*(a*x + b)/x) - 12*(a*x + b)*a^2/x - 6*a*cos(2*(a*x + b)/x
) - 12*(a*x + b)*a*sin(2*(a*x + b)/x)/x + 12*(a*x + b)^2*a/x^2 + 6*(a*x + b
)*cos(2*(a*x + b)/x)/x + 6*(a*x + b)^2*sin(2*(a*x + b)/x)/x^2 - 4*(a*x + b)
^3/x^3 - 3*sin(2*(a*x + b)/x))/b^3
```

Mupad [B]
time = 4.70, size = 64, normalized size = 0.74

$$\frac{\frac{b\,x^2\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{4}-\frac{b^3}{6}+\frac{b^2\,x\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{4}}{b^3\,x^3}-\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)^2/x^4,x)
```

```
[Out] ((b*x^2*cos(2*a + (2*b)/x))/4 - b^3/6 + (b^2*x*sin(2*a + (2*b)/x))/4)/(b^3*
x^3) - sin(2*a + (2*b)/x)/(8*b^3)
```

3.118 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5} dx$

Optimal. Leaf size=107

$$-\frac{1}{8x^4}+\frac{3}{8b^2x^2}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3}-\frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x}+\frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}$$

```
[Out] -1/8/x^4+3/8/b^2/x^2+1/2*cos(a+b/x)*sin(a+b/x)/b/x^3-3/4*cos(a+b/x)*sin(a+b
/x)/b^3/x+3/8*sin(a+b/x)^2/b^4-3/4*sin(a+b/x)^2/b^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3460, 3392, 30, 3391}

$$\frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4}-\frac{3\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3x}-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}+\frac{3}{8b^2x^2}-\frac{1}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^5,x]
```

```
[Out] -1/8*1/x^4 + 3/(8*b^2*x^2) + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x^3) - (3*Cos
[a + b/x]*Sin[a + b/x])/(4*b^3*x) + (3*Sin[a + b/x]^2)/(8*b^4) - (3*Sin[a +
 b/x]^2)/(4*b^2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx &= -\text{Subst}\left(\int x^3\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2} - \frac{1}{2}\text{Subst}\left(\int x^3\,dx, x, \frac{1}{x}\right) + \frac{3\text{Subst}\left(\int x\sin^2 \right.}{}\\
&= -\frac{1}{8x^4} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x} + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin^2}{4b}\\
&= -\frac{1}{8x^4} + \frac{3}{8b^2x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x} + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 65, normalized size = 0.61

$$
-\frac{3(-2b^2x^2+x^4)\cos\left(2\left(a+\frac{b}{x}\right)\right)+2b\left(b^3+(-2b^2x+3x^3)\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{16b^4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^5,x]
```

```
[Out] -1/16*(3*(-2*b^2*x^2 + x^4)*Cos[2*(a + b/x)] + 2*b*(b^3 + (-2*b^2*x + 3*x^3
)*Sin[2*(a + b/x)]))/(b^4*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(95) = 190$.
time = 0.07, size = 334, normalized size = 3.12

method	result
risch	$-\frac{1}{8x^4}+\frac{3(2b^2-x^2)\cos\left(\frac{2ax+2b}{x}\right)}{16x^2b^4}+\frac{(2b^2-3x^2)\sin\left(\frac{2ax+2b}{x}\right)}{8b^3x^3}$
norman	$\frac{-\frac{1}{8}+\frac{x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}-\frac{\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{4}-\frac{\left(\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{8}+\frac{3x^2}{8b^2}-\frac{3x^3\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{2b^3}+\frac{3x^3\left(\tan^3\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{2b^3}-\frac{9x^2\left(\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{4b^2}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)^2x^4}$

derivativedivides	$-\frac{-a^3\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)+3a^2\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)-3a}{}$
default	$-\frac{-a^3\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)+3a^2\left(\left(a+\frac{b}{x}\right)\left(-\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2}+\frac{a}{2}+\frac{b}{2x}\right)-\frac{\left(a+\frac{b}{x}\right)^2}{4}+\frac{\left(\sin^2\left(a+\frac{b}{x}\right)\right)}{4}\right)-3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-a^3*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)+3*a^2*((a+b/x)*(-1/
2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/4*(a+b/x)^2+1/4*sin(a+b/x)^2)-3*a*
((a+b/x)^2*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/2*(a+b/x)*cos(a+b/x
)^2+1/4*cos(a+b/x)*sin(a+b/x)+1/4*b/x+1/4*a-1/3*(a+b/x)^3)+(a+b/x)^3*(-1/2*
cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-3/4*(a+b/x)^2*cos(a+b/x)^2+3/2*(a+b/x)
*(1/2*cos(a+b/x)*sin(a+b/x)+1/2*b/x+1/2*a)-3/8*(a+b/x)^2-3/8*sin(a+b/x)^2-3
/8*(a+b/x)^4)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 68, normalized size = 0.64

$$-\frac{\left(\left(\Gamma\left(4, \frac{2i\,b}{x}\right)+\Gamma\left(4, -\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(i\,\Gamma\left(4, \frac{2i\,b}{x}\right)-i\,\Gamma\left(4, -\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)x^4+8\,b^4}{64\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/64*(((gamma(4, 2*I*b/x) + gamma(4, -2*I*b/x))*cos(2*a) - (I*gamma(4, 2*I
*b/x) - I*gamma(4, -2*I*b/x))*sin(2*a))*x^4 + 8*b^4)/(b^4*x^4)
```

Fricas [A]
time = 0.37, size = 90, normalized size = 0.84

$$-\frac{2\,b^4+6\,b^2x^2-3\,x^4-6\left(2\,b^2x^2-x^4\right)\cos\left(\frac{ax+b}{x}\right)^2-4\left(2\,b^3x-3\,bx^3\right)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{16\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4 + 6*b^2*x^2 - 3*x^4 - 6*(2*b^2*x^2 - x^4)*cos((a*x + b)/x)^2 -
 4*(2*b^3*x - 3*b*x^3)*cos((a*x + b)/x)*sin((a*x + b)/x))/(b^4*x^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(92) = 184.
time = 3.43, size = 726, normalized size = 6.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**5,x)
```

```
[Out] Piecewise((-b**4*tan(a/2 + b/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 +
 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 2*b**4*tan(a/2 + b/(2*
x))**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))
**2 + 8*b**4*x**4) - b**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4
*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 8*b**3*x*tan(a/2 + b/(2*x))**3/(8*b
**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**
4*x**4) + 8*b**3*x*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 +
16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3*b**2*x**2*tan(a/2 + b
/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4) - 18*b**2*x**2*tan(a/2 + b/(2*x))**2/(8*b**4*x**4*ta
n(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3
*b**2*x**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4) + 12*b*x**3*tan(a/2 + b/(2*x))**3/(8*b**4*x**4*tan(a
/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 12*b
*x**3*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*
tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 12*x**4*tan(a/2 + b/(2*x))**2/(8*b**
4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*
x**4), Ne(b, 0)), (-sin(a)**2/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(95) = 190$.
time = 3.76, size = 255, normalized size = 2.38

$$-\frac{4\,a^3\sin\left(\frac{2\,(ax+b)}{x}\right)-\frac{8\,(ax+b)a^3}{x}-6\,a^2\cos\left(\frac{2\,(ax+b)}{x}\right)-\frac{12\,(ax+b)a^2\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{12\,(ax+b)^2a^2}{x^2}+\frac{12\,(ax+b)a\cos\left(\frac{2\,(ax+b)}{x}\right)}{x}-6\,a\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{12\,(ax+b)^2a\sin\left(\frac{2\,(ax+b)}{x}\right)}{x^2}-\frac{8\,(ax+b)^3a}{x^3}-\frac{6\,(ax+b)^2\cos\left(\frac{2\,(ax+b)}{x}\right)}{x^2}-\frac{4\,(ax+b)^3\sin\left(\frac{2\,(ax+b)}{x}\right)}{x^3}+\frac{6\,(ax+b)\sin\left(\frac{2\,(ax+b)}{x}\right)}{x}+\frac{2\,(ax+b)^4}{x^4}+3\,\cos\left(\frac{2\,(ax+b)}{x}\right)}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/16*(4*a^3*sin(2*(a*x + b)/x) - 8*(a*x + b)*a^3/x - 6*a^2*cos(2*(a*x + b)
/x) - 12*(a*x + b)*a^2*sin(2*(a*x + b)/x)/x + 12*(a*x + b)^2*a^2/x^2 + 12*(
a*x + b)*a*cos(2*(a*x + b)/x)/x - 6*a*sin(2*(a*x + b)/x) + 12*(a*x + b)^2*a
*sin(2*(a*x + b)/x)/x^2 - 8*(a*x + b)^3*a/x^3 - 6*(a*x + b)^2*cos(2*(a*x +
b)/x)/x^2 - 4*(a*x + b)^3*sin(2*(a*x + b)/x)/x^3 + 6*(a*x + b)*sin(2*(a*x +
 b)/x)/x + 2*(a*x + b)^4/x^4 + 3*cos(2*(a*x + b)/x))/b^4
```

Mupad [B]
time = 4.72, size = 84, normalized size = 0.79

$$-\frac{3\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{16\,b^4}-\frac{\frac{b^4}{8}-\frac{3\,b^2\,x^2\,\cos\left(2\,a+\frac{2\,b}{x}\right)}{8}+\frac{3\,b\,x^3\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{8}-\frac{b^3\,x\,\sin\left(2\,a+\frac{2\,b}{x}\right)}{4}}{b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x)^2/x^5,x)
```

```
[Out] - (3*cos(2*a + (2*b)/x))/(16*b^4) - (b^4/8 - (3*b^2*x^2*cos(2*a + (2*b)/x))
/8 + (3*b*x^3*sin(2*a + (2*b)/x))/8 - (b^3*x*sin(2*a + (2*b)/x))/4)/(b^4*x^
4)
```

3.119 $\int \sin\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=80

$$-\sqrt{b}\,\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{b}\,\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)+x\sin\left(a+\frac{b}{x^2}\right)$$

```
[Out] x*sin(a+b/x^2)-cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*b^(1/2)*2^(1/2)*
Pi^(1/2)+FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/
2)
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3440, 3468, 3435, 3433, 3432}

$$\sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}}{x}\right)+\sqrt{2\pi}\,\sqrt{b}\,\sin(a)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)+x\sin\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2],x]
```

```
[Out] -(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]) + Sqrt[b]*Sqr
t[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a] + x*Sin[a + b/x^2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+\frac{b}{x^2}\right)\,dx &= -\text{Subst}\left(\int \frac{\sin\left(a+bx^2\right)}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= x\sin\left(a+\frac{b}{x^2}\right)-(2b)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)\\ &= x\sin\left(a+\frac{b}{x^2}\right)-(2b\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,\frac{1}{x}\right)+(2b\sin(a))\text{Subst}\left(\int \sin\right.\\ &= -\sqrt{b}\,\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{b}\,\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)+x\sin\left(a+\frac{b}{x^2}\right.\end{aligned}$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 1.01

$$x\cos\left(\frac{b}{x^2}\right)\sin(a)-\sqrt{b}\,\sqrt{2\pi}\left(\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)-S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right)+x\cos(a)\sin\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2],x]
```

```
[Out] x*Cos[b/x^2]*Sin[a] - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/x] - FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) + x*Cos[a]*Sin[b/x^2]
```

Maple [A]
time = 0.07, size = 59, normalized size = 0.74

method	result
derivativedivides	$x\sin\left(a+\frac{b}{x^2}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin\left(a\right)\operatorname{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$
default	$x\sin\left(a+\frac{b}{x^2}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin\left(a\right)\operatorname{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$
risch	$-\frac{\mathrm{e}^{ia}b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{2\sqrt{-ib}}-\frac{\mathrm{e}^{-ia}b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{ib}}{x}\right)}{2\sqrt{ib}}+x\sin\left(\frac{a\,x^2+b}{x^2}\right)$
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\sqrt{2}\,\sqrt{b}\,\left(-\frac{4\sqrt{2}\,x\sin\left(\frac{b}{x^2}\right)}{\sqrt{b}\,\sqrt{\pi}}+8\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{8}-\frac{\sqrt{\pi}\,\sin(a)\sqrt{2}\,(b^2)^{\frac{1}{4}}\left(-\frac{4x\sqrt{2}}{\sqrt{\pi}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*sin(a+b/x^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 127, normalized size = 1.59

$$\frac{\sqrt{2}\left(2\sqrt{2}\,bx^2\sqrt{\frac{1}{x^4}}\sin\left(\frac{ax^2+b}{x^2}\right)+\left(\left((i-1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right)-(i+1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos(a)+\left((i+1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right)-(i-1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\sin(a)\right)b\left(\frac{b^2}{x^4}\right)^{\frac{1}{4}}\right)\sqrt{x^4}}{4bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*b*x^2*sqrt(x^(-4))*sin((a*x^2 + b)/x^2) + (((I - 1)*
sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) -
 1))*cos(a) + ((I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1)*sqrt(pi)
*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*b*(b^2/x^4)^(1/4))*sqrt(x^4)/(b*x)
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.92

$$-\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(a\right)\operatorname{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\operatorname{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)+x\sin\left(\frac{ax^2+b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) + sqrt(2)*p
i*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) + x*sin((a*x^2 + b)/x
^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2),x)
```

```
[Out] Integral(sin(a + b/x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x^2),x)
```

```
[Out] int(sin(a + b/x^2), x)
```

3.120 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x} dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\text{Ci}\left(\frac{b}{x^2}\right)\sin(a)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*cos(a)*Si(b/x^2)-1/2*Ci(b/x^2)*sin(a)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3458, 3457, 3456}

$$-\frac{1}{2}\sin(a)\text{CosIntegral}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x,x]
```

```
[Out] -1/2*(CosIntegral[b/x^2]*Sin[a]) - (Cos[a]*SinIntegral[b/x^2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx &= \cos(a)\int \frac{\sin\left(\frac{b}{x^2}\right)}{x}\,dx+\sin(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}\,dx\\ &= -\frac{1}{2}\text{Ci}\left(\frac{b}{x^2}\right)\sin(a)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.00

$$\frac{1}{2}\left(-\text{Ci}\left(\frac{b}{x^2}\right)\sin(a)-\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x,x]
[Out] (-(CosIntegral[b/x^2]*Sin[a]) - Cos[a]*SinIntegral[b/x^2])/2
```

Maple [A]
time = 0.04, size = 22, normalized size = 0.88

method	result	s
derivativedivides	$-\frac{\cos(a)\,\text{sinIntegral}\left(\frac{b}{x^2}\right)}{2}-\frac{\text{cosineIntegral}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	2
default	$-\frac{\cos(a)\,\text{sinIntegral}\left(\frac{b}{x^2}\right)}{2}-\frac{\text{cosineIntegral}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	2
risch	$-\frac{i\mathrm{e}^{ia}\,\text{expIntegral}\left(1,-\frac{ib}{x^2}\right)}{4}+\frac{\mathrm{e}^{-ia}\pi\,\text{csgn}\left(\frac{b}{x^2}\right)}{4}-\frac{\mathrm{e}^{-ia}\,\text{sinIntegral}\left(\frac{b}{x^2}\right)}{2}+\frac{i\,\text{expIntegral}\left(1,-\frac{ib}{x^2}\right)\mathrm{e}^{-ia}}{4}$	6
meijerg	$-\frac{\cos(a)\,\text{sinIntegral}\left(\frac{b}{x^2}\right)}{2}-\frac{\sqrt{\pi}\,\sin(a)\left(\frac{2\gamma-4\ln(x)+\ln(b^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x^2}\right)}{\sqrt{\pi}}+\frac{2\,\text{cosineIntegral}\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{4}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*cos(a)*Si(b/x^2)-1/2*Ci(b/x^2)*sin(a)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 43, normalized size = 1.72

$$\frac{1}{4}\left(i\,\text{Ei}\left(\frac{i\,b}{x^2}\right)-i\,\text{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\cos\left(a\right)-\frac{1}{4}\left(\text{Ei}\left(\frac{i\,b}{x^2}\right)+\text{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="maxima")
[Out] 1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x^2))*cos(a) - 1/4*(Ei(I*b/x^2) + Ei(-I*b/x^
2))*sin(a)
```

Fricas [A]
time = 0.44, size = 29, normalized size = 1.16

$$-\frac{1}{4}\left(\text{Ci}\left(\frac{b}{x^2}\right)+\text{Ci}\left(-\frac{b}{x^2}\right)\right)\sin\left(a\right)-\frac{1}{2}\cos\left(a\right)\text{Si}\left(\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(b/x^2) + cos_integral(-b/x^2))*sin(a) - 1/2*cos(a)*sin_i
ntegral(b/x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x,x)
```

```
[Out] Integral(sin(a + b/x**2)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$-\frac{\sin\left(a\right)\,\mathrm{cosint}\left(\frac{b}{x^2}\right)}{2}-\frac{\cos\left(a\right)\,\mathrm{sinint}\left(\frac{b}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x^2)/x,x)
```

```
[Out] - (sin(a)*cosint(b/x^2))/2 - (cos(a)*sinint(b/x^2))/2
```

3.121 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}$$

```
[Out] -1/2*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/2)*Pi^(1/2)/b^(1/2)-1
/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3464, 3434, 3433, 3432}

$$-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) - (Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3464

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= -\left(\cos(a)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\right) - \sin(a)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 0.81

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.63

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	47

default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	47
meijerg	$-\frac{\cos(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}-\frac{\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sin(a)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}$	56
risch	$\frac{ie^{ia}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{4\sqrt{-ib}}-\frac{ie^{-ia}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{ib}}{x}\right)}{4\sqrt{ib}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)+
sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 98, normalized size = 1.31

$$-\frac{\sqrt{2}\,\sqrt{x^4}\left(\left((i+1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{i\,b}{x^2}}\right)-1\right)-(i-1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{i\,b}{x^2}}\right)-1\right)\right)\cos\left(a\right)+\left(-(i-1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{i\,b}{x^2}}\right)-1\right)+(i+1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{i\,b}{x^2}}\right)-1\right)\right)\sin\left(a\right)\right)\left(\frac{b^2}{x^4}\right)^{\frac{1}{4}}}{8\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(x^4)*(((I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1
)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt
(I*b/x^2)) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*(b^2/
x^4)^(1/4)/(b*x)
```

Fricas [A]
time = 0.38, size = 64, normalized size = 0.85

$$-\frac{\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqrt
(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x**2,x)
```

```
[Out] Integral(sin(a + b/x**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x^2, x)
```

Mupad [B]
time = 4.80, size = 55, normalized size = 0.73

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\cos\left(a\right)}{2\,\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\sin\left(a\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x^2)/x^2,x)
```

```
[Out] - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*cos(a))/(2*b^(
1/2)) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*sin(a))/
(2*b^(1/2))
```

3.122 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$

Optimal. Leaf size=15

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

[Out] 1/2*cos(a+b/x^2)/b

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3460, 2718}

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b/x^2]/x^3,x]

[Out] Cos[a + b/x^2]/(2*b)

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x^3,x]
```

```
[Out] Cos[a + b/x^2]/(2*b)
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$	14
risch	$\frac{\cos\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$	18
norman	$\frac{1}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x^2}\right)\right)}$	22
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}-\frac{\sin(a)\sin\left(\frac{b}{x^2}\right)}{2b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cos(a+b/x^2)/b
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.87

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*cos(a + b/x^2)/b
```

Fricas [A]
time = 0.37, size = 17, normalized size = 1.13

$$\frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos((a*x^2 + b)/x^2)/b
```

Sympy [A]
time = 0.63, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x**3,x)
[Out] Piecewise((cos(a + b/x**2)/(2*b), Ne(b, 0)), (-sin(a)/(2*x**2), True))
```

Giac [A]
time = 4.68, size = 17, normalized size = 1.13

$$\frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="giac")
[Out] 1/2*cos((a*x^2 + b)/x^2)/b
```

Mupad [B]
time = 4.67, size = 13, normalized size = 0.87

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x^2)/x^3,x)
[Out] cos(a + b/x^2)/(2*b)
```

3.123 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=97

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}$$

```
[Out] 1/2*cos(a+b/x^2)/b/x-1/4*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/2
)*Pi^(1/2)/b^(3/2)+1/4*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)*
Pi^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3490, 3466, 3435, 3433, 3432}

$$-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x^4,x]
```

```
[Out] Cos[a + b/x^2]/(2*b*x) - (Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x
])/(2*b^(3/2)) + (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(2*b^
(3/2))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\text{Subst}\left(\int\cos\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}\\
&= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\cos(a)\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}+\frac{\sin(a)\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}\\
&= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 89, normalized size = 0.92

$$\frac{2\sqrt{b}\cos\left(a+\frac{b}{x^2}\right)-\sqrt{2\pi}\,x\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{2\pi}\,xS\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{4b^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x^4,x]
[Out] (2*Sqrt[b]*Cos[a + b/x^2] - Sqrt[2*Pi]*x*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
])/x] + Sqrt[2*Pi]*x*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(4*b^(3/2)*x)
```

Maple [A]
time = 0.05, size = 65, normalized size = 0.67

method	result
derivativedivides	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$
default	$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$
risch	$-\frac{e^{ia}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{8b\sqrt{-ib}}-\frac{e^{-ia}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{ib}}{x}\right)}{8b\sqrt{ib}}+\frac{\cos\left(\frac{a\,x^2+b}{x^2}\right)}{2bx}$
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\sqrt{2}\left(-\frac{\sqrt{2}\,\sqrt{b}\,\cos\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,x}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)}{2}\right)}{2b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\,\sin(a)\sqrt{2}\,(b^2)^{\frac{1}{4}}\left(\frac{\sqrt{2}\,(b^2)^{\frac{3}{4}}}{2\sqrt{\pi}} \cdots\right.}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cos(a+b/x^2)/b/x-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*
2^(1/2)/Pi^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 74, normalized size = 0.76

$$-\frac{\sqrt{2}\,(x^4)^{\frac{3}{2}}\left(\left((i-1)\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)-(i+1)\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\cos(a)+\left((i+1)\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)-(i-1)\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\sin(a)\right)\left(\frac{b^2}{x^4}\right)^{\frac{3}{4}}}{8\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(((I - 1)*gamma(3/2, I*b/x^2) - (I + 1)*gamma(3/2, -I*b/x^2))*
cos(a) + ((I + 1)*gamma(3/2, I*b/x^2) - (I - 1)*gamma(3/2, -I*b/x^2))*sin(a
))*(x^4)^(3/2)*(b^2/x^4)^(3/4)/(b^3*x^3)
```

Fricas [A]
time = 0.39, size = 85, normalized size = 0.88

$$-\frac{\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)-\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)-2\,b\cos\left(\frac{ax^2+b}{x^2}\right)}{4\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) - sq
rt(2)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) - 2*b*cos((a
*x^2 + b)/x^2))/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x**4,x)
```

```
[Out] Integral(sin(a + b/x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/x^2)/x^4,x)
```

```
[Out] int(sin(a + b/x^2)/x^4, x)
```

3.124 $\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\cos\left(\sqrt{x}\right)$$

```
[Out] -2*cos(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3460, 2718}

$$-2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Cos[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= -2\cos\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Cos[Sqrt[x]]
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-2\cos\left(\sqrt{x}\right)$	7
default	$-2\cos\left(\sqrt{x}\right)$	7
meijerg	$2\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\sqrt{x}\right)}{\sqrt{\pi}}\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*cos(x^(1/2))
```

Maxima [A]
time = 0.28, size = 6, normalized size = 0.75

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*cos(sqrt(x))
```

Fricas [A]
time = 0.37, size = 6, normalized size = 0.75

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*cos(sqrt(x))
```

Sympy [A]
time = 0.10, size = 8, normalized size = 1.00

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2))/x**(1/2),x)
```

```
[Out] -2*cos(sqrt(x))
```

Giac [A]
time = 4.49, size = 6, normalized size = 0.75

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*cos(sqrt(x))
```

Mupad [B]
time = 4.57, size = 6, normalized size = 0.75

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*cos(x^(1/2))
```

3.125 $\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$-2\cos\left(\sqrt{x}\right)+\frac{2}{3}\cos^3\left(\sqrt{x}\right)$$

```
[Out] -2*cos(x^(1/2))+2/3*cos(x^(1/2))^3
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3460, 2713}

$$\frac{2}{3}\cos^3\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]]^3/Sqrt[x],x]
```

```
[Out] -2*Cos[Sqrt[x]] + (2*Cos[Sqrt[x]]^3)/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \sin^3(x)\,dx,x,\sqrt{x}\right)\\ &= -\left(2\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos\left(\sqrt{x}\right)\right)\right)\\ &= -2\cos\left(\sqrt{x}\right)+\frac{2}{3}\cos^3\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.10

$$-\frac{3}{2}\cos\left(\sqrt{x}\right)+\frac{1}{6}\cos\left(3\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]]^3/Sqrt[x],x]
[Out] (-3*Cos[Sqrt[x]])/2 + Cos[3*Sqrt[x]]/6
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2\left(2+\sin^{2}\left(\sqrt{x}\right)\right)\cos\left(\sqrt{x}\right)}{3}$	15
default	$-\frac{2\left(2+\sin^{2}\left(\sqrt{x}\right)\right)\cos\left(\sqrt{x}\right)}{3}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(2+sin(x^(1/2))^2)*cos(x^(1/2))
```

Maxima [A]
time = 0.31, size = 15, normalized size = 0.71

$$\frac{2}{3}\cos\left(\sqrt{x}\right)^{3}-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="maxima")
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Fricas [A]
time = 0.38, size = 15, normalized size = 0.71

$$\frac{2}{3}\cos\left(\sqrt{x}\right)^{3}-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="fricas")
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Sympy [A]
time = 0.18, size = 29, normalized size = 1.38

$$-2\sin^2\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)-\frac{4\cos^3\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2))**3/x**(1/2),x)
[Out] -2*sin(sqrt(x))**2*cos(sqrt(x)) - 4*cos(sqrt(x))**3/3
```

Giac [A]
time = 4.02, size = 15, normalized size = 0.71

$$\frac{2}{3}\cos\left(\sqrt{x}\right)^3-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="giac")
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Mupad [B]
time = 4.74, size = 14, normalized size = 0.67

$$\frac{2\cos\left(\sqrt{x}\right)\left(\cos\left(\sqrt{x}\right)^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2))^3/x^(1/2),x)
[Out] (2*cos(x^(1/2))*(cos(x^(1/2))^2 - 3))/3
```

3.126 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3442, 3377, 2717}

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \sin\left(\sqrt{x}\right)\,dx&=2\text{Subst}\left(\int x\sin(x)\,dx,x,\sqrt{x}\right)\\&=-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\text{Subst}\left(\int \cos(x)\,dx,x,\sqrt{x}\right)\\&=-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\,\left(-\frac{\sqrt{x}\,\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 0.34, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A]
time = 0.41, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A]
time = 0.09, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A]
time = 4.51, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B]
time = 4.64, size = 16, normalized size = 0.73

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

3.127 $\int \sin^2\left(\sqrt[3]{x}\right) dx$

Optimal. Leaf size=69

$$-\frac{3\sqrt[3]{x}}{4}+\frac{x}{2}+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)-\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)$$

```
[Out] -3/4*x^(1/3)+1/2*x+3/4*cos(x^(1/3))*sin(x^(1/3))-3/2*x^(2/3)*cos(x^(1/3))*s
in(x^(1/3))+3/2*x^(1/3)*sin(x^(1/3))^2
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3442, 3392, 30, 2715, 8}

$$-\frac{3}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{2}-\frac{3\sqrt[3]{x}}{4}+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)+\frac{3}{4}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(1/3)]^2,x]
```

```
[Out] (-3*x^(1/3))/4 + x/2 + (3*Cos[x^(1/3)]*Sin[x^(1/3)])/4 - (3*x^(2/3)*Cos[x^(
1/3)]*Sin[x^(1/3)])/2 + (3*x^(1/3)*Sin[x^(1/3)]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin^2\left(\sqrt[3]{x}\right)\,dx = 3\text{Subst}\left(\int x^2\sin^2(x)\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) + \frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right) + \frac{3}{2}\text{Subst}\left(\int x^2\,dx, x, \sqrt[3]{x}\right) - \frac{3}{2}\text{Subs}$$
$$= \frac{x}{2} + \frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) - \frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) + \frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right) - \frac{3}{4}\text{Subst}$$
$$= -\frac{3\sqrt[3]{x}}{4} + \frac{x}{2} + \frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) - \frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) + \frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.59

$$\frac{1}{8}\left(4x - 6\sqrt[3]{x}\cos\left(2\sqrt[3]{x}\right) + \left(3 - 6x^{2/3}\right)\sin\left(2\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(1/3)]^2,x]
[Out] (4*x - 6*x^(1/3)*Cos[2*x^(1/3)] + (3 - 6*x^(2/3))*Sin[2*x^(1/3)])/8
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.75

method	result	size
meijerg	$\frac{3x^{\frac{5}{3}}\,\text{hypergeom}\left(\left[1,\frac{5}{2}\right],\left[\frac{3}{2},2,\frac{7}{2}\right],-x^{\frac{2}{3}}\right)}{5}$	19
derivativedivides	$3x^{\frac{2}{3}}\left(-\frac{\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{x^{\frac{1}{3}}}{2}\right)-\frac{3x^{\frac{1}{3}}\left(\cos^2\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}}{4}-x$	52
default	$3x^{\frac{2}{3}}\left(-\frac{\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{2}+\frac{x^{\frac{1}{3}}}{2}\right)-\frac{3x^{\frac{1}{3}}\left(\cos^2\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)}{4}+\frac{3x^{\frac{1}{3}}}{4}-x$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(2/3)*(-1/2*cos(x^(1/3))*sin(x^(1/3))+1/2*x^(1/3))-3/2*x^(1/3)*cos(x^(1
/3))^2+3/4*cos(x^(1/3))*sin(x^(1/3))+3/4*x^(1/3)-x
```

Maxima [A]
time = 0.31, size = 30, normalized size = 0.43

$$-\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] -3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Fricas [A]
time = 0.37, size = 37, normalized size = 0.54

$$-\frac{3}{4}\left(2\,x^{\frac{2}{3}}-1\right)\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)-\frac{3}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}+\frac{1}{2}\,x+\frac{3}{4}\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -3/4*(2*x^(2/3) - 1)*cos(x^(1/3))*sin(x^(1/3)) - 3/2*x^(1/3)*cos(x^(1/3))^2
 + 1/2*x + 3/4*x^(1/3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(66) = 132.
time = 0.52, size = 379, normalized size = 5.49

$$\frac{12x^{\frac{2}{3}}\tan^{3}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}-\frac{12x^{\frac{2}{3}}\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}-\frac{3\sqrt[3]{x}\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}+\frac{18\sqrt[3]{x}\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}-\frac{3\sqrt[3]{x}}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}+\frac{2x\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}+\frac{4x\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}+\frac{2x}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}-\frac{6\tan^{3}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}+\frac{6\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/3))**2,x)
```

```
[Out] 12*x**(2/3)*tan(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2
 + 4) - 12*x**(2/3)*tan(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/
2)**2 + 4) - 3*x**(1/3)*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(x*
*(1/3)/2)**2 + 4) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 +
8*tan(x**(1/3)/2)**2 + 4) - 3*x**(1/3)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 2*x*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 4*x*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
```

```
3)/2)**2 + 4) + 2*x/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4) - 6*t
an(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4) + 6*tan
(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4)
```

Giac [A]
time = 5.46, size = 30, normalized size = 0.43

$$-\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="giac")
[Out] -3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Mupad [B]
time = 4.77, size = 34, normalized size = 0.49

$$\frac{x}{2}+\frac{3\sin\left(2\,x^{1/3}\right)}{8}-\frac{3\,x^{1/3}\cos\left(2\,x^{1/3}\right)}{4}-\frac{3\,x^{2/3}\sin\left(2\,x^{1/3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3))^2,x)
[Out] x/2 + (3*sin(2*x^(1/3)))/8 - (3*x^(1/3)*cos(2*x^(1/3)))/4 - (3*x^(2/3)*sin(
2*x^(1/3)))/4
```

3.128 $\int \sin^3\left(\sqrt[3]{x}\right) dx$

Optimal. Leaf size=87

$$\frac{14}{3}\cos\left(\sqrt[3]{x}\right)-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)$$

```
[Out] 14/3*cos(x^(1/3))-2*x^(2/3)*cos(x^(1/3))-2/9*cos(x^(1/3))^3+4*x^(1/3)*sin(x
^(1/3))-x^(2/3)*cos(x^(1/3))*sin(x^(1/3))^2+2/3*x^(1/3)*sin(x^(1/3))^3
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3442, 3392, 3377, 2718, 2713}

$$-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+\frac{14}{3}\cos\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(1/3)]^3,x]
```

```
[Out] (14*Cos[x^(1/3)])/3 - 2*x^(2/3)*Cos[x^(1/3)] - (2*Cos[x^(1/3)]^3)/9 + 4*x^(
1/3)*Sin[x^(1/3)] - x^(2/3)*Cos[x^(1/3)]*Sin[x^(1/3)]^2 + (2*x^(1/3)*Sin[x^
(1/3)]^3)/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \sin^3\left(\sqrt[3]{x}\right)\,dx &= 3\text{Subst}\left(\int x^2\sin^3(x)\,dx, x, \sqrt[3]{x}\right)\\ &= -x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)-\frac{2}{3}\text{Subst}\left(\int \sin^3(x)\,dx, x, \sqrt[3]{x}\right)+2\text{S}\\ &= -2x^{2/3}\cos\left(\sqrt[3]{x}\right)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+\frac{2}{3}\text{Subst}\left(\int (1-x\right.\\ &= \frac{2}{3}\cos\left(\sqrt[3]{x}\right)-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\\ &= \frac{14}{3}\cos\left(\sqrt[3]{x}\right)-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\end{aligned}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.71

$$\frac{1}{36}\left(-81\left(-2+x^{2/3}\right)\cos\left(\sqrt[3]{x}\right)+\left(-2+9x^{2/3}\right)\cos\left(3\sqrt[3]{x}\right)-6\sqrt[3]{x}\left(-27\sin\left(\sqrt[3]{x}\right)+\sin\left(3\sqrt[3]{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(1/3)]^3,x]
```

```
[Out] (-81*(-2 + x^(2/3))*Cos[x^(1/3)] + (-2 + 9*x^(2/3))*Cos[3*x^(1/3)] - 6*x^(1
/3)*(-27*Sin[x^(1/3)] + Sin[3*x^(1/3)]))/36
```

Maple [A]
time = 0.02, size = 59, normalized size = 0.68

method	result
derivativedivides	$-x^{\frac{2}{3}}\left(2+\sin^2\left(x^{\frac{1}{3}}\right)\right)\cos\left(x^{\frac{1}{3}}\right)+4\cos\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\left(\sin^3\left(x^{\frac{1}{3}}\right)\right)}{3}+\frac{2\left(2+\sin^2\right.}{}$

default	$-x^{\frac{2}{3}}\left(2+\sin^2\left(x^{\frac{1}{3}}\right)\right)\cos\left(x^{\frac{1}{3}}\right)+4\cos\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\left(\sin^3\left(x^{\frac{1}{3}}\right)\right)}{3}+\frac{2\left(2+\sin^2\left(x\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(2/3)*(2+sin(x^(1/3))^2)*cos(x^(1/3))+4*cos(x^(1/3))+4*x^(1/3)*sin(x^(1/
3))+2/3*x^(1/3)*sin(x^(1/3))^3+2/9*(2+sin(x^(1/3))^2)*cos(x^(1/3))
```

Maxima [A]
time = 0.30, size = 47, normalized size = 0.54

$$\frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(3\,x^{\frac{1}{3}}\right)-\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6*
x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Fricas [A]
time = 0.39, size = 51, normalized size = 0.59

$$\frac{1}{9}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)^{3}-\frac{1}{3}\left(9\,x^{\frac{2}{3}}-14\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{2}{3}\left(x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}-7\,x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/9*(9*x^(2/3) - 2)*cos(x^(1/3))^3 - 1/3*(9*x^(2/3) - 14)*cos(x^(1/3)) - 2/
3*(x^(1/3)*cos(x^(1/3))^2 - 7*x^(1/3))*sin(x^(1/3))
```

Sympy [A]
time = 2.96, size = 80, normalized size = 0.92

$$-\frac{9x^{\frac{2}{3}}\cos\left(\sqrt[3]{x}\right)}{4}+\frac{x^{\frac{2}{3}}\cos\left(3\sqrt[3]{x}\right)}{4}+\frac{9\sqrt[3]{x}\,\sin\left(\sqrt[3]{x}\right)}{2}-\frac{\sqrt[3]{x}\,\sin\left(3\sqrt[3]{x}\right)}{6}+\frac{9\cos\left(\sqrt[3]{x}\right)}{2}-\frac{\cos\left(3\sqrt[3]{x}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/3))**3,x)
```

```
[Out] -9*x**(2/3)*cos(x**(1/3))/4 + x**(2/3)*cos(3*x**(1/3))/4 + 9*x**(1/3)*sin(x
**(1/3))/2 - x**(1/3)*sin(3*x**(1/3))/6 + 9*cos(x**(1/3))/2 - cos(3*x**(1/3
))/18
```

Giac [A]
time = 5.37, size = 47, normalized size = 0.54

$$\frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(3\,x^{\frac{1}{3}}\right)-\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6*
x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Mupad [B]
time = 4.78, size = 58, normalized size = 0.67

$$\frac{14\cos\left(x^{1/3}\right)}{3}-3\,x^{2/3}\cos\left(x^{1/3}\right)+\frac{14\,x^{1/3}\sin\left(x^{1/3}\right)}{3}-\frac{2\cos\left(x^{1/3}\right)^3}{9}+x^{2/3}\cos\left(x^{1/3}\right)^3-\frac{2\,x^{1/3}\cos\left(x^{1/3}\right)^2\sin\left(x^{1/3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3))^3,x)
```

```
[Out] (14*cos(x^(1/3)))/3 - 3*x^(2/3)*cos(x^(1/3)) + (14*x^(1/3)*sin(x^(1/3)))/3
- (2*cos(x^(1/3))^3)/9 + x^(2/3)*cos(x^(1/3))^3 - (2*x^(1/3)*cos(x^(1/3))^2
*sin(x^(1/3)))/3
```

3.129 $\int (ex)^m (b\sin(c+dx^n))^p \, dx$

Optimal. Leaf size=21

$$\mathrm{Int}((ex)^m (b\sin(c+dx^n))^p, x)$$

```
[Out] Unintegrable((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^m (b\sin(c+d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*(b*sin(d*x^n + c))^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*(b*sin(d*x^n + c))^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(c+dx^n))^p (ex)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*sin(c+d*x**n))**p,x)
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((x*e)^m*(b*sin(d*x^n + c))^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (b\,\sin(c+d\,x^n))^p\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(c + d*x^n))^p*(e*x)^m,x)
[Out] int((b*sin(c + d*x^n))^p*(e*x)^m, x)
```

3.130 $\int (ex)^m (a+b\sin (c+dx^n))^p \, dx$

Optimal. Leaf size=23

$$\text{Int}((ex)^m (a+b\sin (c+dx^n))^p, x)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\sin (c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a+b\sin (c+dx^n))^p \, dx = \int (ex)^m (a+b\sin (c+dx^n))^p \, dx$$

Mathematica [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int (ex)^m (a+b\sin (c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m (a+b\sin (c+d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**n))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((x*e)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e\,x\right)}^m\,{\left(a+b\,\sin\left(c+d\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*x^n))^p, x)
```

3.131 $\int (ex)^{-1+n} (b\sin(c+dx^n))^p \, dx$

Optimal. Leaf size=92

$$\frac{x^{-n}(ex)^n\cos(c+dx^n)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cos^2(c+dx^n)}}$$

```
[Out] (e*x)^n*cos(c+d*x^n)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],sin(c+d*x^n)^2)
*(b*sin(c+d*x^n))^(1+p)/b/d/e/n/(1+p)/(x^n)/(cos(c+d*x^n)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3462, 3460, 2722}

$$\frac{x^{-n}(ex)^n\cos(c+dx^n)(b\sin(c+dx^n))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\sin^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cos^2(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*Cos[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[c
+ d*x^n]^2]*(b*Sin[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cos[c + d
*x^n]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n} (b\sin (c+dx^n))^p \, dx = \frac{(x^{-n}(ex)^n) \int x^{-1+n}(b\sin (c+dx^n))^p \, dx}{e}$$
$$= \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int (b\sin(c+dx))^p \, dx, x, x^n\right)}{en}$$
$$= \frac{x^{-n}(ex)^n \cos (c+dx^n) \, {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \sin^2 (c+dx^n)\right) (b\sin (c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cos^2 (c+dx^n)}}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.96

$$\frac{x^{1-n}(ex)^{-1+n}\sqrt{\cos^2 (c+dx^n)} \, {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \sin^2 (c+dx^n)\right) (b\sin (c+dx^n))^p \tan (c+dx^n)}{dn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
[Out] (x^(1 - n)*(e*x)^(-1 + n)*Sqrt[Cos[c + d*x^n]^2]*Hypergeometric2F1[1/2, (1
+ p)/2, (3 + p)/2, Sin[c + d*x^n]^2]*(b*Sin[c + d*x^n])^p*Tan[c + d*x^n])/(
d*n*(1 + p))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} (b\sin (c+d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((x*e)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((x*e)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sin (c + dx^n))^p (ex)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(b*sin(c+d*x**n))**p,x)
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**(n - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((x*e)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b \sin (c + d\, x^n))^p (e\, x)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(c + d*x^n))^p*(e*x)^(n - 1),x)
[Out] int((b*sin(c + d*x^n))^p*(e*x)^(n - 1), x)
```

3.132 $\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx$

Optimal. Leaf size=39

$$\frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(b\sin(c+dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*sin(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(b\sin(c+dx^n))^p \, dx}{e}$$

Mathematica [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b\sin(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b\sin(c+d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((x*e)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((x*e)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin (c + dx^{n}))^{p} (ex)^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(b*sin(c+d*x**n))**p,x)
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((x*e)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(b\,\sin \left(c+d\,x^n\right)\right)}^p\,{\left(e\,x\right)}^{2\,n-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(c + d*x^n))^p*(e*x)^(2*n - 1),x)
[Out] int((b*sin(c + d*x^n))^p*(e*x)^(2*n - 1), x)
```

3.133 $\int (ex)^{-1+n} (a+b\sin(c+dx^n))^p \, dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{2}\, x^{-n}(ex)^n F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx^n)),\frac{b(1-\sin(c+dx^n))}{a+b}\right)\cos(c+dx^n)(a+b\sin(c+dx^n))^p \left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p}}{den\sqrt{1+\sin(c+dx^n)}}$$

```
[Out] -(e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-sin(c+d*x^n))/(a+b),1/2-1/2*sin(c+d*x
^n))*cos(c+d*x^n)*(a+b*sin(c+d*x^n))^p*2^(1/2)/d/e/n/(x^n)/(((a+b*sin(c+d*x
^n))/(a+b))^p)/(1+sin(c+d*x^n))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3462, 3460, 2744, 144, 143}

$$-\frac{\sqrt{2}\, x^{-n}(ex)^n \cos(c+dx^n)(a+b\sin(c+dx^n))^p \left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\sin(dx^n+c)),\frac{b(1-\sin(dx^n+c))}{a+b}\right)}{den\sqrt{\sin(c+dx^n)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] -((Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Sin[c + d*x^n])/2, (b*(
1 - Sin[c + d*x^n]))/(a + b)]*Cos[c + d*x^n]*(a + b*Sin[c + d*x^n])^p)/(d*e
*n*x^n*Sqrt[1 + Sin[c + d*x^n]]*((a + b*Sin[c + d*x^n])/(a + b))^p))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx=\frac{\left(x^{-n}(ex)^n\right)\int x^{-1+n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx}{e}$$

$$=\frac{\left(x^{-n}(ex)^n\right)\text{Subst}\left(\int (a+b\sin(c+dx))^p\,dx,x,x^n\right)}{en}$$

$$=\frac{\left(x^{-n}(ex)^n\cos\left(c+dx^n\right)\right)\text{Subst}\left(\int\frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sin\left(c+dx^n\right)\right)}{den\sqrt{1-\sin\left(c+dx^n\right)}\sqrt{1+\sin\left(c+dx^n\right)}}$$

$$=\frac{\left(x^{-n}(ex)^n\cos\left(c+dx^n\right)\left(a+b\sin\left(c+dx^n\right)\right)^p\left(-\frac{a+b\sin(c+dx^n)}{-a-b}\right)^{-p}\right)\text{Su}}{den\sqrt{1-\sin\left(c+dx^n\right)}\sqrt{1+\sin}}$$

$$=-\frac{\sqrt{2}\,x^{-n}(ex)^nF_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}\left(1-\sin\left(c+dx^n\right)\right),\frac{b(1-\sin(c+dx^n))}{a+b}\right)\text{co}}{den\sqrt{1+\sin\left(c+dx\right.}}$$

Mathematica [A]
time = 0.31, size = 148, normalized size = 1.12

$$\frac{x^{-n}(ex)^nF_1\left(1+p;\frac{1}{2},\frac{1}{2};2+p;\frac{a+b\sin(c+dx^n)}{a-b},\frac{a+b\sin(c+dx^n)}{a+b}\right)\sec\left(c+dx^n\right)\sqrt{-\frac{b\left(-1+\sin\left(c+dx^n\right)\right)}{a+b}}\sqrt{\frac{b\left(1+\sin\left(c+dx^n\right)\right)}{-a+b}}\left(a+b\sin\left(c+dx^n\right)\right)^{1+p}}{bden(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sin[c + d*x^n])/(a - b), (
a + b*Sin[c + d*x^n])/(a + b)]*Sec[c + d*x^n]*Sqrt[-((b*(-1 + Sin[c + d*x^n
]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x^n]))/(-a + b)]*(a + b*Sin[c + d*x^n]
)^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} (a + b\sin (c + d\,x^{n}))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((x*e)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{n-1}\,\left(a+b\,\sin\left(c+d\,x^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)*(a + b*sin(c + d*x^n))^p,x)
```

```
[Out] int((e*x)^(n - 1)*(a + b*sin(c + d*x^n))^p, x)
```

3.134 $\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx$

Optimal. Leaf size=41

$$\frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(a+b\sin(c+dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x])/(e*x^(2*
n))
```

Rubi steps

$$\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a+b\sin(c+dx^n))^p \, dx}{e}$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b\sin(c + dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b\sin(c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((x*e)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int (e\,x)^{2\,n-1}\,\left(a+b\,\sin\left(c+d\,x^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(a + b*sin(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*sin(c + d*x^n))^p, x)
```

3.135 $\int \frac{\sin(a+bx^n)}{x} \, dx$

Optimal. Leaf size=25

$$\frac{\text{Ci}(bx^n)\sin(a)}{n}+\frac{\cos(a)\text{Si}(bx^n)}{n}$$

```
[Out] cos(a)*Si(b*x^n)/n+Ci(b*x^n)*sin(a)/n
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3458, 3457, 3456}

$$\frac{\sin(a)\text{CosIntegral}(bx^n)}{n}+\frac{\cos(a)\text{Si}(bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]/x,x]
```

```
[Out] (CosIntegral[b*x^n]*Sin[a])/n + (Cos[a]*SinIntegral[b*x^n])/n
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx^n)}{x} \, dx &= \cos(a)\int \frac{\sin(bx^n)}{x} \, dx+\sin(a)\int \frac{\cos(bx^n)}{x} \, dx \\
&= \frac{\text{Ci}(bx^n)\sin(a)}{n}+\frac{\cos(a)\text{Si}(bx^n)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.92

$$\frac{\text{Ci}(bx^n)\sin(a)+\cos(a)\text{Si}(bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]/x,x]
[Out] (CosIntegral[b*x^n]*Sin[a] + Cos[a]*SinIntegral[b*x^n])/n
```

Maple [A]
time = 0.07, size = 24, normalized size = 0.96

method	result	s
derivativedivides	$\frac{\text{sinIntegral}(b\,x^{n})\cos(a)+\text{cosineIntegral}(b\,x^{n})\sin(a)}{n}$	2
default	$\frac{\text{sinIntegral}(b\,x^{n})\cos(a)+\text{cosineIntegral}(b\,x^{n})\sin(a)}{n}$	2
risch	$\frac{i\mathrm{e}^{ia}\,\text{expIntegral}(1,-ib\,x^{n})}{2n}-\frac{\mathrm{e}^{-ia}\pi\,\text{csgn}(b\,x^{n})}{2n}+\frac{\mathrm{e}^{-ia}\,\text{sinIntegral}(b\,x^{n})}{n}-\frac{i\mathrm{e}^{-ia}\,\text{expIntegral}(1,-ib\,x^{n})}{2n}$	[illegible]
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma+2n\ln(x)+\ln\left(b^{2}\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b\,x^{n}}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{cosineIntegral}(b\,x^{n})}{\sqrt{\pi}}\right)\sin(a)}{2n}+\frac{\cos(a)\,\text{sinIntegral}(b\,x^{n})}{n}$	[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(Si(b*x^n)*cos(a)+Ci(b*x^n)*sin(a))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 91, normalized size = 3.64

$$-\frac{\left(i\,\text{Ei}(i\,bx^n)-i\,\text{Ei}(-i\,bx^n)+i\,\text{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\,\text{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a)-\left(\text{Ei}(i\,bx^n)+\text{Ei}(-i\,bx^n)+\text{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)+\text{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(a)}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="maxima")
[Out] -1/4*((I*Ei(I*b*x^n) - I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x)))) -
 I*Ei(-I*b*e^(n*conjugate(log(x)))))*cos(a) - (Ei(I*b*x^n) + Ei(-I*b*x^n) +
 Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjugate(log(x)))))*sin(a)
)/n
```

Fricas [A]
time = 0.38, size = 35, normalized size = 1.40

$$\frac{\text{Ci}\left(bx^{n}\right)\sin\left(a\right)+\text{Ci}\left(-bx^{n}\right)\sin\left(a\right)+2\,\cos\left(a\right)\text{Si}\left(bx^{n}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(cos_integral(b*x^n)*sin(a) + cos_integral(-b*x^n)*sin(a) + 2*cos(a)*si
n_integral(b*x^n))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)/x,x)
```

```
[Out] Integral(sin(a + b*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin(a+b\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)/x,x)
```

```
[Out] int(sin(a + b*x^n)/x, x)
```

3.136 $\int \frac{\sin^2(a+bx^n)}{x} dx$

Optimal. Leaf size=43

$$-\frac{\cos(2a)\mathrm{Ci}(2bx^n)}{2n}+\frac{\log(x)}{2}+\frac{\sin(2a)\mathrm{Si}(2bx^n)}{2n}$$

```
[Out] -1/2*Ci(2*b*x^n)*cos(2*a)/n+1/2*ln(x)+1/2*Si(2*b*x^n)*sin(2*a)/n
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 3459, 3457, 3456}

$$-\frac{\cos(2a)\mathrm{CosIntegral}(2bx^n)}{2n}+\frac{\sin(2a)\mathrm{Si}(2bx^n)}{2n}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^2/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x^n])/n + Log[x]/2 + (Sin[2*a]*SinIntegral[2
*b*x^n])/(2*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(a+bx^n)}{x}\,dx &= \int \left(\frac{1}{2x}-\frac{\cos(2a+2bx^n)}{2x}\right)dx \\
&= \frac{\log(x)}{2}-\frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx \\
&= \frac{\log(x)}{2}-\frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx+\frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\,dx \\
&= -\frac{\cos(2a)\text{Ci}(2bx^n)}{2n}+\frac{\log(x)}{2}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 37, normalized size = 0.86

$$\frac{-\cos(2a)\text{Ci}(2bx^n)+n\log(x)+\sin(2a)\text{Si}(2bx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^2/x,x]
```

```
[Out] (-(Cos[2*a]*CosIntegral[2*b*x^n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n
])/(2*n)
```

Maple [A]
time = 0.06, size = 40, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{\ln(b\,x^{n})}{2}+\frac{\text{sinIntegral}(2b\,x^{n})\sin(2a)}{2}-\frac{\text{cosineIntegral}(2b\,x^{n})\cos(2a)}{2}}{n}$
default	$\frac{\frac{\ln(b\,x^{n})}{2}+\frac{\text{sinIntegral}(2b\,x^{n})\sin(2a)}{2}-\frac{\text{cosineIntegral}(2b\,x^{n})\cos(2a)}{2}}{n}$
risch	$-\frac{i\mathrm{e}^{-2ia}\pi\,\text{csgn}(b\,x^{n})}{4n}+\frac{i\mathrm{e}^{-2ia}\,\text{sinIntegral}(2b\,x^{n})}{2n}+\frac{\mathrm{e}^{-2ia}\,\text{expIntegral}(1,-2ib\,x^{n})}{4n}+\frac{\mathrm{e}^{2ia}\,\text{expIntegral}(1,-2ib\,x^{n})}{4n}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/2*ln(b*x^n)+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 100, normalized size = 2.33

$$-\frac{\left(\text{Ei}(2i\,bx^n)+\text{Ei}(-2i\,bx^n)+\text{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\text{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(2\,a)-4\,n\log(x)-\left(-i\,\text{Ei}(2i\,bx^n)+i\,\text{Ei}(-2i\,bx^n)-i\,\text{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+i\,\text{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(2\,a)}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="maxima")
```

```
[Out] -1/8*((Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) +
 Ei(-2*I*b*e^(n*conjugate(log(x)))))*cos(2*a) - 4*n*log(x) - (-I*Ei(2*I*b*x
^n) + I*Ei(-2*I*b*x^n) - I*Ei(2*I*b*e^(n*conjugate(log(x)))) + I*Ei(-2*I*b*
e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Fricas [A]
time = 0.39, size = 48, normalized size = 1.12

$$-\frac{\cos(2\,a)\,\mathrm{Ci}\left(2\,bx^n\right)+\cos(2\,a)\,\mathrm{Ci}\left(-2\,bx^n\right)-2\,n\log(x)-2\,\sin(2\,a)\,\mathrm{Si}\left(2\,bx^n\right)}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(2*a)*cos_integral(2*b*x^n) + cos(2*a)*cos_integral(-2*b*x^n) - 2*
n*log(x) - 2*sin(2*a)*sin_integral(2*b*x^n))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^2\left(a+bx^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**2/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\sin\left(a+b\,x^n\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^2/x,x)
```

```
[Out] int(sin(a + b*x^n)^2/x, x)
```

3.137 $\int \frac{\sin^3(a+bx^n)}{x}\,dx$

Optimal. Leaf size=67

$$\frac{3\text{Ci}(bx^n)\sin(a)}{4n}-\frac{\text{Ci}(3bx^n)\sin(3a)}{4n}+\frac{3\cos(a)\text{Si}(bx^n)}{4n}-\frac{\cos(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] 3/4*cos(a)*Si(b*x^n)/n-1/4*cos(3*a)*Si(3*b*x^n)/n+3/4*Ci(b*x^n)*sin(a)/n-1/
4*Ci(3*b*x^n)*sin(3*a)/n
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3506, 3458, 3457, 3456}

$$\frac{3\sin(a)\text{CosIntegral}(bx^n)}{4n}-\frac{\sin(3a)\text{CosIntegral}(3bx^n)}{4n}+\frac{3\cos(a)\text{Si}(bx^n)}{4n}-\frac{\cos(3a)\text{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^n]*Sin[a])/(4*n) - (CosIntegral[3*b*x^n]*Sin[3*a])/(4*n)
 + (3*Cos[a]*SinIntegral[b*x^n])/(4*n) - (Cos[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3\left(a+bx^n\right)}{x}\,dx &= \int \left(\frac{3\sin\left(a+bx^n\right)}{4x}-\frac{\sin\left(3a+3bx^n\right)}{4x}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\sin\left(3a+3bx^n\right)}{x}\,dx\right)+\frac{3}{4}\int \frac{\sin\left(a+bx^n\right)}{x}\,dx\\
&= \frac{1}{4}(3\cos(a))\int \frac{\sin\left(bx^n\right)}{x}\,dx-\frac{1}{4}\cos(3a)\int \frac{\sin\left(3bx^n\right)}{x}\,dx+\frac{1}{4}(3\sin(a))\int \frac{\cos\left(bx^n\right)}{x}\,dx\\
&= \frac{3\mathrm{Ci}(bx^n)\sin(a)}{4n}-\frac{\mathrm{Ci}(3bx^n)\sin(3a)}{4n}+\frac{3\cos(a)\mathrm{Si}(bx^n)}{4n}-\frac{\cos(3a)\mathrm{Si}(3bx^n)}{4n}
\end{aligned}
$$

Mathematica [**A**]
time = 0.08, size = 54, normalized size = 0.81

$$\frac{3\mathrm{Ci}(bx^n)\sin(a)-\mathrm{Ci}(3bx^n)\sin(3a)+3\cos(a)\mathrm{Si}(bx^n)-\cos(3a)\mathrm{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^n]*Sin[a] - CosIntegral[3*b*x^n]*Sin[3*a] + 3*Cos[a]*Sin
Integral[b*x^n] - Cos[3*a]*SinIntegral[3*b*x^n])/(4*n)
```

Maple [**A**]
time = 0.06, size = 52, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{\mathrm{sinIntegral}(3b\,x^{n})\cos(3a)}{4}-\frac{\mathrm{cosineIntegral}(3b\,x^{n})\sin(3a)}{4}+\frac{3\,\mathrm{sinIntegral}(b\,x^{n})\cos(a)}{4}+\frac{3\,\mathrm{cosineIntegral}(b\,x^{n})\sin(a)}{4}}{n}$
default	$\frac{-\frac{\mathrm{sinIntegral}(3b\,x^{n})\cos(3a)}{4}-\frac{\mathrm{cosineIntegral}(3b\,x^{n})\sin(3a)}{4}+\frac{3\,\mathrm{sinIntegral}(b\,x^{n})\cos(a)}{4}+\frac{3\,\mathrm{cosineIntegral}(b\,x^{n})\sin(a)}{4}}{n}$
risch	$-\frac{i\mathrm{e}^{3ia}\,\mathrm{expIntegral}(1,-3ib\,x^{n})}{8n}+\frac{\mathrm{e}^{-3ia}\pi\,\mathrm{csgn}(b\,x^{n})}{8n}-\frac{\mathrm{e}^{-3ia}\,\mathrm{sinIntegral}(3b\,x^{n})}{4n}+\frac{i\mathrm{e}^{-3ia}\,\mathrm{expIntegral}(1,-3ib\,x^{n})}{8n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/4*Si(3*b*x^n)*cos(3*a)-1/4*Ci(3*b*x^n)*sin(3*a)+3/4*Si(b*x^n)*cos(a
)+3/4*Ci(b*x^n)*sin(a))
```

Maxima [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 181, normalized size = 2.70

$$\frac{\left(i\,\mathrm{Ei}(3i\,bx^n)-i\,\mathrm{Ei}(-3i\,bx^n)+i\,\mathrm{Ei}\left(3i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\,\mathrm{Ei}\left(-3i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(3a)-3\left(i\,\mathrm{Ei}(i\,bx^n)-i\,\mathrm{Ei}(-i\,bx^n)+i\,\mathrm{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\,\mathrm{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a)-\left(\mathrm{Ei}(3i\,bx^n)+\mathrm{Ei}(-3i\,bx^n)+\mathrm{Ei}\left(3i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-3i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(3a)+3\left(\mathrm{Ei}(i\,bx^n)+\mathrm{Ei}(-i\,bx^n)+\mathrm{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(a)}{16n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*((I*Ei(3*I*b*x^n) - I*Ei(-3*I*b*x^n) + I*Ei(3*I*b*e^(n*conjugate(log(x
)))) - I*Ei(-3*I*b*e^(n*conjugate(log(x)))))*cos(3*a) - 3*(I*Ei(I*b*x^n) -
I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x)))) - I*Ei(-I*b*e^(n*conjuga
te(log(x)))))*cos(a) - (Ei(3*I*b*x^n) + Ei(-3*I*b*x^n) + Ei(3*I*b*e^(n*conj
ugate(log(x)))) + Ei(-3*I*b*e^(n*conjugate(log(x)))))*sin(3*a) + 3*(Ei(I*b*
x^n) + Ei(-I*b*x^n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjug
ate(log(x)))))*sin(a))/n
```

Fricas [A]
time = 0.35, size = 74, normalized size = 1.10

$$-\frac{\operatorname{Ci}\left(3\,bx^{n}\right)\sin\left(3\,a\right)+\operatorname{Ci}\left(-3\,bx^{n}\right)\sin\left(3\,a\right)-3\,\operatorname{Ci}\left(bx^{n}\right)\sin\left(a\right)-3\,\operatorname{Ci}\left(-bx^{n}\right)\sin\left(a\right)+2\,\cos\left(3\,a\right)\operatorname{Si}\left(3\,bx^{n}\right)-6\,\cos\left(a\right)\operatorname{Si}\left(bx^{n}\right)}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] -1/8*(cos_integral(3*b*x^n)*sin(3*a) + cos_integral(-3*b*x^n)*sin(3*a) - 3*
cos_integral(b*x^n)*sin(a) - 3*cos_integral(-b*x^n)*sin(a) + 2*cos(3*a)*sin
_integral(3*b*x^n) - 6*cos(a)*sin_integral(b*x^n))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**3/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,x^{n}\right)}^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^3/x,x)
```

```
[Out] int(sin(a + b*x^n)^3/x, x)
```

3.138 $\int \frac{\sin^4(a+bx^n)}{x} dx$

Optimal. Leaf size=79

$$-\frac{\cos(2a)\text{Ci}(2bx^n)}{2n}+\frac{\cos(4a)\text{Ci}(4bx^n)}{8n}+\frac{3\log(x)}{8}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}-\frac{\sin(4a)\text{Si}(4bx^n)}{8n}$$

```
[Out] -1/2*Ci(2*b*x^n)*cos(2*a)/n+1/8*Ci(4*b*x^n)*cos(4*a)/n+3/8*ln(x)+1/2*Si(2*b
*x^n)*sin(2*a)/n-1/8*Si(4*b*x^n)*sin(4*a)/n
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3506, 3459, 3457, 3456}

$$-\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n}+\frac{\cos(4a)\text{CosIntegral}(4bx^n)}{8n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}-\frac{\sin(4a)\text{Si}(4bx^n)}{8n}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^4/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x^n])/n + (Cos[4*a]*CosIntegral[4*b*x^n])/(8
*n) + (3*Log[x])/8 + (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*SinI
ntegral[4*b*x^n])/(8*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(a+bx^n)}{x}\,dx &= \int \left(\frac{3}{8x}-\frac{\cos(2a+2bx^n)}{2x}+\frac{\cos(4a+4bx^n)}{8x}\right)dx\\ &= \frac{3\log(x)}{8}+\frac{1}{8}\int \frac{\cos(4a+4bx^n)}{x}\,dx-\frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx\\ &= \frac{3\log(x)}{8}-\frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx+\frac{1}{8}\cos(4a)\int \frac{\cos(4bx^n)}{x}\,dx+\frac{1}{2}\sin(2a)\int \text{sin}\\ &= -\frac{\cos(2a)\text{Ci}(2bx^n)}{2n}+\frac{\cos(4a)\text{Ci}(4bx^n)}{8n}+\frac{3\log(x)}{8}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}-\frac{\sin(4a)\text{Si}(4b}{8n}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.84

$$\frac{3\log(x)}{8}+\frac{-4\cos(2a)\text{Ci}(2bx^n)+\cos(4a)\text{Ci}(4bx^n)+4\sin(2a)\text{Si}(2bx^n)-\sin(4a)\text{Si}(4bx^n)}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (-4*Cos[2*a]*CosIntegral[2*b*x^n] + Cos[4*a]*CosIntegral[4*b
*x^n] + 4*Sin[2*a]*SinIntegral[2*b*x^n] - Sin[4*a]*SinIntegral[4*b*x^n])/(8
*n)
```

Maple [A]
time = 0.06, size = 66, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{3\ln(b\,x^n)}{8}-\frac{\text{sinIntegral}(4b\,x^n)\sin(4a)}{8}+\frac{\text{cosineIntegral}(4b\,x^n)\cos(4a)}{8}+\frac{\text{sinIntegral}(2b\,x^n)\sin(2a)}{2}-\frac{\text{cosineIntegral}(2b\,x^n)\cos(2a)}{2}}{n}$
default	$\frac{\frac{3\ln(b\,x^n)}{8}-\frac{\text{sinIntegral}(4b\,x^n)\sin(4a)}{8}+\frac{\text{cosineIntegral}(4b\,x^n)\cos(4a)}{8}+\frac{\text{sinIntegral}(2b\,x^n)\sin(2a)}{2}-\frac{\text{cosineIntegral}(2b\,x^n)\cos(2a)}{2}}{n}$
risch	$\frac{i\mathrm{e}^{-4ia}\pi\,\text{csgn}(b\,x^n)}{16n}-\frac{i\mathrm{e}^{-4ia}\,\text{sinIntegral}(4b\,x^n)}{8n}-\frac{\mathrm{e}^{-4ia}\,\text{expIntegral}(1,-4ib\,x^n)}{16n}-\frac{\mathrm{e}^{4ia}\,\text{expIntegral}(1,-4ib\,x^n)}{16n}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(3/8*ln(b*x^n)-1/8*Si(4*b*x^n)*sin(4*a)+1/8*Ci(4*b*x^n)*cos(4*a)+1/2*Si
(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 189, normalized size = 2.39

$$\frac{\left(\text{Ei}(4i\,bx^n)+\text{Ei}(-4i\,bx^n)+\text{Ei}\left(4i\,be^{\left(n\overline{\log(x)}\right)}\right)+\text{Ei}\left(-4i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(4\,a)-4\left(\text{Ei}(2i\,bx^n)+\text{Ei}(-2i\,bx^n)+\text{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\text{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(2\,a)+12\,n\log(x)+\left(i\,\text{Ei}(4i\,bx^n)-i\,\text{Ei}(-4i\,bx^n)+i\,\text{Ei}\left(4i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\,\text{Ei}\left(-4i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(4\,a)-4\left(i\,\text{Ei}(2i\,bx^n)-i\,\text{Ei}(-2i\,bx^n)+i\,\text{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)-i\,\text{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(2\,a)}{32\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="maxima")
```

```
[Out] 1/32*((Ei(4*I*b*x^n) + Ei(-4*I*b*x^n) + Ei(4*I*b*e^(n*conjugate(log(x)))) +
 Ei(-4*I*b*e^(n*conjugate(log(x)))))*cos(4*a) - 4*(Ei(2*I*b*x^n) + Ei(-2*I*
b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) + Ei(-2*I*b*e^(n*conjugate(log(x
)))))*cos(2*a) + 12*n*log(x) + (I*Ei(4*I*b*x^n) - I*Ei(-4*I*b*x^n) + I*Ei(4
*I*b*e^(n*conjugate(log(x)))) - I*Ei(-4*I*b*e^(n*conjugate(log(x)))))*sin(4
*a) - 4*(I*Ei(2*I*b*x^n) - I*Ei(-2*I*b*x^n) + I*Ei(2*I*b*e^(n*conjugate(log
(x)))) - I*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Fricas [A]
time = 0.38, size = 87, normalized size = 1.10

$$\frac{\cos(4a)\operatorname{Ci}(4bx^n) - 4\cos(2a)\operatorname{Ci}(2bx^n) - 4\cos(2a)\operatorname{Ci}(-2bx^n) + \cos(4a)\operatorname{Ci}(-4bx^n) + 6n\log(x) - 2\sin(4a)\operatorname{Si}(4bx^n) + 8\sin(2a)\operatorname{Si}(2bx^n)}{16n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(4*a)*cos_integral(4*b*x^n) - 4*cos(2*a)*cos_integral(2*b*x^n) - 4
*cos(2*a)*cos_integral(-2*b*x^n) + cos(4*a)*cos_integral(-4*b*x^n) + 6*n*lo
g(x) - 2*sin(4*a)*sin_integral(4*b*x^n) + 8*sin(2*a)*sin_integral(2*b*x^n))
/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(a + bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**4/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**4/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^4/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x^n\right)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^4/x,x)
```

```
[Out] int(sin(a + b*x^n)^4/x, x)
```

3.139 $\int \sin(a+bx^n)\, dx$

Optimal. Leaf size=87

$$\frac{ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-1/2*I*x*GAMMA(1/n
,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3446, 2239}

$$\frac{ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - ((I/2
)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx^n)\, dx &= \frac{1}{2}i\int e^{-ia-ibx^n}\, dx-\frac{1}{2}i\int e^{ia+ibx^n}\, dx\\ &= \frac{ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 1.09

$$\frac{ix(b^2x^{2n})^{-1/n}\left(-(-ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n],x]
[Out] ((I/2)*x*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[a] - I*Sin[a]))
 + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Sin[a])))/(n*(b^2
*x^(2*n))^n^(-1))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.07, size = 74, normalized size = 0.85

method	result	si
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{1}{2n}\right],-\frac{x^{2n}b^2}{4}\right)\sin(a)+\frac{b\,x^{1+n}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{2n}\right],-\frac{x^{2n}b^2}{4}\right)\cos(a)}{1+n}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)+b/(1+n)*x^(1+n)*
hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n),x, algorithm="maxima")
[Out] integrate(sin(b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n),x, algorithm="fricas")
[Out] integral(sin(b*x^n + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin{\left(a + bx^{n} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n),x)
```

```
[Out] Integral(sin(a + b*x**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n),x)
```

```
[Out] int(sin(a + b*x^n), x)
```

3.140 $\int \sin^2(a+bx^n)\,dx$

Optimal. Leaf size=100

$$\frac{x}{2}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}$$

```
[Out] 1/2*x+2^(-2-1/n)*exp(2*I*a)*x*GAMMA(1/n,-2*I*b*x^n)/n/((-I*b*x^n)^(1/n))+2^
(-2-1/n)*x*GAMMA(1/n,2*I*b*x^n)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3448, 3447, 2239}

$$\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\,\mathrm{Gamma}\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\,\mathrm{Gamma}\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^2,x]
```

```
[Out] x/2 + (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)*
b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)*a
)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(a+bx^n)\,dx &= \int \left(\frac{1}{2}-\frac{1}{2}\cos(2a+2bx^n)\right)dx \\
&= \frac{x}{2}-\frac{1}{2}\int \cos(2a+2bx^n)\,dx \\
&= \frac{x}{2}-\frac{1}{4}\int e^{-2ia-2ibx^n}\,dx-\frac{1}{4}\int e^{2ia+2ibx^n}\,dx \\
&= \frac{x}{2}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 0.94

$$\frac{x\left(2n+2^{-1/n}e^{2ia}(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)+2^{-1/n}e^{-2ia}(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^2,x]
[Out] (x*(2*n + (E^((2*I)*a)*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b*x^n)^
n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^n^(-1)
)))/(4*n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sin^2(a+b\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^2,x)
[Out] int(sin(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/2*integrate(cos(2*b*x^n + 2*a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(-cos(b*x^n + a)^2 + 1, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2 (a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**2,x)
[Out] Integral(sin(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(sin(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (a + b\, x^n)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^2,x)
[Out] int(sin(a + b*x^n)^2, x)
```

3.141 $\int \sin^3(a+bx^n)\,dx$

Optimal. Leaf size=187

$$\frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}-\frac{i3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n}+\frac{i3^{-1/}}{}$$

```
[Out] 3/8*I*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-3/8*I*x*GAMMA(1/n
,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))-1/8*I*exp(3*I*a)*x*GAMMA(1/n,-3*I*b*
x^n)/(3^(1/n))/n/((-I*b*x^n)^(1/n))+1/8*I*x*GAMMA(1/n,3*I*b*x^n)/(3^(1/n))/
exp(3*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A]

time = 0.06, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3448, 3446, 2239}

$$\frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right)}{8n}+\frac{ie^{-3ia}3^{-1/n}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},3ibx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^3,x]
```

```
[Out] (((3*I)/8)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - (
((3*I)/8)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1)) - ((I/8)*E
^((3*I)*a)*x*Gamma[n^(-1), (-3*I)*b*x^n])/(3^n^(-1)*n*((-I)*b*x^n)^n^(-1))
+ ((I/8)*x*Gamma[n^(-1), (3*I)*b*x^n])/(3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\int \sin^3\left(a+bx^n\right)\,dx &= \int \left(\frac{3}{4}\sin\left(a+bx^n\right)-\frac{1}{4}\sin\left(3a+3bx^n\right)\right)\,dx \\
&= -\left(\frac{1}{4}\int \sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int \sin\left(a+bx^n\right)\,dx \\
&= -\left(\frac{1}{8}i\int e^{-3ia-3ibx^n}\,dx\right)+\frac{1}{8}i\int e^{3ia+3ibx^n}\,dx+\frac{3}{8}i\int e^{-ia-ibx^n}\,dx-\frac{3}{8}i\int e^{ia+ibx^n}\,dx \\
&= \frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}-\frac{i3^{-1/n}e^{3ia}x(-ibx^n}{8}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 177, normalized size = 0.95

$$
\frac{i3^{-1/n}e^{-3ia}x(b^2x^{2n})^{-1/n}\left(3^{1+\frac{1}{n}}e^{4ia}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-ibx^n\right)-3^{1+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},ibx^n\right)-e^{6ia}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-3ibx^n\right)+(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},3ibx^n\right)\right)}{8n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^3,x]
```

```
[Out] ((I/8)*x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*
x^n] - 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n
] - E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x^n)
^n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*n))^
n^(-1))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int \sin^3\left(a+b\,x^n\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^3,x)
```

```
[Out] int(sin(a+b*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x^n + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^3(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**3,x)
```

```
[Out] Integral(sin(a + b*x**n)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + b\, x^n\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^3,x)
```

```
[Out] int(sin(a + b*x^n)^3, x)
```

3.142 $\int x^m \sin(a+bx^n)\, dx$

Optimal. Leaf size=109

$$\frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-1/2
*I*x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3504, 2250}

$$\frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^((1
+ m)/n)) - ((I/2)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)
^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int x^m\sin(a+bx^n)\,dx &= \frac{1}{2}i\int e^{-ia-ibx^n}x^m\,dx-\frac{1}{2}i\int e^{ia+ibx^n}x^m\,dx\\ &= \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 118, normalized size = 1.08

$$\frac{i x^{1+m} (b^2 x^{2n})^{-\frac{1+m}{n}} \left(-(-ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right)(\cos(a) - i\sin(a)) + (ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right)(\cos(a)+i\sin(a))\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*x^(1 + m)*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n]*(Cos
[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n]*(Cos[
a] + I*Sin[a])))/(n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 110, normalized size = 1.01

method	result
meijerg	$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],-\frac{x^{2n}b^{2}}{4}\right)\sin(a)}{1+m}+\frac{b\,x^{n+m+1}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n}\right],-\frac{x^{2n}b^{2}}{4}\right)\cos(a)}{n+m+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],-1/4*x^(2*n
)*b^2)*sin(a)+b/(n+m+1)*x^(n+m+1)*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+1/
2/n*m+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*sin(b*x^n + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sin{(a + bx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+b*x**n),x)
```

```
[Out] Integral(x**m*sin(a + b*x**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sin\left(a + b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + b*x^n),x)
```

```
[Out] int(x^m*sin(a + b*x^n), x)
```

3.143 $\int x^m \sin^2(a+bx^n)\,dx$

Optimal. Leaf size=139

$$\frac{x^{1+m}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)}{n}+\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2ibx^n\right)}{n}$$

```
[Out] 1/2*x^(1+m)/(1+m)+exp(2*I*a)*x^(1+m)*GAMMA((1+m)/n,-2*I*b*x^n)/(2^((1+m+2*n
)/n))/n/((-I*b*x^n)^((1+m)/n))+x^(1+m)*GAMMA((1+m)/n,2*I*b*x^n)/(2^((1+m+2*
n)/n))/exp(2*I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3506, 3505, 2250}

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2ibx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x^n]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^
n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1 + m)*Gamma[(1
+ m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^m \sin^2\left(a+bx^n\right)\,dx = \int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^n\right)\right)dx$$
$$= \frac{x^{1+m}}{2(1+m)}-\frac{1}{2}\int x^m\cos\left(2a+2bx^n\right)dx$$
$$= \frac{x^{1+m}}{2(1+m)}-\frac{1}{4}\int e^{-2ia-2ibx^n}x^m\,dx-\frac{1}{4}\int e^{2ia+2ibx^n}x^m\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}\left(-ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)}{n}+\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+}}{}$$

Mathematica [**A**]
time = 0.38, size = 129, normalized size = 0.93

$$\frac{x^{1+m}\left(2n+2^{-\frac{1+m}{n}}e^{2ia}(1+m)\left(-ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)+2^{-\frac{1+m}{n}}e^{-2ia}(1+m)\left(ibx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2ibx^n\right)\right)}{4(1+m)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x^n]^2,x]
```

```
[Out] (x^(1 + m)*(2*n + (E^((2*I)*a)*(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^n])/(2^(
(1 + m)/n)*((-I)*b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, (2*I)*b*x^
n])/(2^((1 + m)/n)*E^((2*I)*a)*(I*b*x^n)^((1 + m)/n))))/(4*(1 + m)*n)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m\left(\sin^2\left(a+b\,x^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+b*x^n)^2,x)
```

```
[Out] int(x^m*sin(a+b*x^n)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))/(m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(-x^m*cos(b*x^n + a)^2 + x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sin^2\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+b*x**n)**2,x)
[Out] Integral(x**m*sin(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^m*sin(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sin\left(a + b\,x^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + b*x^n)^2,x)
[Out] int(x^m*sin(a + b*x^n)^2, x)
```

3.144 $\int x^m \sin^3(a + bx^n)\, dx$

Optimal. Leaf size=237

$$\frac{3ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n}, -ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n}, ibx^n\right)}{8n} - \frac{i3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}}{8n}$$

```
[Out] 3/8*I*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-3/8
*I*x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))-1/8*I*ex
p(3*I*a)*x^(1+m)*GAMMA((1+m)/n,-3*I*b*x^n)/(3^((1+m)/n))/n/((-I*b*x^n)^((1+
m)/n))+1/8*I*x^(1+m)*GAMMA((1+m)/n,3*I*b*x^n)/(3^((1+m)/n))/exp(3*I*a)/n/((
I*b*x^n)^((1+m)/n))
```

Rubi [A]

time = 0.14, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3506, 3504, 2250}

$$\frac{3ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n}, -ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n}, ibx^n\right)}{8n} - \frac{ie^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n}, -3ibx^n\right)}{8n} + \frac{ie^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n}, 3ibx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x^n]^3,x]
```

```
[Out] (((3*I)/8)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^
((1 + m)/n)) - (((3*I)/8)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(
I*b*x^n)^((1 + m)/n)) - ((I/8)*E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-3*I
)*b*x^n])/(3^((1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + ((I/8)*x^(1 + m)*Gam
ma[(1 + m)/n, (3*I)*b*x^n])/(3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \sin^3(a+bx^n)\,dx &= \int \left(\frac{3}{4}x^m \sin(a+bx^n) - \frac{1}{4}x^m \sin(3a+3bx^n)\right)dx\\ &= -\left(\frac{1}{4}\int x^m \sin(3a+3bx^n)\,dx\right) + \frac{3}{4}\int x^m \sin(a+bx^n)\,dx\\ &= -\left(\frac{1}{8}i\int e^{-3ia-3ibx^n}x^m\,dx\right) + \frac{1}{8}i\int e^{3ia+3ibx^n}x^m\,dx + \frac{3}{8}i\int e^{-ia-ibx^n}x^m\,dx - \frac{3}{8}i\int\\ &= \frac{3ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n} - \frac{i3}{}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 225, normalized size = 0.95

$$\frac{i3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\left(3^{\frac{1+m+n}{n}}e^{4ia}(ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right) - 3^{\frac{1+m+n}{n}}e^{2ia}(-ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right) - e^{6ia}(ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3ibx^n\right) + (-ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3ibx^n\right)\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x^n]^3,x]
[Out] ((I/8)*x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma
[(1 + m)/n, (-I)*b*x^n] - 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1 +
m)/n)*Gamma[(1 + m)/n, I*b*x^n] - E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(
1 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*I)*b
*x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^m \left(\sin^3(a+b\,x^n)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+b*x^n)^3,x)
[Out] int(x^m*sin(a+b*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x^n + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(-(x^m*cos(b*x^n + a)^2 - x^m)*sin(b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sin^3(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+b*x**n)**3,x)
```

```
[Out] Integral(x**m*sin(a + b*x**n)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \sin\left(a + b\,x^n\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + b*x^n)^3,x)
```

```
[Out] int(x^m*sin(a + b*x^n)^3, x)
```

3.145 $\int x^{-1+2n} \sin(a+bx^n)\,dx$

Optimal. Leaf size=35

$$-\frac{x^n \cos(a+bx^n)}{bn}+\frac{\sin(a+bx^n)}{b^2 n}$$

```
[Out] -x^n*cos(a+b*x^n)/b/n+sin(a+b*x^n)/b^2/n
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3460, 3377, 2717}

$$\frac{\sin(a+bx^n)}{b^2 n}-\frac{x^n \cos(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -((x^n*Cos[a + b*x^n])/(b*n)) + Sin[a + b*x^n]/(b^2*n)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^{-1+2n}\sin\left(a+bx^n\right)\,dx=\frac{\text{Subst}\left(\int x\sin(a+bx)\,dx,x,x^n\right)}{n}$$
$$=-\frac{x^n\cos\left(a+bx^n\right)}{bn}+\frac{\text{Subst}\left(\int\cos(a+bx)\,dx,x,x^n\right)}{bn}$$
$$=-\frac{x^n\cos\left(a+bx^n\right)}{bn}+\frac{\sin\left(a+bx^n\right)}{b^2n}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.86

$$\frac{-bx^n\cos\left(a+bx^n\right)+\sin\left(a+bx^n\right)}{b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sin[a + b*x^n],x]
[Out] (-(b*x^n*Cos[a + b*x^n]) + Sin[a + b*x^n])/(b^2*n)
```

Maple [A]
time = 0.05, size = 44, normalized size = 1.26

method	result	size
risch	$-\frac{x^n\cos(a+b\,x^n)}{bn}+\frac{\sin(a+b\,x^n)}{b^2n}$	36
default	$\frac{\sin(a+b\,x^n)-(a+b\,x^n)\cos(a+b\,x^n)+a\cos(a+b\,x^n)}{n\,b^2}$	44
meijerg	`error in int/gproduct: numeric exception: division by zero\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n/b^2*(sin(a+b*x^n)-(a+b*x^n)*cos(a+b*x^n)+a*cos(a+b*x^n))
```

Maxima [A]
time = 0.32, size = 32, normalized size = 0.91

$$-\frac{bx^n\cos\left(bx^n+a\right)-\sin\left(bx^n+a\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="maxima")
[Out] -(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Fricas [A]
time = 0.37, size = 32, normalized size = 0.91

$$-\frac{bx^n\cos(bx^n+a)-\sin(bx^n+a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Sympy [A]
time = 29.25, size = 53, normalized size = 1.51

$$\begin{cases} \log(x)\sin(a) & \text{for}\ b=0\wedge n=0 \\ \frac{x^{2n}\sin(a)}{2n} & \text{for}\ b=0 \\ \log(x)\sin(a+b) & \text{for}\ n=0 \\ -\frac{x^n\cos(a+bx^n)}{bn}+\frac{\sin(a+bx^n)}{b^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*sin(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)*sin(a), Eq(b, 0) & Eq(n, 0)), (x**(2*n)*sin(a)/(2*n), Eq(
b, 0)), (log(x)*sin(a + b), Eq(n, 0)), (-x**n*cos(a + b*x**n)/(b*n) + sin(a
 + b*x**n)/(b**2*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)*sin(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^{2\,n-1}\,\sin\left(a+b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*sin(a + b*x^n),x)
```

```
[Out] int(x^(2*n - 1)*sin(a + b*x^n), x)
```

3.146 $\int x^{-1+2n} \cos(a + bx^n) \, dx$

Optimal. Leaf size=34

$$\frac{\cos(a + bx^n)}{b^2 n} + \frac{x^n \sin(a + bx^n)}{bn}$$

```
[Out] cos(a+b*x^n)/b^2/n+x^n*sin(a+b*x^n)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3461, 3377, 2718}

$$\frac{\cos(a + bx^n)}{b^2 n} + \frac{x^n \sin(a + bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Cos[a + b*x^n],x]
```

```
[Out] Cos[a + b*x^n]/(b^2*n) + (x^n*Sin[a + b*x^n])/(b*n)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^{-1+2n}\cos\left(a+bx^n\right)\,dx = \frac{\text{Subst}\left(\int x\cos(a+bx)\,dx,x,x^n\right)}{n}$$
$$= \frac{x^n\sin\left(a+bx^n\right)}{bn} - \frac{\text{Subst}\left(\int \sin(a+bx)\,dx,x,x^n\right)}{bn}$$
$$= \frac{\cos\left(a+bx^n\right)}{b^2n} + \frac{x^n\sin\left(a+bx^n\right)}{bn}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.85

$$\frac{\cos\left(a+bx^n\right)+bx^n\sin\left(a+bx^n\right)}{b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Cos[a + b*x^n],x]
[Out] (Cos[a + b*x^n] + b*x^n*Sin[a + b*x^n])/(b^2*n)
```

Maple [A]
time = 0.06, size = 44, normalized size = 1.29

method	result	size
risch	$\frac{\cos(a+b\,x^n)}{b^2n}+\frac{x^n\sin(a+b\,x^n)}{bn}$	35
default	$\frac{\cos(a+b\,x^n)+\sin(a+b\,x^n)(a+b\,x^n)-\sin(a+b\,x^n)a}{n\,b^2}$	44
meijerg	error in int/gproduct: numeric exception: division by zero\	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n/b^2*(cos(a+b*x^n)+sin(a+b*x^n)*(a+b*x^n)-sin(a+b*x^n)*a)
```

Maxima [A]
time = 0.29, size = 29, normalized size = 0.85

$$\frac{bx^n\sin\left(bx^n+a\right)+\cos\left(bx^n+a\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="maxima")
[Out] (b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.85

$$\frac{bx^n\sin\left(bx^n+a\right)+\cos\left(bx^n+a\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="fricas")
[Out] (b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Sympy [A]
time = 29.00, size = 53, normalized size = 1.56

$$\begin{cases}\log\left(x\right)\cos\left(a\right) & \text{for } b=0\wedge n=0\\ \frac{x^{2n}\cos\left(a\right)}{2n} & \text{for } b=0\\ \log\left(x\right)\cos\left(a+b\right) & \text{for } n=0\\ \frac{x^n\sin\left(a+bx^n\right)}{bn}+\frac{\cos\left(a+bx^n\right)}{b^2n} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*cos(a+b*x**n),x)
[Out] Piecewise((log(x)*cos(a), Eq(b, 0) & Eq(n, 0)), (x**(2*n)*cos(a)/(2*n), Eq(
b, 0)), (log(x)*cos(a + b), Eq(n, 0)), (x**n*sin(a + b*x**n)/(b*n) + cos(a
+ b*x**n)/(b**2*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)*cos(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^{2\,n-1}\,\cos\left(a+b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*cos(a + b*x^n),x)
[Out] int(x^(2*n - 1)*cos(a + b*x^n), x)
```

3.147 $\int x^{-1-n} \sin(a+bx^n)\,dx$

Optimal. Leaf size=46

$$\frac{b\cos(a)\text{Ci}(bx^n)}{n} - \frac{x^{-n}\sin(a+bx^n)}{n} - \frac{b\sin(a)\text{Si}(bx^n)}{n}$$

```
[Out] b*Ci(b*x^n)*cos(a)/n-b*Si(b*x^n)*sin(a)/n-sin(a+b*x^n)/n/(x^n)
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3460, 3378, 3384, 3380, 3383}

$$\frac{b\cos(a)\text{CosIntegral}(bx^n)}{n} - \frac{b\sin(a)\text{Si}(bx^n)}{n} - \frac{x^{-n}\sin(a+bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n],x]
```

```
[Out] (b*Cos[a]*CosIntegral[b*x^n])/n - Sin[a + b*x^n]/(n*x^n) - (b*Sin[a]*SinInt
egral[b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\sin(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}\sin(a+bx^n)}{n}+\frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}\sin(a+bx^n)}{n}+\frac{(b\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,x^n\right)}{n}-\frac{(b\sin(a))\text{Subst}\left(\int\right.}{n}\\
&= \frac{b\cos(a)\text{Ci}(bx^n)}{n}-\frac{x^{-n}\sin(a+bx^n)}{n}-\frac{b\sin(a)\text{Si}(bx^n)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 1.02

$$\frac{x^{-n}(bx^n\cos(a)\text{Ci}(bx^n)-\sin(a+bx^n)-bx^n\sin(a)\text{Si}(bx^n))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n],x]
```

```
[Out] (b*x^n*Cos[a]*CosIntegral[b*x^n] - Sin[a + b*x^n] - b*x^n*Sin[a]*SinIntegra
l[b*x^n])/(n*x^n)
```

Maple [A]
time = 0.08, size = 44, normalized size = 0.96

method	result
default	$\frac{b\left(-\frac{\sin(a+b\,x^n)\,x^{-n}}{b}-\text{sinIntegral}(b\,x^n)\sin(a)+\text{cosineIntegral}(b\,x^n)\cos(a)\right)}{n}$
risch	$-\frac{b\,e^{ia}\,\text{expIntegral}(1,-ib\,x^n)}{2n}+\frac{ib\,e^{-ia}\pi\,\text{csgn}(b\,x^n)}{2n}-\frac{ib\,e^{-ia}\,\text{sinIntegral}(b\,x^n)}{n}-\frac{b\,e^{-ia}\,\text{expIntegral}(1,-ib\,x^n)}{2n}-\frac{\sin(a+b\,x^n)}{n}$
meijerg	`error in int/gproduct: numeric exception: division by zero\`

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*b*(-sin(a+b*x^n)/b/(x^n)-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Fricas [A]
time = 0.37, size = 62, normalized size = 1.35

$$\frac{bx^n \cos(a) \operatorname{Ci}(bx^n) + bx^n \cos(a) \operatorname{Ci}(-bx^n) - 2\,bx^n \sin(a) \operatorname{Si}(bx^n) - 2\,\sin(bx^n + a)}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^n*cos(a)*cos_integral(b*x^n) + b*x^n*cos(a)*cos_integral(-b*x^n) -
 2*b*x^n*sin(a)*sin_integral(b*x^n) - 2*sin(b*x^n + a))/(n*x^n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1} \sin(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sin(a+b*x**n),x)
```

```
[Out] Integral(x**(-n - 1)*sin(a + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x^n\right)}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)/x^(n + 1),x)
```

```
[Out] int(sin(a + b*x^n)/x^(n + 1), x)
```

3.148 $\int x^{-1-n} \sin^2(a+bx^n)\, dx$

Optimal. Leaf size=67

$$-\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{b\mathrm{Ci}(2bx^n)\sin(2a)}{n}+\frac{b\cos(2a)\mathrm{Si}(2bx^n)}{n}$$

```
[Out] -1/2/n/(x^n)+1/2*cos(2*a+2*b*x^n)/n/(x^n)+b*cos(2*a)*Si(2*b*x^n)/n+b*Ci(2*b
*x^n)*sin(2*a)/n
```

Rubi [A]
time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3506, 3461, 3378, 3384, 3380, 3383}

$$\frac{b\sin(2a)\mathrm{CosIntegral}(2bx^n)}{n}+\frac{b\cos(2a)\mathrm{Si}(2bx^n)}{n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

```
[Out] -1/2*1/(n*x^n) + Cos[2*(a + b*x^n)]/(2*n*x^n) + (b*CosIntegral[2*b*x^n]*Sin
[2*a])/n + (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\sin^2(a+bx^n)\,dx &= \int\left(\frac{x^{-1-n}}{2}-\frac{1}{2}x^{-1-n}\cos(2a+2bx^n)\right)dx\\
&= -\frac{x^{-n}}{2n}-\frac{1}{2}\int x^{-1-n}\cos(2a+2bx^n)\,dx\\
&= -\frac{x^{-n}}{2n}-\frac{\operatorname{Subst}\left(\int\frac{\cos(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}\\
&= -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{b\operatorname{Subst}\left(\int\frac{\sin(2a+2bx)}{x}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{(b\cos(2a))\operatorname{Subst}\left(\int\frac{\sin(2bx)}{x}\,dx,x,x^n\right)}{n}+\frac{(b\sin
\\
&= -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{b\operatorname{Ci}(2bx^n)\sin(2a)}{n}+\frac{b\cos(2a)\operatorname{Si}(2bx^n)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 0.87

$$\frac{x^{-n}(-1+\cos(2(a+bx^n))+2bx^n\operatorname{Ci}(2bx^n)\sin(2a)+2bx^n\cos(2a)\operatorname{Si}(2bx^n))}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

[Out] (-1 + Cos[2*(a + b*x^n)] + 2*b*x^n*CosIntegral[2*b*x^n]*Sin[2*a] + 2*b*x^n*Cos[2*a]*SinIntegral[2*b*x^n])/(2*n*x^n)

Maple [A]
time = 0.07, size = 66, normalized size = 0.99

method	result
default	$-\frac{x^{-n}}{2n}-\frac{b\left(-\frac{\cos(2a+2b\,x^{n})x^{-n}}{2b}-\mathrm{sinIntegral}(2b\,x^{n})\cos(2a)-\mathrm{cosineIntegral}(2b\,x^{n})\sin(2a)\right)}{n}$
risch	$-\frac{b\,\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(b\,x^{n})}{2n}+\frac{b\,\mathrm{e}^{-2ia}\,\mathrm{sinIntegral}(2b\,x^{n})}{n}-\frac{ib\,\mathrm{e}^{-2ia}\,\mathrm{expIntegral}(1,-2ib\,x^{n})}{2n}+\frac{ib\,\mathrm{e}^{2ia}\,\mathrm{expIntegral}(1,-2ib\,x^{n})}{2n}-\frac{x^{-n}}{2n}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-n)*sin(a+b*x^n)^2,x,method=_RETURNVERBOSE)

[Out] -1/2/n/(x^n)-1/n*b*(-1/2*cos(2*a+2*b*x^n)/b/(x^n)-Si(2*b*x^n)*cos(2*a)-Ci(2*b*x^n)*sin(2*a))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="maxima")

[Out] -1/2*(n*x^n*integrate(cos(2*b*x^n + 2*a)/(x*x^n), x) + 1)/(n*x^n)

Fricas [A]
time = 0.52, size = 73, normalized size = 1.09

$$\frac{bx^n \operatorname{Ci}\left(2\,bx^n\right)\sin\left(2\,a\right)+bx^n \operatorname{Ci}\left(-2\,bx^n\right)\sin\left(2\,a\right)+2\,bx^n\cos\left(2\,a\right)\operatorname{Si}\left(2\,bx^n\right)+2\,\cos\left(bx^n+a\right)^2-2}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="fricas")

[Out] 1/2*(b*x^n*cos_integral(2*b*x^n)*sin(2*a) + b*x^n*cos_integral(-2*b*x^n)*sin(2*a) + 2*b*x^n*cos(2*a)*sin_integral(2*b*x^n) + 2*cos(b*x^n + a)^2 - 2)/(n*x^n)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\sin^2\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sin(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-n - 1)*sin(a + b*x**n)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x^n\right)^2}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^2/x^(n + 1),x)
```

```
[Out] int(sin(a + b*x^n)^2/x^(n + 1), x)
```

3.149 $\int x^{-1-n} \sin^3(a+bx^n)\,dx$

Optimal. Leaf size=113

$$\frac{3b\cos(a)\text{Ci}(bx^n)}{4n}-\frac{3b\cos(3a)\text{Ci}(3bx^n)}{4n}-\frac{3x^{-n}\sin(a+bx^n)}{4n}+\frac{x^{-n}\sin(3(a+bx^n))}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3}{}$$

```
[Out] 3/4*b*Ci(b*x^n)*cos(a)/n-3/4*b*Ci(3*b*x^n)*cos(3*a)/n-3/4*b*Si(b*x^n)*sin(a
)/n+3/4*b*Si(3*b*x^n)*sin(3*a)/n-3/4*sin(a+b*x^n)/n/(x^n)+1/4*sin(3*a+3*b*x
^n)/n/(x^n)
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3506, 3460, 3378, 3384, 3380, 3383}

$$\frac{3b\cos(a)\text{CosIntegral}(bx^n)}{4n}-\frac{3b\cos(3a)\text{CosIntegral}(3bx^n)}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\text{Si}(3bx^n)}{4n}-\frac{3x^{-n}\sin(a+bx^n)}{4n}+\frac{x^{-n}\sin(3(a+bx^n))}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (3*b*Cos[a]*CosIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*CosIntegral[3*b*x^n])
/(4*n) - (3*Sin[a + b*x^n])/(4*n*x^n) + Sin[3*(a + b*x^n)]/(4*n*x^n) - (3*b
*Sin[a]*SinIntegral[b*x^n])/(4*n) + (3*b*Sin[3*a]*SinIntegral[3*b*x^n])/(4*
n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\sin^3\left(a+bx^n\right)\,dx &= \int \left(\frac{3}{4}x^{-1-n}\sin\left(a+bx^n\right)-\frac{1}{4}x^{-1-n}\sin\left(3a+3bx^n\right)\right)\,dx \\
&= -\left(\frac{1}{4}\int x^{-1-n}\sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int x^{-1-n}\sin\left(a+bx^n\right)\,dx \\
&= -\frac{\mathrm{Subst}\left(\int \frac{\sin(3a+3bx)}{x^2}\,dx,x,x^n\right)}{4n}+\frac{3\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,x^n\right)}{4n} \\
&= -\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3\left(a+bx^n\right)\right)}{4n}+\frac{(3b)\mathrm{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,x^n\right)}{4n} \\
&= -\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3\left(a+bx^n\right)\right)}{4n}+\frac{(3b\cos(a))\mathrm{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x \right.}{4n} \\
&= \frac{3b\cos(a)\mathrm{Ci}(bx^n)}{4n}-\frac{3b\cos(3a)\mathrm{Ci}(3bx^n)}{4n}-\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3(a+\right.}{4n}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 95, normalized size = 0.84

$$\frac{x^{-n}\left(3bx^n\cos(a)\mathrm{Ci}(bx^n)-3bx^n\cos(3a)\mathrm{Ci}(3bx^n)-3\sin\left(a+bx^n\right)+\sin\left(3\left(a+bx^n\right)\right)-3bx^n\sin(a)\mathrm{Si}(bx^n)+3bx^n\sin(3a)\mathrm{Si}(3bx^n)\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (3*b*x^n*Cos[a]*CosIntegral[b*x^n] - 3*b*x^n*Cos[3*a]*CosIntegral[3*b*x^n]
- 3*Sin[a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b*x^n*Sin[a]*SinIntegral[b*x^n]
 + 3*b*x^n*Sin[3*a]*SinIntegral[3*b*x^n])/(4*n*x^n)
```

Maple [A]
time = 0.08, size = 99, normalized size = 0.88

method	result
default	$\frac{3b\left(-\frac{\sin(a+b\,x^{n})x^{-n}}{b}-\text{sinIntegral}(b\,x^{n})\sin(a)+\text{cosineIntegral}(b\,x^{n})\cos(a)\right)}{4n}-\frac{3b\left(-\frac{\sin(3a+3b\,x^{n})x^{-n}}{3b}-\text{sinIntegral}(3b\,x^{n})\sin(3a)+\right.}{4n}$
risch	$\frac{3b\,\mathrm{e}^{3ia}\,\text{expIntegral}(1,-3ib\,x^{n})}{8n}-\frac{3ib\,\mathrm{e}^{-3ia}\pi\,\text{csgn}(b\,x^{n})}{8n}+\frac{3ib\,\mathrm{e}^{-3ia}\,\text{sinIntegral}(3b\,x^{n})}{4n}+\frac{3b\,\mathrm{e}^{-3ia}\,\text{expIntegral}(1,-3ib\,x^{n})}{8n}+3ib\,\mathrm{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sin(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b*(-sin(a+b*x^n)/b/(x^n)-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))-3/4/n*b*(
-1/3*sin(3*a+3*b*x^n)/b/(x^n)-Si(3*b*x^n)*sin(3*a)+Ci(3*b*x^n)*cos(3*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Fricas [A]
time = 0.59, size = 127, normalized size = 1.12

$$-\frac{3\,bx^{n}\cos\left(3\,a\right)\operatorname{Ci}\left(3\,bx^{n}\right)-3\,bx^{n}\cos\left(a\right)\operatorname{Ci}\left(bx^{n}\right)-3\,bx^{n}\cos\left(a\right)\operatorname{Ci}\left(-bx^{n}\right)+3\,bx^{n}\cos\left(3\,a\right)\operatorname{Ci}\left(-3\,bx^{n}\right)-6\,bx^{n}\sin\left(3\,a\right)\operatorname{Si}\left(3\,bx^{n}\right)+6\,bx^{n}\sin\left(a\right)\operatorname{Si}\left(bx^{n}\right)-8\,\left(\cos\left(bx^{n}+a\right)^{2}-1\right)\sin\left(bx^{n}+a\right)}{8\,nx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*b*x^n*cos(3*a)*cos_integral(3*b*x^n) - 3*b*x^n*cos(a)*cos_integral(
b*x^n) - 3*b*x^n*cos(a)*cos_integral(-b*x^n) + 3*b*x^n*cos(3*a)*cos_integra
l(-3*b*x^n) - 6*b*x^n*sin(3*a)*sin_integral(3*b*x^n) + 6*b*x^n*sin(a)*sin_i
ntegral(b*x^n) - 8*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(n*x^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sin(a+b*x**n)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x^n\right)^3}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^3/x^(n + 1),x)
```

```
[Out] int(sin(a + b*x^n)^3/x^(n + 1), x)
```

3.150 $\int x^{-1-2n} \sin(a + bx^n)\, dx$

Optimal. Leaf size=78

$$-\frac{bx^{-n}\cos(a+bx^n)}{2n}-\frac{b^2\text{Ci}(bx^n)\sin(a)}{2n}-\frac{x^{-2n}\sin(a+bx^n)}{2n}-\frac{b^2\cos(a)\text{Si}(bx^n)}{2n}$$

```
[Out] -1/2*b*cos(a+b*x^n)/n/(x^n)-1/2*b^2*cos(a)*Si(b*x^n)/n-1/2*b^2*Ci(b*x^n)*si
n(a)/n-1/2*sin(a+b*x^n)/n/(x^(2*n))
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3460, 3378, 3384, 3380, 3383}

$$-\frac{b^2\sin(a)\text{CosIntegral}(bx^n)}{2n}-\frac{b^2\cos(a)\text{Si}(bx^n)}{2n}-\frac{x^{-2n}\sin(a+bx^n)}{2n}-\frac{bx^{-n}\cos(a+bx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -1/2*(b*Cos[a + b*x^n])/(n*x^n) - (b^2*CosIntegral[b*x^n]*Sin[a])/(2*n) - S
in[a + b*x^n]/(2*n*x^(2*n)) - (b^2*Cos[a]*SinIntegral[b*x^n])/(2*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\sin(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-2n}\sin(a+bx^n)}{2n}+\frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, x^n\right)}{2n}\\
&= -\frac{bx^{-n}\cos(a+bx^n)}{2n}-\frac{x^{-2n}\sin(a+bx^n)}{2n}-\frac{b^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, x^n\right)}{2n}\\
&= -\frac{bx^{-n}\cos(a+bx^n)}{2n}-\frac{x^{-2n}\sin(a+bx^n)}{2n}-\frac{(b^2\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x\right.}{2n}\\
&= -\frac{bx^{-n}\cos(a+bx^n)}{2n}-\frac{b^2\text{Ci}(bx^n)\sin(a)}{2n}-\frac{x^{-2n}\sin(a+bx^n)}{2n}-\frac{b^2\cos(a)\text{Si}(bx}{2n}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 68, normalized size = 0.87

$$-\frac{x^{-2n}(bx^n\cos(a+bx^n)+b^2x^{2n}\text{Ci}(bx^n)\sin(a)+\sin(a+bx^n)+b^2x^{2n}\cos(a)\text{Si}(bx^n))}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -1/2*(b*x^n*Cos[a + b*x^n] + b^2*x^(2*n)*CosIntegral[b*x^n]*Sin[a] + Sin[a
+ b*x^n] + b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n])/(n*x^(2*n))
```

Maple [A]
time = 0.06, size = 65, normalized size = 0.83

method	result

default	$\frac{b^2\left(-\frac{x^{-2n}\sin(a+b\,x^n)}{2b^2}-\frac{\cos(a+b\,x^n)x^{-n}}{2b}-\frac{\mathrm{sinIntegral}(b\,x^n)\cos(a)}{2}-\frac{\mathrm{cosineIntegral}(b\,x^n)\sin(a)}{2}\right)}{n}$
risch	$\frac{b^2\mathrm{e}^{-ia}\pi\,\mathrm{csgn}(b\,x^n)}{4n}-\frac{b^2\mathrm{e}^{-ia}\,\mathrm{sinIntegral}(b\,x^n)}{2n}+\frac{ib^2\mathrm{e}^{-ia}\,\mathrm{expIntegral}(1,-ib\,x^n)}{4n}-\frac{ib^2\mathrm{e}^{ia}\,\mathrm{expIntegral}(1,-ib\,x^n)}{4n}-\frac{b\cos(a+b\,x^n)}{2n}$
meijerg	$\frac{b^2\sqrt{\pi}\left(-\frac{x^{2\left(\frac{-1-2n}{2n}+\frac{1}{2n}\right)n}2^{-\frac{-1-2n}{n}-\frac{1}{n}}}{\sqrt{\pi}\,b^2}+\frac{(-1)^{-\frac{-1-2n}{2n}-\frac{1}{2n}}\left(-\Psi\left(1-\frac{-1-2n}{2n}-\frac{1}{2n}\right)-\Psi\left(\frac{1}{2}-\frac{-1-2n}{2n}-\frac{1}{2n}\right)+2n\ln(x)-2\ln(2)+\ln\left(b^2\right)\right)\sqrt{2}}{2\sqrt{\pi}\,\Gamma\left(-\frac{-1-2n}{n}-\frac{1}{n}\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*sin(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*b^2*(-1/2/b^2/(x^n)^2*sin(a+b*x^n)-1/2*cos(a+b*x^n)/b/(x^n)-1/2*Si(b*x^
n)*cos(a)-1/2*Ci(b*x^n)*sin(a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Fricas [A]
time = 0.53, size = 90, normalized size = 1.15

$$-\frac{b^2x^{2\,n}\,\mathrm{Ci}\left(bx^n\right)\sin\left(a\right)+b^2x^{2\,n}\,\mathrm{Ci}\left(-bx^n\right)\sin\left(a\right)+2\,b^2x^{2\,n}\cos\left(a\right)\mathrm{Si}\left(bx^n\right)+2\,bx^n\cos\left(bx^n+a\right)+2\,\sin\left(bx^n+a\right)}{4\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*x^(2*n)*cos_integral(b*x^n)*sin(a) + b^2*x^(2*n)*cos_integral(-b*
x^n)*sin(a) + 2*b^2*x^(2*n)*cos(a)*sin_integral(b*x^n) + 2*b*x^n*cos(b*x^n
+ a) + 2*sin(b*x^n + a))/(n*x^(2*n))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x^n\right)}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)/x^(2*n + 1),x)
```

```
[Out] int(sin(a + b*x^n)/x^(2*n + 1), x)
```

3.151 $\int x^{-1-2n}\sin^2(a+bx^n)\,dx$

Optimal. Leaf size=95

$$-\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}+\frac{b^2\cos(2a)\text{Ci}(2bx^n)}{n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}$$

```
[Out] -1/4/n/(x^(2*n))+b^2*Ci(2*b*x^n)*cos(2*a)/n+1/4*cos(2*a+2*b*x^n)/n/(x^(2*n)
)-b^2*Si(2*b*x^n)*sin(2*a)/n-1/2*b*sin(2*a+2*b*x^n)/n/(x^n)
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3506, 3461, 3378, 3384, 3380, 3383}

$$\frac{b^2\cos(2a)\text{CosIntegral}(2bx^n)}{n}-\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{x^{-2n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

```
[Out] -1/4*1/(n*x^(2*n)) + Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) + (b^2*Cos[2*a]*CosIn
tegral[2*b*x^n])/n - (b*Sin[2*(a + b*x^n)])/(2*n*x^n) - (b^2*Sin[2*a]*SinIn
tegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-2n}\sin^2\left(a+bx^n\right)\,dx &= \int\left(\frac{1}{2}x^{-1-2n}-\frac{1}{2}x^{-1-2n}\cos\left(2a+2bx^n\right)\right)\,dx \\
&= -\frac{x^{-2n}}{4n}-\frac{1}{2}\int x^{-1-2n}\cos\left(2a+2bx^n\right)\,dx \\
&= -\frac{x^{-2n}}{4n}-\frac{\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x^3}\,dx,x,x^n\right)}{2n} \\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n}+\frac{b\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n} \\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n}-\frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n}+\frac{b^2\text{Subst}\left(\int\frac{\cos(2a+2}{x}\right.}{n} \\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n}-\frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n}+\frac{(b^2\cos(2a))\,\text{Subst}}{} \\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n}+\frac{b^2\cos(2a)\text{Ci}(2bx^n)}{n}-\frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.86

$$\frac{x^{-2n}\left(-1+\cos\left(2(a+bx^n)\right)+4b^2x^{2n}\cos(2a)\text{Ci}(2bx^n)-2bx^n\sin\left(2(a+bx^n)\right)-4b^2x^{2n}\sin(2a)\text{Si}(2bx^n)\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

```
[Out] (-1 + Cos[2*(a + b*x^n)] + 4*b^2*x^(2*n)*Cos[2*a]*CosIntegral[2*b*x^n] - 2*
b*x^n*Sin[2*(a + b*x^n)] - 4*b^2*x^(2*n)*Sin[2*a]*SinIntegral[2*b*x^n])/(4*
n*x^(2*n))
```

Maple [A]
time = 0.07, size = 89, normalized size = 0.94

method	result
default	$-\frac{x^{-2n}}{4n}-\frac{2b^{2}\left(-\frac{x^{-2n}\cos\left(2a+2b\,x^{n}\right)}{8b^{2}}+\frac{\sin\left(2a+2b\,x^{n}\right)x^{-n}}{4b}+\frac{\operatorname{sinIntegral}\left(2b\,x^{n}\right)\sin\left(2a\right)}{2}-\frac{\operatorname{cosineIntegral}\left(2b\,x^{n}\right)\cos\left(2a\right)}{2}\right)}{n}$
risch	$\frac{i b^{2} e^{-2ia}\pi\,\operatorname{csgn}\left(b\,x^{n}\right)}{2n}-\frac{i b^{2} e^{-2ia}\operatorname{sinIntegral}\left(2b\,x^{n}\right)}{n}-\frac{b^{2} e^{-2ia}\operatorname{expIntegral}\left(1,-2ib\,x^{n}\right)}{2n}-\frac{b^{2} e^{2ia}\operatorname{expIntegral}\left(1,-2ib\,x^{n}\right)}{2n}-\frac{x^{-2n}}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*sin(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x^n)^2/n-2/n*b^2*(-1/8/b^2/(x^n)^2*cos(2*a+2*b*x^n)+1/4*sin(2*a+2*b*x
^n)/b/(x^n)+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*n*x^(2*n)*integrate(cos(2*b*x^n + 2*a)/(x*x^(2*n)), x) + 1)/(n*x^(2
*n))
```

Fricas [A]
time = 0.43, size = 107, normalized size = 1.13

$$\frac{b^{2}x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^{n}\right)+b^{2}x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,bx^{n}\right)-2\,b^{2}x^{2\,n}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^{n}\right)-2\,bx^{n}\cos\left(bx^{n}+a\right)\sin\left(bx^{n}+a\right)+\cos\left(bx^{n}+a\right)^{2}-1}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^(2*n)*cos(2*a)*cos_integral(2*b*x^n) + b^2*x^(2*n)*cos(2*a)*cos_
integral(-2*b*x^n) - 2*b^2*x^(2*n)*sin(2*a)*sin_integral(2*b*x^n) - 2*b*x^n
*cos(b*x^n + a)*sin(b*x^n + a) + cos(b*x^n + a)^2 - 1)/(n*x^(2*n))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n)**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x^n\right)^2}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^2/x^(2*n + 1),x)
[Out] int(sin(a + b*x^n)^2/x^(2*n + 1), x)
```

3.152 $\int x^{-1-2n} \sin^3(a+bx^n)\,dx$

Optimal. Leaf size=165

$$-\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n}-\frac{3b^2\text{Ci}(bx^n)\sin(a)}{8n}+\frac{9b^2\text{Ci}(3bx^n)\sin(3a)}{8n}-\frac{3x^{-2n}\sin(a+bx^n)}{8n}$$

```
[Out] -3/8*b*cos(a+b*x^n)/n/(x^n)+3/8*b*cos(3*a+3*b*x^n)/n/(x^n)-3/8*b^2*cos(a)*S
i(b*x^n)/n+9/8*b^2*cos(3*a)*Si(3*b*x^n)/n-3/8*b^2*Ci(b*x^n)*sin(a)/n+9/8*b^
2*Ci(3*b*x^n)*sin(3*a)/n-3/8*sin(a+b*x^n)/n/(x^(2*n))+1/8*sin(3*a+3*b*x^n)/
n/(x^(2*n))
```

Rubi [A]
time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3506, 3460, 3378, 3384, 3380, 3383}

$$-\frac{3b^2\sin(a)\text{CosIntegral}(bx^n)}{8n}+\frac{9b^2\sin(3a)\text{CosIntegral}(3bx^n)}{8n}-\frac{3b^2\cos(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n}-\frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3(a+bx^n))}{8n}-\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (-3*b*Cos[a + b*x^n])/(8*n*x^n) + (3*b*Cos[3*(a + b*x^n)])/(8*n*x^n) - (3*b
^2*CosIntegral[b*x^n]*Sin[a])/(8*n) + (9*b^2*CosIntegral[3*b*x^n]*Sin[3*a])
/(8*n) - (3*Sin[a + b*x^n])/(8*n*x^(2*n)) + Sin[3*(a + b*x^n)]/(8*n*x^(2*n)
) - (3*b^2*Cos[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Cos[3*a]*SinIntegral[3
*b*x^n])/(8*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-2n}\sin^3\left(a+bx^n\right)\,dx &= \int \left(\frac{3}{4}x^{-1-2n}\sin\left(a+bx^n\right)-\frac{1}{4}x^{-1-2n}\sin\left(3a+3bx^n\right)\right)\,dx\\
&= -\left(\frac{1}{4}\int x^{-1-2n}\sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sin(3a+3bx)}{x^3}\,dx,x,x^n\right)}{4n}+\frac{3\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx,x,x^n\right)}{4n}\\
&= -\frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n}+\frac{x^{-2n}\sin\left(3\left(a+bx^n\right)\right)}{8n}+\frac{(3b)\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx,x,x\right.}{8n}\\
&= -\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\cos\left(3\left(a+bx^n\right)\right)}{8n}-\frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n}+\frac{x^{-2n}}{}\\
&= -\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\cos\left(3\left(a+bx^n\right)\right)}{8n}-\frac{3x^{-2n}\sin\left(a+bx^n\right)}{8n}+\frac{x^{-2n}}{}\\
&= -\frac{3bx^{-n}\cos\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\cos\left(3\left(a+bx^n\right)\right)}{8n}-\frac{3b^2\operatorname{Ci}(bx^n)\sin(a)}{8n}+\frac{9b^2\operatorname{Ci}}{}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 141, normalized size = 0.85

$$\frac{x^{-2n}\left(-3bx^n\cos\left(a+bx^n\right)+3bx^n\cos\left(3\left(a+bx^n\right)\right)-3b^2x^{2n}\operatorname{Ci}(bx^n)\sin(a)+9b^2x^{2n}\operatorname{Ci}(3bx^n)\sin(3a)-3\sin\left(a+bx^n\right)+\sin\left(3\left(a+bx^n\right)\right)-3b^2x^{2n}\cos(a)\operatorname{Si}(bx^n)+9b^2x^{2n}\cos(3a)\operatorname{Si}(3bx^n)\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (-3*b*x^n*Cos[a + b*x^n] + 3*b*x^n*Cos[3*(a + b*x^n)] - 3*b^2*x^(2*n)*CosIn
tegral[b*x^n]*Sin[a] + 9*b^2*x^(2*n)*CosIntegral[3*b*x^n]*Sin[3*a] - 3*Sin[
a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n] +
 9*b^2*x^(2*n)*Cos[3*a]*SinIntegral[3*b*x^n])/(8*n*x^(2*n))
```

Maple [A]
time = 0.08, size = 144, normalized size = 0.87

method	result
default	$\frac{3b^2\left(-\frac{x^{-2n}\sin(a+b\,x^n)}{2b^2}-\frac{\cos(a+b\,x^n)x^{-n}}{2b}-\frac{\mathrm{sinIntegral}(b\,x^n)\cos(a)}{2}-\frac{\mathrm{cosineIntegral}(b\,x^n)\sin(a)}{2}\right)}{4n}-\frac{9b^2\left(-\frac{\sin(3a+3b\,x^n)x^{-2n}}{18b^2}-\frac{\cos(3a+3\ldots}{6\ldots}\right.}{\ldots}$
risch	$\frac{9ib^2\mathrm{e}^{3ia}\,\mathrm{expIntegral}(1,-3ib\,x^n)}{16n}-\frac{9b^2\mathrm{e}^{-3ia}\pi\,\mathrm{csgn}(b\,x^n)}{16n}+\frac{9b^2\mathrm{e}^{-3ia}\,\mathrm{sinIntegral}(3b\,x^n)}{8n}-\frac{9ib^2\mathrm{e}^{-3ia}\,\mathrm{expIntegral}(1,-3ib\,x^n)}{16n}+\frac{3\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*sin(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b^2*(-1/2/b^2/(x^n)^2*sin(a+b*x^n)-1/2*cos(a+b*x^n)/b/(x^n)-1/2*Si(b*
x^n)*cos(a)-1/2*Ci(b*x^n)*sin(a))-9/4/n*b^2*(-1/18*sin(3*a+3*b*x^n)/b^2/(x^
n)^2-1/6*cos(3*a+3*b*x^n)/b/(x^n)-1/2*Si(3*b*x^n)*cos(3*a)-1/2*Ci(3*b*x^n)*
sin(3*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Fricas [A]
time = 0.56, size = 183, normalized size = 1.11

$$\frac{24\,bx^n\cos\left(bx^n+a\right)^3+9\,b^2x^{2\,n}\,\mathrm{Ci}\left(3\,bx^n\right)\sin\left(3\,a\right)+9\,b^2x^{2\,n}\,\mathrm{Ci}\left(-3\,bx^n\right)\sin\left(3\,a\right)-3\,b^2x^{2\,n}\,\mathrm{Ci}\left(bx^n\right)\sin\left(a\right)-3\,b^2x^{2\,n}\,\mathrm{Ci}\left(-bx^n\right)\sin\left(a\right)+18\,b^2x^{2\,n}\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx^n\right)-6\,b^2x^{2\,n}\cos\left(a\right)\mathrm{Si}\left(bx^n\right)-24\,bx^n\cos\left(bx^n+a\right)+8\,\left(\cos\left(bx^n+a\right)^2-1\right)\sin\left(bx^n+a\right)}{16\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*b*x^n*cos(b*x^n + a)^3 + 9*b^2*x^(2*n)*cos_integral(3*b*x^n)*sin(3
*a) + 9*b^2*x^(2*n)*cos_integral(-3*b*x^n)*sin(3*a) - 3*b^2*x^(2*n)*cos_int
egral(b*x^n)*sin(a) - 3*b^2*x^(2*n)*cos_integral(-b*x^n)*sin(a) + 18*b^2*x^
```

```
(2*n)*cos(3*a)*sin_integral(3*b*x^n) - 6*b^2*x^(2*n)*cos(a)*sin_integral(b*
x^n) - 24*b*x^n*cos(b*x^n + a) + 8*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(
n*x^(2*n))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x^n\right)^3}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x^n)^3/x^(2*n + 1),x)
```

```
[Out] int(sin(a + b*x^n)^3/x^(2*n + 1), x)
```

3.153 $\int (e+fx)^3 \sin \left(b(c+dx)^2\right) \, dx$

Optimal. Leaf size=223

$$-\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\cos\left(b(c+dx)^2\right)}{2bd^4}+\frac{3f^2(d$$

```
[Out] -3/2*f*(-c*f+d*e)^2*cos(b*(d*x+c)^2)/b/d^4-3/2*f^2*(-c*f+d*e)*(d*x+c)*cos(b
*(d*x+c)^2)/b/d^4-1/2*f^3*(d*x+c)^2*cos(b*(d*x+c)^2)/b/d^4+1/2*f^3*sin(b*(d
*x+c)^2)/b^2/d^4+3/4*f^2*(-c*f+d*e)*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/
2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d^4+1/2*(-c*f+d*e)^3*FresnelS((d*x+c)*b^(1/2)*
2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^4/b^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717}

$$\frac{3\sqrt{\frac{\pi}{2}}f^2(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(b(c+dx)^2\right)}{2b^2d^4}-\frac{3f^2(c+dx)(de-cf)\cos\left(b(c+dx)^2\right)}{2bd^4}+\frac{\sqrt{\frac{\pi}{2}}(de-cf)^3S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4}-\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\cos\left(b(c+dx)^2\right)}{2bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-3*f*(d*e - c*f)^2*Cos[b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*(c +
 d*x)*Cos[b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[b*(c + d*x)^2])/
(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(Sqrt[b]*d^4) + (f^3*Sin[b*(c + d*x)^2])/(2*b^2*d^4)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(b(c+dx)^2\right)\,dx = \frac{\text{Subst}\Big(\int \Big(d^3e^3\Big(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\Big)\sin(bx^2)+3d^2e^2f\Big(1+\frac{cf(-2de+cf}{d^2e^2}}{d}$$

$$= \frac{f^3\text{Subst}\left(\int x^3\sin(bx^2)\,dx,x,c+dx\right)}{d^4} + \frac{(3f^2(de-cf))\,\text{Subst}\left(\int x^2\sin(bx\right.}{d^4}$$

$$= -\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} + \frac{(de-cf)^3\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c\right.}{\sqrt{b}\,d^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} -$$

$$= -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4} -$$

Mathematica [A]
time = 0.66, size = 173, normalized size = 0.78

$$\frac{-4bf(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2))\cos\left(b(c+dx)^2\right)-6\sqrt{b}\,f^2(-de+cf)\sqrt{2\pi}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+4b^{3/2}(de-cf)^3\sqrt{2\pi}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+4f^3\sin\left(b(c+dx)^2\right)}{8b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos
[b*(c + d*x)^2] - 6*Sqrt[b]*f^2*(-(d*e) + c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)] + 4*b^(3/2)*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelS[Sqrt[b]
*Sqrt[2/Pi]*(c + d*x)] + 4*f^3*Sin[b*(c + d*x)^2])/(8*b^2*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(194) = 388.
time = 0.15, size = 586, normalized size = 2.63

method	result

default	$-\frac{f^3 x^2 \cos(d^2x^2b+2cdxb+b\,c^2)}{2b\,d^2} - \frac{f^3 c\left(-\frac{x\cos\left(d^2x^2b+2cdxb+b\,c^2\right)}{2b\,d^2} - \frac{c\left(-\frac{\cos\left(d^2x^2b+2cdxb+b\,c^2\right)}{2b\,d^2} - \frac{c\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(b\,d^2x+bcd}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right.}{2d\sqrt{b\,d^2}}\right.}{d}\right.}{d}$
risch	$\frac{ie^3\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}} - \frac{if^3c^3\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^4\sqrt{-ib}} + \frac{3f^3c\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{i}{\sqrt{-}}\right.}{8b\,d^4\sqrt{-ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f^3/b/d^2*x^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^3*c/d*(-1/2/b/d^2*x*cos
(b*d^2*x^2+2*b*c*d*x+b*c^2)-c/d*(-1/2/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-
1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1
/2)*(b*d^2*x+b*c*d)))+1/4/b/d^2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelC(2^(
1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))+f^3/b/d^2*(1/2/b/d^2*sin(b*d^
2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelC(2^(1
/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))-3/2*e*f^2/b/d^2*x*cos(b*d^2*x^
2+2*b*c*d*x+b*c^2)-3*e*f^2*c/d*(-1/2/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1
/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/
2)*(b*d^2*x+b*c*d)))+3/4*e*f^2/b/d^2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-3/2*e^2*f/b/d^2*cos(b*d^2
*x^2+2*b*c*d*x+b*c^2)-3/2*e^2*f*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS
(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*Pi^(1/2)/(b*d^
2)^(1/2)*e^3*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.51, size = 973, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(6*b*c^3*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2)) + 2*(3*b*c^2*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^
2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - I*gamma(2, I*b*d^2*x^2 + 2
*I*b*c*d*x + I*b*c^2) + I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*d
```

```
*x + 2*c*(-I*gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*gamma(2, -I*
b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - ((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt
(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf
(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^3 - 3*((I - 1)*sqrt(
2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - (I + 1)*sqrt(2)*gamma(
3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*c)*sqrt(b*d^2*x^2 + 2*b*c*d*x +
 b*c^2))*f^3/(b^2*d^5*x + b^2*c*d^4) + 3/8*(4*b*c*d*x*(e^(I*b*d^2*x^2 + 2*I
*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 4*b*c^2*(
e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I
*b*c^2)) - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*
(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt
(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2 - (I - 1)
*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*
gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*f^2*e/(b^2*d^4*x + b^2*c
*d^3) - 3/8*(2*d*x*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x
^2 - 2*I*b*c*d*x - I*b*c^2)) - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1
)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (
I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
1))*c + 2*c*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2)))*f*e^2/(b*d^3*x + b*c*d^2) + 1/8*sqrt(2)*sqrt(pi)*((I
 + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)*erf((I*b*d*x + I*b*c)/sqrt
(-I*b)))*e^3/(sqrt(b)*d)
```

Fricas [A]
time = 0.38, size = 259, normalized size = 1.16

$$\frac{2\,df^3\sin\left(bd^2x^2+2\,bcdx+bc^2\right)-3\,\sqrt{2}\,(\pi cf^3-\pi df^2e)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)-2\,\sqrt{2}\,(\pi bc^3f^3-3\,\pi bc^2df^2e+3\,\pi bcd^2fe^2-\pi bd^3e^3)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)-2\,(bd^3f^3x^2-bcd^2f^3x+bc^2df^3+3\,bd^3fe^2+3\,(bd^3f^2x-bcd^2f^2)e)\cos\left(bd^2x^2+2\,bcdx+bc^2\right)}{4\,b^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2) - 3*sqrt(2)*(pi*c*f^3 - pi*
d*f^2*e)*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2
*sqrt(2)*(pi*b*c^3*f^3 - 3*pi*b*c^2*d*f^2*e + 3*pi*b*c*d^2*f*e^2 - pi*b*d^3
*e^3)*sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b
*d^3*f^3*x^2 - b*c*d^2*f^3*x + b*c^2*d*f^3 + 3*b*d^3*f*e^2 + 3*(b*d^3*f^2*x
 - b*c*d^2*f^2)*e)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3\sin\left(bc^2+2bcdx+bd^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**3*sin(b*(d*x+c)**2),x)

[Out] Integral((e + f*x)**3*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)

Giac [C] Result contains complex when optimal does not.
time = 5.62, size = 1021, normalized size = 4.58

$$-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^3}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^3}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{3\left(-\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^2}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-ibd^2x^2-2ibcdx-ibc^2+2\right)}}{bd}\right)}{4d}-\frac{3\left(\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^2}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(ibd^2x^2+2ibcdx+ibc^2+2\right)}}{bd}\right)}{4d}-\frac{3\left(\frac{i\sqrt{2}\sqrt{\pi}\left(2bc^2f^2-if^2\right)\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2i\left(df^2\left(ix+\frac{ic}{d}\right)-2icf^2\right)e^{\left(-ibd^2x^2-2ibcdx-ibc^2+1\right)}}{bd}\right)}{8d^2}-\frac{3\left(-\frac{i\sqrt{2}\sqrt{\pi}\left(2bc^2f^2+if^2\right)\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2i\left(df^2\left(ix+\frac{ic}{d}\right)-2icf^2\right)e^{\left(ibd^2x^2+2ibcdx+ibc^2+1\right)}}{bd}\right)}{8d^2}+\frac{\frac{\sqrt{2}\sqrt{\pi}\left(2ibc^3f^3+3cf^3\right)\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2\left(bd^2f^3\left(x+\frac{c}{d}\right)^2-3bcdf^3\left(x+\frac{c}{d}\right)+3bc^2f^3-if^3\right)e^{\left(-ibd^2x^2-2ibcdx-ibc^2\right)}}{b^2d}}{8d^3}+\frac{\frac{\sqrt{2}\sqrt{\pi}\left(-2ibc^3f^3+3cf^3\right)\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}-\frac{2\left(bd^2f^3\left(x+\frac{c}{d}\right)^2-3bcdf^3\left(x+\frac{c}{d}\right)+3bc^2f^3+if^3\right)e^{\left(ibd^2x^2+2ibcdx+ibc^2\right)}}{b^2d}}{8d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="giac")

[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^3/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^3/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 3/4*(-I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^2/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 + 2)/(b*d))/d - 3/4*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^2/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + 2)/(b*d))/d - 3/8*(I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 - I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*I*(d*f^2*(I*x + I*c/d) - 2*I*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 + 1)/(b*d))/d^2 - 3/8*(-I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 + I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*I*(d*f^2*(I*x + I*c/d) - 2*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + 1)/(b*d))/d^2 + 1/8*(sqrt(2)*sqrt(pi)*(2*I*b*c^3*f^3 + 3*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)^2 - 3*b*c*d*f^3*(x + c/d) + 3*b*c^2*f^3 - I*f^3)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)/(b^2*d))/d^3 + 1/8*(sqrt(2)*sqrt(pi)*(-2*I*b*c^3*f^3 + 3*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)^2 - 3*b*c*d*f^3*(x + c/d) + 3*b*c^2*f^3 + I*f^3)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b^2*d))/d^3

Mupad [B]
time = 4.71, size = 231, normalized size = 1.04

$$\frac{f^3\sin\left(b\,{\left(c+d\,x\right)}^2\right)}{2\,b^2\,d^4}-\frac{\cos\left(b\,{\left(c+d\,x\right)}^2\right)\,\left(c^2\,f^3-3\,c\,d\,e\,f^2+3\,d^2\,e^2\,f\right)}{2\,b\,d^4}-\frac{f^3\,x^2\,\cos\left(b\,{\left(c+d\,x\right)}^2\right)}{2\,b\,d^2}+\frac{x\,\cos\left(b\,{\left(c+d\,x\right)}^2\right)\,\left(c\,f^3-3\,d\,e\,f^2\right)}{2\,b\,d^3}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi}}\right)\,\left(c^3\,f^3-3\,c^2\,d\,e\,f^2+3\,c\,d^2\,e^2\,f-d^3\,e^3\right)}{2\,\sqrt{b}\,d^4}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi}}\right)\,\left(3\,c\,f^3-3\,d\,e\,f^2\right)}{4\,b^{3/2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*(c + d*x)^2)*(e + f*x)^3,x)

[Out] (f^3*sin(b*(c + d*x)^2))/(2*b^2*d^4) - (cos(b*(c + d*x)^2)*(c^2*f^3 + 3*d^2*e^2*f - 3*c*d*e*f^2))/(2*b*d^4) - (f^3*x^2*cos(b*(c + d*x)^2))/(2*b*d^2) +

```
 (x*cos(b*(c + d*x)^2)*(c*f^3 - 3*d*e*f^2))/(2*b*d^3) - (2^(1/2)*pi^(1/2)*f
resnels((2^(1/2)*b^(1/2)*(c + d*x))/pi^(1/2))*(c^3*f^3 - d^3*e^3 + 3*c*d^2*
e^2*f - 3*c^2*d*e*f^2))/(2*b^(1/2)*d^4) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/
2)*b^(1/2)*(c + d*x))/pi^(1/2))*(3*c*f^3 - 3*d*e*f^2))/(4*b^(3/2)*d^4)
```

3.154 $\int (e+fx)^2 \sin\left(b(c+dx)^2\right)\,dx$

Optimal. Leaf size=150

$$-\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3}+\frac{f^2\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3}+\frac{(de-cf)^2\sqrt{\cdot}}{}$$

```
[Out] -f*(-c*f+d*e)*cos(b*(d*x+c)^2)/b/d^3-1/2*f^2*(d*x+c)*cos(b*(d*x+c)^2)/b/d^3
+1/4*f^2*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2
)/d^3+1/2*(-c*f+d*e)^2*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*P
i^(1/2)/d^3/b^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3514, 3432, 3460, 2718, 3466, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,f^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,(c+dx)\right)}{2b^{3/2}d^3}+\frac{\sqrt{\frac{\pi}{2}}\,(de-cf)^2 S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^3}-\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

```
[Out] -((f*(d*e - c*f)*Cos[b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[b*(c + d
*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])
/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)])/(Sqrt[b]*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(b(c+dx)^2\right)\,dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(bx^2\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(bx^2\right)+f^2x\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(bx^2\right)\right.}{d^3}$$

$$= -\frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3}+\frac{(de-cf)^2\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d^3}+$$

$$= -\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3}+\frac{f^2\sqrt{\frac{\pi}{2}}\,C\left(\right.}{}$$

Mathematica [A]
time = 0.41, size = 117, normalized size = 0.78

$$\frac{-2\sqrt{b}\,f(2de-cf+dfx)\cos\left(b(c+dx)^2\right)+f^2\sqrt{2\pi}\,C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)+2b(de-cf)^2\sqrt{2\pi}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{4b^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-2*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[b*(c + d*x)^2] + f^2*Sqrt[2*Pi]*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + 2*b*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(4*b^(3/2)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(127) = 254$.
time = 0.08, size = 291, normalized size = 1.94

method	result
default	$-\frac{f^2 x \cos\left(d^2x^2b+2cdxb+b\,c^2\right)}{2b\,d^2}-\frac{f^2 c\left(-\frac{\cos\left(d^2x^2b+2cdxb+b\,c^2\right)}{2b\,d^2}-\frac{c\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2d\sqrt{b\,d^2}}\right)}{d}+\frac{f^2\sqrt{2}\,\sqrt{\pi}\,\text{Fresne}}{4b\,d^2}$
risch	$\frac{ie^2\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}+\frac{if^2c^2\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^3\sqrt{-ib}}-\frac{f^2\sqrt{\pi}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-i}}\right)}{8b\,d^3\sqrt{-ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f^2/b/d^2*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^2*c/d*(-1/2/b/d^2*cos(b*d
^2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(
1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))+1/4*f^2/b/d^2*2^(1/2)*Pi^(1/2
)/(b*d^2)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-e*
f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-e*f*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/
2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*Pi^
(1/2)/(b*d^2)^(1/2)*e^2*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*
c*d))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.08, size = 564, normalized size = 3.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b*c*d*x*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2)) + 4*b*c^2*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)
 + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - sqrt(b*d^2*x^2 + 2*b*c*d*x +
```

```
b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d
*x - I*b*c^2)) - 1))*b*c^2 - (I - 1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b
*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x -
 I*b*c^2)))*f^2/(b^2*d^4*x + b^2*c*d^3) - 1/4*(2*d*x*(e^(I*b*d^2*x^2 + 2*I*
b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - sqrt(b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x
 + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*f*e/(b*d^3*x + b*c
*d^2) + 1/8*sqrt(2)*sqrt(pi)*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I
 - 1)*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))*e^2/(sqrt(b)*d)
```

Fricas [A]
time = 0.48, size = 165, normalized size = 1.10

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{bd^2}{\pi}}\,f^2\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)+2\,\sqrt{2}\,\left(\pi bc^2f^2-2\,\pi bcdfe+\pi bd^2e^2\right)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)-2\,\left(bd^2f^2x-bcdf^2+2\,bd^2fe\right)\cos\left(bd^2x^2+2\,bcdx+bc^2\right)}{4\,b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(b*d^2/pi)*f^2*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x
+ c)/d) + 2*sqrt(2)*(pi*b*c^2*f^2 - 2*pi*b*c*d*f*e + pi*b*d^2*e^2)*sqrt(b*d
^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*f^2*x - b
*c*d*f^2 + 2*b*d^2*f*e)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2\sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.59, size = 669, normalized size = 4.46

$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^2}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^2}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\,cf\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-ibd^2x^2-2ibcdx-ibc^2+1\right)}}{bd}}{2d}-\frac{\frac{i\sqrt{2}\sqrt{\pi}\,cf\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(ibd^2x^2+2ibcdx+ibc^2+1\right)}}{bd}}{2d}-\frac{\frac{i\sqrt{2}\sqrt{\pi}\left(2bc^2f^2-if^2\right)\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(df^2\left(-ix-\frac{ic}{d}\right)+2icf^2\right)e^{\left(-ibd^2x^2-2ibcdx-ibc^2\right)}}{bd}}{8d^2}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\left(2bc^2f^2+if^2\right)\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(df^2\left(-ix-\frac{ic}{d}\right)+2icf^2\right)e^{\left(ibd^2x^2+2ibcdx+ibc^2\right)}}{bd}}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^2/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt
(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x +
 c/d))*e^2/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/2*(-I*sqrt(2)*sqr
t(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d
))*e/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*
c*d*x - I*b*c^2 + 1)/(b*d))/d - 1/2*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2
)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e/(sqrt(b*d^2)*(-I*b*
d^2/sqrt(b^2*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + 1)/(b*
d))/d - 1/8*(I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 - I*f^2)*erf(-1/2*sqrt(2)*sqrt
(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b
^2*d^4) + 1)*b) + 2*I*(d*f^2*(-I*x - I*c/d) + 2*I*c*f^2)*e^(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2)/(b*d))/d^2 - 1/8*(-I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 +
 I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d)
)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*I*(d*f^2*(-I*x - I*c/d)
+ 2*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d^2
```

Mupad [B]
time = 0.19, size = 136, normalized size = 0.91

$$\frac{\cos\left(b\,{\left(c+d\,x\right)}^2\right)\,\left(c\,f^2-2\,d\,e\,f\right)}{2\,b\,d^3}-\frac{f^2\,x\,\cos\left(b\,{\left(c+d\,x\right)}^2\right)}{2\,b\,d^2}+\frac{\sqrt{2}\,f^2\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi}}\right)}{4\,b^{3/2}\,d^3}+\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi}}\right)\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2\right)}{2\,\sqrt{b}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] (cos(b*(c + d*x)^2)*(c*f^2 - 2*d*e*f))/(2*b*d^3) - (f^2*x*cos(b*(c + d*x)^2
))/(2*b*d^2) + (2^(1/2)*f^2*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2)*(c + d*x))/p
i^(1/2)))/(4*b^(3/2)*d^3) + (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2)*(c
+ d*x))/pi^(1/2))*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f))/(2*b^(1/2)*d^3)
```

3.155 $\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx$

Optimal. Leaf size=69

$$-\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d^2}$$

```
[Out] -1/2*f*cos(b*(d*x+c)^2)/b/d^2+1/2*(-c*f+d*e)*FresnelS((d*x+c)*b^(1/2)*2^(1/
2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^2/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3514, 3432, 3460, 2718}

$$\frac{\sqrt{\frac{\pi}{2}}\,(de-cf)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d^2}-\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

```
[Out] -1/2*(f*Cos[b*(c + d*x)^2])/(b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt
[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(bx^2\right)+fx\sin\left(bx^2\right)\right)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{f\text{Subst}\left(\int x\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,c
\end{aligned}$$

$$= \frac{(de-cf)\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^2}+\frac{f\text{Subst}\left(\int \sin(bx)\,dx,x,(c+dx)\right.}{2d^2}$$

$$= -\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^2}$$

Mathematica [**A**]
time = 0.13, size = 66, normalized size = 0.96

$$\frac{-f\cos\left(b(c+dx)^2\right)+\sqrt{b}\,(de-cf)\sqrt{2\pi}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-(f*Cos[b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)])/(2*b*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.
time = 0.04, size = 120, normalized size = 1.74

method	result

default	$-\frac{f\cos\left(d^2x^2b+2cdxb+b\,c^2\right)}{2b\,d^2}-\frac{fc\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2d\sqrt{b\,d^2}}+\frac{\sqrt{2}\,\sqrt{\pi}\,e\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2\sqrt{b\,d^2}}$
risch	$\frac{ie\sqrt{\pi}\,\operatorname{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}-\frac{ifc\sqrt{\pi}\,\operatorname{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^2\sqrt{-ib}}+\frac{ie\sqrt{\pi}\,\operatorname{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}-\frac{ifc}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1/2*f*c/d*2^(1/2)*Pi^(1/2)/(b*d
^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1
/2)*Pi^(1/2)/(b*d^2)^(1/2)*e*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2
*x+b*c*d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.76, size = 272, normalized size = 3.94

$$\frac{\sqrt{2}\,\sqrt{\pi}\left((i+1)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+(i-1)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)e}{8\,\sqrt{b}\,d}-\frac{\left(2\,dx\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)-\sqrt{bd^2x^2+2\,bcdx+bc^2}\left(-(i+1)\,\sqrt{2}\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{i\,bd^2x^2+2i\,bcdx+i\,bc^2}\right)-1\right)+(i-1)\,\sqrt{2}\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-i\,bd^2x^2-2i\,bcdx-i\,bc^2}\right)-1\right)\right)c+2\,c\left(e^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)}+e^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2\right)}\right)\right)f}{8\,\left(bd^3x+bcd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*sqrt(pi)*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)*er
f((I*b*d*x + I*b*c)/sqrt(-I*b)))*e/(sqrt(b)*d) - 1/8*(2*d*x*(e^(I*b*d^2*x^2
 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - sqr
t(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(
-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*c + 2*c*(e^(I*b*d^2*x^2 + 2*I*
b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*f/(b*d^3*x
+ b*c*d^2)
```

Fricas [A]
time = 0.42, size = 81, normalized size = 1.17

$$-\frac{df\cos\left(bd^2x^2+2\,bcdx+bc^2\right)+\sqrt{2}\,\left(\pi cf-\pi de\right)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)}{2\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(d*f*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2) + sqrt(2)*(pi*c*f - pi*d*e)*sq
rt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin \left(bc^2+2bcdx+bd^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 2.72, size = 367, normalized size = 5.32

$$-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-ibd^2x^2-2ibcdx-ibc^2\right)}}{bd}}{4d}-\frac{\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(ibd^2x^2+2ibcdx+ibc^2\right)}}{bd}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt(2
)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c
/d))*e/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4*(-I*sqrt(2)*sqrt(pi
)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(
sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x
- I*b*c^2)/(b*d))/d - 1/4*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d
^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2
*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d
```

Mupad [B]
time = 0.11, size = 58, normalized size = 0.84

$$-\frac{f\cos\left(b\,{\left(c+d\,x\right)}^2\right)}{2\,b\,d^2}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,\left(c+d\,x\right)}{\sqrt{\pi}}\right)\,\left(c\,f-d\,e\right)}{2\,\sqrt{b}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(c + d*x)^2)*(e + f*x),x)
```

```
[Out] - (f*cos(b*(c + d*x)^2))/(2*b*d^2) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^
(1/2)*(c + d*x))/pi^(1/2))*(c*f - d*e))/(2*b^(1/2)*d^2)
```

3.156 $\int \sin\left(b(c+dx)^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d}$$

```
[Out] 1/2*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{3432\}$

$$\frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(b(c+dx)^2\right)\,dx = \frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.08

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(b\,d^2x+bcd)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)}{2\sqrt{b\,d^2}}$	42
risch	$\frac{i\sqrt{\pi}\,\operatorname{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}+\frac{i\sqrt{\pi}\,\operatorname{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*
(b*d^2*x+b*c*d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 53, normalized size = 1.36

$$\frac{\sqrt{2}\,\sqrt{\pi}\left((i+1)\,\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+(i-1)\,\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*sqrt(pi)*((I + 1)*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I - 1)*er
f((I*b*d*x + I*b*c)/sqrt(-I*b)))/(sqrt(b)*d)
```

Fricas [A]
time = 0.39, size = 45, normalized size = 1.15

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{bd^2}{\pi}}\ \mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/
d)/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin \left(b(c+dx)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)**2),x)
```

```
[Out] Integral(sin(b*(c + d*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.28, size = 143, normalized size = 3.67

$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt(2)*
sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d
))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1))
```

Mupad [B]
time = 0.08, size = 41, normalized size = 1.05

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,b\,d\,\sqrt{\frac{1}{b\,d^2}}\,(c+d\,x)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(c + d*x)^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b*d*(1/(b*d^2))^(1/2)*(c + d*x))/pi^(1/
2))*(1/(b*d^2))^(1/2))/2
```

3.157 $\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sin\left(b(c+dx)^2\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(b*(d*x+c)^2)/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[b*(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Mathematica [A]
time = 3.42, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[b*(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Integrate[Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(b(dx+c)^2\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(d*x+c)^2)/(f*x+e),x)
[Out] int(sin(b*(d*x+c)^2)/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bc^2 + 2bcdx + bd^2x^2\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)**2)/(f*x+e),x)
[Out] Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sin\left(b\,{\left(c+d\,x\right)}^2\right)}{e+f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(c + d*x)^2)/(e + f*x),x)
[Out] int(sin(b*(c + d*x)^2)/(e + f*x), x)
```

3.158 $\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} dx = \int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} dx$$

Mathematica [A]
time = 5.94, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(b(dx+c)^2\right)}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

```
[Out] int(sin(b*(d*x+c)^2)/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bc^2 + 2bcdx + bd^2x^2\right)}{\left(e + fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sin\left(b\,{\left(c+d\,x\right)}^2\right)}{{\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*(c + d*x)^2)/(e + f*x)^2,x)
```

```
[Out] int(sin(b*(c + d*x)^2)/(e + f*x)^2, x)
```

3.159 $\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=337

$$\frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4}+\frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4}-\frac{3bf(de-cf)^2\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^4}-\frac{\sqrt{b}\,(de-cf}{}$$

```
[Out] -3/2*b*f*(-c*f+d*e)^2*Ci(b/(d*x+c)^2)/d^4+2*b*f^2*(-c*f+d*e)*(d*x+c)*cos(b/
(d*x+c)^2)/d^4+1/4*b*f^3*(d*x+c)^2*cos(b/(d*x+c)^2)/d^4+1/4*b^2*f^3*Si(b/(d
*x+c)^2)/d^4+(-c*f+d*e)^3*(d*x+c)*sin(b/(d*x+c)^2)/d^4+3/2*f*(-c*f+d*e)^2*(
d*x+c)^2*sin(b/(d*x+c)^2)/d^4+f^2*(-c*f+d*e)*(d*x+c)^3*sin(b/(d*x+c)^2)/d^4
+1/4*f^3*(d*x+c)^4*sin(b/(d*x+c)^2)/d^4+2*b^(3/2)*f^2*(-c*f+d*e)*FresnelS(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*2^(1/2)*Pi^(1/2)/d^4-(-c*f+d*e)^3*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^4
```

Rubi [A]
time = 0.29, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3514, 3440, 3468, 3433, 3460, 3378, 3383, 3490, 3469, 3432, 3380}

$$\frac{2\sqrt{2\pi}\,b^{3/2}f^2(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4}+\frac{b^2f^3\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}-\frac{3bf(de-cf)^2\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4}+\frac{f^2(c+dx)^3(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4}+\frac{2bf^2(c+dx)(de-cf)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4}-\frac{\sqrt{2\pi}\sqrt{b}\,(de-cf)^3\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^4}+\frac{3f(c+dx)^2(de-cf)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4}+\frac{(c+dx)(de-cf)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4}+\frac{f^3(c+dx)^4\sin\left(\frac{b}{(c+dx)^2}\right)}{4d^4}+\frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*f^2*(d*e - c*f)*(c + d*x)*Cos[b/(c + d*x)^2])/d^4 + (b*f^3*(c + d*x)^2
*Cos[b/(c + d*x)^2])/(4*d^4) - (3*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)
^2])/(2*d^4) - (Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)])/d^4 + (2*b^(3/2)*f^2*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi])/(c + d*x)])/d^4 + ((d*e - c*f)^3*(c + d*x)*Sin[b/(c + d*x)^2
])/d^4 + (3*f*(d*e - c*f)^2*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^4) + (f^2*
(d*e - c*f)*(c + d*x)^3*Sin[b/(c + d*x)^2])/d^4 + (f^3*(c + d*x)^4*Sin[b/(c
 + d*x)^2])/(4*d^4) + (b^2*f^3*SinIntegral[b/(c + d*x)^2])/(4*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
```

```
LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3\sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \frac{\text{Subst}\left(\int\left(d^3e^3\left(1-\frac{cf\left(3d^2e^2-3cdef+c^2f^2\right)}{d^3e^3}\right)\sin\left(\frac{b}{x^2}\right)+3d^2e^2f\left(1+\frac{cf(-2de+c}{d^2e^2}\right.\right.\right.}{}$$

$$=\frac{f^3\text{Subst}\left(\int x^3\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^4}+\frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x^2\sin\left(\right.\right.}{d^4}$$

$$=-\frac{f^3\text{Subst}\left(\int\frac{\sin(bx)}{x^3}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^4}-\frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int\frac{\sin(bx^2)}{x^4}\,dx,\right.}{d^4}$$

$$=\frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4}+\frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4}$$

$$=\frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4}+\frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4}-\frac{3bf}{}$$

$$=\frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4}+\frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4}-\frac{3bf}{}$$

Mathematica [A]
time = 0.51, size = 440, normalized size = 1.31

$$\frac{8bcdef^2\cos\left(\frac{b}{(c+dx)^2}\right)-7bc^2f^3\cos\left(\frac{b}{(c+dx)^2}\right)+8bd^2ef^2x\cos\left(\frac{b}{(c+dx)^2}\right)-6bcdf^3x\cos\left(\frac{b}{(c+dx)^2}\right)+bd^2f^3x^2\cos\left(\frac{b}{(c+dx)^2}\right)-6bf(de-cf)^2\text{Ci}\left(\frac{b}{(c+dx)^2}\right)-4\sqrt{b}\,(de-cf)^3\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+8b^{3/2}def^2\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)-8b^{3/2}cf^3\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+4cd^3e^3\sin\left(\frac{b}{(c+dx)^2}\right)-6c^2d^2e^2f\sin\left(\frac{b}{(c+dx)^2}\right)+4c^3def^2\sin\left(\frac{b}{(c+dx)^2}\right)-c^4f^3\sin\left(\frac{b}{(c+dx)^2}\right)+4d^4e^3x\sin\left(\frac{b}{(c+dx)^2}\right)+6d^4e^2fx^2\sin\left(\frac{b}{(c+dx)^2}\right)+4d^4ef^2x^3\sin\left(\frac{b}{(c+dx)^2}\right)+d^4f^3x^4\sin\left(\frac{b}{(c+dx)^2}\right)+b^2f^3\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

```
[Out] (8*b*c*d*e*f^2*Cos[b/(c + d*x)^2] - 7*b*c^2*f^3*Cos[b/(c + d*x)^2] + 8*b*d^
2*e*f^2*x*Cos[b/(c + d*x)^2] - 6*b*c*d*f^3*x*Cos[b/(c + d*x)^2] + b*d^2*f^3
*x^2*Cos[b/(c + d*x)^2] - 6*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)^2] -
4*Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]
 + 8*b^(3/2)*d*e*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] -
8*b^(3/2)*c*f^3*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 4*c*d
^3*e^3*Sin[b/(c + d*x)^2] - 6*c^2*d^2*e^2*f*Sin[b/(c + d*x)^2] + 4*c^3*d*e*
f^2*Sin[b/(c + d*x)^2] - c^4*f^3*Sin[b/(c + d*x)^2] + 4*d^4*e^3*x*Sin[b/(c
+ d*x)^2] + 6*d^4*e^2*f*x^2*Sin[b/(c + d*x)^2] + 4*d^4*e*f^2*x^3*Sin[b/(c +
 d*x)^2] + d^4*f^3*x^4*Sin[b/(c + d*x)^2] + b^2*f^3*SinIntegral[b/(c + d*x)
^2])/(4*d^4)
```

Maple [**A**]
time = 0.13, size = 365, normalized size = 1.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-(c^3*f^3-3*c^2*d*e*f^2+3*c*d^2*e^2*f-d^3*e^3)*(d*x+c)*sin(b/(d*x+c)
^2)+(c^3*f^3-3*c^2*d*e*f^2+3*c*d^2*e^2*f-d^3*e^3)*b^(1/2)*2^(1/2)*Pi^(1/2)*
FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-1/2*(-3*c^2*f^3+6*c*d*e*f^2-3*d^
2*e^2*f)*(d*x+c)^2*sin(b/(d*x+c)^2)+1/2*(-3*c^2*f^3+6*c*d*e*f^2-3*d^2*e^2*f
)*b*Ci(b/(d*x+c)^2)-1/3*(3*c*f^3-3*d*e*f^2)*(d*x+c)^3*sin(b/(d*x+c)^2)+2/3*
(3*c*f^3-3*d*e*f^2)*b*(-(d*x+c)*cos(b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*F
resnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/4*f^3*(d*x+c)^4*sin(b/(d*x+c)^
2)-1/2*f^3*b*(-1/2*(d*x+c)^2*cos(b/(d*x+c)^2)-1/2*b*Si(b/(d*x+c)^2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*d^3*integrate(-1/4*((3*b*c^4*f^3 - 4*b*c^3*d*f^2*e + 6*(b*c^2*d^2*f
^3 - 2*b*c*d^3*f^2*e + b*d^4*f*e^2)*x^2 + 4*(2*b*c^3*d*f^3 - 3*b*c^2*d^2*f^
2*e + b*d^4*e^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*d^2*f^3*x^2 - 2
*(3*b^2*c*d*f^3 - 4*b^2*d^2*f^2*e)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/(d^
6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3), x) + 4*d^3*integrate(-1/4*((3
*b*c^4*f^3 - 4*b*c^3*d*f^2*e + 6*(b*c^2*d^2*f^3 - 2*b*c*d^3*f^2*e + b*d^4*f
*e^2)*x^2 + 4*(2*b*c^3*d*f^3 - 3*b*c^2*d^2*f^2*e + b*d^4*e^3)*x)*cos(b/(d^2
*x^2 + 2*c*d*x + c^2)) - (b^2*d^2*f^3*x^2 - 2*(3*b^2*c*d*f^3 - 4*b^2*d^2*f^
```

```
2*e)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d
^4*x + c^3*d^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^6*x^3 + 3*c*d^5*x^2
 + 3*c^2*d^4*x + c^3*d^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) - (b*d*f^
3*x^2 - 2*(3*b*c*f^3 - 4*b*d*f^2*e)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - (
d^3*f^3*x^4 + 4*d^3*f^2*x^3*e + 6*d^3*f*x^2*e^2 + 4*d^3*x*e^3)*sin(b/(d^2*x
^2 + 2*c*d*x + c^2)))/d^3
```

Fricas [A]
time = 0.42, size = 448, normalized size = 1.33

$$\frac{b^2 f^3 \operatorname{Si}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right) + 4\,\sqrt{2}\,(\pi c^3 d f^3 - 3\,\pi c^2 d^2 f^2 e + 3\,\pi c d^3 f e^2 - \pi d^4 e^3)\sqrt{\frac{b}{\pi d^2}}\,\operatorname{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right) - 8\,\sqrt{2}\,(\pi b c d f^3 - \pi b d^2 f^2 e)\sqrt{\frac{b}{\pi d^2}}\,\operatorname{S}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right) + (b d^2 f^3 x^2 - 6\,b c d f^3 x - 7\,b c^2 f^3 + 8\,(b d^2 f^2 x + b c d f^2)e)\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right) - 3\,(b c^2 f^3 - 2\,b c d f^2 e + b d^2 f e^2)\operatorname{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right) - 3\,(b c^2 f^3 - 2\,b c d f^2 e + b d^2 f e^2)\operatorname{Ci}\left(-\frac{b}{d^2x^2+2\,cdx+c^2}\right) + (d^4 f^3 x^4 - c^4 f^3 + 4\,(d^4 x + c d^3)e^3 + 6\,(d^4 f x^2 - c^2 d^2 f)e^2 + 4\,(d^4 f^2 x^3 + c^3 d f^2)e)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*f^3*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + 4*sqrt(2)*(pi*c^3*
d*f^3 - 3*pi*c^2*d^2*f^2*e + 3*pi*c*d^3*f*e^2 - pi*d^4*e^3)*sqrt(b/(pi*d^2)
)*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) - 8*sqrt(2)*(pi*b*c*d*f
^3 - pi*b*d^2*f^2*e)*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2)
)/(d*x + c)) + (b*d^2*f^3*x^2 - 6*b*c*d*f^3*x - 7*b*c^2*f^3 + 8*(b*d^2*f^2*
x + b*c*d*f^2)*e)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*c^2*f^3 - 2*b*c*d
*f^2*e + b*d^2*f*e^2)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*c^2*
f^3 - 2*b*c*d*f^2*e + b*d^2*f*e^2)*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2
)) + (d^4*f^3*x^4 - c^4*f^3 + 4*(d^4*x + c*d^3)*e^3 + 6*(d^4*f*x^2 - c^2*d^
2*f)*e^2 + 4*(d^4*f^2*x^3 + c^3*d*f^2)*e)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))
/d^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(b/(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(c + d*x)^2)*(e + f*x)^3,x)
[Out] int(sin(b/(c + d*x)^2)*(e + f*x)^3, x)
```

3.160 $\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=233

$$\frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3}-\frac{bf(de-cf)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{b}\,(de-cf)^2\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}+\frac{2b^{3/2}f^2\sqrt{2\pi}\,S}{3d}$$

```
[Out] -b*f*(-c*f+d*e)*Ci(b/(d*x+c)^2)/d^3+2/3*b*f^2*(d*x+c)*cos(b/(d*x+c)^2)/d^3+
(-c*f+d*e)^2*(d*x+c)*sin(b/(d*x+c)^2)/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(b/(d*x
+c)^2)/d^3+1/3*f^2*(d*x+c)^3*sin(b/(d*x+c)^2)/d^3+2/3*b^(3/2)*f^2*FresnelS(
b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*2^(1/2)*Pi^(1/2)/d^3-(-c*f+d*e)^2*Fresnel
C(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3
```

Rubi [A]
time = 0.18, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3514, 3440, 3468, 3433, 3460, 3378, 3383, 3490, 3469, 3432}

$$\frac{2\sqrt{2\pi}\,b^{3/2}f^2S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\,\sqrt{b}\,(de-cf)^2\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^3}+\frac{f(c+dx)^2(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{(c+dx)(de-cf)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3}+\frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*f^2*(c + d*x)*Cos[b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*CosIntegr
al[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[b/(c + d*x)
^2])/d^3 + (f*(d*e - c*f)*(c + d*x)^2*Sin[b/(c + d*x)^2])/d^3 + (f^2*(c + d
*x)^3*Sin[b/(c + d*x)^2])/(3*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right)\,dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(\frac{b}{x^2}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(\frac{b}{x^2}\right)+f^2 \ldots\right.\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right) \ldots\right.}{d^3}$$

$$= -\frac{f^2\text{Subst}\left(\int \frac{\sin(bx^2)}{x^4}\,dx,x,\frac{1}{c+dx}\right)}{d^3} - \frac{(f(de-cf))\text{Subst}\left(\int \frac{\sin(bx)}{x^2}\,dx,x,\frac{\ldots}{(c\ldots}\right.}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \ldots$$

$$= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}\,(de-cf)^2 \ldots}{\ldots}$$

$$= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}\,(de-cf)^2 \ldots}{\ldots}$$

Mathematica [A]
time = 0.30, size = 265, normalized size = 1.14

$$\frac{2bcf^2\cos\left(\frac{b}{(c+dx)^2}\right)+2bdf^2x\cos\left(\frac{b}{(c+dx)^2}\right)+3bf(-de+cf)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)-3\sqrt{b}\,(de-cf)^2\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+2b^{3/2}f^2\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+3cd^2e^2\sin\left(\frac{b}{(c+dx)^2}\right)-3c^2def\sin\left(\frac{b}{(c+dx)^2}\right)+c^3f^2\sin\left(\frac{b}{(c+dx)^2}\right)+3d^3e^2x\sin\left(\frac{b}{(c+dx)^2}\right)+3d^3efx^2\sin\left(\frac{b}{(c+dx)^2}\right)+d^3f^2x^3\sin\left(\frac{b}{(c+dx)^2}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*c*f^2*Cos[b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[b/(c + d*x)^2] + 3*b*f*(-(d
*e) + c*f)*CosIntegral[b/(c + d*x)^2] - 3*Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*
```

```
FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*b^(3/2)*f^2*Sqrt[2*Pi]*Fresnel
S[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*c*d^2*e^2*Sin[b/(c + d*x)^2] - 3*c^2*
d*e*f*Sin[b/(c + d*x)^2] + c^3*f^2*Sin[b/(c + d*x)^2] + 3*d^3*e^2*x*Sin[b/(
c + d*x)^2] + 3*d^3*e*f*x^2*Sin[b/(c + d*x)^2] + d^3*f^2*x^3*Sin[b/(c + d*x
)^2])/(3*d^3)
```

Maple [A]
time = 0.09, size = 225, normalized size = 0.97

method	result
derivativedivides	$-\frac{-(c^2f^2-2cdef+d^2e^2)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(c^2f^2-2cdef+d^2e^2)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\frac{(-2c}{}}{}$
default	$-\frac{-(c^2f^2-2cdef+d^2e^2)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(c^2f^2-2cdef+d^2e^2)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\frac{(-2c}{}}{}$
risch	$-\frac{e^2b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}-\frac{if^2b^2\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{3d^3\sqrt{-ib}}-\frac{f^2c^2b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d^3\sqrt{-ib}}-\frac{f^2cb\,\operatorname{expIntegral}(1,-}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*(-(c^2*f^2-2*c*d*e*f+d^2*e^2)*(d*x+c)*sin(b/(d*x+c)^2)+(c^2*f^2-2*c*
d*e*f+d^2*e^2)*b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(
d*x+c))-1/2*(-2*c*f^2+2*d*e*f)*(d*x+c)^2*sin(b/(d*x+c)^2)+1/2*(-2*c*f^2+2*d
*e*f)*b*Ci(b/(d*x+c)^2)-1/3*f^2*(d*x+c)^3*sin(b/(d*x+c)^2)+2/3*f^2*b*(-(d*x
+c)*cos(b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(
1/2)/(d*x+c))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*b*f^2*x*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*d^2*integrate(1/3*(2*b^
2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 + 3*(b*c*d^2*f^2 -
b*d^3*f*e)*x^2 + 3*(b*c^2*d*f^2 - b*d^3*e^2)*x)*cos(b/(d^2*x^2 + 2*c*d*x +
c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2), x) - 3*d^2*integrat
e(1/3*(2*b^2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 + 3*(b*c
*d^2*f^2 - b*d^3*f*e)*x^2 + 3*(b*c^2*d*f^2 - b*d^3*e^2)*x)*cos(b/(d^2*x^2 +
```

```
2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*cos(b/(d
^2*x^2 + 2*c*d*x + c^2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2
)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d^2*f*x^2*e +
3*d^2*x*e^2)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Fricas [A]
time = 0.40, size = 303, normalized size = 1.30

$$\frac{4\sqrt{2}\,\pi bdf^2\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-6\sqrt{2}\,(\pi c^2df^2-2\,\pi cd^2fe+\pi d^3e^2)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+4\,(bdf^2x+bcf^2)\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+3\,(bcf^2-bdfe)\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+3\,(bcf^2-bdfe)\,\mathrm{Ci}\left(-\frac{b}{d^2x^2+2\,cdx+c^2}\right)+2\,(d^3f^2x^3+c^3f^2+3\,(d^3x+cd^2)e^2+3\,(d^3fx^2-c^2df)e)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(2)*pi*b*d*f^2*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi
*d^2))/(d*x + c)) - 6*sqrt(2)*(pi*c^2*d*f^2 - 2*pi*c*d^2*f*e + pi*d^3*e^2)*
sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 4*(b*d
*f^2*x + b*c*f^2)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) + 3*(b*c*f^2 - b*d*f*e)*
cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + 3*(b*c*f^2 - b*d*f*e)*cos_integ
ral(-b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d^3*f^2*x^3 + c^3*f^2 + 3*(d^3*x + c
*d^2)*e^2 + 3*(d^3*f*x^2 - c^2*d*f)*e)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^
3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(b/(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(\frac{b}{{\left(c+d\,x\right)}^2}\right)\,{\left(e+f\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] int(sin(b/(c + d*x)^2)*(e + f*x)^2, x)
```

3.161 $\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$

Optimal. Leaf size=120

$$-\frac{bf\,\mathrm{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{b}\,(de-cf)\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}+\frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

```
[Out] -1/2*b*f*Ci(b/(d*x+c)^2)/d^2+(-c*f+d*e)*(d*x+c)*sin(b/(d*x+c)^2)/d^2+1/2*f*
(d*x+c)^2*sin(b/(d*x+c)^2)/d^2-(-c*f+d*e)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)
/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^2
```

Rubi [A]

time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3514, 3440, 3468, 3433, 3460, 3378, 3383}

$$-\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{2\pi}\sqrt{b}\,(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^2}+\frac{(c+dx)(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

```
[Out] -1/2*(b*f*CosIntegral[b/(c + d*x)^2])/d^2 - (Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + ((d*e - c*f)*(c + d*x)*Sin
[b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(\frac{b}{x^2}\right)+fx\sin\left(\frac{b}{x^2}\right)\right)dx,x,c+dx\right)}{d^2}$$
$$= \frac{f\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^2} + \frac{(de-cf)\text{Subst}\left(\int \sin\left(\frac{b}{x^2}\right)dx,x,c\right.}{d^2}$$
$$= -\frac{f\text{Subst}\left(\int \frac{\sin(bx)}{x^2}dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2} - \frac{(de-cf)\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}dx,x,\frac{1}{c+dx}\right.}{d^2}$$
$$= \frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{(bf)\text{Subst}\left(\right.}{}$$
$$= -\frac{bf\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{\sqrt{b}(de-cf)\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{(de-cf)(c+}{}$$

Mathematica [A]
time = 0.20, size = 95, normalized size = 0.79

$$-\frac{bf\text{Ci}\left(\frac{b}{(c+dx)^2}\right)+2\sqrt{b}(de-cf)\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+(c+dx)(-2de+cf-dfx)\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

```
[Out] -1/2*(b*f*CosIntegral[b/(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Fre
snelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + (c + d*x)*(-2*d*e + c*f - d*f*x)*Si
n[b/(c + d*x)^2])/d^2
```

Maple [A]
time = 0.05, size = 101, normalized size = 0.84

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\sqrt{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)+\frac{f(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{fb\cos}{}}{d^2}$
default	$\frac{-(cf-de)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\sqrt{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}(dx+c)}\right)+\frac{f(dx+c)^2\sin\left(\frac{b}{(dx+c)^2}\right)}{2}-\frac{fb\cos}{}}{d^2}$

risch	$-\frac{eb\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}+\frac{fb\,\mathrm{expIntegral}\left(1,-\frac{ib}{(dx+c)^2}\right)}{4d^2}+\frac{cfb\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d^2\sqrt{-ib}}-\frac{eb\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)}{2d\sqrt{ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-(c*f-d*e)*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(1/
2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+1/2*f*(d*x+c)^2*sin(b/(d*x+c)
^2)-1/2*f*b*Ci(b/(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(1/2*(b*d*f
*x^2 + 2*b*d*x*e)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*x*e)*cos(b/(d^2*x^
2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x
^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d
^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Fricas [A]
time = 0.37, size = 158, normalized size = 1.32

$$-\frac{bf\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+bf\,\mathrm{Ci}\left(-\frac{b}{d^2x^2+2\,cdx+c^2}\right)-4\,\sqrt{2}\,(\pi cdf-\pi d^2e)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-2\,(d^2fx^2-c^2f+2\,(d^2x+cd)e)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(b*f*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + b*f*cos_integral(-b/(
d^2*x^2 + 2*c*d*x + c^2)) - 4*sqrt(2)*(pi*c*d*f - pi*d^2*e)*sqrt(b/(pi*d^2)
)*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) - 2*(d^2*f*x^2 - c^2*f
+ 2*(d^2*x + c*d)*e)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(b/(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(c + d*x)^2)*(e + f*x),x)
```

```
[Out] int(sin(b/(c + d*x)^2)*(e + f*x), x)
```

3.162 $\int \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\,\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}+\frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

[Out] (d*x+c)*sin(b/(d*x+c)^2)/d-FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3440, 3468, 3433}

$$\frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}-\frac{\sqrt{2\pi}\,\sqrt{b}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[b/(c + d*x)^2],x]

[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c + d*x)*Sin[b/(c + d*x)^2])/d

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin\left(\frac{b}{(c+dx)^2}\right) dx &= -\frac{\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}\\ &= \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b)\text{Subst}\left(\int \cos(bx^2)\ dx, x, \frac{1}{c+dx}\right)}{d}\\ &= -\frac{\sqrt{b}\sqrt{2\pi}\, C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{2\pi}\, C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b/(c + d*x)^2],x]
[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
d*x)*Sin[b/(c + d*x)^2])/d
```

Maple [A]
time = 0.04, size = 52, normalized size = 0.87

method	result	size
derivativedivides	$-\frac{-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\ \text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)}{d}$	52
default	$-\frac{-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\ \text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)}{d}$	52

risch	$-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)}{2d\sqrt{ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}-\frac{(-dx-c)\sin\left(\frac{b}{(dx+c)^2}\right)}{d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-(d*x+c)*sin(b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3
*c^2*d*x + c^3), x) + b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/((d^
3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 +
 (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
^2), x) + x*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
```

Fricas [A]
time = 0.37, size = 73, normalized size = 1.22

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d^2}}\;\mathrm{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-(dx+c)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x
 + c)) - (d*x + c)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(\frac{b}{(c+dx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)**2),x)
```

```
[Out] Integral(sin(b/(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2), x)
```

Mupad [B]
time = 5.20, size = 52, normalized size = 0.87

$$\frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)\,(c+d\,x)}{d}-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{\pi}\,(c+d\,x)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(c + d*x)^2),x)
```

```
[Out] (sin(b/(c + d*x)^2)*(c + d*x))/d - (2^(1/2)*b^(1/2)*pi^(1/2)*fresnelc((2^(1
/2)*b^(1/2))/(pi^(1/2)*(c + d*x))))/d
```

3.163 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

[Out] Unintegrable(sin(b/(d*x+c)^2)/(f*x+e),x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx$$

Verification is not applicable to the result.

[In] Int[Sin[b/(c + d*x)^2]/(e + f*x),x]

[Out] Defer[Int][Sin[b/(c + d*x)^2]/(e + f*x), x]

Rubi steps

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx$$

Mathematica [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx} \, dx$$

Verification is not applicable to the result.

[In] Integrate[Sin[b/(c + d*x)^2]/(e + f*x),x]

[Out] Integrate[Sin[b/(c + d*x)^2]/(e + f*x), x]

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

```
[Out] int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)**2)/(f*x+e),x)
```

```
[Out] Integral(sin(b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(c + d*x)^2)/(e + f*x),x)
```

```
[Out] int(sin(b/(c + d*x)^2)/(e + f*x), x)
```

3.164 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
[Out] Defer[Int][Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Mathematica [A]
time = 14.51, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
[Out] Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

```
[Out] int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sin\left(\frac{b}{(c+d\,x)^2}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(c + d*x)^2)/(e + f*x)^2,x)
[Out] int(sin(b/(c + d*x)^2)/(e + f*x)^2, x)
```

3.165 $\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal. Leaf size=341

$$-\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4}-\frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\cos\left(a+b(c+\right.}{2bd^4}$$

```
[Out] -3/2*f*(-c*f+d*e)^2*cos(a+b*(d*x+c)^2)/b/d^4-3/2*f^2*(-c*f+d*e)*(d*x+c)*cos
(a+b*(d*x+c)^2)/b/d^4-1/2*f^3*(d*x+c)^2*cos(a+b*(d*x+c)^2)/b/d^4+1/2*f^3*si
n(a+b*(d*x+c)^2)/b^2/d^4+3/4*f^2*(-c*f+d*e)*cos(a)*FresnelC((d*x+c)*b^(1/2)
*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d^4-3/4*f^2*(-c*f+d*e)*FresnelS
((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)/d^4+1/2*
(-c*f+d*e)^3*cos(a)*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(
1/2)/d^4/b^(1/2)+1/2*(-c*f+d*e)^3*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2)
)*sin(a)*2^(1/2)*Pi^(1/2)/d^4/b^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3514, 3434, 3433, 3432, 3460, 2718, 3466, 3435, 3377, 2717}

$$\frac{3\sqrt{\frac{\pi}{2}}f^2\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,(c+dx)\right)}{2b^{3/2}d^4}-\frac{3\sqrt{\frac{\pi}{2}}f^2\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(a+b(c+dx)^2\right)}{2b^2d^4}-\frac{3f^2(c+dx)(de-cf)\cos\left(a+b(c+dx)^2\right)}{2bd^4}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^3\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,(c+dx)\right)}{\sqrt{b}\,d^4}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^3S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^4}-\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-3*f*(d*e - c*f)^2*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*
(c + d*x)*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[a + b*(c
 + d*x)^2])/(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*Cos[a
]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^4) + ((d*e - c*f)^3*Sq
rt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^4) - (3*
f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(
2*b^(3/2)*d^4) + (f^3*Sin[a + b*(c + d*x)^2])/(2*b^2*d^4)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
```

```
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx = \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(a+bx^2\right)+3d^2e^2f\left(1+\frac{c}{\cdots}\right.\right.\right.}{\cdots}$$

$$= \frac{f^3\text{Subst}\left(\int x^3\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^4} + \frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x\cdots\right.}{\cdots}$$

$$= -\frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4} + \frac{f^3\text{Subst}\left(\int x\sin(a+bx\cdots\right.}{2d^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(a+b\cdots\right.}{2bd^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(a+b\cdots\right.}{2bd^4}$$

Mathematica [A]
time = 1.84, size = 218, normalized size = 0.64

$$\frac{-4bf\left(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2)\right)\cos\left(a+b(c+dx)^2\right)+2\sqrt{b}\,(de-cf)\sqrt{2\pi}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)\left(2b(de-cf)^2\cos(a)-3f^2\sin(a)\right)+2\sqrt{b}\,(de-cf)\sqrt{2\pi}\,C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right)\left(3f^2\cos(a)+2b(de-cf)^2\sin(a)\right)+4f^3\sin\left(a+b(c+dx)^2\right)}{8b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos
[a + b*(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqr
t[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - 3*f^2*Sin[a]) + 2*Sqrt[b]*(d
*e - c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(3*f^2*Cos[a] +
 2*b*(d*e - c*f)^2*Sin[a]) + 4*f^3*Sin[a + b*(c + d*x)^2])/(8*b^2*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(291) = 582$.
time = 0.12, size = 1248, normalized size = 3.66

method	result

risch	$\frac{ie^3\sqrt{\pi}\,e^{ia}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}-\frac{if^3c^3\sqrt{\pi}\,e^{ia}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^4\sqrt{-ib}}+\frac{3f^3c\sqrt{\pi}\,e^{ia}\,\mathrm{erf}\left(-d\sqrt{-ib}\right.}{8b\,d^4\sqrt{-ib}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f^3/b/d^2*x^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-f^3*c/d*(-1/2/b/d^2*x*c
os(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-c/d*(-1/2/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c
^2+a)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2
+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b
^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*
(b*d^2*x+b*c*d))))+1/4/b/d^2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d
^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x
+b*c*d))+sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)
/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))))+f^3/b/d^2*(1/2/b/d^2*sin(b*d^2*x^2+2*b*c*
d*x+b*c^2+a)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2
*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))
+sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)
^(1/2)*(b*d^2*x+b*c*d))))-3/2*f^2*e/b/d^2*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a
)-3*f^2*e*c/d*(-1/2/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-1/2*c/d*2^(1/2)*
Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2
^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-b*d^2*(b*c^
2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))))+3/4
*f^2*e/b/d^2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+
a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+sin((b^
2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(
b*d^2*x+b*c*d)))-3/2*e^2*f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-3/2*e^2*f
*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^
2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^
2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+
b*c*d)))+1/2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*e^3*(cos((b^2*c^2*d^2-b*d^2*(b*
c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin
((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/
2)*(b*d^2*x+b*c*d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 2.31, size = 1823, normalized size = 5.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(6*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b
*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I
*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^3 + 2*(3*((e^(I*b*d^
2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*
cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 - (I*gamma(2, I*b*d^2*x^2 + 2*I*b*c*d
*x + I*b*c^2) - I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) -
(gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(2, -I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2))*sin(a))*d*x - 2*((I*gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2) - I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (
gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(2, -I*b*d^2*x^2 - 2*I
*b*c*d*x - I*b*c^2))*sin(a))*c - (((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqr
t(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I - 1)*sqrt(2)*sq
rt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (I + 1)*sqrt(
2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*
b*c^3 + 3*((-(I - 1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2
) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a
) + (-(I + 1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I
- 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c)*s
qrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2))*f^3/(b^2*d^5*x + b^2*c*d^4) + 3/8*(4*((
e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I
*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d
^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d*x + 4*((e^(I*b*d^2*x^2 + 2*I
*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-
I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x
 - I*b*c^2))*sin(a))*b*c^2 - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(((-(I + 1
)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (
I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
1))*cos(a) + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*
c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2 + (-(I - 1)*sqrt(2)*gamma(3/2, I*b*d^
2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2))*cos(a) + (-(I + 1)*sqrt(2)*gamma(3/2, I*b*d^2*x^2 +
 2*I*b*c*d*x + I*b*c^2) + (I - 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c
*d*x - I*b*c^2))*sin(a)))*f^2*e/(b^2*d^4*x + b^2*c*d^3) - 3/8*(2*((e^(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2))*sin(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
 - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2)) - 1))*cos(a) + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I
*b*c*d*x + I*b*c^2)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d
*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I
```

```
*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2))*sin(a))*c)*f*e^2/(b*d^3*x + b*c*d^2) - 1/8*sqrt(2)*sqrt(pi)*((-(I +
1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (-(I - 1)*co
s(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))*e^3/(sqrt(b)*d)
```

Fricas [A]
time = 0.38, size = 338, normalized size = 0.99

$$\frac{2\,df^3\sin\left(bd^2x^2+2\,bcdx+bc^2+a\right)-\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\left(3\left(\pi cf^3-\pi df^2e\right)\cos\left(a\right)+2\left(\pi bc^3f^3-3\,\pi bc^2df^2e+3\,\pi bcd^2fe^2-\pi bd^3e^3\right)\sin\left(a\right)\right)\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)-\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\left(2\left(\pi bc^3f^3-3\,\pi bc^2df^2e+3\,\pi bcd^2fe^2-\pi bd^3e^3\right)\cos\left(a\right)-3\left(\pi cf^3-\pi df^2e\right)\sin\left(a\right)\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)-2\left(bd^3f^3x^2-bcd^2f^3x+bc^2df^3+3\,bd^3fe^2+3\left(bd^3f^2x-bcd^2f^2\right)e\right)\cos\left(bd^2x^2+2\,bcdx+bc^2+a\right)}{4\,b^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a) - sqrt(2)*sqrt(b*d^2/pi
)*(3*(pi*c*f^3 - pi*d*f^2*e)*cos(a) + 2*(pi*b*c^3*f^3 - 3*pi*b*c^2*d*f^2*e
+ 3*pi*b*c*d^2*f*e^2 - pi*b*d^3*e^3)*sin(a))*fresnel_cos(sqrt(2)*sqrt(b*d^2
/pi)*(d*x + c)/d) - sqrt(2)*sqrt(b*d^2/pi)*(2*(pi*b*c^3*f^3 - 3*pi*b*c^2*d*
f^2*e + 3*pi*b*c*d^2*f*e^2 - pi*b*d^3*e^3)*cos(a) - 3*(pi*c*f^3 - pi*d*f^2*
e)*sin(a))*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3*x
^2 - b*c*d^2*f^3*x + b*c^2*d*f^3 + 3*b*d^3*f*e^2 + 3*(b*d^3*f^2*x - b*c*d^2
*f^2)*e)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^2*d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)^3\sin\left(a+bc^2+2bcdx+bd^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 6.83, size = 1071, normalized size = 3.14

$$\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia+3)}}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia+3)}}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a + 3)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)*(x + c/d))*e^(-I*a + 3)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) - 3/4
*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^
```

```
4) + 1)*(x + c/d))*e^(I*a + 2)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) +
 f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a + 2)/(b*d))/d - 3/4*(-I*sqr
t(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*
(x + c/d))*e^(-I*a + 2)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I
*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a + 2)/(b*d))/d - 3/8*(-I*sqrt(2)*sq
rt(pi)*(2*b*c^2*f^2 + I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^
2*d^4) + 1)*(x + c/d))*e^(I*a + 1)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1
)*b) - 2*I*(d*f^2*(I*x + I*c/d) - 2*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x +
 I*b*c^2 + I*a + 1)/(b*d))/d^2 - 3/8*(I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 - I*f
^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(
-I*a + 1)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*I*(d*f^2*(I*x + I
*c/d) - 2*I*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a + 1)/(b*d)
)/d^2 + 1/8*(sqrt(2)*sqrt(pi)*(-2*I*b*c^3*f^3 + 3*c*f^3)*erf(-1/2*sqrt(2)*s
qrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I
*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)^2 - 3*b*c*d*f^3*(x +
c/d) + 3*b*c^2*f^3 + I*f^3)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(
b^2*d))/d^3 + 1/8*(sqrt(2)*sqrt(pi)*(2*I*b*c^3*f^3 + 3*c*f^3)*erf(-1/2*sqrt
(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(sqrt(b*d^2
)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)^2 - 3*b*c*d*f^3*(
x + c/d) + 3*b*c^2*f^3 - I*f^3)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I
*a)/(b^2*d))/d^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin \left(a+b\left(c+d\,x\right)^2\right)\,\left(e+f\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^2)*(e + f*x)^3,x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)*(e + f*x)^3, x)
```

3.166 $\int (e+fx)^2 \sin \left(a+b(c+dx)^2\right) \, dx$

Optimal. Leaf size=256

$$-\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3}+\frac{f^2\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3}$$

```
[Out] -f*(-c*f+d*e)*cos(a+b*(d*x+c)^2)/b/d^3-1/2*f^2*(d*x+c)*cos(a+b*(d*x+c)^2)/b
/d^3+1/4*f^2*cos(a)*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/d^3-1/4*f^2*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*
2^(1/2)*Pi^(1/2)/b^(3/2)/d^3+1/2*(-c*f+d*e)^2*cos(a)*FresnelS((d*x+c)*b^(1/
2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)+1/2*(-c*f+d*e)^2*FresnelC
((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3514, 3434, 3433, 3432, 3460, 2718, 3466, 3435}

$$\frac{\sqrt{\frac{\pi}{2}}f^2\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^3}-\frac{\sqrt{\frac{\pi}{2}}f^2\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{\sqrt{b}d^3}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}-\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] -((f*(d*e - c*f)*Cos[a + b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[a +
b*(c + d*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*F
resnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^3) - (f^2*Sqrt[Pi/
2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b^(3/2)*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^2\right) \, dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right) \sin \left(a+bx^2\right)+2def\left(1-\frac{cf}{de}\right) x \sin (\right.\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2 \sin \left(a+bx^2\right) \, dx, x, c+dx\right)}{d^3} + \frac{(2f(de-cf))\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{f^2(c+dx) \cos \left(a+b(c+dx)^2\right)}{2bd^3} + \frac{f^2\text{Subst}\left(\int \cos \left(a+bx^2\right) \, dx, x, c\right.}{2bd^3}$$

$$= -\frac{f(de-cf) \cos \left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx) \cos \left(a+b(c+dx)^2\right)}{2bd^3}$$

$$= -\frac{f(de-cf) \cos \left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx) \cos \left(a+b(c+dx)^2\right)}{2bd^3}$$

Mathematica [A]
time = 1.14, size = 151, normalized size = 0.59

$$\frac{-4\sqrt{b}\, f(2de-cf+dfx) \cos \left(a+b(c+dx)^2\right)+2\sqrt{2\pi}\, S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right) \left(2b(de-cf)^2 \cos(a)-f^2 \sin(a)\right)+2\sqrt{2\pi}\, C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,(c+dx)\right) \left(f^2 \cos(a)+2b(de-cf)^2 \sin(a)\right)}{8b^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-4*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[a + b*(c + d*x)^2] + 2*Sqrt[2*Pi]*F
resnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - f^2*Sin[a
]) + 2*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(f^2*Cos[a] + 2*b*
(d*e - c*f)^2*Sin[a]))/(8*b^(3/2)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(212) = 424$.
time = 0.09, size = 669, normalized size = 2.61

method	result
risch	$\frac{i e^2 \sqrt{\pi}\, e^{ia} \operatorname{erf}\left(-d\sqrt{-ib}\, x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}} + \frac{i f^2 c^2 \sqrt{\pi}\, e^{ia} \operatorname{erf}\left(-d\sqrt{-ib}\, x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^3\sqrt{-ib}} - \frac{f^2 \sqrt{\pi}\, e^{ia} \operatorname{erf}\left(-d\sqrt{-ib}\, x\right.}{8b\, d^3\sqrt{-ib}}$

default	$-\frac{f^2 x\cos\left(d^2x^2b+2cdxb+b\,c^2+a\right)}{2b\,d^2}-\frac{f^2c\left(-\frac{\cos\left(d^2x^2b+2cdxb+b\,c^2+a\right)}{2b\,d^2}-\frac{c\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{b^2c^2d^2-b\,d^2\left(b\,c^2+a\right)}{b\,d^2}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right.}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right.\right.}{2d\sqrt{}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f^2/b/d^2*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-f^2*c/d*(-1/2/b/d^2*cos(b
*d^2*x^2+2*b*c*d*x+b*c^2+a)-1/2*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^
2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(
b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/P
i^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))))+1/4*f^2/b/d^2*2^(1/2)*Pi^(1/2)/(b*
d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1
/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))+sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)
*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))-e*f/b/d^2*cos(b*
d^2*x^2+2*b*c*d*x+b*c^2+a)-e*f*c/d*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2
*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b
*d^2*x+b*c*d))-sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi
^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))+1/2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*e
^2*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d
^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*Fresnel
C(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.34, size = 1038, normalized size = 4.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*
c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*
e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d*x + 4*((e^(I*b*d^2*
x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*co
s(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*(
((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
 - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2)) - 1))*cos(a) + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I
*b*c*d*x + I*b*c^2)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2 + (-(I - 1)*sqrt(2)*gamma(3/
```

```
2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I + 1)*sqrt(2)*gamma(3/2, -I*b*d
^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (-(I + 1)*sqrt(2)*gamma(3/2, I*b*
d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + (I - 1)*sqrt(2)*gamma(3/2, -I*b*d^2*x^2
- 2*I*b*c*d*x - I*b*c^2))*sin(a)))*f^2/(b^2*d^4*x + b^2*c*d^3) - 1/4*(2*((e
^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*
x - I*b*c^2)) - 1))*cos(a) + ((I - 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*
I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (
-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*
x - I*b*c^2))*sin(a))*c)*f*e/(b*d^3*x + b*c*d^2) - 1/8*sqrt(2)*sqrt(pi)*((-
(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (-(I -
1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))*e^2/(sqrt(b)
*d)
```

Fricas [**A**]
time = 0.36, size = 213, normalized size = 0.83

$$\frac{\sqrt{2}\,\left(\pi f^2 \cos\left(a\right) + 2\left(\pi b c^2 f^2 - 2\,\pi b c d f e + \pi b d^2 e^2\right)\sin\left(a\right)\right)\sqrt{\frac{b d^2}{\pi}}\;\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{b d^2}{\pi}}\,(dx+c)}{d}\right) - \sqrt{2}\,\left(\pi f^2 \sin\left(a\right) - 2\left(\pi b c^2 f^2 - 2\,\pi b c d f e + \pi b d^2 e^2\right)\cos\left(a\right)\right)\sqrt{\frac{b d^2}{\pi}}\;\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{b d^2}{\pi}}\,(dx+c)}{d}\right) - 2\left(b d^2 f^2 x - b c d f^2 + 2\,b d^2 f e\right)\cos\left(b d^2 x^2 + 2\,b c d x + b c^2 + a\right)}{4\,b^2 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(pi*f^2*cos(a) + 2*(pi*b*c^2*f^2 - 2*pi*b*c*d*f*e + pi*b*d^2*e
^2)*sin(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d)
- sqrt(2)*(pi*f^2*sin(a) - 2*(pi*b*c^2*f^2 - 2*pi*b*c*d*f*e + pi*b*d^2*e^2)
*cos(a))*sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2
*(b*d^2*f^2*x - b*c*d*f^2 + 2*b*d^2*f*e)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2
+ a))/(b^2*d^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e + f x\right)^{2} \sin\left(a + b c^{2} + 2 b c d x + b d^{2} x^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.00, size = 705, normalized size = 2.75

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia+2)}}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia+2)}}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia+1)}}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{(ibd^2x^2+2ibcdx+ibc^2+ia+1)}}{bd}}{2d}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia+1)}}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{(-ibd^2x^2-2ibcdx-ibc^2-ia+1)}}{bd}}{2d}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}(2bc^2f^2+if^2)\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia)}}{\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(df^2\left(-ix-\frac{ic}{d}\right)+2icf^2\right)e^{(ibd^2x^2+2ibcdx+ibc^2+ia)}}{bd}}{8d^2}-\frac{\frac{i\sqrt{2}\sqrt{\pi}(2bc^2f^2-if^2)\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia)}}{\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)b}+\frac{2i\left(df^2\left(-ix-\frac{ic}{d}\right)+2icf^2\right)e^{(-ibd^2x^2-2ibcdx-ibc^2-ia)}}{bd}}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a + 2)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)*(x + c/d))*e^(-I*a + 2)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/2
*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^
4) + 1)*(x + c/d))*e^(I*a + 1)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) +
 f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a + 1)/(b*d))/d - 1/2*(-I*sqr
t(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*
(x + c/d))*e^(-I*a + 1)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I
*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a + 1)/(b*d))/d - 1/8*(-I*sqrt(2)*sq
rt(pi)*(2*b*c^2*f^2 + I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^
2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b)
 + 2*I*(d*f^2*(-I*x - I*c/d) + 2*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2 + I*a)/(b*d))/d^2 - 1/8*(I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 - I*f^2)*erf
(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(
sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*I*(d*f^2*(-I*x - I*c/d) + 2*
I*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,(c+d\,x)^2\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)*(e + f*x)^2, x)
```

3.167 $\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$

Optimal. Leaf size=122

$$-\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}\,d^2}$$

```
[Out] -1/2*f*cos(a+b*(d*x+c)^2)/b/d^2+1/2*(-c*f+d*e)*cos(a)*FresnelS((d*x+c)*b^(1
/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d^2/b^(1/2)+1/2*(-c*f+d*e)*FresnelC(
(d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/d^2/b^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3514, 3434, 3433, 3432, 3460, 2718}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,(c+dx)\right)}{\sqrt{b}\,d^2}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^2}-\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] -1/2*(f*Cos[a + b*(c + d*x)^2])/(b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*Cos[a]*Fr
esnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2) + ((d*e - c*f)*Sqrt[Pi/
2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+bx^2\right)+fx\sin\left(a+bx^2\right)\right)\,dx,x,c+dx\right)}{d^2}\\ &= \frac{f\text{Subst}\left(\int x\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(a+b\right.\right.}{d^2}\\ &= \frac{f\text{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^2\right)}{2d^2}+\frac{((de-cf)\cos(a))\text{Subst}\left(\int \sin\right.}{d^2}\\ &= -\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d^2}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 114, normalized size = 0.93

$$\frac{-f\cos\left(a+b(c+dx)^2\right)+\sqrt{b}\,(de-cf)\sqrt{2\pi}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+\sqrt{b}\,(de-cf)\sqrt{2\pi}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-(f*Cos[a + b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fresne
lS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelC[
Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(102) = 204$.
time = 0.05, size = 309, normalized size = 2.53

method	result
risch	$\frac{ie\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}-\frac{ifc\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d^2\sqrt{-ib}}+\frac{ie\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{i}}\right.}{4d\sqrt{ib}}$
default	$-\frac{f\cos\left(d^2x^2b+2cdxb+b\,c^2+a\right)}{2b\,d^2}-\frac{fc\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{b^2c^2d^2-b\,d^2\left(b\,c^2+a\right)}{b\,d^2}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)-\sin\left(\frac{b^2c^2d^2-b\,d^2\left(b\,c^2+a\right)}{b\,d^2}\right.\right.}{2d\sqrt{b\,d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f/b/d^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-1/2*f*c/d*2^(1/2)*Pi^(1/2)/(b
*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^(1/2)/Pi^(
1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2
)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))+1/2*2^(1/2)*Pi^
(1/2)/(b*d^2)^(1/2)*e*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2)*FresnelS(2^
(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-b*d^2*(b*c^2
+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.91, size = 484, normalized size = 3.97

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\cos(a)+(i-1)\sin(a)\right)\mathrm{erf}\left(\frac{ibdx+ibc}{\sqrt{ib}}\right)+\left(-(i-1)\cos(a)+(i+1)\sin(a)\right)\mathrm{erf}\left(\frac{ibdx+ibc}{\sqrt{-ib}}\right)\right)e}{8\sqrt{b}\,d}-\frac{\left(2\left(\left(e^{(ibd^2x^2+2ibcdx+ibc^2)}+e^{(-ibd^2x^2-2ibcdx-ibc^2)}\right)\cos(a)-\left(-ie^{(ibd^2x^2+2ibcdx+ibc^2)}+ie^{(-ibd^2x^2-2ibcdx-ibc^2)}\right)\sin(a)\right)dx-\sqrt{bd^2x^2+2bcdx+bc^2}\left(\left(-(i+1)\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{ibd^2x^2+2ibcdx+ibc^2}\right)-1\right)+(i-1)\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-ibd^2x^2-2ibcdx-ibc^2}\right)-1\right)\right)\cos(a)+\left((i-1)\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{ibd^2x^2+2ibcdx+ibc^2}\right)-1\right)-(i+1)\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-ibd^2x^2-2ibcdx-ibc^2}\right)-1\right)\right)\sin(a)\right)c+2\left(\left(e^{(ibd^2x^2+2ibcdx+ibc^2)}+e^{(-ibd^2x^2-2ibcdx-ibc^2)}\right)\cos(a)-\left(-ie^{(ibd^2x^2+2ibcdx+ibc^2)}+ie^{(-ibd^2x^2-2ibcdx-ibc^2)}\right)\sin(a)\right)c\right)f}{8\left(bd^3x+bcd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*
b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/
sqrt(-I*b)))*e/(sqrt(b)*d) - 1/8*(2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^
2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(
a))*d*x - sqrt(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*((-(I + 1)*sqrt(2)*sqrt(pi)*(
erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + (I - 1)*sqrt(2)*sqrt(
pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + ((I - 1
)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - (
I + 1)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
1))*sin(a))*c + 2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x
^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (-I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) + I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*c)*f/(b*d^3*x
+ b*c*d^2)
```

Fricas [A]
time = 0.37, size = 134, normalized size = 1.10

$$-\frac{\sqrt{2}\,(\pi cf-\pi de)\sqrt{\frac{bd^2}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)+\sqrt{2}\,(\pi cf-\pi de)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)\sin(a)+df\cos\left(bd^2x^2+2\,bcdx+bc^2+a\right)}{2\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(pi*c*f - pi*d*e)*sqrt(b*d^2/pi)*cos(a)*fresnel_sin(sqrt(2)*s
qrt(b*d^2/pi)*(d*x + c)/d) + sqrt(2)*(pi*c*f - pi*d*e)*sqrt(b*d^2/pi)*fresn
el_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d)*sin(a) + d*f*cos(b*d^2*x^2 + 2*b
*c*d*x + b*c^2 + a))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.83, size = 389, normalized size = 3.19

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a+1)}}{4\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a+1)}}{4\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{\frac{i\sqrt{2}\sqrt{\pi}\,cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2+i\,a\right)}}{bd}}{4\,d}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\,cf\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{fe^{\left(-i\,bd^2x^2-2i\,bcdx-i\,bc^2-i\,a\right)}}{bd}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a + 1)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)*(x + c/d))*e^(-I*a + 1)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4
*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^
4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e
^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b*d))/d - 1/4*(-I*sqrt(2)*sqr
t(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d
))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^2\right)\,\left(e+f\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^2)*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)*(e + f*x), x)
```

3.168 $\int \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}\,d}$$

```
[Out] 1/2*cos(a)*FresnelS((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/d/b^
(1/2)+1/2*FresnelC((d*x+c)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2
)/d/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3434, 3433, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d) + (S
qrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a+b(c+dx)^2\right)\,dx = \cos(a)\int \sin\left(b(c+dx)^2\right)\,dx + \sin(a)\int \cos\left(b(c+dx)^2\right)\,dx$$

$$= \frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}\,d} + \frac{\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}\,d}$$

Mathematica [**A**]
time = 0.04, size = 67, normalized size = 0.81

$$\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)+C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a]))/(Sqrt[b]*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(65) = 130.
time = 0.04, size = 136, normalized size = 1.64

method	result	s
risch	$\frac{i\sqrt{\pi}\,e^{-ia}\operatorname{erf}\left(d\sqrt{ib}\,x+\frac{ibc}{\sqrt{ib}}\right)}{4d\sqrt{ib}}+\frac{i\sqrt{\pi}\,e^{ia}\operatorname{erf}\left(-d\sqrt{-ib}\,x+\frac{ibc}{\sqrt{-ib}}\right)}{4d\sqrt{-ib}}$	8
default	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{b^2c^2d^2-b\,d^2\left(b\,c^2+a\right)}{b\,d^2}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)-\sin\left(\frac{b^2c^2d^2-b\,d^2\left(b\,c^2+a\right)}{b\,d^2}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\left(b\,d^2x+bcd\right)}{\sqrt{\pi}\,\sqrt{b\,d^2}}\right)\right)}{2\sqrt{b\,d^2}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/(b*d^2)^(1/2)*(cos((b^2*c^2*d^2-b*d^2*(b*c^2+a))/b/d^2
)*FresnelS(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2
-b*d^2*(b*c^2+a))/b/d^2)*FresnelC(2^(1/2)/Pi^(1/2)/(b*d^2)^(1/2)*(b*d^2*x+b
*c*d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 69, normalized size = 0.83

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\,\cos\left(a\right)+(i-1)\,\sin\left(a\right)\right)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{i\,b}}\right)+\left(-(i-1)\,\cos\left(a\right)+(i+1)\,\sin\left(a\right)\right)\operatorname{erf}\left(\frac{i\,bdx+i\,bc}{\sqrt{-i\,b}}\right)\right)}{8\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf((I*b*d*x + I*
b*c)/sqrt(I*b)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf((I*b*d*x + I*b*c)/
sqrt(-I*b)))/(sqrt(b)*d)
```

Fricas [A]
time = 0.35, size = 89, normalized size = 1.07

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{bd^2}{\pi}}\,\cos\left(a\right)\operatorname{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)+\sqrt{2}\,\pi\sqrt{\frac{bd^2}{\pi}}\,\operatorname{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{bd^2}{\pi}}\,(dx+c)}{d}\right)\sin\left(a\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d
*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(
d*x + c)/d)*sin(a))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(a+b(c+dx)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.38, size = 151, normalized size = 1.82

$$\frac{i\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(i\,a)}}{4\,\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{i\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-i\,a)}}{4\,\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4*I
*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*
(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1))
```

Mupad [B]
time = 0.05, size = 95, normalized size = 1.14

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\cos\left(a\right)\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{b\,d^2}}\,\left(b\,x\,d^2+b\,c\,d\right)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}+\frac{\sqrt{2}\,\sqrt{\pi}\,\sin\left(a\right)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{b\,d^2}}\,\left(b\,x\,d^2+b\,c\,d\right)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b\,d^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*cos(a)*fresnels((2^(1/2)*(1/(b*d^2))^(1/2)*(b*c*d + b*d^2
*x))/pi^(1/2))*(1/(b*d^2))^(1/2))/2 + (2^(1/2)*pi^(1/2)*sin(a)*fresnelc((2^
(1/2)*(1/(b*d^2))^(1/2)*(b*c*d + b*d^2*x))/pi^(1/2))*(1/(b*d^2))^(1/2))/2
```

3.169 $\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sin\left(a+b(c+dx)^2\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx = \int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

Mathematica [A]
time = 9.11, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^2\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
[Out] int(sin(a+b*(d*x+c)^2)/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bc^2 + 2bcdx + bd^2x^2\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**2)/(f*x+e),x)
[Out] Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^2\right)}{e+f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^2)/(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^2)/(e + f*x), x)
```

3.170 $\int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sin(a+b(c+dx)^2)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2} dx = \int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2} dx$$

Mathematica [A]
time = 13.75, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+b(c+dx)^2)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^2\right)}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^2*x^2 + 2*f*x*e + e^2),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bc^2 + 2bcdx + bd^2x^2\right)}{\left(e + fx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+b\,(c+d\,x)^2\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^2)/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^2)/(e + f*x)^2, x)
```

3.171 $\int (e+fx)^3 \sin \left(a+b(c+dx)^3\right) \, dx$

Optimal. Leaf size=434

$$-\frac{f^2(de-cf)\cos \left(a+b(c+dx)^3\right)}{bd^4}-\frac{f^3(c+dx)\cos \left(a+b(c+dx)^3\right)}{3bd^4}-\frac{e^{ia}f^3(c+dx)\Gamma \left(\frac{1}{3},-ib(c+dx)^3\right)}{18bd^4\sqrt[3]{-ib(c+dx)^3}}+\frac{ie^{ia}(}{}$$

```
[Out] -f^2*(-c*f+d*e)*cos(a+b*(d*x+c)^3)/b/d^4-1/3*f^3*(d*x+c)*cos(a+b*(d*x+c)^3)
/b/d^4-1/18*exp(I*a)*f^3*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/b/d^4/(-I*b*(d*x
+c)^3)^(1/3)+1/6*I*exp(I*a)*(-c*f+d*e)^3*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/
d^4/(-I*b*(d*x+c)^3)^(1/3)-1/18*f^3*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/b/d^4/
exp(I*a)/(I*b*(d*x+c)^3)^(1/3)-1/6*I*(-c*f+d*e)^3*(d*x+c)*GAMMA(1/3,I*b*(d*
x+c)^3)/d^4/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)+1/2*I*exp(I*a)*f*(-c*f+d*e)^2*(d
*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/d^4/(-I*b*(d*x+c)^3)^(2/3)-1/2*I*f*(-c*f+
d*e)^2*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^4/exp(I*a)/(I*b*(d*x+c)^3)^(2/3
)
```

Rubi [A]
time = 0.32, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3514, 3436, 2239, 3470, 2250, 3460, 2718, 3466, 3437}

$$\frac{ie^{ia}f(c+dx)^2(de-cf)^2\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)^2\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de-cf)^3\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^4\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)(de-cf)^3\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^4\sqrt[3]{ib(c+dx)^3}}-\frac{e^{ia}f^3(c+dx)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{18bd^4\sqrt[3]{-ib(c+dx)^3}}-\frac{e^{-ia}f^3(c+dx)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{18bd^4\sqrt[3]{ib(c+dx)^3}}-\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4}-\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] -((f^2*(d*e - c*f)*Cos[a + b*(c + d*x)^3])/(b*d^4)) - (f^3*(c + d*x)*Cos[a
+ b*(c + d*x)^3])/(3*b*d^4) - (E^(I*a)*f^3*(c + d*x)*Gamma[1/3, (-I)*b*(c +
 d*x)^3])/(18*b*d^4*((-I)*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*(d*e - c*f
)^3*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(1/
3)) - (f^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(18*b*d^4*E^(I*a)*(I*b*(c
 + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)
^3])/(d^4*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/2)*E^(I*a)*f*(d*e - c*f)^2
*(c + d*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(2/3
)) - ((I/2)*f*(d*e - c*f)^2*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^4*E
^(I*a)*(I*b*(c + d*x)^3)^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
```

```
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right)\,dx = \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(a+bx^3\right)+3d^2e^2f\left(1+\frac{c}{}\right.\right.\right.}{}$$

$$=\frac{f^3\text{Subst}\left(\int x^3\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^4}+\frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x\right.}{}$$

$$=-\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4}+\frac{f^3\text{Subst}\left(\int\cos\left(a+bx^3\right)\,dx,x,c+\right.}{3bd^4}$$

$$=-\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4}-\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4}$$

$$=-\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4}-\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4}$$

Mathematica [F]
time = 101.48, size = 0, normalized size = 0.00

$$\int (e+fx)^3\sin\left(a+b(c+dx)^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
[Out] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^3], x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx+e)^3\sin\left(a+b(dx+c)^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
[Out] int((f*x+e)^3*sin(a+b*(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Fricas [A]
time = 0.11, size = 424, normalized size = 0.98

$$\frac{(3\,bc^3f^3-9\,bc^2df^2e+9\,bcd^2fe^2-3\,bd^3e^3+i\,f^3)(i\,bd^3)^{\frac{2}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+(3\,bc^3f^3-9\,bc^2df^2e+9\,bcd^2fe^2-3\,bd^3e^3-i\,f^3)(-i\,bd^3)^{\frac{2}{3}}\,e^{(i\,a)}\Gamma\left(\frac{1}{3},-i\,bd^3x^3-3i\,bcd^2x^2-3i\,bc^2dx-i\,bc^3\right)-9\,(bc^2df^3-2\,bcd^2f^2e+bd^3fe^2)(i\,bd^3)^{\frac{1}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{2}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)-9\,(bc^2df^3-2\,bcd^2f^2e+bd^3fe^2)(-i\,bd^3)^{\frac{1}{3}}\,e^{(i\,a)}\Gamma\left(\frac{2}{3},-i\,bd^3x^3-3i\,bcd^2x^2-3i\,bc^2dx-i\,bc^3\right)-6\,(bd^3f^3x-2\,bcd^2f^3+3\,bd^3f^2e)\cos\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{18\,b^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/18*((3*b*c^3*f^3 - 9*b*c^2*d*f^2*e + 9*b*c*d^2*f*e^2 - 3*b*d^3*e^3 + I*f^
3)*(I*b*d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*
b*c^2*d*x + I*b*c^3) + (3*b*c^3*f^3 - 9*b*c^2*d*f^2*e + 9*b*c*d^2*f*e^2 - 3
*b*d^3*e^3 - I*f^3)*(-I*b*d^3)^(2/3)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3 - 3*I*
b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) - 9*(b*c^2*d*f^3 - 2*b*c*d^2*f^2*e +
 b*d^3*f*e^2)*(I*b*d^3)^(1/3)*e^(-I*a)*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2
*x^2 + 3*I*b*c^2*d*x + I*b*c^3) - 9*(b*c^2*d*f^3 - 2*b*c*d^2*f^2*e + b*d^3*
f*e^2)*(-I*b*d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2 -
 3*I*b*c^2*d*x - I*b*c^3) - 6*(b*d^3*f^3*x - 2*b*c*d^2*f^3 + 3*b*d^3*f^2*e)
*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(b^2*d^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)**3*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d
**3*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^3)*(e + f*x)^3,x)
[Out] int(sin(a + b*(c + d*x)^3)*(e + f*x)^3, x)
```

3.172 $\int (e+fx)^2 \sin\left(a+b(c+dx)^3\right)\, dx$

Optimal. Leaf size=280

$$-\frac{f^2\cos\left(a+b(c+dx)^3\right)}{3bd^3}+\frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},ib(c+d$$

```
[Out] -1/3*f^2*cos(a+b*(d*x+c)^3)/b/d^3+1/6*I*exp(I*a)*(-c*f+d*e)^2*(d*x+c)*GAMMA
(1/3,-I*b*(d*x+c)^3)/d^3/(-I*b*(d*x+c)^3)^(1/3)-1/6*I*(-c*f+d*e)^2*(d*x+c)*
GAMMA(1/3,I*b*(d*x+c)^3)/d^3/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)+1/3*I*exp(I*a)*
f*(-c*f+d*e)*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/d^3/(-I*b*(d*x+c)^3)^(2/3)
-1/3*I*f*(-c*f+d*e)*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^3/exp(I*a)/(I*b*(d
*x+c)^3)^(2/3)
```

Rubi [A]

time = 0.18, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3514, 3436, 2239, 3470, 2250, 3460, 2718}

$$\frac{ie^{ia}f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{3d^3\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{3d^3\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de-cf)^2\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)(de-cf)^2\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^3\sqrt[3]{ib(c+dx)^3}}-\frac{f^2\cos\left(a+b(c+dx)^3\right)}{3bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] -1/3*(f^2*Cos[a + b*(c + d*x)^3])/(b*d^3) + ((I/6)*E^(I*a)*(d*e - c*f)^2*(c
 + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(1/3)) -
((I/6)*(d*e - c*f)^2*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I
*b*(c + d*x)^3)^(1/3)) + ((I/3)*E^(I*a)*f*(d*e - c*f)*(c + d*x)^2*Gamma[2/3
, (-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(2/3)) - ((I/3)*f*(d*e - c
*f)*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I*b*(c + d*x)^3)
^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^3\right)\,dx = \frac{\text{Subst}\Big(\int \Big(d^2e^2\Big(1+\frac{cf(-2de+cf)}{d^2e^2}\Big)\sin\left(a+bx^3\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\text{Subst}\left(\int x\,\text{s}\right.}{d}$$

$$= \frac{f^2\text{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^3\right)}{3d^3}+\frac{(if(de-cf))\text{Subst}\Big(\int e^{-ia}}{d^3}$$

$$= -\frac{f^2\cos\left(a+b(c+dx)^3\right)}{3bd^3}+\frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3\sqrt[3]{-ib(c+dx)^3}}$$

Mathematica [F]
time = 42.13, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^3], x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (fx+e)^2 \sin\left(a+b(dx+c)^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)
```

```
[Out] int((f*x+e)^2*sin(a+b*(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Fricas [A]
time = 0.11, size = 321, normalized size = 1.15

$$-\frac{2\,d^2f^2\cos\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)+\left(i\,bd^3\right)^{\frac{2}{3}}\left(c^2f^2-2\,cdfe+d^2e^2\right)e^{(-i\,a)}\Gamma\left(\frac{1}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+\left(-i\,bd^3\right)^{\frac{2}{3}}\left(c^2f^2-2\,cdfe+d^2e^2\right)e^{(i\,a)}\Gamma\left(\frac{1}{3},-i\,bd^3x^3-3i\,bcd^2x^2-3i\,bc^2dx-i\,bc^3\right)-2\left(i\,bd^3\right)^{\frac{1}{3}}\left(cdf^2-d^2fe\right)e^{(-i\,a)}\Gamma\left(\frac{2}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)-2\left(-i\,bd^3\right)^{\frac{1}{3}}\left(cdf^2-d^2fe\right)e^{(i\,a)}\Gamma\left(\frac{2}{3},-i\,bd^3x^3-3i\,bcd^2x^2-3i\,bc^2dx-i\,bc^3\right)}{6\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*d^2*f^2*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) +
(I*b*d^3)^(2/3)*(c^2*f^2 - 2*c*d*f*e + d^2*e^2)*e^(-I*a)*gamma(1/3, I*b*d^3
*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*(c^2*f
^2 - 2*c*d*f*e + d^2*e^2)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2
 - 3*I*b*c^2*d*x - I*b*c^3) - 2*(I*b*d^3)^(1/3)*(c*d*f^2 - d^2*f*e)*e^(-I*a
)*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) - 2*(
```

```
-I*b*d^3)^(1/3)*(c*d*f^2 - d^2*f*e)*e^(I*a)*gamma(2/3, -I*b*d^3*x^3 - 3*I*b
*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d
**3*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,(c+d\,x)^3\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^3)*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b*(c + d*x)^3)*(e + f*x)^2, x)
```

3.173 $\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$

Optimal. Leaf size=235

$$\frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}+\frac{ie^{ia}f(c+dx)^2\Gamma\left(\frac{2}{3},-ib\right.}{6d^2\left(-ib(c+dx\right.}$$

```
[Out] 1/6*I*exp(I*a)*(-c*f+d*e)*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/d^2/(-I*b*(d*x+
c)^3)^(1/3)-1/6*I*(-c*f+d*e)*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/d^2/exp(I*a)/
(I*b*(d*x+c)^3)^(1/3)+1/6*I*exp(I*a)*f*(d*x+c)^2*GAMMA(2/3,-I*b*(d*x+c)^3)/
d^2/(-I*b*(d*x+c)^3)^(2/3)-1/6*I*f*(d*x+c)^2*GAMMA(2/3,I*b*(d*x+c)^3)/d^2/e
xp(I*a)/(I*b*(d*x+c)^3)^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3514, 3436, 2239, 3470, 2250}

$$\frac{ie^{ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}+\frac{ie^{ia}f(c+dx)^2\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{6d^2\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{6d^2\left(ib(c+dx)^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*E^(I*a)*(d*e - c*f)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^2*(
(-I)*b*(c + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)*(c + d*x)*Gamma[1/3, I*b*(c
 + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*f*(c + d
*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^2*((-I)*b*(c + d*x)^3)^(2/3)) - ((
I/6)*f*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)
^3)^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
```

```
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx &= \frac{\text{Subst}\left(\int\left(de\left(1-\frac{cf}{de}\right)\sin\left(a+bx^3\right)+fx\sin\left(a+bx^3\right)\right)\,dx,x,c+dx\right)}{d^2}\\ &= \frac{f\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int\sin\left(a+b\right.\right.}{d^2}\\ &= \frac{(if)\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx,x,c+dx\right)}{2d^2}-\frac{(if)\text{Subst}\left(\int e^{ia+ibx^3}x\,dx,x,c\right.}{2d^2}\\ &= \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3}\right.}{6d^2\sqrt[3]{ib(c+dx)}}\end{aligned}$$

Mathematica [F]
time = 33.86, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
[Out] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^3], x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx+e)\sin\left(a+b(dx+c)^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

```
[Out] int((f*x+e)*sin(a+b*(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Fricas [A]
time = 0.10, size = 229, normalized size = 0.97

$$-\frac{\left(i\,bd^{3}\right)^{\frac{1}{3}}\,dfe^{(-i\,a)}\Gamma\left(\frac{2}{3}, i\,bd^{3}x^{3}+3i\,bcd^{2}x^{2}+3i\,bc^{2}dx+i\,bc^{3}\right)+\left(-i\,bd^{3}\right)^{\frac{1}{3}}\,dfe^{(i\,a)}\Gamma\left(\frac{2}{3}, -i\,bd^{3}x^{3}-3i\,bcd^{2}x^{2}-3i\,bc^{2}dx-i\,bc^{3}\right)-\left(i\,bd^{3}\right)^{\frac{2}{3}}\,(cf-de)e^{(-i\,a)}\Gamma\left(\frac{1}{3}, i\,bd^{3}x^{3}+3i\,bcd^{2}x^{2}+3i\,bc^{2}dx+i\,bc^{3}\right)-\left(-i\,bd^{3}\right)^{\frac{2}{3}}\,(cf-de)e^{(i\,a)}\Gamma\left(\frac{1}{3}, -i\,bd^{3}x^{3}-3i\,bcd^{2}x^{2}-3i\,bc^{2}dx-i\,bc^{3}\right)}{6\,bd^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*((I*b*d^3)^(1/3)*d*f*e^(-I*a)*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2
 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(1/3)*d*f*e^(I*a)*gamma(2/3, -I*b*
d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) - (I*b*d^3)^(2/3)*(c*f
 - d*e)*e^(-I*a)*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x +
 I*b*c^3) - (-I*b*d^3)^(2/3)*(c*f - d*e)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3 -
3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+bc^{3}+3bc^{2}dx+3bcd^{2}x^{2}+bd^{3}x^{3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3
*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^3\right)\,\left(e+f\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^3)*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^3)*(e + f*x), x)
```

3.174 $\int \sin\left(a+b(c+dx)^3\right)\,dx$

Optimal. Leaf size=107

$$\frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

```
[Out] 1/6*I*exp(I*a)*(d*x+c)*GAMMA(1/3,-I*b*(d*x+c)^3)/d/(-I*b*(d*x+c)^3)^(1/3)-1
/6*I*(d*x+c)*GAMMA(1/3,I*b*(d*x+c)^3)/d/exp(I*a)/(I*b*(d*x+c)^3)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3436, 2239}

$$\frac{ie^{ia}(c+dx)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*E^(I*a)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d*((-I)*b*(c + d*
x)^3)^(1/3)) - ((I/6)*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d*E^(I*a)*(I*
b*(c + d*x)^3)^(1/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+b(c+dx)^3\right)\,dx &= \frac{1}{2}i\int e^{-ia-ib(c+dx)^3}\,dx-\frac{1}{2}i\int e^{ia+ib(c+dx)^3}\,dx\\ &= \frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.07

$$\frac{i(c+dx)\left(-\sqrt[3]{-ib(c+dx)^3}\,\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)(\cos(a)-i\sin(a))+\sqrt[3]{ib(c+dx)^3}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)(\cos(a)+i\sin(a))\right)}{6d\sqrt[3]{b^2(c+dx)^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*(c + d*x)*(-(((-I)*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3]*
(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x)^
3]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^6)^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \sin\left(a+b(dx+c)^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a), x)
```

Fricas [A]
time = 0.09, size = 107, normalized size = 1.00

$$-\frac{(i\,bd^3)^{\frac{2}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{3},i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc^2dx+i\,bc^3\right)+(-i\,bd^3)^{\frac{2}{3}}\,e^{(i\,a)}\Gamma\left(\frac{1}{3},-i\,bd^3x^3-3i\,bcd^2x^2-3i\,bc^2dx-i\,bc^3\right)}{6\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*((I*b*d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3
*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3
- 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a + b(c + dx)^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**3),x)
[Out] Integral(sin(a + b*(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^3*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + b\,(c + d\,x)^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^3),x)
[Out] int(sin(a + b*(c + d*x)^3), x)
```

3.175 $\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Mathematica [A]
time = 45.14, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^3\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

```
[Out] int(sin(a+b*(d*x+c)^3)/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f*x + e)
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bc^3 + 3bc^2dx + 3bcd^2x^2 + bd^3x^3\right)}{e + fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**3)/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
+ f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin \left(a+b\,(c+d\,x)^3\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^3)/(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^3)/(e + f*x), x)
```

3.176 $\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sin(a+b(c+dx)^3)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2} dx = \int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2} dx$$

Mathematica [A]
time = 93.29, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+b(c+dx)^3)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^3\right)}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f^2*x^2
+ 2*f*x*e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bc^3 + 3bc^2dx + 3bcd^2x^2 + bd^3x^3\right)}{\left(e + fx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**3)/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
+ f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin \left(a+b\,(c+d\,x)^3\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^3)/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^3)/(e + f*x)^2, x)
```

3.177 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\, dx$

Optimal. Leaf size=371

$$\frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}-\frac{bf(de-cf)\cos(a)\mathrm{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{b}\,(de-cf)^2\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}$$

```
[Out] -b*f*(-c*f+d*e)*Ci(b/(d*x+c)^2)*cos(a)/d^3+2/3*b*f^2*(d*x+c)*cos(a+b/(d*x+c
)^2)/d^3+b*f*(-c*f+d*e)*Si(b/(d*x+c)^2)*sin(a)/d^3+(-c*f+d*e)^2*(d*x+c)*sin
(a+b/(d*x+c)^2)/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^2)/d^3+1/3*f^2*(
d*x+c)^3*sin(a+b/(d*x+c)^2)/d^3+2/3*b^(3/2)*f^2*cos(a)*FresnelS(b^(1/2)*2^(
1/2)/Pi^(1/2)/(d*x+c))*2^(1/2)*Pi^(1/2)/d^3+2/3*b^(3/2)*f^2*FresnelC(b^(1/2
)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)*2^(1/2)*Pi^(1/2)/d^3-(-c*f+d*e)^2*cos(a)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3+(-
c*f+d*e)^2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(a)*b^(1/2)*2^(1/2
)*Pi^(1/2)/d^3
```

Rubi [A]
time = 0.36, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3514, 3440, 3468, 3435, 3433, 3432, 3460, 3378, 3384, 3380, 3383, 3490, 3469, 3434}

$$\frac{2\sqrt{2\pi}\, b^{3/2} f^2 \sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{3d^3}+\frac{2\sqrt{2\pi}\, b^{3/2} f^2 \cos(a) S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf\cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)^2\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^3}+\frac{\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)^2 S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3}+\frac{bf\sin(a)(de-cf)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f(c+dx)^2(de-cf)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{(c+dx)(de-cf)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}+\frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (2*b*f^2*(c + d*x)*Cos[a + b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*Cos[a
]*CosIntegral[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*Cos[a
]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]
*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + (2*b^(3/2)*f^2*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/(3*d^3) + (Sqrt
[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a
])/d^3 + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^3 + (f*(d*e - c
*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^2])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/(c
 + d*x)^2])/(3*d^3) + (b*f*(d*e - c*f)*Sin[a]*SinIntegral[b/(c + d*x)^2])/d
^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
```

```
+ f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+\frac{b}{x^2}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(\right.\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{f^2\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}\,dx,x,\frac{1}{c+dx}\right)}{d^3}-\frac{(f(de-cf))\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\right.}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f(de-cf)(c+dx)^2\sin\left(a-\right.}{d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}+\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)}\right.}{d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}-\frac{bf(de-cf)\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-$$

$$= \frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}-\frac{bf(de-cf)\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-$$

Mathematica [A]
time = 0.91, size = 467, normalized size = 1.26

$$\frac{2bcf^2\cos\left(a+\frac{b}{(c+dx)^2}\right)+2bdf^2x\cos\left(a+\frac{b}{(c+dx)^2}\right)+3bf(-de+cf)\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)+2b^{3/2}f^2\sqrt{2\pi}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+3\sqrt{b}\,d^2e^2\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)-6\sqrt{b}\,cdef\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)+3\sqrt{b}\,c^2f^2\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)+\sqrt{b}\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\left(-3(de-cf)^2\cos(a)+2bf^2\sin(a)\right)+3cd^2e^2\sin\left(a+\frac{b}{(c+dx)^2}\right)-3c^2def\sin\left(a+\frac{b}{(c+dx)^2}\right)+c^3f^2\sin\left(a+\frac{b}{(c+dx)^2}\right)+3d^3e^2x\sin\left(a+\frac{b}{(c+dx)^2}\right)+3d^3efx^2\sin\left(a+\frac{b}{(c+dx)^2}\right)+d^3f^2x^3\sin\left(a+\frac{b}{(c+dx)^2}\right)+3bdef\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)-3bcf^2\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (2*b*c*f^2*Cos[a + b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[a + b/(c + d*x)^2] + 3*
b*f*(-(d*e) + c*f)*Cos[a]*CosIntegral[b/(c + d*x)^2] + 2*b^(3/2)*f^2*Sqrt[2
*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*Sqrt[b]*d^2*e^2*Sq
rt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 6*Sqrt[b]*c*d*e*
f*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 3*Sqrt[b]*c^
2*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + Sqrt[b]*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*(-3*(d*e - c*f)^2*Cos[a
] + 2*b*f^2*Sin[a]) + 3*c*d^2*e^2*Sin[a + b/(c + d*x)^2] - 3*c^2*d*e*f*Sin[
a + b/(c + d*x)^2] + c^3*f^2*Sin[a + b/(c + d*x)^2] + 3*d^3*e^2*x*Sin[a + b
/(c + d*x)^2] + 3*d^3*e*f*x^2*Sin[a + b/(c + d*x)^2] + d^3*f^2*x^3*Sin[a +
```

```
b/(c + d*x)^2] + 3*b*d*e*f*Sin[a]*SinIntegral[b/(c + d*x)^2] - 3*b*c*f^2*Si
n[a]*SinIntegral[b/(c + d*x)^2])/(3*d^3)
```

Maple [A]
time = 0.13, size = 302, normalized size = 0.81

method	result
derivativedivides	$-\frac{-(c^2f^2-2cdef+d^2e^2)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(c^2f^2-2cdef+d^2e^2)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx-\cdots)}\right.\right.\cdots}{\cdots}$
default	$-\frac{-(c^2f^2-2cdef+d^2e^2)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(c^2f^2-2cdef+d^2e^2)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx-\cdots)}\right.\right.\cdots}{\cdots}$
risch	$-\frac{e^2 e^{ia} b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}-\frac{i f^2 e^{ia} b^2\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{3d^3\sqrt{-ib}}-\frac{f^2c^2e^{ia}b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d^3\sqrt{-ib}}-\frac{c\,f^2e^{ia}b\exp\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*(-(c^2*f^2-2*c*d*e*f+d^2*e^2)*(d*x+c)*sin(a+b/(d*x+c)^2)+(c^2*f^2-2*
c*d*e*f+d^2*e^2)*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/
Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))-1/2*(-
2*c*f^2+2*d*e*f)*(d*x+c)^2*sin(a+b/(d*x+c)^2)+(-2*c*f^2+2*d*e*f)*b*(1/2*cos
(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2))-1/3*f^2*(d*x+c)^3*sin(a+b/(
d*x+c)^2)+2/3*f^2*b*(-(d*x+c)*cos(a+b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(
cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+sin(a)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)/(d*x+c)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*b*f^2*x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2)) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 + 3*(b*c*d^2*f^2 - b*d^3*f
*e)*x^2 + 3*(b*c^2*d*f^2 - b*d^3*e^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^
3*d^2), x) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 + 3*(b*c*d^2*f^2 - b*d
```

```
^3*f*e)*x^2 + 3*(b*c^2*d*f^2 - b*d^3*e^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x
 + c^3*d^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*sin((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d
^2*f*x^2*e + 3*d^2*x*e^2)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
+ 2*c*d*x + c^2)))/d^2
```

Fricas [A]
time = 0.41, size = 436, normalized size = 1.18

$$\frac{2\sqrt{2}\left(2\pi bdf^2\sin\left(a\right)-3\left(\pi c^2df^2-2\pi cd^2fe+\pi d^3e^2\right)\cos\left(a\right)\right)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+2\sqrt{2}\left(2\pi bdf^2\cos\left(a\right)+3\left(\pi c^2df^2-2\pi cd^2fe+\pi d^3e^2\right)\sin\left(a\right)\right)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-6\left(bcf^2-bdfe\right)\sin\left(a\right)\mathrm{Si}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+3\left(\left(bcf^2-bdfe\right)\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+\left(bcf^2-bdfe\right)\mathrm{Ci}\left(-\frac{b}{d^2x^2+2\,cdx+c^2}\right)\right)\cos\left(a\right)+4\left(bdf^2x+bcf^2\right)\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)+2\left(d^3f^2x^3+c^3f^2+3\left(d^3x+cd^2\right)e^2+3\left(d^3fx^2-c^2df\right)e\right)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(2)*(2*pi*b*d*f^2*sin(a) - 3*(pi*c^2*d*f^2 - 2*pi*c*d^2*f*e + pi
*d^3*e^2)*cos(a))*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(
d*x + c)) + 2*sqrt(2)*(2*pi*b*d*f^2*cos(a) + 3*(pi*c^2*d*f^2 - 2*pi*c*d^2*f
*e + pi*d^3*e^2)*sin(a))*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*
d^2))/(d*x + c)) - 6*(b*c*f^2 - b*d*f*e)*sin(a)*sin_integral(b/(d^2*x^2 + 2
*c*d*x + c^2)) + 3*((b*c*f^2 - b*d*f*e)*cos_integral(b/(d^2*x^2 + 2*c*d*x +
 c^2)) + (b*c*f^2 - b*d*f*e)*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2)))*co
s(a) + 4*(b*d*f^2*x + b*c*f^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2
*x^2 + 2*c*d*x + c^2)) + 2*(d^3*f^2*x^3 + c^3*f^2 + 3*(d^3*x + c*d^2)*e^2 +
 3*(d^3*f*x^2 - c^2*d*f)*e)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^
2 + 2*c*d*x + c^2)))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)^2\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{(c+d\,x)^2}\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^2)*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^2)*(e + f*x)^2, x)
```

3.178 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal. Leaf size=198

$$-\frac{bf\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{b}\,(de-cf)\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}+\frac{\sqrt{b}\,(de-cf)\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(}{d^2}$$

```
[Out] -1/2*b*f*Ci(b/(d*x+c)^2)*cos(a)/d^2+1/2*b*f*Si(b/(d*x+c)^2)*sin(a)/d^2+(-c*
f+d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)/d^
2-(-c*f+d*e)*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*b^(1/2)*2^(1
/2)*Pi^(1/2)/d^2+(-c*f+d*e)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*sin(
a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^2
```

Rubi [A]
time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.611$, Rules used = {3514, 3440, 3468, 3435, 3433, 3432, 3460, 3378, 3384, 3380, 3383}

$$-\frac{bf\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{2\pi}\,\sqrt{b}\,\cos(a)(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}}{c+dx}\right)}{d^2}+\frac{\sqrt{2\pi}\,\sqrt{b}\,\sin(a)(de-cf)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}+\frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{bf\sin(a)\text{Si}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] -1/2*(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2])/d^2 - (Sqrt[b]*(d*e - c*f)*Sqr
t[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + (Sqrt[b]*(d*
e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d^2 +
((d*e - c*f)*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[a +
 b/(c + d*x)^2])/(2*d^2) + (b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+\frac{b}{x^2}\right)+fx\sin\left(a+\frac{b}{x^2}\right)\right)\,dx,x,c+dx\right)}{d^2} \\
&= \frac{f\operatorname{Subst}\left(\int x\sin\left(a+\frac{b}{x^2}\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\operatorname{Subst}\left(\int \sin\left(a+\right.\right.}{d^2} \\
&= -\frac{f\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}-\frac{(de-cf)\operatorname{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx\right.}{d^2} \\
&= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}- \\
&= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2}- \\
&= -\frac{bf\cos(a)\operatorname{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{b}\,(de-cf)\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 242, normalized size = 1.22

$$
\frac{-bf\cos(a)\operatorname{Ci}\left(\frac{b}{(c+dx)^2}\right)-2\sqrt{b}\,(de-cf)\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+2\sqrt{b}\,de\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)-2\sqrt{b}\,cf\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)+2cde\sin\left(a+\frac{b}{(c+dx)^2}\right)-c^2f\sin\left(a+\frac{b}{(c+dx)^2}\right)+2d^2ex\sin\left(a+\frac{b}{(c+dx)^2}\right)+d^2fx^2\sin\left(a+\frac{b}{(c+dx)^2}\right)+bf\sin(a)\operatorname{Si}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (-(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2]) - 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi
]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*Sqrt[b]*d*e*Sqrt[2*Pi
```

```
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 2*Sqrt[b]*c*f*Sqrt[2*Pi
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 2*c*d*e*Sin[a + b/(c +
d*x)^2] - c^2*f*Sin[a + b/(c + d*x)^2] + 2*d^2*e*x*Sin[a + b/(c + d*x)^2] +
 d^2*f*x^2*Sin[a + b/(c + d*x)^2] + b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/
(2*d^2)
```

Maple [A]
time = 0.06, size = 150, normalized size = 0.76

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c}\right.\right.}{d^2}$
default	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+(cf-de)\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c}\right.\right.}{d^2}$
risch	$-\frac{e\,\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d\sqrt{-ib}}+\frac{f\,\mathrm{e}^{ia}b\,\mathrm{expIntegral}\left(1,-\frac{ib}{(dx+c)^2}\right)}{4d^2}+\frac{cf\,\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)}{2d^2\sqrt{-ib}}-\frac{e\,\mathrm{e}^{-ia}b\sqrt{\pi}}{2d\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(
1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)-f*b*(1/2
*cos(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*
d*x + c^2)) + integrate(1/2*(b*d*f*x^2 + 2*b*d*x*e)*cos((a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*
d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*x*e)*cos((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*
c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Fricas [A]
time = 0.37, size = 266, normalized size = 1.34

$$\frac{4\sqrt{2}\left(\pi cdf-\pi d^{2}e\right)\sqrt{\frac{b}{\pi d^{2}}}\cos\left(a\right)\mathrm{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)-4\sqrt{2}\left(\pi cdf-\pi d^{2}e\right)\sqrt{\frac{b}{\pi d^{2}}}\,\mathrm{S}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^{2}}}}{dx+c}\right)\sin\left(a\right)+2\,bf\sin\left(a\right)\mathrm{Si}\left(\frac{b}{d^{2}x^{2}+2\,cdx+c^{2}}\right)-\left(bf\,\mathrm{Ci}\left(\frac{b}{d^{2}x^{2}+2\,cdx+c^{2}}\right)+bf\,\mathrm{Ci}\left(-\frac{b}{d^{2}x^{2}+2\,cdx+c^{2}}\right)\right)\cos\left(a\right)+2\left(d^{2}fx^{2}-c^{2}f+2\left(d^{2}x+cd\right)e\right)\sin\left(\frac{ad^{2}x^{2}+2\,acdx+ac^{2}+b}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{4\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(pi*c*d*f - pi*d^2*e)*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sq
rt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) - 4*sqrt(2)*(pi*c*d*f - pi*d^2*e)*sqrt(
b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))*sin(a) + 2*b*
f*sin(a)*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - (b*f*cos_integral(b/(d
^2*x^2 + 2*c*d*x + c^2)) + b*f*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2)))*
cos(a) + 2*(d^2*f*x^2 - c^2*f + 2*(d^2*x + c*d)*e)*sin((a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)\sin\left(a+\frac{b}{c^{2}+2cdx+d^{2}x^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^2)*(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^2)*(e + f*x), x)
```

3.179 $\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{b}\,\sqrt{2\pi}\,\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}+\frac{\sqrt{b}\,\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

```
[Out] (d*x+c)*sin(a+b/(d*x+c)^2)/d-cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c))*b^(1/2)*2^(1/2)*Pi^(1/2)/d+FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))*s
in(a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3440, 3468, 3435, 3433, 3432}

$$-\frac{\sqrt{2\pi}\,\sqrt{b}\,\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}}{c+dx}\right)}{d}+\frac{\sqrt{2\pi}\,\sqrt{b}\,\sin(a)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) +
 (Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d + (
(c + d*x)*Sin[a + b/(c + d*x)^2])/d
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] && EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx = -\frac{\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b\cos(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{c+dx}\right)}{d} + \frac{(2b\,\text{s}}{}$$
$$= -\frac{\sqrt{b}\sqrt{2\pi}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{\sqrt{b}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d} + \frac{(c+}{}$$

Mathematica [A]
time = 0.12, size = 100, normalized size = 0.95

$$\frac{-\sqrt{b}\sqrt{2\pi}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+\sqrt{b}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (-(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]) + Sq
rt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + (c + d*x
)*Sin[a + b/(c + d*x)^2])/d
```

Maple [A]
time = 0.04, size = 80, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{-(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d}$	80
default	$-\frac{-(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)-\sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\right)\right)}{d}$	80
risch	$-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{dx+c}\right)\mathrm{e}^{-ia}}{2d\sqrt{ib}}-\frac{b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{dx+c}\right)\mathrm{e}^{ia}}{2d\sqrt{-ib}}-\frac{(-dx-c)\sin\left(\frac{a\,d^2x^2+2acdx+a\,c^2+b}{(dx+c)^2}\right)}{d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-(d*x+c)*sin(a+b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
/(d*x+c))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*d*integrate(x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
+ c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x) + b*d*integrate(x*cos
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^
2*x^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((
a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + x*si
n((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
```

Fricas [A]

time = 0.37, size = 137, normalized size = 1.30

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d^2}}\,\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}\,d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)\sin(a)-(dx+c)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2
))/(d*x + c)) - sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/
(pi*d^2))/(d*x + c))*sin(a) - (d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(a+\frac{b}{(c+dx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(a+\frac{b}{(c+d\,x)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^2),x)
```

```
[Out] int(sin(a + b/(c + d*x)^2), x)
```

3.180 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Mathematica [A]
time = 3.82, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**2)/(f*x+e),x)
```

```
[Out] Integral(sin(a + b/(c**2 + 2*c*d*x + d**2*x**2))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^2}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^2)/(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^2)/(e + f*x), x)
```

3.181 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Mathematica [A]
time = 17.99, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^2}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^2)/(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^2)/(e + f*x)^2, x)
```

3.182 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right) dx$

Optimal. Leaf size=330

$$-\frac{bf^2\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}-\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right.}{}$$

```
[Out] -1/3*b*f^2*Ci(b/(d*x+c)^3)*cos(a)/d^3-1/3*I*exp(I*a)*f*(-c*f+d*e)*(-I*b/(d*
x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-I*b/(d*x+c)^3)/d^3+1/3*I*f*(-c*f+d*e)*(
I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,I*b/(d*x+c)^3)/d^3/exp(I*a)-1/6*I
*exp(I*a)*(-c*f+d*e)^2*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-I*b/(d*x+
c)^3)/d^3+1/6*I*(-c*f+d*e)^2*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,I*b/(
d*x+c)^3)/d^3/exp(I*a)+1/3*b*f^2*Si(b/(d*x+c)^3)*sin(a)/d^3+1/3*f^2*(d*x+c)
^3*sin(a+b/(d*x+c)^3)/d^3
```

Rubi [A]
time = 0.20, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3514, 3446, 2239, 3504, 2250, 3460, 3378, 3384, 3380, 3383}

$$-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3d^3}-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)^2\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^3}+\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)^2\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^3}-\frac{bf^2\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}+\frac{bf^2\sin(a)\text{Si}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^3}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] -1/3*(b*f^2*Cos[a]*CosIntegral[b/(c + d*x)^3])/d^3 - ((I/3)*E^(I*a)*f*(d*e
- c*f)*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I)*b)/(c + d
*x)^3])/d^3 + ((I/3)*f*(d*e - c*f)*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Ga
mma[-2/3, (I*b)/(c + d*x)^3])/(d^3*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)^2*
(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d
^3 + ((I/6)*(d*e - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (
I*b)/(c + d*x)^3])/(d^3*E^(I*a)) + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^3])
/(3*d^3) + (b*f^2*Sin[a]*SinIntegral[b/(c + d*x)^3])/(3*d^3)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right) dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+\frac{b}{x^3}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a\right.\right.\right.}{d^3}$$

$$= \frac{f^2\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\text{Subst}\left(\int x\,\text{s}\right.}{d}$$

$$= -\frac{f^2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{(c+dx)^3}\right)}{3d^3}+\frac{(if(de-cf))\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}\right.}{d^3}$$

$$= -\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(d}{}$$

$$= -\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(d}{}$$

$$= -\frac{bf^2\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}-\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(\right.}{3d^3}$$

Mathematica [A]
time = 1.59, size = 405, normalized size = 1.23

$$\frac{\frac{3bf(de-cf)\left(\sqrt[3]{-\frac{ib}{(c+dx)^3}}\,\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)(\cos(a)-i\sin(a))+\sqrt[3]{\frac{ib}{(c+dx)^3}}\,\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)(\cos(a)+i\sin(a))\right)}{2\sqrt[3]{\frac{b^2}{(c+dx)^6}}\,(c+dx)}+\frac{3b(de-cf)^2\left(\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)(\cos(a)-i\sin(a))+\left(\frac{ib}{(c+dx)^3}\right)^{2/3}\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)(\cos(a)+i\sin(a))\right)}{2\left(\frac{b^2}{(c+dx)^6}\right)^{2/3}(c+dx)^2}+(c+dx)\left(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2)\right)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)+(c+dx)\left(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2)\right)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)-bf^2\left(\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^3}\right)-\sin(a)\text{Si}\left(\frac{b}{(c+dx)^3}\right)\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((3*b*f*(d*e - c*f)*((((-I)*b)/(c + d*x)^3)^(1/3)*Gamma[1/3, (I*b)/(c + d*x
)^3]*(Cos[a] - I*Sin[a]) + ((I*b)/(c + d*x)^3)^(1/3)*Gamma[1/3, ((-I)*b)/(c
```

```
 + d*x)^3]*(Cos[a] + I*Sin[a])))/(2*(b^2/(c + d*x)^6)^(1/3)*(c + d*x)) + (3
*b*(d*e - c*f)^2*((((-I)*b)/(c + d*x)^3)^(2/3)*Gamma[2/3, (I*b)/(c + d*x)^3
]*(Cos[a] - I*Sin[a]) + ((I*b)/(c + d*x)^3)^(2/3)*Gamma[2/3, ((-I)*b)/(c +
d*x)^3]*(Cos[a] + I*Sin[a])))/(2*(b^2/(c + d*x)^6)^(2/3)*(c + d*x)^2) + (c
+ d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos[
b/(c + d*x)^3]*Sin[a] + (c + d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2
 + 3*e*f*x + f^2*x^2))*Cos[a]*Sin[b/(c + d*x)^3] - b*f^2*(Cos[a]*CosIntegra
l[b/(c + d*x)^3] - Sin[a]*SinIntegral[b/(c + d*x)^3]))/(3*d^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

```
[Out] int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^
2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(1
/2*(b*d*f^2*x^3 + 3*b*d*f*x^2*e + 3*b*d*x*e^2)*cos((a*d^3*x^3 + 3*a*c*d^2*x
^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d
^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + integrate(1/2
*(b*d*f^2*x^3 + 3*b*d*f*x^2*e + 3*b*d*x*e^2)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2
 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((d^
4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a*d^3*x^3 + 3*a
*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*sin((a*
d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3))^2), x)
```

Fricas [A]
time = 0.13, size = 489, normalized size = 1.48

$$-\frac{bf^2\mathrm{Ei}\left(\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)e^{(ia)}+bf^2\mathrm{Ei}\left(-\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)e^{(-ia)}+3\left(-i\,cd^2f^2+i\,d^3fe\right)\left(\frac{ib}{d^3}\right)^{\frac{2}{3}}e^{(-ia)}\Gamma\left(\frac{1}{3},\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+3\left(i\,cd^2f^2-i\,d^3fe\right)\left(-\frac{ib}{d^3}\right)^{\frac{2}{3}}e^{(ia)}\Gamma\left(\frac{1}{3},-\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+3\left(i\,c^2df^2-2i\,cd^2fe+i\,d^3e^2\right)\left(\frac{ib}{d^3}\right)^{\frac{1}{3}}e^{(-ia)}\Gamma\left(\frac{2}{3},\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+3\left(-i\,c^2df^2+2i\,cd^2fe-i\,d^3e^2\right)\left(-\frac{ib}{d^3}\right)^{\frac{1}{3}}e^{(ia)}\Gamma\left(\frac{2}{3},-\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-2\left(d^3f^2x^3+c^3f^2+3\left(d^3x+cd^2\right)e^2+3\left(d^3fx^2-c^2df\right)e\right)\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(b*f^2*Ei(I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*e^(I*a) + b*f
^2*Ei(-I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*e^(-I*a) + 3*(-I*c*d^
2*f^2 + I*d^3*f*e)*(I*b/d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b/(d^3*x^3 + 3*c*d
^2*x^2 + 3*c^2*d*x + c^3)) + 3*(I*c*d^2*f^2 - I*d^3*f*e)*(-I*b/d^3)^(2/3)*e
^(I*a)*gamma(1/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 3*(I*c^
2*d*f^2 - 2*I*c*d^2*f*e + I*d^3*e^2)*(I*b/d^3)^(1/3)*e^(-I*a)*gamma(2/3, I*
b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 3*(-I*c^2*d*f^2 + 2*I*c*d^2*
f*e - I*d^3*e^2)*(-I*b/d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b/(d^3*x^3 + 3*c*d^
2*x^2 + 3*c^2*d*x + c^3)) - 2*(d^3*f^2*x^3 + c^3*f^2 + 3*(d^3*x + c*d^2)*e^
2 + 3*(d^3*f*x^2 - c^2*d*f)*e)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x
 + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)\,{\left(e+f\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^3)*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^3)*(e + f*x)^2, x)
```

3.183 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal. Leaf size=235

$$-\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}-\frac{ie^{ia}(de-cf}{}$$

```
[Out] -1/6*I*exp(I*a)*f*(-I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-I*b/(d*x+c)^
3)/d^2+1/6*I*f*(I*b/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,I*b/(d*x+c)^3)/d^
2/exp(I*a)-1/6*I*exp(I*a)*(-c*f+d*e)*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-
1/3,-I*b/(d*x+c)^3)/d^2+1/6*I*(-c*f+d*e)*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMM
A(-1/3,I*b/(d*x+c)^3)/d^2/exp(I*a)
```

Rubi [A]
time = 0.10, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3514, 3446, 2239, 3504, 2250}

$$-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^3}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((-1/6*I)*E^(I*a)*f*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((
-I)*b)/(c + d*x)^3])/d^2 + ((I/6)*f*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*G
amma[-2/3, (I*b)/(c + d*x)^3])/(d^2*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)*(
((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d^
2 + ((I/6)*(d*e - c*f)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (I*b
)/(c + d*x)^3])/(d^2*E^(I*a))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx &= \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+\frac{b}{x^3}\right)+fx\sin\left(a+\frac{b}{x^3}\right)\right)\,dx,x,c+dx\right)}{d^2}\\ &= \frac{f\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)\,dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\text{Subst}\left(\int \sin\left(a+\frac{b}{x}\right.\right.}{d^2}\\ &= \frac{(if)\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{2d^2}-\frac{(if)\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx,x,c+\right.}{2d^2}\\ &= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 700 vs. $2(235) = 470$.
time = 1.49, size = 700, normalized size = 2.98

$$\frac{e(c+dx)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)}{d}+\frac{f(-c+dx)(c+dx)\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)}{2d^2}+\frac{3bf\left(\frac{1}{2}\cos(a)\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)}+\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)}\right)+\frac{1}{2}i\left(\frac{\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)}-\frac{\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)}\right)\sin(a)\right)}{2d^2}+\frac{3be\left(\frac{1}{2}\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)+\frac{1}{2}i\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)\sin(a)\right)}{d}-\frac{3bcf\left(\frac{1}{2}\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)+\frac{1}{2}i\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{3\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2}\right)\sin(a)\right)}{d^2}+\frac{e(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{d}+\frac{f(-c+dx)(c+dx)\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] (e*(c + d*x)*Cos[b/(c + d*x)^3]*Sin[a])/d + (f*(-c + d*x)*(c + d*x)*Cos[b/(
c + d*x)^3]*Sin[a])/(2*d^2) + (3*b*f*((Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x
```

```
)^3]/(3*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)) + Gamma[1/3, (I*b)/(c + d*x
)^3]/(3*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x))))/2 + (I/2)*(Gamma[1/3, ((-I)*
b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)) - Gamma[1/3, (I*
b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)))*Sin[a]))/(2*d^2) +
 (3*b*e*((Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3
)^(2/3)*(c + d*x)^2) + Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)
^(2/3)*(c + d*x)^2)))/2 + (I/2)*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)
*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*
b)/(c + d*x)^3)^(2/3)*(c + d*x)^2))*Sin[a]))/d - (3*b*c*f*((Cos[a]*(Gamma[2
/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) + Ga
mma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2)))/2 +
 (I/2)*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c
 + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c
+ d*x)^2))*Sin[a]))/d^2 + (e*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^3])/d + (f*(-
c + d*x)*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^3])/(2*d^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

```
[Out] int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 +
b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(3/4*(b*d*f*x^2 +
2*b*d*x*e)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x
^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2
 + 4*c^3*d*x + c^4), x) + integrate(3/4*(b*d*f*x^2 + 2*b*d*x*e)*cos((a*d^3*
x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c
^2*d*x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*c
os((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2
*x^2 + 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3
*d*x + c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*
x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x)
```

Fricas [A]
time = 0.12, size = 326, normalized size = 1.39

$$\frac{-i\,d^2f\left(\frac{ib}{d^3}\right)^{\frac{2}{3}}e^{(-ia)}\Gamma\left(\frac{1}{3},\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+i\,d^2f\left(-\frac{ib}{d^3}\right)^{\frac{2}{3}}e^{(ia)}\Gamma\left(\frac{1}{3},-\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-2\left(-i\,cdf+i\,d^2e\right)\left(\frac{ib}{d^3}\right)^{\frac{1}{3}}e^{(-ia)}\Gamma\left(\frac{2}{3},\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-2\left(i\,cdf-i\,d^2e\right)\left(-\frac{ib}{d^3}\right)^{\frac{1}{3}}e^{(ia)}\Gamma\left(\frac{2}{3},-\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+2\left(d^2fx^2-c^2f+2\left(d^2x+cd\right)e\right)\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*d^2*f*(I*b/d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b/(d^3*x^3 + 3*c*d^2*x^
2 + 3*c^2*d*x + c^3)) + I*d^2*f*(-I*b/d^3)^(2/3)*e^(I*a)*gamma(1/3, -I*b/(d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*(-I*c*d*f + I*d^2*e)*(I*b/d^3)
^(1/3)*e^(-I*a)*gamma(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) -
 2*(I*c*d*f - I*d^2*e)*(-I*b/d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b/(d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d^2*f*x^2 - c^2*f + 2*(d^2*x + c*d)*e)
*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d
^2*x^2 + 3*c^2*d*x + c^3)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+\frac{b}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^3}\right)\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^3)*(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^3)*(e + f*x), x)
```

3.184 $\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal. Leaf size=107

$$-\frac{ie^{ia}\sqrt[3]{-\frac{ib}{(c+dx)^3}}\,(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}+\frac{ie^{-ia}\sqrt[3]{\frac{ib}{(c+dx)^3}}\,(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}$$

```
[Out] -1/6*I*exp(I*a)*(-I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-I*b/(d*x+c)^3)/d
+1/6*I*(I*b/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,I*b/(d*x+c)^3)/d/exp(I*a)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3446, 2239}

$$\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}\,\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}\,\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((-1/6*I)*E^(I*a)*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*
b)/(c + d*x)^3])/d + ((I/6)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3,
 (I*b)/(c + d*x)^3])/(d*E^(I*a))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx = \frac{1}{2}i\int e^{-ia-\frac{ib}{(c+dx)^3}}\,dx - \frac{1}{2}i\int e^{ia+\frac{ib}{(c+dx)^3}}\,dx$$

$$= -\frac{ie^{ia}\sqrt[3]{-\frac{ib}{(c+dx)^3}}\,(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d} + \frac{ie^{-ia}\sqrt[3]{\frac{ib}{(c+dx)^3}}\,(c+dx)\Gamma\left(\right.}{6d}$$

Mathematica [A]
time = 0.30, size = 203, normalized size = 1.90

$$\frac{b\cos(a)\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}+\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)+2(c+dx)^3\cos\left(\frac{b}{(c+dx)^3}\right)\sin(a)+ib\left(\frac{\Gamma\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}-\frac{\Gamma\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)\sin(a)+2(c+dx)^3\cos(a)\sin\left(\frac{b}{(c+dx)^3}\right)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^3],x]
```

```
[Out] (b*Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(2/3) +
Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)) + 2*(c + d*x)^3*Co
s[b/(c + d*x)^3]*Sin[a] + I*b*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(
c + d*x)^3)^(2/3) - Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)
)*Sin[a] + 2*(c + d*x)^3*Cos[a]*Sin[b/(c + d*x)^3])/(2*d*(c + d*x)^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 3*b*d*integrate(1/2*x*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3
+ b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*
```

```
c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + 3*b*d*integrate(1/2*x*cos((a*d^3*x^3 +
 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a
*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
+ 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
+ c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x) + x*sin((a*d^3*x^3 + 3*a*c*d^2*x^2
+ 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(77) = 154$.
time = 0.10, size = 175, normalized size = 1.64

$$\frac{-i\,d\left(\frac{i\,b}{d^3}\right)^{\frac{1}{3}}e^{(-i\,a)}\Gamma\left(\frac{2}{3},\frac{i\,b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+i\,d\left(-\frac{i\,b}{d^3}\right)^{\frac{1}{3}}e^{(i\,a)}\Gamma\left(\frac{2}{3},-\frac{i\,b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+2\,(dx+c)\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*(I*b/d^3)^(1/3)*e^(-I*a)*gamma(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3)) + I*d*(-I*b/d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d*x + c)*sin((a*d^3*x^3 + 3*a*c*d^2*x
^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+\frac{b}{(c+dx)^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{(c+d\,x)^3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^3), x)
```

3.185 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Mathematica [A]
time = 3.94, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**3)/(f*x+e),x)
```

```
[Out] Integral(sin(a + b/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(e + f*
x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^3}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^3)/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^3)/(e + f*x), x)
```

3.186 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2} dx$$

Mathematica [A]
time = 20.46, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^3}\right)}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**3)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^3}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^3)/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^3)/(e + f*x)^2, x)
```

3.187 $\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=410

$$-\frac{240f^2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3}+\frac{24f(de-cf)\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}-\frac{2(de-cf)^2\sqrt{c+dx}}{}$$

```
[Out] 40*f^2*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b^3/d^3-4*f*(-c*f+d*e)*(d*x+c)^
(3/2)*cos(a+b*(d*x+c)^(1/2))/b/d^3-2*f^2*(d*x+c)^(5/2)*cos(a+b*(d*x+c)^(1/2
))/b/d^3+240*f^2*sin(a+b*(d*x+c)^(1/2))/b^6/d^3-24*f*(-c*f+d*e)*sin(a+b*(d*
x+c)^(1/2))/b^4/d^3+2*(-c*f+d*e)^2*sin(a+b*(d*x+c)^(1/2))/b^2/d^3-120*f^2*(
d*x+c)*sin(a+b*(d*x+c)^(1/2))/b^4/d^3+12*f*(-c*f+d*e)*(d*x+c)*sin(a+b*(d*x+
c)^(1/2))/b^2/d^3+10*f^2*(d*x+c)^2*sin(a+b*(d*x+c)^(1/2))/b^2/d^3-240*f^2*c
os(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3+24*f*(-c*f+d*e)*cos(a+b*(d*x+c)
^(1/2))*(d*x+c)^(1/2)/b^3/d^3-2*(-c*f+d*e)^2*cos(a+b*(d*x+c)^(1/2))*(d*x+c)
^(1/2)/b/d^3
```

Rubi [A]
time = 0.28, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3512, 3377, 2717}

$$\frac{240f^2\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{240f^2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3}-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}-\frac{120f^2(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{24f\sqrt{c+dx}\,(de-cf)\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}+\frac{40f^2(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}+\frac{12f(c+dx)(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10f^2(c+dx)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{4f(c+dx)^{3/2}(de-cf)\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{2\sqrt{c+dx}\,(de-cf)^2\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{2f^2(c+dx)^{5/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-240*f^2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^5*d^3) + (24*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (2*(d*e - c*f)^2*S
qrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^3) + (40*f^2*(c + d*x)^(3/2)*Co
s[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (4*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^3) - (2*f^2*(c + d*x)^(5/2)*Cos[a + b*Sqrt[c + d*x
]])/(b*d^3) + (240*f^2*Sin[a + b*Sqrt[c + d*x]])/(b^6*d^3) - (24*f*(d*e - c
*f)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3) + (2*(d*e - c*f)^2*Sin[a + b*Sqrt[c
 + d*x]])/(b^2*d^3) - (120*f^2*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3
) + (12*f*(d*e - c*f)*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3) + (10*f
^2*(c + d*x)^2*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{2\text{Subst}\left(\int\left(\frac{(de-cf)^2 x\sin(a+bx)}{d^2}+\frac{2f(de-cf)x^3\sin(a+bx)}{d^2}+\frac{f^2x^5\sin(a+bx)}{d^2}\right)dx,\right.}{d} \\
&= \frac{(2f^2)\,\text{Subst}\left(\int x^5\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}+\frac{(4f(de-cf))\text{Subs}}{} \\
&= -\frac{2(de-cf)^2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{4f(de-cf)(c+dx)^3}{} \\
&= -\frac{2(de-cf)^2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{4f(de-cf)(c+dx)^3}{} \\
&= \frac{24f(de-cf)\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}-\frac{2(de-cf)^2\sqrt{c+dx}}{b} \\
&= \frac{24f(de-cf)\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3}-\frac{2(de-cf)^2\sqrt{c+dx}}{b} \\
&= -\frac{240f^2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3}+\frac{24f(de-cf)\sqrt{c+dx}\,\cos}{b^3d^3} \\
&= -\frac{240f^2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3}+\frac{24f(de-cf)\sqrt{c+dx}\,\cos}{b^3d^3}
\end{aligned}$$

Mathematica [A]
time = 1.00, size = 138, normalized size = 0.34

$$\frac{-2b\sqrt{c+dx}\,(120f^2+b^4d^2(e+fx)^2-4b^2f(3de+2cf+5dfx))\cos\left(a+b\sqrt{c+dx}\right)+2(120f^2-12b^2f(4cf+d(e+5fx))+b^4d(e+fx)(4cf+d(e+5fx)))\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*(120*f^2 + b^4*d^2*(e + f*x)^2 - 4*b^2*f*(3*d*e + 2*c*f
 + 5*d*f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(120*f^2 - 12*b^2*f*(4*c*f + d*(e
 + 5*f*x)) + b^4*d*(e + f*x)*(4*c*f + d*(e + 5*f*x)))*Sin[a + b*Sqrt[c + d*
x]])/(b^6*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(374) = 748$.
time = 0.06, size = 1246, normalized size = 3.04

method	result	size
derivativedivides	Expression too large to display	1246
default	Expression too large to display	1246

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3/b^2*(a*c^2*f^2*cos(a+b*(d*x+c)^(1/2))-2*a*c*d*e*f*cos(a+b*(d*x+c)^(1/
2))+a*d^2*e^2*cos(a+b*(d*x+c)^(1/2))+c^2*f^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(
d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-2*c*d*e*f*(sin(a+b*(d*x+c)^(1/2))-(a+
b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+d^2*e^2*(sin(a+b*(d*x+c)^(1/2))-(a
+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-2/b^2*a^3*c*f^2*cos(a+b*(d*x+c)^(
1/2))+2/b^2*a^3*d*e*f*cos(a+b*(d*x+c)^(1/2))-6/b^2*a^2*c*f^2*(sin(a+b*(d*x+
c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+6/b^2*a^2*d*e*f*(sin(
a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*f^
2*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+2*cos(a+b*(d*x+c)^(1/2))+2
*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-6/b^2*a*d*e*f*(-(a+b*(d*x+c)^(
1/2))^2*cos(a+b*(d*x+c)^(1/2))+2*cos(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2
))*sin(a+b*(d*x+c)^(1/2)))-2/b^2*c*f^2*(-(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x
+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-6*sin(a+b*(d*x+c)
^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+2/b^2*d*e*f*(-(a+b*(d
*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+
c)^(1/2))-6*sin(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1
/2)))+1/b^4*a^5*f^2*cos(a+b*(d*x+c)^(1/2))+5/b^4*a^4*f^2*(sin(a+b*(d*x+c)^(
1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-10/b^4*a^3*f^2*(-(a+b*(d*
x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+2*cos(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c
)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*f^2*(-(a+b*(d*x+c)^(1/2))^3*cos
(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-6*sin(a+
b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-5/b^4*a*f^2*
(-(a+b*(d*x+c)^(1/2))^4*cos(a+b*(d*x+c)^(1/2))+4*(a+b*(d*x+c)^(1/2))^3*sin(
a+b*(d*x+c)^(1/2))+12*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))-24*cos(a
+b*(d*x+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+1/b^4*f^2*
(-(a+b*(d*x+c)^(1/2))^5*cos(a+b*(d*x+c)^(1/2))+5*(a+b*(d*x+c)^(1/2))^4*sin(
a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))-60*(a+b*
(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))+120*sin(a+b*(d*x+c)^(1/2))-120*(a+b
*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(380) = 760$.
time = 0.37, size = 1105, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*c^2*f^2*cos(sqrt(d*x + c)*b + a)/d^2 - 2*a*c*f*cos(sqrt(d*x + c)*b + a
)*e/d - ((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)
*b + a))*c^2*f^2/d^2 - 2*a^3*c*f^2*cos(sqrt(d*x + c)*b + a)/(b^2*d^2) + a*c
os(sqrt(d*x + c)*b + a)*e^2 + 2*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b
+ a) - sin(sqrt(d*x + c)*b + a))*c*f*e/d + 2*a^3*f*cos(sqrt(d*x + c)*b + a)
*e/(b^2*d) + 6*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d
*x + c)*b + a))*a^2*c*f^2/(b^2*d^2) + a^5*f^2*cos(sqrt(d*x + c)*b + a)/(b^4
*d^2) - ((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)
*b + a))*e^2 - 6*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt
(d*x + c)*b + a))*a^2*f*e/(b^2*d) - 5*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x +
 c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^4*f^2/(b^4*d^2) - 6*(((sqrt(d*x +
c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqr
t(d*x + c)*b + a))*a*c*f^2/(b^2*d^2) + 6*(((sqrt(d*x + c)*b + a)^2 - 2)*cos
(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a
*f*e/(b^2*d) + 10*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) -
 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a^3*f^2/(b^4*d^2) + 2*((
(sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b + a)
 - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*c*f^2/(b^2*d^2
) - 2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*x + c
)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*f*e/(b
^2*d) - 10*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*
x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*a
^2*f^2/(b^4*d^2) + 5*(((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2
 + 24)*cos(sqrt(d*x + c)*b + a) - 4*((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x +
 c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))*a*f^2/(b^4*d^2) - (((sqrt(d*x + c)*b
 + a)^5 - 20*(sqrt(d*x + c)*b + a)^3 + 120*sqrt(d*x + c)*b + 120*a)*cos(sqr
t(d*x + c)*b + a) - 5*((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2
 + 24)*sin(sqrt(d*x + c)*b + a))*f^2/(b^4*d^2))/(b^2*d)
```

Fricas [A]
time = 0.35, size = 196, normalized size = 0.48

$$-\frac{2\left(\left(b^5d^2f^2x^2+b^5d^2e^2-20\,b^3df^2x-8\,(b^3c-15\,b)f^2+2\,(b^5d^2fx-6\,b^3df)e\right)\sqrt{dx+c}\,\cos\left(\sqrt{dx+c}\,b+a\right)-\left(5\,b^4d^2f^2x^2+b^4d^2e^2+4\,(b^4c-15\,b^2)df^2x-24\,(2\,b^2c-5)f^2+2\,(3\,b^4d^2fx+2\,(b^4c-3\,b^2)df)e\right)\sin\left(\sqrt{dx+c}\,b+a\right)\right)}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b^5*d^2*f^2*x^2 + b^5*d^2*e^2 - 20*b^3*d*f^2*x - 8*(b^3*c - 15*b)*f^2
+ 2*(b^5*d^2*f*x - 6*b^3*d*f)*e)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) - (
5*b^4*d^2*f^2*x^2 + b^4*d^2*e^2 + 4*(b^4*c - 15*b^2)*d*f^2*x - 24*(2*b^2*c
- 5)*f^2 + 2*(3*b^4*d^2*f*x + 2*(b^4*c - 3*b^2)*d*f)*e)*sin(sqrt(d*x + c)*b
 + a))/(b^6*d^3)
```

Sympy [A]
time = 0.38, size = 529, normalized size = 1.29

$$\begin{cases} \left(e^2x + efx^2 + \frac{f^2x^3}{3}\right)\sin(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \left(e^2x + efx^2 + \frac{f^2x^3}{3}\right)\sin\left(a + b\sqrt{c}\right) & \text{for } d = 0 \\ -\frac{2e^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{4efx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2f^2x^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{8cef\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{8cf^2x\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{2e^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{12efx\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{10f^2x^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{16cf^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{24ef\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} + \frac{40f^2x\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{96cf^2\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{24ef\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} - \frac{120f^2x\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} - \frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3} + \frac{240f^2\sin\left(a+b\sqrt{c+dx}\right)}{b^6d^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise(((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a), Eq(b, 0) & (Eq(b, 0) |
 Eq(d, 0))), ((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a + b*sqrt(c)), Eq(d, 0
)), (-2*e**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 4*e*f*x*sqrt(c
+ d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 2*f**2*x**2*sqrt(c + d*x)*cos(a + b
*sqrt(c + d*x))/(b*d) + 8*c*e*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 8*c*
f**2*x*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e**2*sin(a + b*sqrt(c + d*x
))/(b**2*d) + 12*e*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d) + 10*f**2*x**2*sin
(a + b*sqrt(c + d*x))/(b**2*d) + 16*c*f**2*sqrt(c + d*x)*cos(a + b*sqrt(c +
 d*x))/(b**3*d**3) + 24*e*f*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d*
*2) + 40*f**2*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*f
**2*sin(a + b*sqrt(c + d*x))/(b**4*d**3) - 24*e*f*sin(a + b*sqrt(c + d*x))/
(b**4*d**2) - 120*f**2*x*sin(a + b*sqrt(c + d*x))/(b**4*d**2) - 240*f**2*sq
rt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**5*d**3) + 240*f**2*sin(a + b*sqrt(
c + d*x))/(b**6*d**3), True))
```

Giac [A]
time = 2.46, size = 701, normalized size = 1.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(f^2*(((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b +
a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^
2*b^2*c + 2*a^3*b^2*c + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)^4
*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*(s
qrt(d*x + c)*b + a)*a^4 - a^5 + 12*(sqrt(d*x + c)*b + a)*b^2*c - 12*a*b^2*c
 - 20*(sqrt(d*x + c)*b + a)^3 + 60*(sqrt(d*x + c)*b + a)^2*a - 60*(sqrt(d*x
 + c)*b + a)*a^2 + 20*a^3 + 120*sqrt(d*x + c)*b)*cos(sqrt(d*x + c)*b + a)/(
b^4*d^2) - (b^4*c^2 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c + 12*(sqrt(d*x + c)*b
```

```
 + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)^4 - 20*(sqrt(d*x + c)
*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sqrt(d*x + c)*b + a)*a^3
 + 5*a^4 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x + c)*b + a
)*a - 60*a^2 + 120)*sin(sqrt(d*x + c)*b + a)/(b^4*d^2))/b + (sqrt(d*x + c)*
b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e^2/b - 2*f*(((sqrt(
d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c
)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 + 6*sqrt(d*x + c)*b)*cos(s
qrt(d*x + c)*b + a)/b^2 - (b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x
+ c)*b + a)*a - 3*a^2 + 6)*sin(sqrt(d*x + c)*b + a)/b^2)*e/(b*d))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin \left(a + b\sqrt{c + d\,x}\right)\,(e + f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x)^2, x)
```

3.188 $\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=185

$$\frac{12f\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2(de-cf)\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{2f(c+dx)^{3/2}\cos\left(a+b\surd\right.}{bd^2}$$

```
[Out] -2*f*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(1/2))/b/d^2-12*f*sin(a+b*(d*x+c)^(1/2))
/b^4/d^2+2*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/2))/b^2/d^2+6*f*(d*x+c)*sin(a+b*(d
*x+c)^(1/2))/b^2/d^2+12*f*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*(-
c*f+d*e)*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3512, 3377, 2717}

$$-\frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12f\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2}+\frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{2\sqrt{c+dx}\,(de-cf)\cos\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (12*f*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*(d*e - c*f)*Sq
rt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^2) - (2*f*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^2) - (12*f*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (
2*(d*e - c*f)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^2) + (6*f*(c + d*x)*Sin[a +
b*Sqrt[c + d*x]])/(b^2*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\text{Subst}\left(\int\left(\frac{(de-cf)x\sin(a+bx)}{d}+\frac{fx^3\sin(a+bx)}{d}\right)\,dx,x,\sqrt{c+dx}\right)}{d}$$

$$= \frac{(2f)\text{Subst}\left(\int x^3\sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2(de-cf))\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{2f(c+dx)^{3/2}\cos\left(a+\right.}{bd^2}$$

$$= -\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{2f(c+dx)^{3/2}\cos\left(a+\right.}{bd^2}$$

$$= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\right.}{bd^2}$$

$$= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\right.}{bd^2}$$

Mathematica [A]
time = 0.28, size = 85, normalized size = 0.46

$$\frac{-2b\sqrt{c+dx}\,(-6f+b^2d(e+fx))\cos\left(a+b\sqrt{c+dx}\right)+2(-6f+b^2(2cf+d(e+3fx)))\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*(-6*f + b^2*d*(e + f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(
-6*f + b^2*(2*c*f + d*(e + 3*f*x)))*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(167) = 334$.
time = 0.02, size = 366, normalized size = 1.98

method	result
derivativedivides	$\frac{-2acf\cos\left(a+b\sqrt{dx+c}\right)+2ade\cos\left(a+b\sqrt{dx+c}\right)-2cf\left(\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\cos\left(a\right.\right.}{}$
default	$\frac{-2acf\cos\left(a+b\sqrt{dx+c}\right)+2ade\cos\left(a+b\sqrt{dx+c}\right)-2cf\left(\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\cos\left(a\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)

[Out] 2/d^2/b^2*(-a*c*f*cos(a+b*(d*x+c)^(1/2))+a*d*e*cos(a+b*(d*x+c)^(1/2))-c*f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+d*e*(sin (a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+1/b^2*a^3*f *cos(a+b*(d*x+c)^(1/2))+3/b^2*a^2*f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1 /2))*cos(a+b*(d*x+c)^(1/2)))-3/b^2*a*f*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x +c)^(1/2))+2*cos(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+1/b^2*f*(-(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-6*sin(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(169) = 338$.
time = 0.29, size = 350, normalized size = 1.89

$$-\frac{2\left(\frac{acf\cos\left(\sqrt{dx+c}\,b+a\right)}{d}-a\cos\left(\sqrt{dx+c}\,b+a\right)e-\frac{\left(\left(\sqrt{dx+c}\,b+a\right)\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)cf}{d}-\frac{a^3f\cos\left(\sqrt{dx+c}\,b+a\right)}{b^2d}+\left(\left(\sqrt{dx+c}\,b+a\right)\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)e+\frac{3\left(\left(\sqrt{dx+c}\,b+a\right)\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)a^2f}{b^2d}-\frac{3\left(\left(\left(\sqrt{dx+c}\,b+a\right)^2-2\right)\cos\left(\sqrt{dx+c}\,b+a\right)-2\left(\sqrt{dx+c}\,b+a\right)\sin\left(\sqrt{dx+c}\,b+a\right)\right)af}{b^2d}+\frac{\left(\left(\left(\sqrt{dx+c}\,b+a\right)^3-6\sqrt{dx+c}\,b-6a\right)\cos\left(\sqrt{dx+c}\,b+a\right)-3\left(\left(\sqrt{dx+c}\,b+a\right)^2-2\right)\sin\left(\sqrt{dx+c}\,b+a\right)\right)f}{b^2d}\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] -2*(a*c*f*cos(sqrt(d*x + c)*b + a)/d - a*cos(sqrt(d*x + c)*b + a)*e - ((sqr t(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c*f/ d - a^3*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) + ((sqrt(d*x + c)*b + a)*cos(sqr t(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e + 3*((sqrt(d*x + c)*b + a)* cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*f/(b^2*d) - 3*(((s qrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a*f/(b^2*d) + (((sqrt(d*x + c)*b + a)^3 - 6*sq rt(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*f/(b^2*d))/(b^2*d)

Fricas [A]
time = 0.37, size = 88, normalized size = 0.48

$$-\frac{2\left(\left(b^3dfx+b^3de-6\,bf\right)\sqrt{dx+c}\,\cos\left(\sqrt{dx+c}\,b+a\right)-\left(3\,b^2dfx+b^2de+2\,\left(b^2c-3\right)f\right)\sin\left(\sqrt{dx+c}\,b+a\right)\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] -2*((b^3*d*f*x + b^3*d*e - 6*b*f)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) - (3*b^2*d*f*x + b^2*d*e + 2*(b^2*c - 3)*f)*sin(sqrt(d*x + c)*b + a))/(b^4*d^ 2)

Sympy [A]
time = 0.22, size = 221, normalized size = 1.19

$$\begin{cases} \left(ex + \frac{fx^2}{2}\right)\sin(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \left(ex + \frac{fx^2}{2}\right)\sin\left(a + b\sqrt{c}\right) & \text{for } d = 0 \\ -\frac{2e\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2fx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{4cf\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{2e\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{6fx\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise(((e*x + f*x**2/2)*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), ((e*
x + f*x**2/2)*sin(a + b*sqrt(c)), Eq(d, 0)), (-2*e*sqrt(c + d*x)*cos(a + b*
sqrt(c + d*x))/(b*d) - 2*f*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) +
 4*c*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e*sin(a + b*sqrt(c + d*x))/
(b**2*d) + 6*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d) + 12*f*sqrt(c + d*x)*cos
(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*f*sin(a + b*sqrt(c + d*x))/(b**4*d**
2), True))
```

Giac [A]
time = 5.96, size = 219, normalized size = 1.18

$$-\frac{2\left(\frac{\left(\sqrt{dx+c}\,b\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)e}{b}-\frac{f\left(\frac{\left(\left(\sqrt{dx+c}\,b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}\,b+a\right)^3+3\left(\sqrt{dx+c}\,b+a\right)^2a-3\left(\sqrt{dx+c}\,b+a\right)a^2+a^3+6\sqrt{dx+c}\,b\right)\cos\left(\sqrt{dx+c}\,b+a\right)}{b^2}-\frac{\left(b^2c-3\left(\sqrt{dx+c}\,b+a\right)^2+6\left(\sqrt{dx+c}\,b+a\right)a-3a^2+6\right)\sin\left(\sqrt{dx+c}\,b+a\right)}{b^2}\right)}{bd}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e
/b - f*(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 +
3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 + 6*sqrt(d*
x + c)*b)*cos(sqrt(d*x + c)*b + a)/b^2 - (b^2*c - 3*(sqrt(d*x + c)*b + a)^2
 + 6*(sqrt(d*x + c)*b + a)*a - 3*a^2 + 6)*sin(sqrt(d*x + c)*b + a)/b^2)/(b*
d))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + b\sqrt{c + d\,x}\right)\,(e + f\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/2))*(e + f*x), x)
```

3.189 $\int \sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] 2*sin(a+b*(d*x+c)^(1/2))/b^2/d-2*cos(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3442, 3377, 2717}

$$\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}-\frac{2\sqrt{c+dx}\,\cos\left(a+b\sqrt{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d) + (2*Sin[a + b*Sqrt[c + d
*x]])/(b^2*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{2\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd}$$
$$= -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.93

$$\frac{-2b\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)+2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]] + 2*Sin[a + b*Sqrt[c + d*x]])/
(b^2*d)
```

Maple [A]
time = 0.01, size = 61, normalized size = 1.13

method	result	size
derivativedivides	$\frac{2\sin\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)+2a\cos\left(a+b\sqrt{dx+c}\right)}{b^2d}$	61
default	$\frac{2\sin\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)+2a\cos\left(a+b\sqrt{dx+c}\right)}{b^2d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))+
a*cos(a+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.28, size = 62, normalized size = 1.15

$$-\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)\cos\left(\sqrt{dx+c}\,b+a\right)-a\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - a*cos(sqrt(d*x + c)*b
+ a) - sin(sqrt(d*x + c)*b + a))/(b^2*d)
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.81

$$-\frac{2\left(\sqrt{dx+c}\,b\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(b
^2*d)
```

Sympy [A]
time = 0.15, size = 65, normalized size = 1.20

$$\begin{cases} x\sin{(a)} & \text{for}\: b=0\wedge(b=0\vee d=0) \\ x\sin{\left(a+b\sqrt{c}\right)} & \text{for}\: d=0 \\ -\frac{2\sqrt{c+dx}\cos{\left(a+b\sqrt{c+dx}\right)}}{bd}+\frac{2\sin{\left(a+b\sqrt{c+dx}\right)}}{b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sin(a + b*sqrt(c
)), Eq(d, 0)), (-2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) + 2*sin(a +
 b*sqrt(c + d*x))/(b**2*d), True))
```

Giac [A]
time = 6.44, size = 44, normalized size = 0.81

$$-\frac{2\left(\sqrt{dx+c}\,b\cos\left(\sqrt{dx+c}\,b+a\right)-\sin\left(\sqrt{dx+c}\,b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(b
^2*d)
```

Mupad [B]
time = 4.73, size = 43, normalized size = 0.80

$$\frac{2\left(\sin\left(a+b\sqrt{c+d\,x}\right)-b\,\cos\left(a+b\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/2)),x)
[Out] (2*(sin(a + b*(c + d*x)^(1/2)) - b*cos(a + b*(c + d*x)^(1/2))*(c + d*x)^(1/
2)))/(b^2*d)
```

3.190 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$

Optimal. Leaf size=238

$$\frac{\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f}+\frac{\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)\sin\left(a+\frac{b\sqrt{}}{}\right.}{f}$$

```
[Out] cos(a+b*(c*f-d*e)^(1/2)/f^(1/2))*Si(-b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1
/2))/f+cos(a-b*(c*f-d*e)^(1/2)/f^(1/2))*Si(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x
+c)^(1/2))/f+Ci(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))*sin(a-b*(c*f-d*e
)^(1/2)/f^(1/2))/f+Ci(b*(c*f-d*e)^(1/2)/f^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*(c
*f-d*e)^(1/2)/f^(1/2))/f
```

Rubi [A]
time = 0.52, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3512, 3384, 3380, 3383}

$$\frac{\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{f}+\frac{\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f}-\frac{\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f}+\frac{\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{\sqrt{cf-de}\,b}{\sqrt{f}}+\sqrt{c+dx}\,b\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]]*Sin[a - (b*S
qrt[-(d*e) + c*f])/Sqrt[f]])/f + (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f
] - b*Sqrt[c + d*x]]*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]])/f - (Cos[a +
(b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f]
- b*Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegra
l[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

<u>Rule 3512</u>

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

<u>Rubi steps</u>

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \frac{2\text{Subst}\left(\int \left(-\frac{d\sin(a+bx)}{2\sqrt{f}\left(\sqrt{-de+cf}-\sqrt{f}\,x\right)}+\frac{d\sin(a+bx)}{2\sqrt{f}\left(\sqrt{-de+cf}+\sqrt{f}\,x\right)}\right)\right.}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{-de+cf}-\sqrt{f}\,x}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{f}} + \frac{\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{-de+cf}+\sqrt{}}\right.}{\sqrt{f}}$$

$$= \frac{\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}\,x}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{f}} +$$

$$= \frac{\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f} + \frac{\text{Ci}\left(\frac{b\sqrt{-de+c}}{\sqrt{f}}\right.}{}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 0.64, size = 238, normalized size = 1.00

$$\frac{ie^{-i\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}\left(\text{Ei}\left(-ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)-e^{2i\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}\text{Ei}\left(ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)+e^{\frac{2ib\sqrt{-de+cf}}{\sqrt{f}}}\text{Ei}\left(-ib\left(\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)-e^{2ia}\text{Ei}\left(ib\left(\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] ((I/2)*(ExpIntegralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x]
)] - E^((2*I)*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*ExpIntegralEi[I*b*(-(Sq
rt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])] + E^(((2*I)*b*Sqrt[-(d*e) + c*f
])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x
])] - E^((2*I)*a)*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
d*x])]))/(E^(I*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(197) = 394$.
time = 0.06, size = 793, normalized size = 3.33

method	result
derivativedivides	$-\frac{b^2\left(af+\sqrt{b^2cf^2-b^2def}\right)\left(-\operatorname{sinIntegral}\left(-b\sqrt{dx+c}-a+\frac{af+\sqrt{b^2cf^2-b^2def}}{f}\right)\cos\left(\frac{af+\sqrt{b^2cf^2}}{f}\right.\right.}{f^2\left(-\frac{af+\sqrt{b^2cf^2}}{f}\right.}$
default	$-\frac{b^2\left(af+\sqrt{b^2cf^2-b^2def}\right)\left(-\operatorname{sinIntegral}\left(-b\sqrt{dx+c}-a+\frac{af+\sqrt{b^2cf^2-b^2def}}{f}\right)\cos\left(\frac{af+\sqrt{b^2cf^2}}{f}\right.\right.}{f^2\left(-\frac{af+\sqrt{b^2cf^2}}{f}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-1/2*b^2*(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/(-(a*f+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2
))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(a*f+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/2
*b^2*(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos
((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-a*b^2*(-1
/2/f/(-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(a*f+
(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci
(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2
-b^2*d*e*f)^(1/2))/f))-1/2/f/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(
-b*(d*x+c)^(1/2)-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^2*c*f
^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.50, size = 266, normalized size = 1.12

$$\frac{-i\,\mathrm{Ei}\left(i\sqrt{dx+c}\,b-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(ia+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}-i\,\mathrm{Ei}\left(i\sqrt{dx+c}\,b+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(ia-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}+i\,\mathrm{Ei}\left(-i\sqrt{dx+c}\,b-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(-ia+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}+i\,\mathrm{Ei}\left(-i\sqrt{dx+c}\,b+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(-ia-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(I*sqrt(d*x + c)*b - sqrt(-(b^2*c*f - b^2*d*e)/f))*e^(I*a + sqrt(
-(b^2*c*f - b^2*d*e)/f)) - I*Ei(I*sqrt(d*x + c)*b + sqrt(-(b^2*c*f - b^2*d*
e)/f))*e^(I*a - sqrt(-(b^2*c*f - b^2*d*e)/f)) + I*Ei(-I*sqrt(d*x + c)*b - s
qrt(-(b^2*c*f - b^2*d*e)/f))*e^(-I*a + sqrt(-(b^2*c*f - b^2*d*e)/f)) + I*Ei
(-I*sqrt(d*x + c)*b + sqrt(-(b^2*c*f - b^2*d*e)/f))*e^(-I*a - sqrt(-(b^2*c*
f - b^2*d*e)/f)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*sqrt(c + d*x))/(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,\sqrt{c+d\,x}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x), x)
```

3.191 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=339

$$\frac{bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}}-\frac{bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Ci}\left(\frac{b\sqrt{-de+c}}{\sqrt{f}}\right.}{2f^{3/2}\sqrt{-de+cf}}$$

```
[Out] -sin(a+b*(d*x+c)^(1/2))/f/(f*x+e)-1/2*b*d*Ci(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d
*x+c)^(1/2))*cos(a-b*(c*f-d*e)^(1/2)/f^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)+1/2*b
*d*Ci(b*(c*f-d*e)^(1/2)/f^(1/2)-b*(d*x+c)^(1/2))*cos(a+b*(c*f-d*e)^(1/2)/f^
(1/2))/f^(3/2)/(c*f-d*e)^(1/2)+1/2*b*d*Si(b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+
c)^(1/2))*sin(a-b*(c*f-d*e)^(1/2)/f^(1/2))/f^(3/2)/(c*f-d*e)^(1/2)-1/2*b*d*
Si(-b*(c*f-d*e)^(1/2)/f^(1/2)+b*(d*x+c)^(1/2))*sin(a+b*(c*f-d*e)^(1/2)/f^(1
/2))/f^(3/2)/(c*f-d*e)^(1/2)
```

Rubi [A]

time = 0.68, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$\frac{bd\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}-\frac{bd\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}+\frac{bd\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}+\frac{bd\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{Si}\left(\frac{\sqrt{cf-de}\,b}{\sqrt{f}}+\sqrt{c+dx}\,b\right)}{2f^{3/2}\sqrt{cf-de}}-\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] (b*d*Cos[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c
*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - (b*d*Cos[
a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt
[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - Sin[a + b*Sqrt[c +
 d*x]]/(f*(e + f*x)) + (b*d*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinInte
gral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d
*e) + c*f]) + (b*d*Sin[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*S
qrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f
])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x\sin(a+bx)}{\left(e-\frac{cf}{d}+\frac{fx^2}{d}\right)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{e-\frac{cf}{d}+\frac{fx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-de+cf}\cos(a+bx)}{2\left(e-\frac{cf}{d}\right)\left(\sqrt{-de+cf}-\sqrt{f}\,x\right)} + \frac{\sqrt{}}{2\left(e-\frac{cf}{d}\right.}\right.\right.}{f}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{-de+cf}-\sqrt{f}\,x}\,dx, x, \sqrt{c+dx}\right)}{2f\sqrt{-de+cf}}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt{-d}}{\sqrt{}}\right.}{\sqrt{-de+}}\right.}{2f\sqrt{-de+cf}}$$

$$= \frac{bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} - \frac{bd\cos\left(a-\frac{b}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.24, size = 397, normalized size = 1.17

$$\frac{ide^{-ia}\left(-\frac{2e^{-ib\sqrt{c+dx}}\sqrt{f}}{de+dfx}-\frac{ibe^{-\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{Ei}\left(-ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{-de+cf}}+\frac{ibe^{\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{Ei}\left(-ib\left(\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{-de+cf}}+e^{2ia}\left(\frac{2e^{ib\sqrt{c+dx}}\sqrt{f}}{de+dfx}-\frac{ibe^{\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{Ei}\left(ib\left(-\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{-de+cf}}+\frac{ibe^{-\frac{ib\sqrt{-de+cf}}{\sqrt{f}}}\text{Ei}\left(ib\left(\frac{\sqrt{-de+cf}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{-de+cf}}\right)\right)}{4f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] ((I/4)*d*((-2*Sqrt[f])/(E^(I*b*Sqrt[c + d*x])*(d*e + d*f*x)) - (I*b*ExpInte
gralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/(E^((I*b*Sq
rt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]) + (I*b*E^((I*b*Sqrt[-(d*e) +
 c*f])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
 d*x])])/Sqrt[-(d*e) + c*f] + E^((2*I)*a)*((2*E^(I*b*Sqrt[c + d*x])*Sqrt[f]
)/(d*e + d*f*x) - (I*b*E^((I*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*ExpIntegralEi[I
*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/Sqrt[-(d*e) + c*f] + (
```

```
I*b*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x])])/(E^((I
*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]))))/(E^(I*a)*f^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(273) = 546$.
time = 0.07, size = 1831, normalized size = 5.40

method	result	size
derivativedivides	Expression too large to display	1831
default	Expression too large to display	1831

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^2*(sin(a+b*(d*x+c)^(1/2))*(-1/2*a*b^2/(c*f-d*e)*(a+b*(d*x+c)^(1/2))+1
/2*b^2*(-b^2*c*f+b^2*d*e+a^2*f)/(c*f-d*e)/f)/(-c*f*b^2+d*e*b^2+a^2*f-2*a*f*
(a+b*(d*x+c)^(1/2))+f*(a+b*(d*x+c)^(1/2))^2)+1/4*a*b^2/(c*f-d*e)/f/(-(a*f+(
b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(a*f+(b^2*c*f^2-b^
2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1
/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(
1/2))/f))+1/4*a*b^2/(c*f-d*e)/f/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-
Si(-b*(d*x+c)^(1/2)-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^2*
c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)
^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4*b^2*(-c*f*b^2+d*e
*b^2+a^2*f-a*(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2)))/(c*f-d*e)/f^2/(-(a*f+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f+a)*(Si(-b*(d*x+c)^(1/2)-a+(a*f+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-
(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
f))+1/4*b^2*(-c*f*b^2+d*e*b^2+a^2*f+a*(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2)))/(
c*f-d*e)/f^2/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)
-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/f)+Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*
f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-a*b^4*(sin(a+b*(d*x+c)^(1/2))*(-1/2/b^2/
(c*f-d*e)*(a+b*(d*x+c)^(1/2))+1/2*a/b^2/(c*f-d*e))/(-c*f*b^2+d*e*b^2+a^2*f-
2*a*f*(a+b*(d*x+c)^(1/2))+f*(a+b*(d*x+c)^(1/2))^2)+1/4/b^2/(c*f-d*e)/f/(-(a
*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*(-Si(-b*(d*x+c)^(1/2)-a+(a*f+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c
)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f))+1/4/b^2/(c*f-d*e)/f/((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f+a)*
(-Si(-b*(d*x+c)^(1/2)-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^
2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4/f/b^2/(c*f-d*e)
*(Si(-b*(d*x+c)^(1/2)-a+(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*
c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^
(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4/f/b^2/(c*f-d*e)*(-S
```

```
i(-b*(d*x+c)^(1/2)-a-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^
(1/2))/f)*cos((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.47, size = 446, normalized size = 1.32

$$-\frac{(i\,dfx+i\,de)\sqrt{-\frac{b^2cf-b^2de}{f}}\,\mathrm{Ei}\left(i\sqrt{dx+c}\,b-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(ia+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}+(-i\,dfx-i\,de)\sqrt{-\frac{b^2cf-b^2de}{f}}\,\mathrm{Ei}\left(i\sqrt{dx+c}\,b+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(ia-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}+(-i\,dfx-i\,de)\sqrt{-\frac{b^2cf-b^2de}{f}}\,\mathrm{Ei}\left(-i\sqrt{dx+c}\,b-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(-ia+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}+(i\,dfx+i\,de)\sqrt{-\frac{b^2cf-b^2de}{f}}\,\mathrm{Ei}\left(-i\sqrt{dx+c}\,b+\sqrt{-\frac{b^2cf-b^2de}{f}}\right)e^{\left(-ia-\sqrt{-\frac{b^2cf-b^2de}{f}}\right)}+4\,(cf-de)\sin\left(\sqrt{dx+c}\,b+a\right)}{4\,(cf^3x-dfe^2-(df^2x-cf^2)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((I*d*f*x + I*d*e)*sqrt(-(b^2*c*f - b^2*d*e)/f)*Ei(I*sqrt(d*x + c)*b -
 sqrt(-(b^2*c*f - b^2*d*e)/f))*e^(I*a + sqrt(-(b^2*c*f - b^2*d*e)/f)) + (-I
*d*f*x - I*d*e)*sqrt(-(b^2*c*f - b^2*d*e)/f)*Ei(I*sqrt(d*x + c)*b + sqrt(-(
b^2*c*f - b^2*d*e)/f))*e^(I*a - sqrt(-(b^2*c*f - b^2*d*e)/f)) + (-I*d*f*x -
 I*d*e)*sqrt(-(b^2*c*f - b^2*d*e)/f)*Ei(-I*sqrt(d*x + c)*b - sqrt(-(b^2*c*f
 - b^2*d*e)/f))*e^(-I*a + sqrt(-(b^2*c*f - b^2*d*e)/f)) + (I*d*f*x + I*d*e)
*sqrt(-(b^2*c*f - b^2*d*e)/f)*Ei(-I*sqrt(d*x + c)*b + sqrt(-(b^2*c*f - b^2*
d*e)/f))*e^(-I*a - sqrt(-(b^2*c*f - b^2*d*e)/f)) + 4*(c*f - d*e)*sin(sqrt(d
*x + c)*b + a))/(c*f^3*x - d*f*e^2 - (d*f^2*x - c*f^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*sqrt(c + d*x))/(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\sqrt{c+d\,x}\right)}{\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x)^2,x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/2))/(e + f*x)^2, x)
```

3.192 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal. Leaf size=382

$$-\frac{4f(de-cf)\sqrt{c+dx}\,\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}-\frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}-\frac{2e^{ia}f(de-cf)\sqrt{c+dx}}{9bd^3\sqrt[3]{-ib}}$$

```
[Out] -2/3*f^2*(d*x+c)^(3/2)*cos(a+b*(d*x+c)^(3/2))/b/d^3+1/3*I*exp(I*a)*(-c*f+d*
e)^2*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d^3/(-I*b*(d*x+c)^(3/2))^(2/3)-1
/3*I*(-c*f+d*e)^2*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d^3/exp(I*a)/(I*b*(d
*x+c)^(3/2))^(2/3)+2/3*f^2*sin(a+b*(d*x+c)^(3/2))/b^2/d^3-4/3*f*(-c*f+d*e)*
cos(a+b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^3-2/9*exp(I*a)*f*(-c*f+d*e)*GAMMA(
1/3,-I*b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^3/(-I*b*(d*x+c)^(3/2))^(1/3)-2/9*
f*(-c*f+d*e)*GAMMA(1/3,I*b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^3/exp(I*a)/(I*b
*(d*x+c)^(3/2))^(1/3)
```

Rubi [A]
time = 0.21, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3514, 3470, 2250, 3466, 3437, 2239, 3460, 3377, 2717}

$$-\frac{2e^{ia}f\sqrt{c+dx}\,(de-cf)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{2e^{-ia}f\sqrt{c+dx}\,(de-cf)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}}+\frac{ie^{ia}(c+dx)(de-cf)^2\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^3\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)(de-cf)^2\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^3\left(ib(c+dx)^{3/2}\right)^{2/3}}+\frac{2f^2\sin\left(a+b(c+dx)^{3/2}\right)}{3b^2d^3}-\frac{4f\sqrt{c+dx}\,(de-cf)\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}-\frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-4*f*(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*
f^2*(c + d*x)^(3/2)*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*E^(I*a)*f*(d
*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^3*((-I)*
b*(c + d*x)^(3/2))^(1/3)) - (2*f*(d*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, I*b*(
c + d*x)^(3/2)])/(9*b*d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(
I*a)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^3*((-I)
*b*(c + d*x)^(3/2))^(2/3)) - ((I/3)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, I*b*
(c + d*x)^(3/2)])/(d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3)) + (2*f^2*Sin[a
+ b*(c + d*x)^(3/2)])/(3*b^2*d^3)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
```

```
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right) dx = \frac{2\text{Subst}\left(\int \left((de-cf)^2 x \sin\left(a+bx^3\right) - 2f(-de+cf)x^3 \sin\left(a+bx^3\right)\right.\right.}{d^3}$$

$$= \frac{\left(2f^2\right) \text{Subst}\left(\int x^5 \sin\left(a+bx^3\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\text{Su}}{}$$

$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} + \frac{\left(2f^2\right)\text{Subst}\left(\int x \sin\right.}{}$$

$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} - \frac{2f^2(c+dx)^{3/2}\cos}{3b}$$

$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3} - \frac{2f^2(c+dx)^{3/2}\cos}{3b}$$

Mathematica [A]
time = 3.00, size = 383, normalized size = 1.00

$$\frac{i\left(e^{-ia}\left(\frac{3e^{-ib(c+dx)^{3/2}}f\left(f+2ibde\sqrt{c+dx}-ibf(c-dx)\sqrt{c+dx}\right)}{b^2}+\frac{2f(de-cf)\left(ib(c+dx)^{3/2}\right)^{2/3}\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{b^2(c+dx)}-\frac{3(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)-(\cos(a)+i\sin(a))\left(\frac{2f(-de+cf)(c+dx)^2\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{4/3}}-\frac{3(de-cf)^2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{2/3}}+\frac{3f\left(f-2ibde\sqrt{c+dx}+ibf(c-dx)\sqrt{c+dx}\right)\left(\cos\left(b(c+dx)^{3/2}\right)+i\sin\left(b(c+dx)^{3/2}\right)\right)}{b^2}\right)\right)}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/9)*(((3*f*(f + (2*I)*b*d*e*Sqrt[c + d*x] - I*b*f*(c - d*x)*Sqrt[c + d*x
]))/(b^2*E^(I*b*(c + d*x)^(3/2))) + (2*f*(d*e - c*f)*(I*b*(c + d*x)^(3/2))^
(2/3)*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(b^2*(c + d*x)) - (3*(d*e - c*f)^2*(
c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(I*b*(c + d*x)^(3/2))^(2/3))/E^(I
*a) - (Cos[a] + I*Sin[a])*((2*f*(-(d*e) + c*f)*(c + d*x)^2*Gamma[1/3, (-I)*
b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3/2))^(4/3) - (3*(d*e - c*f)^2*(c +
d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3/2))^(2/3) + (
3*f*(f - (2*I)*b*d*e*Sqrt[c + d*x] + I*b*f*(c - d*x)*Sqrt[c + d*x])*(Cos[b*
(c + d*x)^(3/2)] + I*Sin[b*(c + d*x)^(3/2)]))/b^2)))/d^3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (fx+e)^2 \sin\left(a+b(dx+c)^{\frac{3}{2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

<u>Maxima</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(295) = 590.
time = 0.65, size = 695, normalized size = 1.82

$$-\frac{\frac{3\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)c^2f^2}{\sqrt{dx+c}\,bd^2}-\frac{6\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)cfe}{\sqrt{dx+c}\,bd}+\frac{3\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)e^2}{\sqrt{dx+c}\,b}-\frac{2\left(12\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\sqrt{dx+c}\cos\left((dx+c)^{\frac{3}{2}}b+a\right)+\sqrt{dx+c}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)\right)cf^2}{\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}bd^2}+\frac{2\left(12\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\sqrt{dx+c}\cos\left((dx+c)^{\frac{3}{2}}b+a\right)+\sqrt{dx+c}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)\right)fe}{\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}bd}+\frac{12\left((dx+c)^{\frac{3}{2}}b\cos\left((dx+c)^{\frac{3}{2}}b+a\right)-\sin\left((dx+c)^{\frac{3}{2}}b+a\right)\right)f^2}{b^2d^2}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -1/18*(3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(
3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt
(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(
d*x + c)^(3/2)*b))*sin(a))*c^2*f^2/(sqrt(d*x + c)*b*d^2) - 6*((d*x + c)^(3/
2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)
*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(
d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(
a))*c*f*e/(sqrt(d*x + c)*b*d) + 3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)
*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3
/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqr
t(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))*e^2/(sqrt(d*x + c)*b) -
 2*(12*((d*x + c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) +
 sqrt(d*x + c)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) +
 I)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3,
 I*(d*x + c)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*s
in(a)))*c*f^2/(((d*x + c)^(3/2)*b)^(1/3)*b*d^2) + 2*(12*((d*x + c)^(3/2)*b)
^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) + sqrt(d*x + c)*(((sqrt(3)
- I)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c
)^(3/2)*b))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (I
*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(a)))*f*e/(((d*x + c)^(3
/2)*b)^(1/3)*b*d) + 12*((d*x + c)^(3/2)*b*cos((d*x + c)^(3/2)*b + a) - sin(
(d*x + c)^(3/2)*b + a))*f^2/(b^2*d^2))/d
```

<u>Fricas</u> [**A**]
time = 0.13, size = 281, normalized size = 0.74

$$-\frac{2\left(icf^2-idfe\right)(ib)^{\frac{2}{3}}e^{(-ia)}\Gamma\left(\frac{1}{3},\left(ibdx+ibc\right)\sqrt{dx+c}\right)+2\left(-icf^2+idfe\right)(-ib)^{\frac{2}{3}}e^{(ia)}\Gamma\left(\frac{1}{3},\left(-ibdx-ibc\right)\sqrt{dx+c}\right)+3\left(bc^2f^2-2bcdfe+bd^2e^2\right)(ib)^{\frac{1}{3}}e^{(-ia)}\Gamma\left(\frac{2}{3},\left(ibdx+ibc\right)\sqrt{dx+c}\right)+3\left(bc^2f^2-2bcdfe+bd^2e^2\right)(-ib)^{\frac{1}{3}}e^{(ia)}\Gamma\left(\frac{2}{3},\left(-ibdx-ibc\right)\sqrt{dx+c}\right)-6f^2\sin\left(\left(bdx+bc\right)\sqrt{dx+c}+a\right)+6\left(bdf^2x-bcf^2+2bdfe\right)\sqrt{dx+c}\cos\left(\left(bdx+bc\right)\sqrt{dx+c}+a\right)}{9b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(I*c*f^2 - I*d*f*e)*(I*b)^(2/3)*e^(-I*a)*gamma(1/3, (I*b*d*x + I*b*
c)*sqrt(d*x + c)) + 2*(-I*c*f^2 + I*d*f*e)*(-I*b)^(2/3)*e^(I*a)*gamma(1/3,
(-I*b*d*x - I*b*c)*sqrt(d*x + c)) + 3*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)
*(I*b)^(1/3)*e^(-I*a)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) + 3*(b*c^
2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*(-I*b)^(1/3)*e^(I*a)*gamma(2/3, (-I*b*d*x
- I*b*c)*sqrt(d*x + c)) - 6*f^2*sin((b*d*x + b*c)*sqrt(d*x + c) + a) + 6*(b
*d*f^2*x - b*c*f^2 + 2*b*d*f*e)*sqrt(d*x + c)*cos((b*d*x + b*c)*sqrt(d*x +
c) + a))/(b^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin \left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)\,{\left(e+f\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x)^2, x)
```

3.193 $\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal. Leaf size=291

$$-\frac{2f\sqrt{c+dx}\,\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}-\frac{e^{ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{e^{-ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/}\right.}{9bd^2\sqrt[3]{ib(c+dx)^{3/2}}}$$

```
[Out] 1/3*I*exp(I*a)*(-c*f+d*e)*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d^2/(-I*b*(
d*x+c)^(3/2))^(2/3)-1/3*I*(-c*f+d*e)*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d
^2/exp(I*a)/(I*b*(d*x+c)^(3/2))^(2/3)-2/3*f*cos(a+b*(d*x+c)^(3/2))*(d*x+c)^
(1/2)/b/d^2-1/9*exp(I*a)*f*GAMMA(1/3,-I*b*(d*x+c)^(3/2))*(d*x+c)^(1/2)/b/d^
2/(-I*b*(d*x+c)^(3/2))^(1/3)-1/9*f*GAMMA(1/3,I*b*(d*x+c)^(3/2))*(d*x+c)^(1/
2)/b/d^2/exp(I*a)/(I*b*(d*x+c)^(3/2))^(1/3)
```

Rubi [A]

time = 0.13, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3514, 3470, 2250, 3466, 3437, 2239}

$$\frac{ie^{ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}-\frac{e^{ia}f\sqrt{c+dx}\,\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{e^{-ia}f\sqrt{c+dx}\,\text{Gamma}\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{ib(c+dx)^{3/2}}}-\frac{2f\sqrt{c+dx}\,\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^2) - (E^(I*a)*f*Sqrt
[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^2*((-I)*b*(c + d*x)^(3
/2))^(1/3)) - (f*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(9*b*d^2*E^
(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(I*a)*(d*e - c*f)*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^2*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (
(I/3)*(d*e - c*f)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d^2*E^(I*a)*(
I*b*(c + d*x)^(3/2))^(2/3))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx = \frac{2\text{Subst}\left(\int \left((de-cf)x\sin\left(a+bx^3\right)+fx^3\sin\left(a+bx^3\right)\right)\,dx, x, \sqrt{c+\right.}{d^2}$$

$$= \frac{(2f)\text{Subst}\left(\int x^3\sin\left(a+bx^3\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(2(de-cf))\text{Subst}}{}$$

$$= -\frac{2f\sqrt{c+dx}\,\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2} + \frac{(2f)\text{Subst}\left(\int \cos\left(a+bx^3\right)\,dx\right.}{3bd^2}$$

$$= -\frac{2f\sqrt{c+dx}\,\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2} + \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},-i\right.}{3d^2\left(-ib(c+dx)^{3/2}\right.}$$

$$= -\frac{2f\sqrt{c+dx}\,\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2} - \frac{e^{ia}f\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx\right.}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 705 vs. $2(291) = 582$.
time = 1.65, size = 705, normalized size = 2.42

$$-\frac{2f\sqrt{c+dx}\cos(a)\cos\left(b(c+dx)^{3/2}\right)}{3bd^2}+\frac{f\cos(a)\left(-\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)}{6bd^2}-\frac{ie\cos(a)\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}+\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{2d}+\frac{icf\cos(a)\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}+\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)}{2d^2}+\frac{if\left(-\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}}+\frac{2\sqrt{c+dx}\,\Gamma\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)\sin(a)}{6bd^2}+\frac{e\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)\sin(a)}{2d}-\frac{cf\left(-\frac{2(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{2(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}\right)\sin(a)}{2d^2}+\frac{2f\sqrt{c+dx}\sin(a)\sin\left(b(c+dx)^{3/2}\right)}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a]*Cos[b*(c + d*x)^(3/2)])/(3*b*d^2) + (f*Cos[a]*((
-2*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(
3/2))^(1/3)) - (2*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c
 + d*x)^(3/2))^(1/3))))/(6*b*d^2) - ((I/2)*e*Cos[a]*((-2*(c + d*x)*Gamma[2/
3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*
x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d + (
(I/2)*c*f*Cos[a]*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I
)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])
/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d^2 + ((I/6)*f*((-2*Sqrt[c + d*x]*Gamma[
1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(1/3)) + (2*Sqrt[
c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(1/3)))*
Sin[a])/(b*d^2) + (e*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*
((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/
2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))*Sin[a])/(2*d) - (c*f*((-2*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*
(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))
*Sin[a])/(2*d^2) + (2*f*Sqrt[c + d*x]*Sin[a]*Sin[b*(c + d*x)^(3/2)])/(3*b*d
^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (fx+e)\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

Maxima [A]
time = 0.63, size = 377, normalized size = 1.30

$$\frac{\frac{3\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)cf}{\sqrt{dx+c}\,bd}-\frac{3\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)e}{\sqrt{dx+c}\,b}-\frac{\left(12\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\sqrt{dx+c}\cos\left((dx+c)^{\frac{3}{2}}b+a\right)+\sqrt{dx+c}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},i(dx+c)^{\frac{3}{2}}b\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-i(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)\right)f}{\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}bd}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/18*(3*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3
/2)*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(
3) - 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d
*x + c)^(3/2)*b))*sin(a))*c*f/(sqrt(d*x + c)*b*d) - 3*((d*x + c)^(3/2)*b)^(
1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (sqrt(3) - I)*gamma(
2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c
)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(a))*e/(
sqrt(d*x + c)*b) - (12*((d*x + c)^(3/2)*b)^(1/3)*sqrt(d*x + c)*cos((d*x + c
)^(3/2)*b + a) + sqrt(d*x + c)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)^(3/2)
*b) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(a) + ((-I*sqrt(3)
 - 1)*gamma(1/3, I*(d*x + c)^(3/2)*b) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x
+ c)^(3/2)*b))*sin(a)))*f/(((d*x + c)^(3/2)*b)^(1/3)*b*d))/d
```

Fricas [**A**]
time = 0.13, size = 187, normalized size = 0.64

$$\frac{i\,(i\,b)^{\frac{2}{3}}\,fe^{(-i\,a)}\Gamma\left(\frac{1}{3},(i\,bdx+i\,bc)\sqrt{dx+c}\right)-i\,(-i\,b)^{\frac{2}{3}}\,fe^{(i\,a)}\Gamma\left(\frac{1}{3},(-i\,bdx-i\,bc)\sqrt{dx+c}\right)-6\,\sqrt{dx+c}\,bf\cos\left((bdx+bc)\sqrt{dx+c}+a\right)+3\,(bcf-bde)(i\,b)^{\frac{1}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{2}{3},(i\,bdx+i\,bc)\sqrt{dx+c}\right)+3\,(bcf-bde)(-i\,b)^{\frac{1}{3}}\,e^{(i\,a)}\Gamma\left(\frac{2}{3},(-i\,bdx-i\,bc)\sqrt{dx+c}\right)}{9\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/9*(I*(I*b)^(2/3)*f*e^(-I*a)*gamma(1/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) -
 I*(-I*b)^(2/3)*f*e^(I*a)*gamma(1/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)) - 6*
sqrt(d*x + c)*b*f*cos((b*d*x + b*c)*sqrt(d*x + c) + a) + 3*(b*c*f - b*d*e)*
(I*b)^(1/3)*e^(-I*a)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) + 3*(b*c*f
 - b*d*e)*(-I*b)^(1/3)*e^(I*a)*gamma(2/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c))
)/(b^2*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,(c+d\,x)^{3/2}\right)\,(e+f\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(3/2))*(e + f*x), x)
```

3.194 $\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal. Leaf size=115

$$\frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

```
[Out] 1/3*I*exp(I*a)*(d*x+c)*GAMMA(2/3,-I*b*(d*x+c)^(3/2))/d/(-I*b*(d*x+c)^(3/2))
^(2/3)-1/3*I*(d*x+c)*GAMMA(2/3,I*b*(d*x+c)^(3/2))/d/exp(I*a)/(I*b*(d*x+c)^(
3/2))^(2/3)
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3444, 3470, 2250}

$$\frac{ie^{ia}(c+dx)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/3)*E^(I*a)*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d*((-I)*b*(c
+ d*x)^(3/2))^(2/3)) - ((I/3)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d
*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin\left(a+b(c+dx)^{3/2}\right)\,dx &= \frac{2\text{Subst}\left(\int x\sin\left(a+bx^3\right)\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{i\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx, x, \sqrt{c+dx}\right)}{d} - \frac{i\text{Subst}\left(\int e^{ia+ibx^3}x\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3}, -ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3}, ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 123, normalized size = 1.07

$$\frac{i(c+dx)\left(-\left(-ib(c+dx)^{3/2}\right)^{2/3}\Gamma\left(\frac{2}{3}, ib(c+dx)^{3/2}\right)(\cos(a)-i\sin(a))+\left(ib(c+dx)^{3/2}\right)^{2/3}\Gamma\left(\frac{2}{3}, -ib(c+dx)^{3/2}\right)(\cos(a)+i\sin(a))\right)}{3d\left(b^2(c+dx)^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/3)*(c + d*x)*(-(((-I)*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, I*b*(c + d*x)
^(3/2)]*(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, (-I)*
b*(c + d*x)^(3/2)]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^3)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+b(dx+c)^{\frac{3}{2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2)),x)
```

Maxima [A]
time = 0.35, size = 112, normalized size = 0.97

$$-\frac{\left((dx+c)^{\frac{3}{2}}b\right)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3}, i\,(dx+c)^{\frac{3}{2}}b\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3}, -i\,(dx+c)^{\frac{3}{2}}b\right)\right)\cos(a)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3}, i\,(dx+c)^{\frac{3}{2}}b\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3}, -i\,(dx+c)^{\frac{3}{2}}b\right)\right)\sin(a)\right)}{6\sqrt{dx+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -1/6*((d*x + c)^(3/2)*b)^(1/3)*(((sqrt(3) + I)*gamma(2/3, I*(d*x + c)^(3/2)
*b) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(a) - ((I*sqrt(3)
- 1)*gamma(2/3, I*(d*x + c)^(3/2)*b) + (-I*sqrt(3) - 1)*gamma(2/3, -I*(d*x
+ c)^(3/2)*b))*sin(a))/(sqrt(d*x + c)*b*d)
```

Fricas [A]
time = 0.10, size = 69, normalized size = 0.60

$$-\frac{(i\,b)^{\frac{1}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{2}{3},(i\,bdx+i\,bc)\sqrt{dx+c}\right)+(-i\,b)^{\frac{1}{3}}\,e^{(i\,a)}\Gamma\left(\frac{2}{3},(-i\,bdx-i\,bc)\sqrt{dx+c}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
[Out] -1/3*((I*b)^(1/3)*e^(-I*a)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) + (-
I*b)^(1/3)*e^(I*a)*gamma(2/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(a+b(c+dx)^{\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(3/2)),x)
[Out] Integral(sin(a + b*(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
[Out] integrate(sin((d*x + c)^(3/2)*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(3/2)),x)
[Out] int(sin(a + b*(c + d*x)^(3/2)), x)
```

3.195 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

Mathematica [A]
time = 5.89, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bc\sqrt{c + dx} + bdx\sqrt{c + dx}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin \left(a+b\,{\left(c+d\,x\right)}^{3/2}\right)}{e+f\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x), x)
```

3.196 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2},x\right)$$

[Out] Unintegrable(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx$$

Verification is not applicable to the result.

[In] Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]

[Out] Defer[Int][Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx$$

Mathematica [A]
time = 8.43, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2} \, dx$$

Verification is not applicable to the result.

[In] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]

[Out] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{3}{2}}\right)}{(fx+e)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bc\sqrt{c + dx} + bdx\sqrt{c + dx}\right)}{(e + fx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x))/(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin \left(a+b\,(c+d\,x)^{3/2}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(3/2))/(e + f*x)^2, x)
```

3.197 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$

Optimal. Leaf size=611

$$\frac{b^5 f^2 \sqrt{c+dx}\, \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360 d^3}-\frac{b^3 f (de-cf) \sqrt{c+dx}\, \cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6 d^3}+\frac{b (de-cf)^2 \sqrt{c+dx}\, \text{co}}{d^3}$$

```
[Out] -1/180*b^3*f^2*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^3+1/3*b*f*(-c*f+d*e)*
(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^3+1/15*b*f^2*(d*x+c)^(5/2)*cos(a+b/(
d*x+c)^(1/2))/d^3+1/360*b^6*f^2*cos(a)*Si(b/(d*x+c)^(1/2))/d^3-1/6*b^4*f*(-
c*f+d*e)*cos(a)*Si(b/(d*x+c)^(1/2))/d^3+b^2*(-c*f+d*e)^2*cos(a)*Si(b/(d*x+c
)^(1/2))/d^3+1/360*b^6*f^2*Ci(b/(d*x+c)^(1/2))*sin(a)/d^3-1/6*b^4*f*(-c*f+d
*e)*Ci(b/(d*x+c)^(1/2))*sin(a)/d^3+b^2*(-c*f+d*e)^2*Ci(b/(d*x+c)^(1/2))*sin
(a)/d^3+1/360*b^4*f^2*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^3-1/6*b^2*f*(-c*f+d*
e)*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^
(1/2))/d^3-1/60*b^2*f^2*(d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^3+f*(-c*f+d*e)*(
d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(1/2)
)/d^3+1/360*b^5*f^2*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d^3-1/6*b^3*f*(-c*
f+d*e)*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d^3+b*(-c*f+d*e)^2*cos(a+b/(d*x
+c)^(1/2))*(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.55, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3512, 3378, 3384, 3380, 3383}

$$\frac{b^6 f^2 \sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3}+\frac{b^6 f^2 \cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3}+\frac{b^5 f^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3}-\frac{b^4 f\sin(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3}-\frac{b^4 f\cos(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3}+\frac{b^4 f^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3}-\frac{b^3 f\sqrt{c+dx}(de-cf)\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3}-\frac{b^3 f^2(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{180d^3}+\frac{b^2\sin(a)(de-cf)^2\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{b^2\cos(a)(de-cf)^2\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}-\frac{b^2 f(c+dx)(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3}-\frac{b^2 f^2(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{60d^3}+\frac{f(c+dx)^2(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{(c+dx)(de-cf)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{bf(c+dx)^{3/2}(de-cf)\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3}+\frac{b\sqrt{c+dx}(de-cf)^2\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3}+\frac{bf^2(c+dx)^{5/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b^5*f^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(360*d^3) - (b^3*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(6*d^3) + (b*(d*e - c*f)^2*Sqr
t[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^3 - (b^3*f^2*(c + d*x)^(3/2)*Cos[a +
 b/Sqrt[c + d*x]])/(180*d^3) + (b*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a + b/S
qrt[c + d*x]])/(3*d^3) + (b*f^2*(c + d*x)^(5/2)*Cos[a + b/Sqrt[c + d*x]])/(
15*d^3) + (b^6*f^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(360*d^3) - (b^4*f*
(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(6*d^3) + (b^2*(d*e - c*f)
^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^3 + (b^4*f^2*(c + d*x)*Sin[a + b/
Sqrt[c + d*x]])/(360*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)*Sin[a + b/Sqrt[c +
 d*x]])/(6*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d^3 -
(b^2*f^2*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(60*d^3) + (f*(d*e - c*f)*(c
```

```
 + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/Sqrt[c
 + d*x]])/(3*d^3) + (b^6*f^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(360*d^3)
 - (b^4*f*(d*e - c*f)*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(6*d^3) + (b^2*(
d*e - c*f)^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = -\frac{2\text{Subst}\left(\int \left(\frac{f^2\sin(a+bx)}{d^2x^7}+\frac{2f(de-cf)\sin(a+bx)}{d^2x^5}+\frac{(de-cf)^2\sin(a+bx)}{d^2x^3}\right) dx, x}{d}$$

$$= -\frac{(2f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^7}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d^3} - \frac{(4f(de-cf))\text{Subst}\left(\right.}{}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin}{d^3}$$

$$= \frac{b(de-cf)^2\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{3/}}{3}$$

$$= \frac{b(de-cf)^2\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{3/}}{3}$$

$$= -\frac{b^3f(de-cf)\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+}}{}$$

$$= -\frac{b^3f(de-cf)\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+}}{}$$

$$= \frac{b^5f^2\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3f(de-cf)\sqrt{c+dx}\,\cos\left(\right.}{6d^3}$$

$$= \frac{b^5f^2\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3f(de-cf)\sqrt{c+dx}\,\cos\left(\right.}{6d^3}$$

$$= \frac{b^5f^2\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3f(de-cf)\sqrt{c+dx}\,\cos\left(\right.}{6d^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.36, size = 557, normalized size = 0.91

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] ((I/720)*((Sqrt[c + d*x]*((-I)*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] + (2*I)*b^3*
f*(30*d*e - 29*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x
```

```
) + 120*Sqrt[c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x +
 f^2*x^2)) - (24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*
e*f*x + f^2*x^2))))/E^((I*b)/Sqrt[c + d*x]) - E^(I*(2*a + b/Sqrt[c + d*x]))
*Sqrt[c + d*x]*(I*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] - (2*I)*b^3*f*(30*d*e - 2
9*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x) + 120*Sqrt[
c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2)) +
(24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*e*f*x + f^2*x
^2))) + b^2*(360*d^2*e^2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^
2)*f^2)*ExpIntegralEi[((-I)*b)/Sqrt[c + d*x]] - b^2*E^((2*I)*a)*(360*d^2*e^
2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^2)*f^2)*ExpIntegralEi[(
I*b)/Sqrt[c + d*x]]))/(d^3*E^(I*a))
```

Maple [A]
time = 0.51, size = 696, normalized size = 1.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d^3*b^2*(b^4*f^2*(-1/6*sin(a+b/(d*x+c)^(1/2))/b^6*(d*x+c)^3-1/30*cos(a+b
/(d*x+c)^(1/2))/b^5*(d*x+c)^(5/2)+1/120*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^
2+1/360*cos(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)-1/720*sin(a+b/(d*x+c)^(1/2
))/b^2*(d*x+c)-1/720*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/720*Si(b/(d*x
+c)^(1/2))*cos(a)-1/720*Ci(b/(d*x+c)^(1/2))*sin(a))+d^2*e^2*(-1/2*sin(a+b/(
d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si
(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*sin(a))+c^2*f^2*(-1/2*sin(
a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1
/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*sin(a))-2*b^2*c*f^2*(
-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*cos(a+b/(d*x+c)^(1/2))/b^3*(
d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)+1/24*cos(a+b/(d*x+c)^(
1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))*cos(a)+1/24*Ci(b/(d*x+c)^(1/
2))*sin(a))+2*f*b^2*d*e*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*cos
(a+b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+
c)+1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))*cos
(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(a))-2*c*d*e*f*(-1/2*sin(a+b/(d*x+c)^(1/2))
/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1
/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*sin(a)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.78, size = 878, normalized size = 1.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/720*(360*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) +
 (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x
```

```
+ c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x
 + c)*a + b)/sqrt(d*x + c)))*c^2*f^2/d^2 - 720*(((-I*Ei(I*b/sqrt(d*x + c))
+ I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(
d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d
*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*f*e/d -
60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/
sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3
 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x
+ c)*b^2 - 6*(d*x + c)^2)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*f^2/d
^2 + 360*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (
Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x +
c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c)))*e^2 + 60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei(-I*b/
sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*s
in(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(d*x + c)
*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*sin((sqrt(d*x +
c)*a + b)/sqrt(d*x + c)))*f*e/d + (((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/s
qrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*si
n(a))*b^6 + 2*(sqrt(d*x + c)*b^5 - 2*(d*x + c)^(3/2)*b^3 + 24*(d*x + c)^(5/
2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*((d*x + c)*b^4 - 6*(d*x
+ c)^2*b^2 + 120*(d*x + c)^3)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f^2
/d^2)/d
```

Fricas [**A**]
time = 0.43, size = 457, normalized size = 0.75

$$\frac{(360\,b^2d^2e^2 - 60\,(b^4 + 12\,b^2c)dfe + (b^6 + 60\,b^4c + 360\,b^2c^2)f^2)\,\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a) + (360\,b^2d^2e^2 - 60\,(b^4 + 12\,b^2c)dfe + (b^6 + 60\,b^4c + 360\,b^2c^2)f^2)\,\mathrm{Ci}\left(-\frac{b}{\sqrt{dx+c}}\right)\sin(a) + 2\,(360\,b^2d^2e^2 - 60\,(b^4 + 12\,b^2c)dfe + (b^6 + 60\,b^4c + 360\,b^2c^2)f^2)\cos(a)\,\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right) + 2\,(24\,bd^2f^2x^2 - 2\,(b^3 + 36\,bc)df^2x + 360\,bd^2e^2 + (b^5 + 58\,b^3c + 264\,bc^2)f^2 + 60\,(2\,bd^2fx - (b^3 + 10\,bc)df)e)\sqrt{dx+c}\cos\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right) - 2\,(6\,b^2d^2f^2x^2 - 120\,d^3f^2x^3 - (b^4 + 48\,b^2c)df^2x - (b^4c + 54\,b^2c^2 + 120\,c^3)f^2 - 360\,(d^3x + cd^2)e^2 + 60\,(b^2d^2fx - 6\,d^3fx^2 + (b^2c + 6\,c^2)df)e)\sin\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right)}{720\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/720*((360*b^2*d^2*e^2 - 60*(b^4 + 12*b^2*c)*d*f*e + (b^6 + 60*b^4*c + 360
*b^2*c^2)*f^2)*cos_integral(b/sqrt(d*x + c))*sin(a) + (360*b^2*d^2*e^2 - 60
*(b^4 + 12*b^2*c)*d*f*e + (b^6 + 60*b^4*c + 360*b^2*c^2)*f^2)*cos_integral(
-b/sqrt(d*x + c))*sin(a) + 2*(360*b^2*d^2*e^2 - 60*(b^4 + 12*b^2*c)*d*f*e +
 (b^6 + 60*b^4*c + 360*b^2*c^2)*f^2)*cos(a)*sin_integral(b/sqrt(d*x + c)) +
 2*(24*b*d^2*f^2*x^2 - 2*(b^3 + 36*b*c)*d*f^2*x + 360*b*d^2*e^2 + (b^5 + 58
*b^3*c + 264*b*c^2)*f^2 + 60*(2*b*d^2*f*x - (b^3 + 10*b*c)*d*f)*e)*sqrt(d*x
 + c)*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)) - 2*(6*b^2*d^2*f^2*x^2
 - 120*d^3*f^2*x^3 - (b^4 + 48*b^2*c)*d*f^2*x - (b^4*c + 54*b^2*c^2 + 120*c
^3)*f^2 - 360*(d^3*x + c*d^2)*e^2 + 60*(b^2*d^2*f*x - 6*d^3*f*x^2 + (b^2*c
+ 6*c^2)*d*f)*e)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/d^3
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin \left(a+\frac{b}{\sqrt{c+dx}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/2)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6606 vs. $2(548) = 1096$.
time = 5.78, size = 6606, normalized size = 10.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/360*((a^6*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(
a) - a^6*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) -
 6*(sqrt(d*x + c)*a + b)*a^5*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sq
rt(d*x + c))*sin(a)/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)*a^5*b^7*cos(a)*
sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 15*(s
qrt(d*x + c)*a + b)^2*a^4*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(
d*x + c))*sin(a)/(d*x + c) + 60*a^6*b^5*c*cos_integral(-a + (sqrt(d*x + c)*
a + b)/sqrt(d*x + c))*sin(a) - 15*(sqrt(d*x + c)*a + b)^2*a^4*b^7*cos(a)*si
n_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 60*a^6*b^5*
c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 20*(sqrt(d
*x + c)*a + b)^3*a^3*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x +
 c))*sin(a)/(d*x + c)^(3/2) - 360*(sqrt(d*x + c)*a + b)*a^5*b^5*c*cos_integ
ral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 20*(sq
rt(d*x + c)*a + b)^3*a^3*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/
sqrt(d*x + c))/(d*x + c)^(3/2) + 360*(sqrt(d*x + c)*a + b)*a^5*b^5*c*cos(a)
*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - a^5*
b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 15*(sqrt(d*x + c)*a + b)^4*a
^2*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x +
 c)^2 + 900*(sqrt(d*x + c)*a + b)^2*a^4*b^5*c*cos_integral(-a + (sqrt(d*x +
 c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 360*a^6*b^3*c^2*cos_integral(-
a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 15*(sqrt(d*x + c)*a + b)^
4*a^2*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x
 + c)^2 - 900*(sqrt(d*x + c)*a + b)^2*a^4*b^5*c*cos(a)*sin_integral(a - (sq
rt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 360*a^6*b^3*c^2*cos(a)*sin_in
tegral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 5*(sqrt(d*x + c)*a + b)*a
^4*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 6*(sqrt(d*x
```

```
 + c)*a + b)^5*a*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
*sin(a)/(d*x + c)^(5/2) - 1200*(sqrt(d*x + c)*a + b)^3*a^3*b^5*c*cos_integr
al(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(3/2) - 2160*
(sqrt(d*x + c)*a + b)*a^5*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/s
qrt(d*x + c))*sin(a)/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^5*a*b^7*cos(a)
*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) + 12
00*(sqrt(d*x + c)*a + b)^3*a^3*b^5*c*cos(a)*sin_integral(a - (sqrt(d*x + c)
*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 2160*(sqrt(d*x + c)*a + b)*a^5*b^3
*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x
+ c) - 10*(sqrt(d*x + c)*a + b)^2*a^3*b^7*cos((sqrt(d*x + c)*a + b)/sqrt(d*
x + c))/(d*x + c) - 60*a^5*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) +
 (sqrt(d*x + c)*a + b)^6*b^7*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d
*x + c))*sin(a)/(d*x + c)^3 + 900*(sqrt(d*x + c)*a + b)^4*a^2*b^5*c*cos_int
egral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^2 + 5400*(
sqrt(d*x + c)*a + b)^2*a^4*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/
sqrt(d*x + c))*sin(a)/(d*x + c) + a^4*b^7*sin((sqrt(d*x + c)*a + b)/sqrt(d*
x + c)) - (sqrt(d*x + c)*a + b)^6*b^7*cos(a)*sin_integral(a - (sqrt(d*x + c
)*a + b)/sqrt(d*x + c))/(d*x + c)^3 - 900*(sqrt(d*x + c)*a + b)^4*a^2*b^5*c
*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 -
 5400*(sqrt(d*x + c)*a + b)^2*a^4*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x
 + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 10*(sqrt(d*x + c)*a + b)^3*a^2*b^7*
cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) + 300*(sqrt(d*x +
c)*a + b)*a^4*b^5*c*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c)
- 360*(sqrt(d*x + c)*a + b)^5*a*b^5*c*cos_integral(-a + (sqrt(d*x + c)*a +
b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(5/2) - 7200*(sqrt(d*x + c)*a + b)^3*a^3
*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x
 + c)^(3/2) - 4*(sqrt(d*x + c)*a + b)*a^3*b^7*sin((sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/sqrt(d*x + c) + 360*(sqrt(d*x + c)*a + b)^5*a*b^5*c*cos(a)*sin_
integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^(5/2) + 7200*(s
qrt(d*x + c)*a + b)^3*a^3*b^3*c^2*cos(a)*sin_integral(a - (sqrt(d*x + c)*a
+ b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 5*(sqrt(d*x + c)*a + b)^4*a*b^7*cos((
sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 + 2*a^3*b^7*cos((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c)) - 600*(sqrt(d*x + c)*a + b)^2*a^3*b^5*c*cos((sqrt
(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 360*a^5*b^3*c^2*cos((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c)) + 60*(sqrt(d*x + c)*a + b)^6*b^5*c*cos_integral(-
a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^3 + 5400*(sqrt(d*
x + c)*a + b)^4*a^2*b^3*c^2*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*
x + c))*sin(a)/(d*x + c)^2 + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^7*sin((sqrt(d*
x + c)*a + b)/sqrt(d*x + c))/(d*x + c) + 60*a^4*b^5*c*sin((sqrt(d*x + c)*a
+ b)/sqrt(d*x + c)) - 60*(sqrt(d*x + c)*a + b)^6*b^5*c*cos(a)*sin_integral(
a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x +...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)(e+fx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x)^2, x)
```

3.198 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal. Leaf size=301

$$-\frac{b^3 f\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b(de-cf)\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}$$

```
[Out] 1/6*b*f*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(1/2))/d^2-1/12*b^4*f*cos(a)*Si(b/(d*
x+c)^(1/2))/d^2+b^2*(-c*f+d*e)*cos(a)*Si(b/(d*x+c)^(1/2))/d^2-1/12*b^4*f*Ci
(b/(d*x+c)^(1/2))*sin(a)/d^2+b^2*(-c*f+d*e)*Ci(b/(d*x+c)^(1/2))*sin(a)/d^2-
1/12*b^2*f*(d*x+c)*sin(a+b/(d*x+c)^(1/2))/d^2+(-c*f+d*e)*(d*x+c)*sin(a+b/(d
*x+c)^(1/2))/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^(1/2))/d^2-1/12*b^3*f*cos(
a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d^2+b*(-c*f+d*e)*cos(a+b/(d*x+c)^(1/2))*(d
*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.27, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3512, 3378, 3384, 3380, 3383}

$$-\frac{b^4 f\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2}-\frac{b^4 f\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2}-\frac{b^3 f\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b^2\sin(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{b^2\cos(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{b^2 f(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{b\sqrt{c+dx}\,(de-cf)\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2}+\frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] -1/12*(b^3*f*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b*(d*e - c*f)*S
qrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b*f*(c + d*x)^(3/2)*Cos[a + b
/Sqrt[c + d*x]])/(6*d^2) - (b^4*f*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(12*
d^2) + (b^2*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^2 - (b^2*f*(
c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(12*d^2) + ((d*e - c*f)*(c + d*x)*Sin[a
+ b/Sqrt[c + d*x]])/d^2 + (f*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(2*d^2)
- (b^4*f*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(12*d^2) + (b^2*(d*e - c*f)*C
os[a]*SinIntegral[b/Sqrt[c + d*x]])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx = -\frac{2\text{Subst}\left(\int \left(\frac{f\sin(a+bx)}{dx^5}+\frac{(de-cf)\sin(a+bx)}{dx^3}\right)\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{(2f)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d^2} - \frac{(2(de-cf))\text{Subst}\left(\int \text{s}\right.}{}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+}}\right.}{2d^2}$$

$$= \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}\cos\left(a+\right.}{6d^2}$$

$$= \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}\cos\left(a+\right.}{6d^2}$$

$$= -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\right.}{d^2}$$

$$= -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\right.}{d^2}$$

$$= -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\right.}{d^2}$$

Mathematica [A]
time = 0.41, size = 367, normalized size = 1.22

$$\frac{e\sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)\left(b\cos(a)+\sqrt{c+dx}\sin(a)\right)}{d}+\frac{f\sqrt{c+dx}\cos\left(\frac{b}{\sqrt{c+dx}}\right)\left(-b^3\cos(a)-12bc\cos(a)+2b(c+dx)\cos(a)-b^2\sqrt{c+dx}\sin(a)-12c\sqrt{c+dx}\sin(a)+6(c+dx)^{3/2}\sin(a)\right)}{12d^2}+\frac{e\sqrt{c+dx}\left(\sqrt{c+dx}\cos(a)-b\sin(a)\right)\sin\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{f\sqrt{c+dx}\left(-b^2\sqrt{c+dx}\cos(a)-12c\sqrt{c+dx}\cos(a)+6(c+dx)^{3/2}\cos(a)+b^3\sin(a)+12bc\sin(a)-2b(c+dx)\sin(a)\right)\sin\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b^2e\left(\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{d}-\frac{b^2(b^2+12c)f\left(\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{12d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (e*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(b*Cos[a] + Sqrt[c + d*x]*Sin[a]))/d
+ (f*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(-(b^3*Cos[a]) - 12*b*c*Cos[a] + 2*
b*(c + d*x)*Cos[a] - b^2*Sqrt[c + d*x]*Sin[a] - 12*c*Sqrt[c + d*x]*Sin[a] +
 6*(c + d*x)^(3/2)*Sin[a]))/(12*d^2) + (e*Sqrt[c + d*x]*(Sqrt[c + d*x]*Cos[
a] - b*Sin[a])*Sin[b/Sqrt[c + d*x]])/d + (f*Sqrt[c + d*x]*(-(b^2*Sqrt[c + d
*x]*Cos[a]) - 12*c*Sqrt[c + d*x]*Cos[a] + 6*(c + d*x)^(3/2)*Cos[a] + b^3*Si
n[a] + 12*b*c*Sin[a] - 2*b*(c + d*x)*Sin[a])*Sin[b/Sqrt[c + d*x]])/(12*d^2)
 + (b^2*e*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] + Cos[a]*SinIntegral[b/Sqrt[
```

```
c + d*x]]))/d - (b^2*(b^2 + 12*c)*f*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] +
Cos[a]*SinIntegral[b/Sqrt[c + d*x]]))/(12*d^2)
```

Maple [A]
time = 0.03, size = 295, normalized size = 0.98

method	result
derivativedivides	$-\frac{2b^2\left(-cf\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\text{sinIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\cdots\right.\right.}{\cdots}$
default	$-\frac{2b^2\left(-cf\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\text{sinIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d^2*b^2*(-c*f*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+
c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(
1/2))*sin(a))+d*e*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x
+c)^(1/2))/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^
(1/2))*sin(a))+f*b^2*(-1/4*sin(a+b/(d*x+c)^(1/2))/b^4*(d*x+c)^2-1/12*cos(a+
b/(d*x+c)^(1/2))/b^3*(d*x+c)^(3/2)+1/24*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)+
1/24*cos(a+b/(d*x+c)^(1/2))/b*(d*x+c)^(1/2)+1/24*Si(b/(d*x+c)^(1/2))*cos(a)
+1/24*Ci(b/(d*x+c)^(1/2))*sin(a)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 409, normalized size = 1.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) +
(Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x +
 c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x
+ c)*a + b)/sqrt(d*x + c)))*c*f/d - 12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-
I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)
))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*e - (((I*Ei(I*b/sq
rt(d*x + c)) - I*Ei(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) +
Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3
```

```
/2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x
 + c)^2)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f/d)/d
```

Fricas [A]
time = 0.38, size = 243, normalized size = 0.81

$$\frac{\left(12\,b^2de-\left(b^4+12\,b^2c\right)f\right)\operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+\left(12\,b^2de-\left(b^4+12\,b^2c\right)f\right)\operatorname{Ci}\left(-\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+2\left(12\,b^2de-\left(b^4+12\,b^2c\right)f\right)\cos\left(a\right)\operatorname{Si}\left(\frac{b}{\sqrt{dx+c}}\right)+2\left(2\,bdfx+12\,bde-\left(b^3+10\,bc\right)f\right)\sqrt{dx+c}\,\cos\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right)-2\left(b^2dfx-6\,d^2fx^2+\left(b^2c+6\,c^2\right)f-12\left(d^2x+cd\right)e\right)\sin\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right)}{24\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/24*((12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos_integral(b/sqrt(d*x + c))*sin(a
) + (12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos_integral(-b/sqrt(d*x + c))*sin(a)
 + 2*(12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos(a)*sin_integral(b/sqrt(d*x + c))
 + 2*(2*b*d*f*x + 12*b*d*e - (b^3 + 10*b*c)*f)*sqrt(d*x + c)*cos((a*d*x + a
*c + sqrt(d*x + c)*b)/(d*x + c)) - 2*(b^2*d*f*x - 6*d^2*f*x^2 + (b^2*c + 6*
c^2)*f - 12*(d^2*x + c*d)*e)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c))
)/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(1/2)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2159 vs. $2(271) = 542$.
time = 5.17, size = 2159, normalized size = 7.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*si
n(a) - a^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 - 2*(sqrt(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sq
rt(d*x + c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a)*si
n_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (sqrt(d
*x + c)*a + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))
*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(a - (sq
```

```
rt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x + c)*a +
b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*e
/((a^2 - 2*(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c)*a + b)^2/
(d*x + c))*b) - (a^4*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x +
 c))*sin(a) - a^4*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*
x + c)) - 4*(sqrt(d*x + c)*a + b)*a^3*b^5*cos_integral(-a + (sqrt(d*x + c)*
a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(sqrt(d*x + c)*a + b)*a^3*b^
5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c
) + 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*cos_integral(-a + (sqrt(d*x + c)*a +
b)/sqrt(d*x + c))*sin(a)/(d*x + c) + 12*a^4*b^3*c*cos_integral(-a + (sqrt(d
*x + c)*a + b)/sqrt(d*x + c))*sin(a) - 6*(sqrt(d*x + c)*a + b)^2*a^2*b^5*co
s(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 12*a
^4*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 4*(
sqrt(d*x + c)*a + b)^3*a*b^5*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d
*x + c))*sin(a)/(d*x + c)^(3/2) - 48*(sqrt(d*x + c)*a + b)*a^3*b^3*c*cos_in
tegral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/sqrt(d*x + c) + 4*(
sqrt(d*x + c)*a + b)^3*a*b^5*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/
sqrt(d*x + c))/(d*x + c)^(3/2) + 48*(sqrt(d*x + c)*a + b)*a^3*b^3*c*cos(a)*
sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - a^3*b
^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + (sqrt(d*x + c)*a + b)^4*b^5*c
os_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^2 +
72*(sqrt(d*x + c)*a + b)^2*a^2*b^3*c*cos_integral(-a + (sqrt(d*x + c)*a + b
)/sqrt(d*x + c))*sin(a)/(d*x + c) - (sqrt(d*x + c)*a + b)^4*b^5*cos(a)*sin_
integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 - 72*(sqrt(d*
x + c)*a + b)^2*a^2*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/(d*x + c) + 3*(sqrt(d*x + c)*a + b)*a^2*b^5*cos((sqrt(d*x + c)*
a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 48*(sqrt(d*x + c)*a + b)^3*a*b^3*c*co
s_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^(3/2)
 + 48*(sqrt(d*x + c)*a + b)^3*a*b^3*c*cos(a)*sin_integral(a - (sqrt(d*x + c
)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 3*(sqrt(d*x + c)*a + b)^2*a*b^5*c
os((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c) - 12*a^3*b^3*c*cos((sqrt(
d*x + c)*a + b)/sqrt(d*x + c)) + 12*(sqrt(d*x + c)*a + b)^4*b^3*c*cos_integ
ral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(d*x + c)^2 + a^2*b^5*
sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 12*(sqrt(d*x + c)*a + b)^4*b^3*c
*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c)^2 +
 (sqrt(d*x + c)*a + b)^3*b^5*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/(d*x
+ c)^(3/2) + 36*(sqrt(d*x + c)*a + b)*a^2*b^3*c*cos((sqrt(d*x + c)*a + b)/s
qrt(d*x + c))/sqrt(d*x + c) - 2*(sqrt(d*x + c)*a + b)*a*b^5*sin((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + 2*a*b^5*cos((sqrt(d*x + c)*a + b)
/sqrt(d*x + c)) - 36*(sqrt(d*x + c)*a + b)^2*a*b^3*c*cos((sqrt(d*x + c)*a +
 b)/sqrt(d*x + c))/(d*x + c) + (sqrt(d*x + c)*a + b)^2*b^5*sin((sqrt(d*x +
c)*a + b)/sqrt(d*x + c))/(d*x + c) + 12*a^2*b^3*c*sin((sqrt(d*x + c)*a + b)
/sqrt(d*x + c)) - 2*(sqrt(d*x + c)*a + b)*b^5*cos((sqrt(d*x + c)*a + b)/sqr
t(d*x + c))/sqrt(d*x + c) + 12*(sqrt(d*x + c)*a + b)^3*b^3*c*cos((sqrt(d*x
```

```
+ c)*a + b)/sqrt(d*x + c))/(d*x + c)^(3/2) - 24*(sqrt(d*x + c)*a + b)*a*b^3
*c*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) - 6*b^5*sin((sqrt
(d*x + c)*a + b)/sqrt(d*x + c)) + 12*(sqrt(d*x + c)*a + b)^2*b^3*c*sin((sqr
t(d*x + c)*a + b)/sqrt(d*x + c))/(d*x + c))*f/((a^4 - 4*(sqrt(d*x + c)*a +
b)*a^3/sqrt(d*x + c) + 6*(sqrt(d*x + c)*a + b)^2*a^2/(d*x + c) - 4*(sqrt(d*
x + c)*a + b)^3*a/(d*x + c)^(3/2) + (sqrt(d*x + c)*a + b)^4/(d*x + c)^2)*b*
d))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,(e+f\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/2))*(e + f*x), x)
```

3.199 $\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal. Leaf size=94

$$\frac{b\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\cos(a)\text{Si}\left(\frac{}{\sqrt{c}}\right.}{d}$$

```
[Out] b^2*cos(a)*Si(b/(d*x+c)^(1/2))/d+b^2*Ci(b/(d*x+c)^(1/2))*sin(a)/d+(d*x+c)*s
in(a+b/(d*x+c)^(1/2))/d+b*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3442, 3378, 3384, 3380, 3383}

$$\frac{b^2\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d + (b^2*CosIntegral[b/Sqrt[c +
d*x]]*Sin[a])/d + ((c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d + (b^2*Cos[a]*SinI
ntegral[b/Sqrt[c + d*x]])/d
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx = -\frac{2\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{b\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\text{Subst}(}{}$$

$$= \frac{b\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(b^2\cos(a}{}$$

$$= \frac{b\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin(}{}$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 1.05

$$\frac{b\sqrt{c+dx}\,\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)+b^2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)+c\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)+dx\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)+b^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]] + b^2*CosIntegral[b/Sqrt[c + d*x]
]*Sin[a] + c*Sin[a + b/Sqrt[c + d*x]] + d*x*Sin[a + b/Sqrt[c + d*x]] + b^2*
Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d
```

Maple [A]
time = 0.02, size = 84, normalized size = 0.89

method	result
derivativedivides	$-\frac{2b^2\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\text{sinIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\text{cosineInt}}{}\right.}{d}$
default	$-\frac{2b^2\left(-\frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)(dx+c)}{2b^2}-\frac{\cos\left(a+\frac{b}{\sqrt{dx+c}}\right)\sqrt{dx+c}}{2b}-\frac{\text{sinIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)}{2}-\frac{\text{cosineInt}}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b^2*(-1/2*sin(a+b/(d*x+c)^(1/2))/b^2*(d*x+c)-1/2*cos(a+b/(d*x+c)^(1/2)
)/b*(d*x+c)^(1/2)-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*si
n(a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.39, size = 124, normalized size = 1.32

$$\frac{\left(\left(-i\,\mathrm{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right)+i\,\mathrm{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)\right)\cos\left(a\right)+\left(\mathrm{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right)+\mathrm{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)\right)\sin\left(a\right)\right)b^2+2\sqrt{dx+c}\,b\cos\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)+2\left(dx+c\right)\sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*
b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*
cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)))/d
```

Fricas [A]
time = 0.41, size = 125, normalized size = 1.33

$$\frac{b^2\,\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+b^2\,\mathrm{Ci}\left(-\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+2\,b^2\cos\left(a\right)\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}\right)+2\sqrt{dx+c}\,b\cos\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right)+2\left(dx+c\right)\sin\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos_integral(b/sqrt(d*x + c))*sin(a) + b^2*cos_integral(-b/sqrt(d*
x + c))*sin(a) + 2*b^2*cos(a)*sin_integral(b/sqrt(d*x + c)) + 2*sqrt(d*x +
```

```
c)*b*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)) + 2*(d*x + c)*sin((a*d*
x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/2)),x)
```

```
[Out] Integral(sin(a + b/sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(84) = 168$.
time = 4.29, size = 413, normalized size = 4.39

$$\frac{a^2b^3\operatorname{Ci}\left(-a+\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)\sin(a)-a^2b^3\cos(a)\operatorname{Si}\left(a-\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)-\frac{2\left(\sqrt{dx+c}\,a+b\right)ab^3\operatorname{Ci}\left(-a+\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)\sin(a)}{\sqrt{dx+c}}+\frac{2\left(\sqrt{dx+c}\,a+b\right)ab^3\cos(a)\operatorname{Si}\left(a-\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{\sqrt{dx+c}}+\frac{\left(\sqrt{dx+c}\,a+b\right)^2b^3\operatorname{Ci}\left(-a+\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)\sin(a)}{dx+c}-\frac{\left(\sqrt{dx+c}\,a+b\right)^2b^3\cos(a)\operatorname{Si}\left(a-\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{dx+c}-ab^3\cos\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)+\frac{\left(\sqrt{dx+c}\,a+b\right)b^3\cos\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{\sqrt{dx+c}}+b^3\sin\left(\frac{\sqrt{dx+c}\,a+b}{\sqrt{dx+c}}\right)}{\left(a^2-\frac{2\left(\sqrt{dx+c}\,a+b\right)a}{\sqrt{dx+c}}+\frac{\left(\sqrt{dx+c}\,a+b\right)^2}{dx+c}\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a) - a^
2*b^3*cos(a)*sin_integral(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*(sqr
t(d*x + c)*a + b)*a*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x +
c))*sin(a)/sqrt(d*x + c) + 2*(sqrt(d*x + c)*a + b)*a*b^3*cos(a)*sin_integra
l(a - (sqrt(d*x + c)*a + b)/sqrt(d*x + c))/sqrt(d*x + c) + (sqrt(d*x + c)*a
 + b)^2*b^3*cos_integral(-a + (sqrt(d*x + c)*a + b)/sqrt(d*x + c))*sin(a)/(
d*x + c) - (sqrt(d*x + c)*a + b)^2*b^3*cos(a)*sin_integral(a - (sqrt(d*x +
c)*a + b)/sqrt(d*x + c))/(d*x + c) - a*b^3*cos((sqrt(d*x + c)*a + b)/sqrt(d
*x + c)) + (sqrt(d*x + c)*a + b)*b^3*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c
))/sqrt(d*x + c) + b^3*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))/((a^2 - 2*
(sqrt(d*x + c)*a + b)*a/sqrt(d*x + c) + (sqrt(d*x + c)*a + b)^2/(d*x + c))*
b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/2)),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/2)), x)
```

3.200 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$

Optimal. Leaf size=276

$$-\frac{2\mathrm{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f}+\frac{\mathrm{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f}+\frac{\mathrm{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\ldots\right.}{}$$

```
[Out] -cos(a+b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1
/2))/f+cos(a-b*f^(1/2)/(c*f-d*e)^(1/2))*Si(b*f^(1/2)/(c*f-d*e)^(1/2)+b/(d*x
+c)^(1/2))/f-2*cos(a)*Si(b/(d*x+c)^(1/2))/f-2*Ci(b/(d*x+c)^(1/2))*sin(a)/f+
Ci(b*f^(1/2)/(c*f-d*e)^(1/2)+b/(d*x+c)^(1/2))*sin(a-b*f^(1/2)/(c*f-d*e)^(1/
2))/f+Ci(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))*sin(a+b*f^(1/2)/(c*f-d*
e)^(1/2))/f
```

Rubi [A]
time = 0.85, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3512, 3384, 3380, 3383, 3426}

$$\frac{\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\mathrm{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{f}+\frac{\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\mathrm{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{2\sin(a)\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\mathrm{Si}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{f}+\frac{\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\mathrm{Si}\left(\frac{\sqrt{f}b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] (-2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/f + (CosIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] + b/Sqrt[c + d*x]]*Sin[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f
 + (CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c + d*x]]*Sin[a + (
b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f - (2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])
/f - (Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] - b/Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*
f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]])/f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx = -\frac{2\text{Subst}\left(\int \left(\frac{d\sin(a+bx)}{fx}+\frac{d(-de+cf)x\sin(a+bx)}{f(f+(de-cf)x^2)}\right)\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{2\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{f} + \frac{(2(de-cf))\text{Subst}\left(\int \frac{x\sin(a+bx)}{f+(de-cf)x^2}\,dx,\right.}{f}$$

$$= \frac{(2(de-cf))\text{Subst}\left(\int\left(-\frac{\sqrt{-de+cf}\sin(a+bx)}{2(de-cf)\left(\sqrt{f}-\sqrt{-de+cf}\,x\right)}+\frac{\sqrt{-de+cf}\sin}{2(de-cf)\left(\sqrt{f}+\sqrt{-}\right.}\right.\right.}{f}$$

$$= -\frac{2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} - \frac{2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f} - \frac{\sqrt{-de+cf}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} - \frac{2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f} + \frac{\left(\sqrt{-de+cf}\cos\left(a\right.\right.}{}$$

$$= -\frac{2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f} + \frac{\text{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de}}\right.}{f}$$

Mathematica [F]
time = 9.71, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x), x]
```

Maple [A]
time = 0.06, size = 441, normalized size = 1.60

method	result

derivativedivides	$-2b^2\left(\frac{\text{sinIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)+\text{cosineIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{f\,b^2}-\frac{-\text{sinIntegral}\left(-\frac{b}{\sqrt{dx+c}}\right.}{}\right.$
default	$-2b^2\left(\frac{\text{sinIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a)+\text{cosineIntegral}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a)}{f\,b^2}-\frac{-\text{sinIntegral}\left(-\frac{b}{\sqrt{dx+c}}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2*(1/f/b^2*(Si(b/(d*x+c)^(1/2))*cos(a)+Ci(b/(d*x+c)^(1/2))*sin(a))-1/2
/f/b^2*(-Si(-b/(d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c
*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x
+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f
-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-1/2/f/b^2*(Si(b/(d*x+c)^(1/
2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/(c*f-d*e))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 330, normalized size = 1.20

$$\frac{2i\,\mathrm{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right)e^{(ia)}-2i\,\mathrm{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)e^{(-ia)}-i\,\mathrm{Ei}\left(-\frac{\sqrt{-\frac{b^2f}{cf-de}}(dx+c)-i\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(ia+\sqrt{-\frac{b^2f}{cf-de}}\right)}-i\,\mathrm{Ei}\left(\frac{\sqrt{-\frac{b^2f}{cf-de}}(dx+c)+i\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(ia-\sqrt{-\frac{b^2f}{cf-de}}\right)}+i\,\mathrm{Ei}\left(-\frac{\sqrt{-\frac{b^2f}{cf-de}}(dx+c)+i\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(-ia+\sqrt{-\frac{b^2f}{cf-de}}\right)}+i\,\mathrm{Ei}\left(\frac{\sqrt{-\frac{b^2f}{cf-de}}(dx+c)-i\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(-ia-\sqrt{-\frac{b^2f}{cf-de}}\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*I*Ei(I*b/sqrt(d*x + c))*e^(I*a) - 2*I*Ei(-I*b/sqrt(d*x + c))*e^(-I*a
) - I*Ei(-(sqrt(-b^2*f/(c*f - d*e))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c
))*e^(I*a + sqrt(-b^2*f/(c*f - d*e))) - I*Ei((sqrt(-b^2*f/(c*f - d*e))*(d*x
```

```
 + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(-b^2*f/(c*f - d*e))) +
I*Ei(-(sqrt(-b^2*f/(c*f - d*e))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e
^(-I*a + sqrt(-b^2*f/(c*f - d*e))) + I*Ei((sqrt(-b^2*f/(c*f - d*e))*(d*x +
c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(-b^2*f/(c*f - d*e))))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{\sqrt{c + dx}}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b/sqrt(c + d*x))/(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a + \frac{b}{\sqrt{c + d\,x}}\right)}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x), x)
```

3.201 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\,(-de+cf)^{3/2}}+\frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cj}}\right.}{2\sqrt{f}\,(-de+cf)^{3/2}}$$

```
[Out] (d*x+c)*sin(a+b/(d*x+c)^(1/2))/(-c*f+d*e)/(f*x+e)+1/2*b*d*Ci(b*f^(1/2)/(c*f
-d*e)^(1/2)+b/(d*x+c)^(1/2))*cos(a-b*f^(1/2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/
2)/f^(1/2)-1/2*b*d*Ci(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))*cos(a+b*f^
(1/2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/2)/f^(1/2)-1/2*b*d*Si(b*f^(1/2)/(c*f-d*
e)^(1/2)+b/(d*x+c)^(1/2))*sin(a-b*f^(1/2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/2)/
f^(1/2)-1/2*b*d*Si(b*f^(1/2)/(c*f-d*e)^(1/2)-b/(d*x+c)^(1/2))*sin(a+b*f^(1/
2)/(c*f-d*e)^(1/2))/(c*f-d*e)^(3/2)/f^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$-\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\,(cf-de)^{3/2}}+\frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\,(cf-de)^{3/2}}-\frac{bd\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\,(cf-de)^{3/2}}-\frac{bd\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{Si}\left(\frac{\sqrt{f}\,b}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\,(cf-de)^{3/2}}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] -1/2*(b*d*Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/S
qrt[-(d*e) + c*f] - b/Sqrt[c + d*x]])/(Sqrt[f]*(-(d*e) + c*f)^(3/2)) + (b*d
*Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e
) + c*f] + b/Sqrt[c + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) + ((c + d*x)*
Sin[a + b/Sqrt[c + d*x]])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*Sqrt[f]
)/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c
 + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) - (b*d*Sin[a - (b*Sqrt[f])/Sqrt[
-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]
])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{2\text{Subst}\left(\int \frac{x\sin(a+bx)}{\left(\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^2\right)^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \left(\frac{d\cos(a+bx)}{2\sqrt{f}\left(\sqrt{f}-\sqrt{-de+cf}\,x\right)} + \right.\right.}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{f}-\sqrt{-de+cf}\,x}\,dx, x, \frac{1}{\sqrt{c}}\right.}{2\sqrt{f}\,(de-cf)}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\text{Subst}\left(\int \frac{\cos}{\sqrt{}}\right.}{2\sqrt{f}\,(de-}$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}\,(-de+cf)^{3/2}} + \frac{bd\cos\left(a-\right.}{}$$

Mathematica [F]
time = 180.04, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] $Aborted
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2733 vs. 2(284) = 568.
time = 0.06, size = 2734, normalized size = 7.81

method	result	size

derivativedivides	Expression too large to display	2734
default	Expression too large to display	2734

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*d*b^2*(sin(a+b/(d*x+c)^(1/2))*(-1/2*a/f/b^2*(a+b/(d*x+c)^(1/2))+1/2*(a^2
*c*f-a^2*d*e-b^2*f)/f/b^2/(c*f-d*e))/(a^2*c*f-a^2*d*e-2*a*c*f*(a+b/(d*x+c)^
(1/2))+2*a*d*e*(a+b/(d*x+c)^(1/2))+c*f*(a+b/(d*x+c)^(1/2))^2-d*e*(a+b/(d*x+
c)^(1/2))^2-f*b^2)+1/4*a/f/b^2/(a*c*f-a*d*e-c*f*(a*c*f-a*d*e+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/(c*f-d*e)+d*e*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f
-d*e))*(-Si(-b/(d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c
*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x
+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f
-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4*a/f/b^2/(a*c*f-a*d*e+c*
f*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-d*e*(-a*c*f+a*d*e+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(
b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d
*e)))+1/4*(a^2*c*f-a^2*d*e-a*c*f*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
(c*f-d*e)+a*d*e*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-f*b^2)/
f/b^2/(c*f-d*e)/(a*c*f-a*d*e-c*f*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
(c*f-d*e)+d*e*(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(-b/(
d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*
c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c
*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4*(a^2*c*f-a^2*d*e+a*c*f*(-a*c*f+a*d*e+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-a*d*e*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e)-f*b^2)/f/b^2/(c*f-d*e)/(a*c*f-a*d*e+c*f*(-a*c*f+a*d*e+
(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-d*e*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*
e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-
d*e))+Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-
d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-a*(sin(a+b
/(d*x+c)^(1/2))*(-1/2/b^2*(a+b/(d*x+c)^(1/2))/f+1/2*a/f/b^2)/(a^2*c*f-a^2*d
*e-2*a*c*f*(a+b/(d*x+c)^(1/2))+2*a*d*e*(a+b/(d*x+c)^(1/2))+c*f*(a+b/(d*x+c)
^(1/2))^2-d*e*(a+b/(d*x+c)^(1/2))^2-f*b^2)+1/4/f/b^2/(a*c*f-a*d*e-c*f*(a*c*
f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)+d*e*(a*c*f-a*d*e+(b^2*c*f^2-
b^2*d*e*f)^(1/2))/(c*f-d*e))*(-Si(-b/(d*x+c)^(1/2)-a+(a*c*f-a*d*e+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2)
)/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))
/(c*f-d*e))*sin((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4/f
```

```
/b^2/(a*c*f-a*d*e+c*f*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)-
d*e*(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*(Si(b/(d*x+c)^(1/
2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/(c*f-d*e)))+1/4/f/b^2/(c*f-d*e)*(Si(-b/(d*x+c)^(1/2)-a+(a*c*f
-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f-a*d*e+(b^2*c*f^2-
b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b
^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(
c*f-d*e)))+1/4/f/b^2/(c*f-d*e)*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f
^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.48, size = 477, normalized size = 1.36

$$-\frac{(-i\,dfx - i\,de)\sqrt{-\frac{b^2f}{cf-de}}\,\mathrm{Ei}\left(-\frac{\sqrt{-\frac{b^2f}{cf-de}}\,(dx+c) - i\,\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(i\,a+\sqrt{-\frac{b^2f}{cf-de}}\right)} + (i\,dfx + i\,de)\sqrt{-\frac{b^2f}{cf-de}}\,\mathrm{Ei}\left(\frac{\sqrt{-\frac{b^2f}{cf-de}}\,(dx+c) + i\,\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(i\,a-\sqrt{-\frac{b^2f}{cf-de}}\right)} + (i\,dfx + i\,de)\sqrt{-\frac{b^2f}{cf-de}}\,\mathrm{Ei}\left(-\frac{\sqrt{-\frac{b^2f}{cf-de}}\,(dx+c) + i\,\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(-i\,a+\sqrt{-\frac{b^2f}{cf-de}}\right)} + (-i\,dfx - i\,de)\sqrt{-\frac{b^2f}{cf-de}}\,\mathrm{Ei}\left(\frac{\sqrt{-\frac{b^2f}{cf-de}}\,(dx+c) - i\,\sqrt{dx+c}\,b}{dx+c}\right)e^{\left(-i\,a-\sqrt{-\frac{b^2f}{cf-de}}\right)} + 4\,(dfx + cf)\sin\left(\frac{adx+ac+\sqrt{dx+c}\,b}{dx+c}\right)}{4\,(cf^3x - dfe^2 - (df^2x - cf^2)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((-I*d*f*x - I*d*e)*sqrt(-b^2*f/(c*f - d*e))*Ei(-(sqrt(-b^2*f/(c*f - d
*e))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a + sqrt(-b^2*f/(c*f -
d*e))) + (I*d*f*x + I*d*e)*sqrt(-b^2*f/(c*f - d*e))*Ei((sqrt(-b^2*f/(c*f -
d*e))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(-b^2*f/(c*f -
 d*e))) + (I*d*f*x + I*d*e)*sqrt(-b^2*f/(c*f - d*e))*Ei(-(sqrt(-b^2*f/(c*f
- d*e))*(d*x + c) + I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a + sqrt(-b^2*f/(c*
f - d*e))) + (-I*d*f*x - I*d*e)*sqrt(-b^2*f/(c*f - d*e))*Ei((sqrt(-b^2*f/(c
*f - d*e))*(d*x + c) - I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(-b^2*f/
(c*f - d*e))) + 4*(d*f*x + c*f)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x +
c)))/(c*f^3*x - d*f*e^2 - (d*f^2*x - c*f^2)*e)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e)**2,x)
[Out] Integral(sin(a + b/sqrt(c + d*x))/(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+d\,x}}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(1/2))/(e + f*x)^2, x)
```

3.202 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx$

Optimal. Leaf size=390

$$\frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}-\frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{2ie^{-ia}f(d}{}$$

```
[Out] 1/3*b*f^2*(d*x+c)^(3/2)*cos(a+b/(d*x+c)^(3/2))/d^3-2/3*I*exp(I*a)*f*(-c*f+d
*e)*(-I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,-I*b/(d*x+c)^(3/2))/d^3
+2/3*I*f*(-c*f+d*e)*(I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,I*b/(d*x
+c)^(3/2))/d^3/exp(I*a)-1/3*I*exp(I*a)*(-c*f+d*e)^2*(-I*b/(d*x+c)^(3/2))^(2
/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^(3/2))/d^3+1/3*I*(-c*f+d*e)^2*(I*b/(d*x
+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/2))/d^3/exp(I*a)+1/3*b^2
*f^2*cos(a)*Si(b/(d*x+c)^(3/2))/d^3+1/3*b^2*f^2*Ci(b/(d*x+c)^(3/2))*sin(a)/
d^3+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(3/2))/d^3
```

Rubi [A]
time = 0.29, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3514, 3504, 2250, 3460, 3378, 3384, 3380, 3383}

$$-\frac{2ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{2ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)^2\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)^2\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{b^2f^2\sin(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{b^2f^2\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{f^2(c+dx)^3\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (b*f^2*(c + d*x)^(3/2)*Cos[a + b/(c + d*x)^(3/2)])/(3*d^3) - (((2*I)/3)*E^(
I*a)*f*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
 ((-I)*b)/(c + d*x)^(3/2)])/d^3 + (((2*I)/3)*f*(d*e - c*f)*((I*b)/(c + d*x)
^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(I*a))
 - ((I/3)*E^(I*a)*(d*e - c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*
Gamma[-2/3, ((-I)*b)/(c + d*x)^(3/2)])/d^3 + ((I/3)*(d*e - c*f)^2*((I*b)/(c
 + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(
I*a)) + (b^2*f^2*CosIntegral[b/(c + d*x)^(3/2)]*Sin[a])/(3*d^3) + (f^2*(c +
 d*x)^3*Sin[a + b/(c + d*x)^(3/2)])/(3*d^3) + (b^2*f^2*Cos[a]*SinIntegral[b
/(c + d*x)^(3/2)])/(3*d^3)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \frac{2\text{Subst}\left(\int \left((de-cf)^2 x \sin\left(a+\frac{b}{x^3}\right) - 2f(-de+cf)x^3 \sin\left(a+\frac{b}{x^3}\right.\right.\right.}{d^3}$$

$$= \frac{(2f^2)\,\text{Subst}\left(\int x^5 \sin\left(a+\frac{b}{x^3}\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\text{S}}{}$$

$$= -\frac{(2f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3} dx, x, \frac{1}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{(2if(de-cf))\text{Subst}\left(\right.}{}$$

$$= -\frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3}, -\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{2}{}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)}{3}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)}{3}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)}{3}$$

Mathematica [A]
time = 3.10, size = 518, normalized size = 1.33

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (((b*f*Sqrt[c + d*x]*(9*d*e - 8*c*f + d*f*x))/E^((I*b)/(c + d*x)^(3/2)) + (
I*(c + d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))
)/E^((I*b)/(c + d*x)^(3/2)) + I*b^2*f^2*ExpIntegralEi[((-I)*b)/(c + d*x)^(3
/2)] - (3*I)*(d*e - c*f)^2*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[1/
3, (I*b)/(c + d*x)^(3/2)] + 9*b*f*(-(d*e) + c*f)*((I*b)/(c + d*x)^(3/2))^(1
/3)*Sqrt[c + d*x]*Gamma[2/3, (I*b)/(c + d*x)^(3/2)])/E^(I*a) - I*(Cos[a] +
I*Sin[a])*(b^2*f^2*ExpIntegralEi[(I*b)/(c + d*x)^(3/2)] + Sqrt[c + d*x]*(-3
*(d*e - c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*Sqrt[c + d*x]*Gamma[1/3, ((
-I)*b)/(c + d*x)^(3/2)] + (9*I)*b*f*(-(d*e) + c*f)*(((-I)*b)/(c + d*x)^(3/2
))^(1/3)*Gamma[2/3, ((-I)*b)/(c + d*x)^(3/2)] + (I*b*f*(9*d*e - 8*c*f + d*f
*x) + Sqrt[c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f
^2*x^2)))*(Cos[b/(c + d*x)^(3/2)] + I*Sin[b/(c + d*x)^(3/2)]))))/(6*d^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
[Out] int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(300) = 600.
time = 0.78, size = 994, normalized size = 2.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
[Out] 1/12*(3*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a
 + b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) +
(sqrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*
gamma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)
^(3/2)))*sin(a))*b)*c^2*f^2/(sqrt(d*x + c)*d^2*(b/(d*x + c)^(3/2))^(1/3)) -
 6*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)
/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt
(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma
(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2
)))*sin(a))*b)*c*f*e/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3)) + 3*(4*(d*
x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c
)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) + I)*
gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3, I*b
/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(a
))*b)*e^2/(sqrt(d*x + c)*(b/(d*x + c)^(3/2))^(1/3)) + 2*(2*(d*x + c)^3*sin(
((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 2*(d*x + c)^(3/2)*b*cos(((d*x +
c)^(3/2)*a + b)/(d*x + c)^(3/2)) + ((-I*Ei(I*b/(d*x + c)^(3/2)) + I*Ei(-I*b
/(d*x + c)^(3/2)))*cos(a) + (Ei(I*b/(d*x + c)^(3/2)) + Ei(-I*b/(d*x + c)^(3
/2)))*sin(a))*b^2)*f^2/d^2 - 3*(4*(d*x + c)^3*(b/(d*x + c)^(3/2))^(2/3)*sin
(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x + c)^(3/2)*b*(b/(d*x +
c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) - 3*(((sqrt(3)
 + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) - I)*gamma(2/3, -I*b/(d*x
+ c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (
I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(a))*b^2)*c*f^2/((d*x +
 c)*d^2*(b/(d*x + c)^(3/2))^(2/3)) + 3*(4*(d*x + c)^3*(b/(d*x + c)^(3/2))^(
```

```
2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x + c)^(3/2)*b*(b
/(d*x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) - 3*((
(sqrt(3) + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) - I)*gamma(2/3, -I
*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1)*gamma(2/3, I*b/(d*x + c)^(3
/2)) + (I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(a))*b^2)*f*e/(
(d*x + c)*d*(b/(d*x + c)^(3/2))^(2/3)))/d
```

Fricas [A]
time = 0.17, size = 510, normalized size = 1.31

$$\frac{-i b^2 f^2 \mathrm{Ei}\left(\frac{i\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) e^{(ia)} + i b^2 f^2 \mathrm{Ei}\left(-\frac{i\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) e^{(-ia)} - 3\,(i c^2 f^2 - 2i\, cdfe + i\, d^2e^2)(i b)^{\frac{2}{3}}\, e^{(-ia)}\Gamma\left(\frac{1}{3}, \frac{i\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) - 3\,(-i c^2 f^2 + 2i\, cdfe - i\, d^2e^2)(-i b)^{\frac{2}{3}}\, e^{(ia)}\Gamma\left(\frac{1}{3}, -\frac{i\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) + 9\,(bcf^2 - bdfe)(i b)^{\frac{1}{3}}\, e^{(-ia)}\Gamma\left(\frac{2}{3}, \frac{i\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) + 9\,(bcf^2 - bdfe)(-i b)^{\frac{1}{3}}\, e^{(ia)}\Gamma\left(\frac{2}{3}, -\frac{i\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) + 2\,(bdf^2x - 8\,bcf^2 + 9\,bdfe)\sqrt{dx+c}\,\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right) + 2\,(d^3f^2x^3 + c^3f^2 + 3\,(d^3x + cd^2)e^2 + 3\,(d^3fx^2 - c^2df)e)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}\, b}{d^2x^2+2\,cdx+c^2}\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*b^2*f^2*Ei(I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))*e^(I*a) + I
*b^2*f^2*Ei(-I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))*e^(-I*a) - 3*(I*c
^2*f^2 - 2*I*c*d*f*e + I*d^2*e^2)*(I*b)^(2/3)*e^(-I*a)*gamma(1/3, I*sqrt(d*
x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(-I*c^2*f^2 + 2*I*c*d*f*e - I*d^2*e
^2)*(-I*b)^(2/3)*e^(I*a)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x +
 c^2)) + 9*(b*c*f^2 - b*d*f*e)*(I*b)^(1/3)*e^(-I*a)*gamma(2/3, I*sqrt(d*x +
 c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 9*(b*c*f^2 - b*d*f*e)*(-I*b)^(1/3)*e^(I*
a)*gamma(2/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(b*d*f^2*x
- 8*b*c*f^2 + 9*b*d*f*e)*sqrt(d*x + c)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d^3*f^2*x^3 + c^3*f^2 + 3
*(d^3*x + c*d^2)*e^2 + 3*(d^3*f*x^2 - c^2*d*f)*e)*sin((a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin\left(a + \frac{b}{c\sqrt{c+dx} + dx\sqrt{c+dx}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x)^2, x)
```

3.203 $\int (e+fx)\sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) \, dx$

Optimal. Leaf size=251

$$-\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}-\frac{ie^{ia}(d\cdots}{}$$

```
[Out] -1/3*I*exp(I*a)*f*(-I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,-I*b/(d*x
+c)^(3/2))/d^2+1/3*I*f*(I*b/(d*x+c)^(3/2))^(4/3)*(d*x+c)^2*GAMMA(-4/3,I*b/(
d*x+c)^(3/2))/d^2/exp(I*a)-1/3*I*exp(I*a)*(-c*f+d*e)*(-I*b/(d*x+c)^(3/2))^(
2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^(3/2))/d^2+1/3*I*(-c*f+d*e)*(I*b/(d*x+
c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/2))/d^2/exp(I*a)
```

Rubi [A]
time = 0.15, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3514, 3504, 2250}

$$-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}+\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}\text{Gamma}\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}\text{Gamma}\left(-\frac{4}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((-1/3*I)*E^(I*a)*f*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3
, ((-I)*b)/(c + d*x)^(3/2)])/d^2 + ((I/3)*f*((I*b)/(c + d*x)^(3/2))^(4/3)*(
c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a)) - ((I/3)*E^(I*
a)*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)
*b)/(c + d*x)^(3/2)])/d^2 + ((I/3)*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(2/3
)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{2\text{Subst}\left(\int \left((de-cf)x\sin\left(a+\frac{b}{x^3}\right)+fx^3\sin\left(a+\frac{b}{x^3}\right)\right)dx,x,\sqrt{c+d}\right.}{d^2}$$

$$= \frac{(2f)\text{Subst}\left(\int x^3\sin\left(a+\frac{b}{x^3}\right)dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2(de-cf))\text{Subst}}{}$$

$$= \frac{(if)\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x^3\,dx,x,\sqrt{c+dx}\right)}{d^2}-\frac{(if)\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x^3\,dx\right.}{d^2}$$

$$= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}+\frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^{3/}}\right.}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 835 vs. 2(251) = 502.
time = 1.74, size = 835, normalized size = 3.33

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (3*b*e*Cos[a]*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d
*x)^(3/2))^(1/3)*Sqrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*
((I*b)/(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x])))/(4*d) - (3*b*c*f*Cos[a]*((2*
Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*S
qrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(
3/2))^(1/3)*Sqrt[c + d*x])))/(4*d^2) + (((9*I)/8)*b^2*f*Cos[a]*((2*Gamma[2/
3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)
) - (2*Gamma[2/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3)*
(c + d*x))))/d^2 + (e*(c + d*x)*Cos[b/(c + d*x)^(3/2)]*Sin[a])/d + (((3*I)/
4)*b*e*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/
2))^(1/3)*Sqrt[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/
(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d - (((3*I)/4)*b*c*f*((2*Gam
ma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Sqrt
```

```
[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2
))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d^2 - (9*b^2*f*((2*Gamma[2/3, ((-I)*b)/(c
+ d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)) + (2*Gamma[2/
3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)))*Sin
[a])/(8*d^2) + (f*Sqrt[c + d*x]*Cos[b/(c + d*x)^(3/2)]*(3*b*Cos[a] - 2*c*Sq
rt[c + d*x]*Sin[a] + (c + d*x)^(3/2)*Sin[a]))/(2*d^2) + (e*(c + d*x)*Cos[a]
*Sin[b/(c + d*x)^(3/2)])/d + (f*Sqrt[c + d*x]*(-2*c*Sqrt[c + d*x]*Cos[a] +
(c + d*x)^(3/2)*Cos[a] - 3*b*Sin[a])*Sin[b/(c + d*x)^(3/2)])/(2*d^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(f x +e \right) \sin \left(a +\frac{b}{\left(d x +c \right)^{\frac{3}{2}}}\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(178) = 356$.
time = 0.63, size = 505, normalized size = 2.01

$$-\frac{\frac{2\left(4\left(dx+c\right)^{\frac{3}{2}}\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sin\left(\frac{\left(dx+c\right)^{\frac{3}{2}}a+b}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)\right)\cos\left(a\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)\right)\sin\left(a\right)\right)b\right)cf}{\sqrt{dx+c}\,d\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}}-\frac{2\left(4\left(dx+c\right)^{\frac{3}{2}}\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sin\left(\frac{\left(dx+c\right)^{\frac{3}{2}}a+b}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)\right)\cos\left(a\right)+\left(\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)\right)\sin\left(a\right)\right)b\right)e}{\sqrt{dx+c}\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}}-\frac{\left(4\left(dx+c\right)^{3}\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sin\left(\frac{\left(dx+c\right)^{\frac{3}{2}}a+b}{\left(dx+c\right)^{\frac{3}{2}}}\right)+12\left(dx+c\right)^{\frac{3}{2}}b\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\cos\left(\frac{\left(dx+c\right)^{\frac{3}{2}}a+b}{\left(dx+c\right)^{\frac{3}{2}}}\right)-3\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3},-\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)\right)\cos\left(a\right)+\left(\left(-i\sqrt{3}+1\right)\Gamma\left(\frac{2}{3},\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(i\sqrt{3}+1\right)\Gamma\left(\frac{2}{3},-\frac{ib}{\left(dx+c\right)^{\frac{3}{2}}}\right)\right)\sin\left(a\right)\right)b^{2}\right)f}{\left(dx+c\right)d\left(\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -1/8*(2*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a
 + b)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) +
(sqrt(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*
gamma(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)
^(3/2)))*sin(a))*b)*c*f/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3)) - 2*(4*
(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x
+ c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) +
I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamma(1/3,
I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*si
n(a))*b)*e/(sqrt(d*x + c)*(b/(d*x + c)^(3/2))^(1/3)) - (4*(d*x + c)^3*(b/(d
*x + c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x
 + c)^(3/2)*b*(b/(d*x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x +
c)^(3/2)) - 3*(((sqrt(3) + I)*gamma(2/3, I*b/(d*x + c)^(3/2)) + (sqrt(3) -
I)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) + 1)*gamma(2/3,
I*b/(d*x + c)^(3/2)) + (I*sqrt(3) + 1)*gamma(2/3, -I*b/(d*x + c)^(3/2)))*si
n(a))*b^2)*f/((d*x + c)*d*(b/(d*x + c)^(3/2))^(2/3)))/d
```

Fricas [A]
time = 0.11, size = 333, normalized size = 1.33

$$-\frac{3\,(i\,b)^{\frac{1}{3}}\,bfe^{(-i\,a)}\Gamma\left(\frac{2}{3},\frac{i\,\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)+3\,(-i\,b)^{\frac{1}{3}}\,bfe^{(i\,a)}\Gamma\left(\frac{2}{3},-\frac{i\,\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)+2\,(-i\,cf+i\,de)(i\,b)^{\frac{2}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{3},\frac{i\,\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)+2\,(i\,cf-i\,de)(-i\,b)^{\frac{2}{3}}\,e^{(i\,a)}\Gamma\left(\frac{1}{3},-\frac{i\,\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)-6\,\sqrt{dx+c}\,bf\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)-2\,(d^2fx^2-c^2f+2\,(d^2x+cd)e)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(I*b)^(1/3)*b*f*e^(-I*a)*gamma(2/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*
c*d*x + c^2)) + 3*(-I*b)^(1/3)*b*f*e^(I*a)*gamma(2/3, -I*sqrt(d*x + c)*b/(d
^2*x^2 + 2*c*d*x + c^2)) + 2*(-I*c*f + I*d*e)*(I*b)^(2/3)*e^(-I*a)*gamma(1/
3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(I*c*f - I*d*e)*(-I*b)^
(2/3)*e^(I*a)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 6*
sqrt(d*x + c)*b*f*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^
2*x^2 + 2*c*d*x + c^2)) - 2*(d^2*f*x^2 - c^2*f + 2*(d^2*x + c*d)*e)*sin((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/
d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)\,(e+f\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(3/2))*(e + f*x), x)
```

3.204 $\int \sin \left(a+\frac{b}{(c+dx)^{3/2}}\right) dx$

Optimal. Leaf size=115

$$-\frac{ie^{ia}\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}+\frac{ie^{-ia}\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

```
[Out] -1/3*I*exp(I*a)*(-I*b/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,-I*b/(d*x+c)^
(3/2))/d+1/3*I*(I*b/(d*x+c)^(3/2))^(2/3)*(d*x+c)*GAMMA(-2/3,I*b/(d*x+c)^(3/
2))/d/exp(I*a)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3444, 3504, 2250}

$$\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((-1/3*I)*E^(I*a)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((
-I)*b)/(c + d*x)^(3/2)])/d + ((I/3)*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)
*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d*E^(I*a))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx = \frac{2\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{i\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx, x, \sqrt{c+dx}\right)}{d} - \frac{i\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{ie^{ia}\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d} + \frac{ie^{-ia}\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+d}{3d}$$

Mathematica [A]
time = 0.28, size = 166, normalized size = 1.44

$$\frac{b\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\,\Gamma\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)(\cos(a)-i\sin(a))+b\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\,\Gamma\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)(\cos(a)+i\sin(a))+2\sqrt[3]{\frac{b^2}{(c+dx)^3}}\,(c+dx)^{3/2}\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{2d\sqrt[3]{\frac{b^2}{(c+dx)^3}}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (b*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, (I*b)/(c + d*x)^(3/2)]*(Cos[
a] - I*Sin[a]) + b*((I*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, ((-I)*b)/(c + d
*x)^(3/2)]*(Cos[a] + I*Sin[a]) + 2*(b^2/(c + d*x)^3)^(1/3)*(c + d*x)^(3/2)*
Sin[a + b/(c + d*x)^(3/2)])/(2*d*(b^2/(c + d*x)^3)^(1/3)*Sqrt[c + d*x])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(3/2)),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(3/2)),x)
```

Maxima [A]
time = 0.36, size = 151, normalized size = 1.31

$$\frac{4\,(dx+c)^{\frac{3}{2}}\left(\frac{b}{(dx+c)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{3}{2}}a+b}{(dx+c)^{\frac{3}{2}}}\right)+\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right)\right)\cos\left(a\right)+\left(\left(-i\,\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},\frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right)+\left(i\,\sqrt{3}-1\right)\Gamma\left(\frac{1}{3},-\frac{i\,b}{(dx+c)^{\frac{3}{2}}}\right)\right)\sin\left(a\right)\right)b}{4\,\sqrt{dx+c}\,d\left(\frac{b}{(dx+c)^{\frac{3}{2}}}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)^(3/2)*(b/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b
)/(d*x + c)^(3/2)) + (((sqrt(3) - I)*gamma(1/3, I*b/(d*x + c)^(3/2)) + (sqr
t(3) + I)*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(a) + ((-I*sqrt(3) - 1)*gamm
a(1/3, I*b/(d*x + c)^(3/2)) + (I*sqrt(3) - 1)*gamma(1/3, -I*b/(d*x + c)^(3/
2)))*sin(a))*b)/(sqrt(d*x + c)*d*(b/(d*x + c)^(3/2))^(1/3))
```

Fricas [**A**]
time = 0.12, size = 144, normalized size = 1.25

$$\frac{-i\,(i\,b)^{\frac{2}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{3},\frac{i\,\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)+i\,(-i\,b)^{\frac{2}{3}}\,e^{(i\,a)}\Gamma\left(\frac{1}{3},-\frac{i\,\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)+2\,(dx+c)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}\,b}{d^2x^2+2\,cdx+c^2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*(I*b)^(2/3)*e^(-I*a)*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*
x + c^2)) + I*(-I*b)^(2/3)*e^(I*a)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 +
 2*c*d*x + c^2)) + 2*(d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*
x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+\frac{b}{(c+dx)^{\frac{3}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(3/2)),x)
[Out] int(sin(a + b/(c + d*x)^(3/2)), x)
```

3.205 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

Mathematica [A]
time = 9.36, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2
*c*d*x + c^2))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{c\sqrt{c+dx} + dx\sqrt{c+dx}}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x)))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x), x)
```

3.206 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$$

Mathematica [A]
time = 11.73, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
[Out] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2
*c*d*x + c^2))/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{c\sqrt{c+dx}+dx\sqrt{c+dx}}\right)}{(e+fx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e)**2,x)
[Out] Integral(sin(a + b/(c*sqrt(c + d*x) + d*x*sqrt(c + d*x)))/(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{3/2}}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^(3/2))/(e + f*x)^2, x)
```

3.207 $\int (e+fx)^2 \sin \left(a+b\sqrt[3]{c+dx}\right) \, dx$

Optimal. Leaf size=633

$$-\frac{120960f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}+\frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{\right.}}{b^5d^3}$$

```
[Out] -120960*f^2*cos(a+b*(d*x+c)^(1/3))/b^9/d^3+6*(-c*f+d*e)^2*cos(a+b*(d*x+c)^(
1/3))/b^3/d^3-720*f*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d^3
+60480*f^2*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^7/d^3-3*(-c*f+d*e)^2*(d*x
+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d^3+120*f*(-c*f+d*e)*(d*x+c)*cos(a+b*(d*
x+c)^(1/3))/b^3/d^3-5040*f^2*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d^3-6
*f*(-c*f+d*e)*(d*x+c)^(5/3)*cos(a+b*(d*x+c)^(1/3))/b/d^3+168*f^2*(d*x+c)^2*
cos(a+b*(d*x+c)^(1/3))/b^3/d^3-3*f^2*(d*x+c)^(8/3)*cos(a+b*(d*x+c)^(1/3))/b
/d^3+720*f*(-c*f+d*e)*sin(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*f^2*(d*x+c)^(1/
3)*sin(a+b*(d*x+c)^(1/3))/b^8/d^3+6*(-c*f+d*e)^2*(d*x+c)^(1/3)*sin(a+b*(d*x
+c)^(1/3))/b^2/d^3-360*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^
4/d^3+20160*f^2*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^6/d^3+30*f*(-c*f+d*e)*(d*x
+c)^(4/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^3-1008*f^2*(d*x+c)^(5/3)*sin(a+b*(d*
x+c)^(1/3))/b^4/d^3+24*f^2*(d*x+c)^(7/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^3
```

Rubi [A]
time = 0.44, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3512, 3377, 2718, 2717}

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-120960*f^2*Cos[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*(d*e - c*f)^2*Cos[a
 + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a
 + b*(c + d*x)^(1/3)])/(b^5*d^3) + (60480*f^2*(c + d*x)^(2/3)*Cos[a + b*(c
+ d*x)^(1/3)])/(b^7*d^3) - (3*(d*e - c*f)^2*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d^3) + (120*f*(d*e - c*f)*(c + d*x)*Cos[a + b*(c + d*x)^(1/
3)])/(b^3*d^3) - (5040*f^2*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5
*d^3) - (6*f*(d*e - c*f)*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3
) + (168*f^2*(c + d*x)^2*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (3*f^2*(c
+ d*x)^(8/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*f*(d*e - c*f)*Sin[a
 + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*f^2*(c + d*x)^(1/3)*Sin[a + b*(c
 + d*x)^(1/3)])/(b^8*d^3) + (6*(d*e - c*f)^2*(c + d*x)^(1/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^2*d^3) - (360*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^4*d^3) + (20160*f^2*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/
(b^6*d^3) + (30*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(
```

```
b^2*d^3) - (1008*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d^3)
+ (24*f^2*(c + d*x)^(7/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\text{Subst}\left(\int \left(\frac{(de-cf)^2x^2\sin(a+bx)}{d^2}+\frac{2f(de-cf)x^5\sin(a+bx)}{d^2}+\frac{f^2x^8\sin(a+bx)}{d^2}\right) d}{d}$$

$$= \frac{(3f^2)\,\text{Subst}\left(\int x^8\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\text{Sub}}{}$$

$$= -\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6f(de-cf)(c+d}{}$$

$$= -\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6f(de-cf)(c+d}{}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+\right.}{bd^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+\right.}{bd^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a\right.}{b^5d^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a\right.}{b^5d^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a\right.}{b^5d^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a\right.}{b^5d^3}$$

$$= -\frac{120960f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} + \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right.}{b^3d^3}$$

Mathematica [A]
time = 1.52, size = 256, normalized size = 0.40

$$\frac{-3\left(40320f^2-20160b^2f^2(c+dx)^{2/3}+b^8d^2(c+dx)^{2/3}(e+fx)^2+240b^4f\sqrt[3]{c+dx}\,(6cf+d(e+7fx))-2b^6(9c^2f^2+18cdf(e+2fx)+d^2(e^2+20efx+28f^2x^2))\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+6b\left(-20160f^2\sqrt[3]{c+dx}-12b^4f(c+dx)^{2/3}(5de+9cf+14dfx)+b^6d\sqrt[3]{c+dx}\,(e+fx)(3cf+d(e+4fx))+120b^2f(27cf+d(e+28fx))\right)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(40320*f^2 - 20160*b^2*f^2*(c + d*x)^(2/3) + b^8*d^2*(c + d*x)^(2/3)*(e
 + f*x)^2 + 240*b^4*f*(c + d*x)^(1/3)*(6*c*f + d*(e + 7*f*x)) - 2*b^6*(9*c^
2*f^2 + 18*c*d*f*(e + 2*f*x) + d^2*(e^2 + 20*e*f*x + 28*f^2*x^2)))*Cos[a +
b*(c + d*x)^(1/3)] + 6*b*(-20160*f^2*(c + d*x)^(1/3) - 12*b^4*f*(c + d*x)^(
```

```
2/3)*(5*d*e + 9*c*f + 14*d*f*x) + b^6*d*(c + d*x)^(1/3)*(e + f*x)*(3*c*f +
d*(e + 4*f*x)) + 120*b^2*f*(27*c*f + d*(e + 28*f*x)))*Sin[a + b*(c + d*x)^(
1/3)])/(b^9*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2703 vs. $2(573) = 1146$.
time = 0.06, size = 2704, normalized size = 4.27

method	result	size
derivativedivides	Expression too large to display	2704
default	Expression too large to display	2704

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3/b^3*(10/b^3*a^4*d*e*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos
(a+b*(d*x+c)^(1/3)))+c^2*f^2*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))
+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d^2
*e^2*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3)
)+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+1/b^6*f^2*(-(a+b*(d*x+c)^(1
/3))^8*cos(a+b*(d*x+c)^(1/3))+8*(a+b*(d*x+c)^(1/3))^7*sin(a+b*(d*x+c)^(1/3)
)+56*(a+b*(d*x+c)^(1/3))^6*cos(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5
*sin(a+b*(d*x+c)^(1/3))-1680*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+6
720*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+20160*(a+b*(d*x+c)^(1/3))^
2*cos(a+b*(d*x+c)^(1/3))-40320*cos(a+b*(d*x+c)^(1/3))-40320*(a+b*(d*x+c)^(1
/3))*sin(a+b*(d*x+c)^(1/3)))-2/b^3*a^5*c*f^2*cos(a+b*(d*x+c)^(1/3))-2*c*d*e
*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+
2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-10/b^3*a^4*c*f^2*(sin(a+b*(d*
x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+20/b^3*a^3*c*f^2*(-
(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+
b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-20/b^3*a^2*c*f^2*(-(a+b*(d*x+c)^(1
/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3)
)-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+10
/b^3*a*c*f^2*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^
(1/3))^3*sin(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1
/3))-24*cos(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)
))+2/b^3*d*e*f*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c
)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^
(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^
(1/3))-120*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*f^2*(-(a+
b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(
d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+28/b^6*a^2*f^2*(-(a+b*(d*x+c)^(1/3))^
6*cos(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))+30*
(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))^3*sin(
a+b*(d*x+c)^(1/3))-360*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+720*cos
```

```
(a+b*(d*x+c)^(1/3))+720*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-8/b^6*a
*f^2*(-(a+b*(d*x+c)^(1/3))^7*cos(a+b*(d*x+c)^(1/3))+7*(a+b*(d*x+c)^(1/3))^6
*sin(a+b*(d*x+c)^(1/3))+42*(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))-210
*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))-840*(a+b*(d*x+c)^(1/3))^3*cos
(a+b*(d*x+c)^(1/3))+2520*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-5040*
sin(a+b*(d*x+c)^(1/3))+5040*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-1/b
^6*a^8*f^2*cos(a+b*(d*x+c)^(1/3))-a^2*d^2*e^2*cos(a+b*(d*x+c)^(1/3))-a^2*c^
2*f^2*cos(a+b*(d*x+c)^(1/3))-2/b^3*c*f^2*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d
*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)
^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(
1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1
/3)))-8/b^6*a^7*f^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*
x+c)^(1/3)))-2*a*c^2*f^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+
b*(d*x+c)^(1/3)))-2*a*d^2*e^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*c
os(a+b*(d*x+c)^(1/3)))-56/b^6*a^3*f^2*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+
c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1
/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3
))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)
))-56/b^6*a^5*f^2*(-(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*
x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c
)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+70/b^6*a^4*f^2*(-(a+b*(d*x+c)^(1/3))^4*cos
(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+12*(a+b*
(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a+b*(d*x+c)^(1/3))-24*(a+b*(
d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+4*a*c*d*e*f*(sin(a+b*(d*x+c)^(1/3))-(
a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-20/b^3*a^3*d*e*f*(-(a+b*(d*x+c)^
(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/
3))*sin(a+b*(d*x+c)^(1/3)))+20/b^3*a^2*d*e*f*(-(a+b*(d*x+c)^(1/3))^3*cos(a+
b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*sin(a+b*(
d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-10/b^3*a*d*e*f*
(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(
a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a
+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+2*a^2*c*d*
e*f*cos(a+b*(d*x+c)^(1/3))+2/b^3*a^5*d*e*f*cos(a+b*(d*x+c)^(1/3)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2157 vs. $2(582) = 1164$.
time = 0.43, size = 2157, normalized size = 3.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*c^2*f^2*cos((d*x + c)^(1/3)*b + a)/d^2 - 2*a^2*c*f*cos((d*x + c)^(1
/3)*b + a)*e/d - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - si
```

```
n((d*x + c)^(1/3)*b + a))*a*c^2*f^2/d^2 + 2*a^5*c*f^2*cos((d*x + c)^(1/3)*b
 + a)/(b^3*d^2) + a^2*cos((d*x + c)^(1/3)*b + a)*e^2 + 4*(((d*x + c)^(1/3)*
b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a*c*f*e/d -
 2*a^5*f*cos((d*x + c)^(1/3)*b + a)*e/(b^3*d) - 10*(((d*x + c)^(1/3)*b + a)
*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*c*f^2/(b^3*d^
2) + ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x
+ c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c^2*f^2/d^2 + a^8*f^2*cos((d*
x + c)^(1/3)*b + a)/(b^6*d^2) - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1
/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a*e^2 + 10*(((d*x + c)^(1/3)*b + a
)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*f*e/(b^3*d)
- 2*((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x +
 c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c*f*e/d - 8*(((d*x + c)^(1/3)*
b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^7*f^2/(b^
6*d^2) + 20*((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2
*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*a^3*c*f^2/(b^3*d^2) +
((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^
(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*e^2 - 20*((((d*x + c)^(1/3)*b + a)
^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c
)^(1/3)*b + a))*a^3*f*e/(b^3*d) + 28*((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((
d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a
))*a^6*f^2/(b^6*d^2) - 20*((((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b
 - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b + a)^2 - 2)*sin(
(d*x + c)^(1/3)*b + a))*a^2*c*f^2/(b^3*d^2) + 20*((((d*x + c)^(1/3)*b + a)^
3 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(
1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*f*e/(b^3*d) - 56*((((d*x
 + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a
) - 3*(((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^5*f^2/(
b^6*d^2) + 10*((((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 +
24)*cos((d*x + c)^(1/3)*b + a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)
^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a))*a*c*f^2/(b^3*d^2) - 10*((((d*x
+ c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3
)*b + a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d
*x + c)^(1/3)*b + a))*a*f*e/(b^3*d) + 70*((((d*x + c)^(1/3)*b + a)^4 - 12*(
(d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) - 4*(((d*x + c)^(
1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a))*a^4*
f^2/(b^6*d^2) - 2*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^
3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5*(((d*x +
c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1/3)*
b + a))*c*f^2/(b^3*d^2) + 2*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/
3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5
*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x +
 c)^(1/3)*b + a))*f*e/(b^3*d) - 56*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x +
 c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b +
 a) - 5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin
((d*x + c)^(1/3)*b + a))*a^3*f^2/(b^6*d^2) + 28*((((d*x + c)^(1/3)*b + a)^6
```

```
 - 30*((d*x + c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*cos(
(d*x + c)^(1/3)*b + a) - 6*(((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)
*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*a^2*
f^2/(b^6*d^2) - 8*((((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)*b + a)^
5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)*cos((d
*x + c)^(1/3)*b + a) - 7*(((d*x + c)^(1/3)*b + a)^6 - 30*((d*x + c)^(1/3)*b
 + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*sin((d*x + c)^(1/3)*b + a))*
a*f^2/(b^6*d^2) + ((((d*x + c)^(1/3)*b + a)^8 - 56*((d*x + c)^(1/3)*b + a)^
6 + 1680*((d*x + c)^(1/3)*b + a)^4 - 20160*((d*x + c)^(1/3)*b + a)^2 + 4032
0)*cos((d*x + c)^(1/3)*b + a) - 8*(((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c
)^(1/3)*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b -
 5040*a)*sin((d*x + c)^(1/3)*b + a))*f^2/(b^6*d^2))/(b^3*d)
```

Fricas [**A**]
time = 0.36, size = 336, normalized size = 0.53

$$\frac{3\left(\left(56\,b^6d^2f^2x^2+72\,b^6cdf^2x+2\,b^6d^2e^2+18\,(b^6c^2-2240)f^2+4\,(10\,b^6d^2fx+9\,b^6cdf)e-(b^8d^2f^2x^2+2\,b^8d^2fxe+b^8d^2e^2-20160\,b^2f^2)(dx+c)^{\frac{2}{3}}-240\,(7\,b^4df^2x+6\,b^4cf^2+b^4dfe)(dx+c)^{\frac{1}{3}}\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+2\left(3360\,b^3df^2x+3240\,b^3cf^2+120\,b^3dfe-12\,(14\,b^5df^2x+9\,b^5cf^2+5\,b^5dfe)(dx+c)^{\frac{2}{3}}+(4\,b^7d^2f^2x^2+3\,b^7cdf^2x+b^7d^2e^2-20160\,bf^2+(5\,b^7d^2fx+3\,b^7cdf)e)(dx+c)^{\frac{1}{3}}\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^9d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((56*b^6*d^2*f^2*x^2 + 72*b^6*c*d*f^2*x + 2*b^6*d^2*e^2 + 18*(b^6*c^2 - 2
240)*f^2 + 4*(10*b^6*d^2*f*x + 9*b^6*c*d*f)*e - (b^8*d^2*f^2*x^2 + 2*b^8*d^
2*f*x*e + b^8*d^2*e^2 - 20160*b^2*f^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*f^2*x
 + 6*b^4*c*f^2 + b^4*d*f*e)*(d*x + c)^(1/3))*cos((d*x + c)^(1/3)*b + a) + 2
*(3360*b^3*d*f^2*x + 3240*b^3*c*f^2 + 120*b^3*d*f*e - 12*(14*b^5*d*f^2*x +
9*b^5*c*f^2 + 5*b^5*d*f*e)*(d*x + c)^(2/3) + (4*b^7*d^2*f^2*x^2 + 3*b^7*c*d
*f^2*x + b^7*d^2*e^2 - 20160*b*f^2 + (5*b^7*d^2*f*x + 3*b^7*c*d*f)*e)*(d*x
+ c)^(1/3))*sin((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(582) = 1164$.
time = 5.08, size = 1558, normalized size = 2.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")

[Out] -3*(f^2*(((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*c + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a^8 - 2*b^6*c^2 + 40*((d*x + c)^(1/3)*b + a)^3*b^3*c - 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c + 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c - 40*a^3*b^3*c - 56*((d*x + c)^(1/3)*b + a)^6 + 336*((d*x + c)^(1/3)*b + a)^5*a - 840*((d*x + c)^(1/3)*b + a)^4*a^2 + 1120*((d*x + c)^(1/3)*b + a)^3*a^3 - 840*((d*x + c)^(1/3)*b + a)^2*a^4 + 336*((d*x + c)^(1/3)*b + a)*a^5 - 56*a^6 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 20160*((d*x + c)^(1/3)*b + a)^2 + 40320*((d*x + c)^(1/3)*b + a)*a - 20160*a^2 + 40320)*cos((d*x + c)^(1/3)*b + a)/(b^8*d^2) - 2*((((d*x + c)^(1/3)*b + a)*b^6*c^2 - a*b^6*c^2 - 5*((d*x + c)^(1/3)*b + a)^4*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3*a*b^3*c - 30*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)*a^3*b^3*c - 5*a^4*b^3*c + 4*((d*x + c)^(1/3)*b + a)^7 - 28*((d*x + c)^(1/3)*b + a)^6*a + 84*((d*x + c)^(1/3)*b + a)^5*a^2 - 140*((d*x + c)^(1/3)*b + a)^4*a^3 + 140*((d*x + c)^(1/3)*b + a)^3*a^4 - 84*((d*x + c)^(1/3)*b + a)^2*a^5 + 28*((d*x + c)^(1/3)*b + a)*a^6 - 4*a^7 + 60*((d*x + c)^(1/3)*b + a)^2*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a*b^3*c + 60*a^2*b^3*c - 168*((d*x + c)^(1/3)*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^4*a - 1680*((d*x + c)^(1/3)*b + a)^3*a^2 + 1680*((d*x + c)^(1/3)*b + a)^2*a^3 - 840*((d*x + c)^(1/3)*b + a)*a^4 + 168*a^5 - 120*b^3*c + 3360*((d*x + c)^(1/3)*b + a)^3 - 10080*((d*x + c)^(1/3)*b + a)^2*a + 10080*((d*x + c)^(1/3)*b + a)*a^2 - 3360*a^3 - 20160*(d*x + c)^(1/3)*b)*sin((d*x + c)^(1/3)*b + a)/(b^8*d^2)) - (2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - ((((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b^2)*e^2 - 2*f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3 - 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 - 20*a^3 - 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b + a)/b^5 - (2*((d*x + c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x + c)^(1/3)*b + a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/

```
b^5)*e/d)/(b*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x)^2, x)
```

3.208 $\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=288

$$\frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{360f\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c\right.}}{bd^2}$$

```
[Out] 6*(-c*f+d*e)*cos(a+b*(d*x+c)^(1/3))/b^3/d^2-360*f*(d*x+c)^(1/3)*cos(a+b*(d*
x+c)^(1/3))/b^5/d^2-3*(-c*f+d*e)*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d^2
+60*f*(d*x+c)*cos(a+b*(d*x+c)^(1/3))/b^3/d^2-3*f*(d*x+c)^(5/3)*cos(a+b*(d*x
+c)^(1/3))/b/d^2+360*f*sin(a+b*(d*x+c)^(1/3))/b^6/d^2+6*(-c*f+d*e)*(d*x+c)^
(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^2-180*f*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1
/3))/b^4/d^2+15*f*(d*x+c)^(4/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3512, 3377, 2718, 2717}

$$\frac{360f\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}-\frac{360f\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{180f(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{60f(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{6\sqrt[3]{c+dx}\,(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{3(c+dx)^{2/3}(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*(d*e - c*f)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^2) - (360*f*(c + d*x)^(1/
3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*(d*e - c*f)*(c + d*x)^(2/3)*C
os[a + b*(c + d*x)^(1/3)])/(b*d^2) + (60*f*(c + d*x)*Cos[a + b*(c + d*x)^(1
/3)])/(b^3*d^2) - (3*f*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^2)
+ (360*f*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2) + (6*(d*e - c*f)*(c + d*x)^(
1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^2) - (180*f*(c + d*x)^(2/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^4*d^2) + (15*f*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^
(1/3)])/(b^2*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{3\text{Subst}\left(\int \left(\frac{(de-cf)x^2\sin(a+bx)}{d}+\frac{fx^5\sin(a+bx)}{d}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d} \\
&= \frac{(3f)\text{Subst}\left(\int x^5\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2}+\frac{(3(de-cf))\text{Subst}\left(\right.}{} \\
&= -\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{3f(c+dx)^{5/3}\cos\left(a\right.}{bd^2} \\
&= -\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{3f(c+dx)^{5/3}\cos\left(a\right.}{bd^2} \\
&= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\right.}{bd^2} \\
&= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\right.}{bd^2} \\
&= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right.}{b^5d^2} \\
&= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right.}{b^5d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 147, normalized size = 0.51

$$
\frac{-3b\left(120f\sqrt[3]{c+dx}+b^4d(c+dx)^{2/3}(e+fx)-2b^2(9cf+d(e+10fx))\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+3\left(2b^4de\sqrt[3]{c+dx}+f\left(120-60b^2(c+dx)^{2/3}+b^4\sqrt[3]{c+dx}\,(3c+5dx)\right)\right)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*b*(120*f*(c + d*x)^(1/3) + b^4*d*(c + d*x)^(2/3)*(e + f*x) - 2*b^2*(9*c
*f + d*(e + 10*f*x)))*Cos[a + b*(c + d*x)^(1/3)] + 3*(2*b^4*d*e*(c + d*x)^(
1/3) + f*(120 - 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))
)*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(258) = 516$.
time = 0.02, size = 801, normalized size = 2.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2/b^3*(a^2*c*f*cos(a+b*(d*x+c)^(1/3))-a^2*d*e*cos(a+b*(d*x+c)^(1/3))+2*
a*c*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2
*a*d*e*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-
c*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))
+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d*e*(-(a+b*(d*x+c)^(1/3))^2*
cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a
+b*(d*x+c)^(1/3)))+1/b^3*a^5*f*cos(a+b*(d*x+c)^(1/3))+5/b^3*a^4*f*(sin(a+b*
(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*f*(-(
a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b
*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*f*(-(a+b*(d*x+c)^(1/3))^
3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-6*s
in(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-5/b^3*a
*f*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*s
in(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*co
s(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+1/b^3*f
*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin
(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b
*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+
b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(261) = 522$.
time = 0.34, size = 684, normalized size = 2.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(a^2*c*f*cos((d*x + c)^(1/3)*b + a)/d - a^2*cos((d*x + c)^(1/3)*b + a)*e
- 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/
3)*b + a))*a*c*f/d + a^5*f*cos((d*x + c)^(1/3)*b + a)/(b^3*d) + 2*(((d*x +
c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a*
e - 5*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(
```

```
1/3)*b + a))*a^4*f/(b^3*d) + ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)
^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c*f/d
 - ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x +
c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*e + 10*((((d*x + c)^(1/3)*b + a
)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x +
c)^(1/3)*b + a))*a^3*f/(b^3*d) - 10*((((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b + a)^2
 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*f/(b^3*d) + 5*((((d*x + c)^(1/3)*b +
a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) - 4*((
(d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b
 + a))*a*f/(b^3*d) - ((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b +
a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5*(((d*x
 + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1/
3)*b + a))*f/(b^3*d))/(b^3*d)
```

Fricas [A]
time = 0.37, size = 145, normalized size = 0.50

$$\frac{3\left(\left(20\,b^3dfx+18\,b^3cf+2\,b^3de-120\,(dx+c)^{\frac{1}{3}}bf-(b^5dfx+b^5de)(dx+c)^{\frac{2}{3}}\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(60\,(dx+c)^{\frac{2}{3}}b^2f-(5\,b^4dfx+3\,b^4cf+2\,b^4de)(dx+c)^{\frac{1}{3}}-120\,f\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((20*b^3*d*f*x + 18*b^3*c*f + 2*b^3*d*e - 120*(d*x + c)^(1/3)*b*f - (b^5*
d*f*x + b^5*d*e)*(d*x + c)^(2/3))*cos((d*x + c)^(1/3)*b + a) - (60*(d*x + c
)^(2/3)*b^2*f - (5*b^4*d*f*x + 3*b^4*c*f + 2*b^4*d*e)*(d*x + c)^(1/3) - 120
*f)*sin((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [A]
time = 5.12, size = 454, normalized size = 1.58

$$\frac{3\left(\left(\frac{2\,(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}-\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)e+\frac{f\left(\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2b^3c-2\left((dx+c)^{\frac{1}{3}}b+a\right)ab^3c+a^2b^3c-\left((dx+c)^{\frac{1}{3}}b+a\right)^5+5\left((dx+c)^{\frac{1}{3}}b+a\right)^4a-10\left((dx+c)^{\frac{1}{3}}b+a\right)^3a^2+10\left((dx+c)^{\frac{1}{3}}b+a\right)^2a^3-5\left((dx+c)^{\frac{1}{3}}b+a\right)a^4+a^5-2\,b^3c+20\left((dx+c)^{\frac{1}{3}}b+a\right)^3-60\left((dx+c)^{\frac{1}{3}}b+a\right)^2a+60\left((dx+c)^{\frac{1}{3}}b+a\right)a^2-20\,a^3-120\,(dx+c)^{\frac{1}{3}}b\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^5}-\frac{\left(2\left((dx+c)^{\frac{1}{3}}b+a\right)b^3c-2\,ab^3c-5\left((dx+c)^{\frac{1}{3}}b+a\right)^4+20\left((dx+c)^{\frac{1}{3}}b+a\right)^3a-30\left((dx+c)^{\frac{1}{3}}b+a\right)^2a^2+20\left((dx+c)^{\frac{1}{3}}b+a\right)a^3-5\,a^4+60\left((dx+c)^{\frac{1}{3}}b+a\right)^2-120\left((dx+c)^{\frac{1}{3}}b+a\right)a+60\,a^2-120\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^5}\right)}{d}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*((2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*b +
a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b^
2)*e + f*(((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^
3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a
 - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*
((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3
- 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 - 20*a^3
- 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b + a)/b^5 - (2*((d*x + c)^(1/
3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(
1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b
+ a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x + c)^(1/3)*b +
a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/b^5)/d)/(b*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))*(e + f*x), x)
```

3.209 $\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=85

$$\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}$$

```
[Out] 6*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d+6
*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3442, 3377, 2718}

$$\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{6\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d) + (6*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right.}{bd} \cdots$$

$$= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{6\text{Sub}\cdots}{\cdots}$$

$$= \frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\sqrt[3]{c+dx}\sin\left(\cdots\right.}{b\cdots}$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 0.76

$$\frac{\left(6-3b^2(c+dx)^{2/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+6b\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] ((6 - 3*b^2*(c + d*x)^(2/3))*Cos[a + b*(c + d*x)^(1/3)] + 6*b*(c + d*x)^(1/
3)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A]
time = 0.01, size = 134, normalized size = 1.58

method	result
derivativedivides	$\frac{-3a^2\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-\left(a+b(dx+c)^{\frac{1}{3}}\right)\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cos\left(a+b(dx\cdots\right.}{d\,b^3}$
default	$\frac{-3a^2\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-\left(a+b(dx+c)^{\frac{1}{3}}\right)\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\cos\left(a+b(dx\cdots\right.}{d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d/b^3*(-a^2*cos(a+b*(d*x+c)^(1/3))-2*a*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+
c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/
3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))
```

Maxima [A]
time = 0.32, size = 120, normalized size = 1.41

$$-\frac{3\left(a^2\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)a+\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left((dx+c)^{\frac{1}{3}}b+a\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*cos((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x +
c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a + (((d*x + c)^(1/3)*b + a)^
2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)
^(1/3)*b + a))/(b^3*d)
```

Fricas [**A**]
time = 0.35, size = 58, normalized size = 0.68

$$\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*b*sin((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2 -
2)*cos((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [**A**]
time = 0.27, size = 94, normalized size = 1.11

$$\begin{cases} x\sin(a) & \text{for}\ b=0\wedge(b=0\vee d=0) \\ x\sin\left(a+b\sqrt[3]{c}\right) & \text{for}\ d=0 \\ -\frac{3(c+dx)^{\frac{2}{3}}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sin(a + b*c**(1/
3)), Eq(d, 0)), (-3*(c + d*x)**(2/3)*cos(a + b*(c + d*x)**(1/3))/(b*d) + 6*
(c + d*x)**(1/3)*sin(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cos(a + b*(c + d*
x)**(1/3))/(b**3*d), True))
```

Giac [**A**]
time = 4.03, size = 82, normalized size = 0.96

$$\frac{3\left(\frac{2\,(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}-\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - ((((d*x + c)^(1/3)*b + a
)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b^2
)/(b*d)
```

Mupad [B]
time = 4.61, size = 69, normalized size = 0.81

$$\frac{3\left(2\cos\left(a+b\,(c+d\,x)^{1/3}\right)+2\,b\sin\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+d\,x)^{1/3}-b^2\cos\left(a+b\,(c+d\,x)^{1/3}\right)\,(c+d\,x)^{2/3}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (3*(2*cos(a + b*(c + d*x)^(1/3)) + 2*b*sin(a + b*(c + d*x)^(1/3))*(c + d*x)
^(1/3) - b^2*cos(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3)))/(b^3*d)
```

3.210 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$

Optimal. Leaf size=396

$$\frac{\text{Ci}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}+\frac{\text{Ci}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)\sin\left(a+\frac{\sqrt[3]{-}}{}\right.}{f}$$

```
[Out] cos(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(-(-1)^(1/3)*b*(-c*f+d*e)^(1
/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+cos(a-b*(-c*f+d*e)^(1/3)/f^(1/3))*Si(b*(-c*f
+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+cos(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/
f^(1/3))*Si((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))/f+Ci(b*(
-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*sin(a-b*(-c*f+d*e)^(1/3)/f^(1/3))/
f+Ci((-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(1/3
)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f+Ci((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*
(d*x+c)^(1/3))*sin(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f
```

Rubi [A]
time = 0.96, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3512, 3384, 3380, 3383}

$$\frac{\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}+\frac{\sin\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}+\frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}-\frac{\cos\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}+\frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{de-cf}\,b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}\,b\right)}{f}+\frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{de-cf}\,b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}\,b\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] (CosIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - (b*
(d*e - c*f)^(1/3))/f^(1/3)])/f + (CosIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/
3))/f^(1/3) - b*(c + d*x)^(1/3)]*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f
^(1/3)])/f + (CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c +
 d*x)^(1/3)]*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)])/f - (Cos[a
+ ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(1/3)*b*(d*e
- c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/f + (Cos[a - (b*(d*e - c*f)^(1/
3))/f^(1/3)]*SinIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]
)/f + (Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^
(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/f
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx = \frac{3\mathrm{Subst}\left(\int \left(\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left(\sqrt[3]{de-cf}+\sqrt[3]{f}\,x\right)}+\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{-1}\,\sqrt[3]{de-cf}+\cdots\right.}\right.\right.}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{de-cf}+\sqrt[3]{f}\,x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} + \frac{\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{-\sqrt[3]{-1}\,\sqrt[3]{de-cf}+\cdots}\right.}{f^{2/3}}$$

$$= \frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{\sqrt[3]{de-cf}+\sqrt[3]{f}\,x}\,dx, x, \sqrt[3]{c+dx}\right)}{f^{2/3}} - \cdots \cos\cdots$$

$$= \frac{\mathrm{Ci}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f} + \frac{\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-c\cdots}}{\sqrt[3]{f}}\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.36, size = 118, normalized size = 0.30

$$\frac{i\left(\mathrm{RootSum}\left[de-cf+f\#1^3\&, e^{-ia-ib\#1}\mathrm{Ei}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]-\mathrm{RootSum}\left[de-cf+f\#1^3\&, e^{ia+ib\#1}\mathrm{Ei}\left(ib\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] ((I/2)*(RootSum[d*e - c*f + f*#1^3 & , E^((-I)*a - I*b*#1)*ExpIntegralEi[(-
I)*b*((c + d*x)^(1/3) - #1)] & ] - RootSum[d*e - c*f + f*#1^3 & , E^(I*a +
I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #1)] & ]))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 327, normalized size = 0.83

method	result
derivativedivides	$\frac{b^3 a^2 \left(\sum_{\_R1=\mathrm{RootOf}\left(-b^3 c f+b^3 d e+f\_Z^3-3 a f\_Z^2+3 a^2 f\_Z-a^3 f\right)} \frac{-\operatorname{sinIntegral}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\cos\left(\_R1\right)+\mathrm{cosin}\ldots}{\_R1^2-2\_R1\ldots}\ldots\right.}{f}\ldots$
default	$\frac{b^3 a^2 \left(\sum_{\_R1=\mathrm{RootOf}\left(-b^3 c f+b^3 d e+f\_Z^3-3 a f\_Z^2+3 a^2 f\_Z-a^3 f\right)} \frac{-\operatorname{sinIntegral}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\cos\left(\_R1\right)+\mathrm{cosin}\ldots}{\_R1^2-2\_R1\ldots}\ldots\right.}{f}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/3*b^3*a^2/f*sum(1/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)
*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_
Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))-2/3*b^3*a/f*sum(_R1/(_R1^2-2*_R1*a+a^2)
*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),
_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+1/3*b^3/f*
sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d
*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+
3*_Z*a^2*f-a^3*f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.42, size = 460, normalized size = 1.16

$$\frac{-i\,\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{ib^3cf-ib^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{ib^3cf-ib^3de}{f}\right)^{\frac{1}{3}}+ia\right)}-i\,\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{ib^3cf-ib^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(-i\sqrt{3}+1\right)\left(\frac{ib^3cf-ib^3de}{f}\right)^{\frac{1}{3}}+ia\right)}+i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{-ib^3cf+ib^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{-ib^3cf+ib^3de}{f}\right)^{\frac{1}{3}}-ia\right)}+i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{-ib^3cf+ib^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(-i\sqrt{3}+1\right)\left(\frac{-ib^3cf+ib^3de}{f}\right)^{\frac{1}{3}}-ia\right)}-i\,\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b+\left(\frac{ib^3cf-ib^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(ia-\left(\frac{ib^3cf-ib^3de}{f}\right)^{\frac{1}{3}}\right)}+i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\left(\frac{-ib^3cf+ib^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(-ia-\left(\frac{-ib^3cf+ib^3de}{f}\right)^{\frac{1}{3}}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*c*f - I*b^3*d
*e)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*c*f - I*b^3*d*e)/f)^(1/3) + I*
a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*c*f - I*b^3*d*e
)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*c*f - I*b^3*d*e)/f)^(1/3) + I*a
) + I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*c*f + I*b^3*d
*e)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3) - I
*a) + I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*c*f + I*b^3*
d*e)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3) -
 I*a) - I*Ei(I*(d*x + c)^(1/3)*b + ((I*b^3*c*f - I*b^3*d*e)/f)^(1/3))*e^(I*
a - ((I*b^3*c*f - I*b^3*d*e)/f)^(1/3)) + I*Ei(-I*(d*x + c)^(1/3)*b + ((-I*b
^3*c*f + I*b^3*d*e)/f)^(1/3))*e^(-I*a - ((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3))
)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a + b\,{\left(c + d\,x\right)}^{1/3}\right)}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x), x)
```

3.211 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=555

$$-\frac{\sqrt[3]{-1}\,bd\cos\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{C}}{3f^{4/3}(d}$$

```
[Out] 1/3*b*d*Ci(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*cos(a-b*(-c*f+d*e)^(
1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*(-1)^(1/3)*b*d*Ci((-1)^(1/3)*b*(
-c*f+d*e)^(1/3)/f^(1/3)-b*(d*x+c)^(1/3))*cos(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3
)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)+1/3*(-1)^(2/3)*b*d*Ci((-1)^(2/3)*b*(-c*
f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*cos(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f
^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*b*d*Si(b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d
*x+c)^(1/3))*sin(a-b*(-c*f+d*e)^(1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)+1/3
*(-1)^(1/3)*b*d*Si(-(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*
sin(a+(-1)^(1/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-1/3*(
-1)^(2/3)*b*d*Si((-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3)+b*(d*x+c)^(1/3))*sin
(a-(-1)^(2/3)*b*(-c*f+d*e)^(1/3)/f^(1/3))/f^(4/3)/(-c*f+d*e)^(2/3)-sin(a+b*
(d*x+c)^(1/3))/f/(f*x+e)
```

Rubi [A]
time = 1.45, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$-\frac{\sqrt[3]{-1}\,bd\cos\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}+\frac{(-1)^{2/3}bd\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}-\frac{\sqrt[3]{-1}\,bd\sin\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}-\frac{bd\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{\sqrt[3]{de-cf}\,b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}\,b\right)}{3f^{4/3}(de-cf)^{2/3}}-\frac{(-1)^{2/3}bd\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{de-cf}\,b}{\sqrt[3]{f}}+\sqrt[3]{c+dx}\,b\right)}{3f^{4/3}(de-cf)^{2/3}}-\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] -1/3*((-1)^(1/3)*b*d*Cos[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*CosI
ntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(f^(
4/3)*(d*e - c*f)^(2/3)) + (b*d*Cos[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*CosIn
tegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(d*e
- c*f)^(2/3)) + ((-1)^(2/3)*b*d*Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^
(1/3)]*CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(
1/3)])/(3*f^(4/3)*(d*e - c*f)^(2/3)) - Sin[a + b*(c + d*x)^(1/3)]/(f*(e + f
*x)) - ((-1)^(1/3)*b*d*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*Si
nIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(3
*f^(4/3)*(d*e - c*f)^(2/3)) - (b*d*Sin[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*S
inIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(
d*e - c*f)^(2/3)) - ((-1)^(2/3)*b*d*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3)
```

```
)/f^(1/3)]*SinIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*
x)^(1/3)])/(3*f^(4/3)*(d*e - c*f)^(2/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx = \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(e-\frac{cf}{d}+\frac{fx^3}{d}\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{e-\frac{cf}{d}+\frac{fx^3}{d}}\,dx, x, \sqrt[3]{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} + \frac{b\text{Subst}\left(\int \left(-\frac{\sqrt[3]{de-cf}\cos(a+bx)}{3\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{de-cf}-\sqrt[3]{f}x\right)} - \frac{}{3\left(e-\right.}\right.\right.}{}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{-\sqrt[3]{de-cf}-\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt[3]{de-}}{\sqrt[3]{f}}\right.}{-\sqrt[3]{de-c}}\right.}{3f(de-cf)^{2/3}}$$

$$= -\frac{\sqrt[3]{-1}\,bd\cos\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.70, size = 180, normalized size = 0.32

$$\frac{\frac{3ie^{-i\left(a+b\sqrt[3]{c+dx}\right)}\left(-1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)f}{e+fx} + bd\text{RootSum}\left[de-cf+f\#1^3\&, \frac{e^{-ia-ib\#1}\text{Ei}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + bd\text{RootSum}\left[de-cf+f\#1^3\&, \frac{e^{ia+ib\#1}\text{Ei}\left(ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right]}{6f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] (((3*I)*(-1 + E^((2*I)*(a + b*(c + d*x)^(1/3))))*f)/(E^(I*(a + b*(c + d*x)^
(1/3)))*(e + f*x)) + b*d*RootSum[d*e - c*f + f*#1^3 & , (E^((-I)*a - I*b*#1
)*ExpIntegralEi[(-I)*b*((c + d*x)^(1/3) - #1)])/#1^2 & ] + b*d*RootSum[d*e
- c*f + f*#1^3 & , (E^(I*a + I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #
1)])/#1^2 & ])/(6*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 1176, normalized size = 2.12

method	result	size
derivativedivides	Expression too large to display	1176
default	Expression too large to display	1176

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d/b^3*(b^6*a^2*(sin(a+b*(d*x+c)^(1/3))*(1/3/b^3/(c*f-d*e)*(a+b*(d*x+c)^(1
/3))-1/3*a/b^3/(c*f-d*e))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x+c)^(1/3)
)+3*a*f*(a+b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)-2/9/b^3/f*sum(1/(c*f
-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c
)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z
*a^2*f-a^3*f))-1/9/b^3/f*sum(1/(-_RR1+a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/3)+_RR
1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b
^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)))+sin(a+b*(d*x+c)^(1/3))*(-2/3*a
*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))^2+2/3*a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/
3)))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x+c)^(1/3))+3*a*f*(a+b*(d*x+c)^
(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)+2/9*a*b^3/f*sum((_R1+a)/(c*f-d*e)/(_R1^2-
2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a
)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)
)+2/9*a*b^3/f*sum(_RR1/(-_RR1+a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/3)+_RR1-a)*sin
(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b^3*d*e+_
Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+sin(a+b*(d*x+c)^(1/3))*(2/3*a*b^3/(c*f-
d*e)*(a+b*(d*x+c)^(1/3))^2-a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))+1/3*b^3*(b
^3*c*f-b^3*d*e+a^3*f)/f/(c*f-d*e))/(b^3*c*f-b^3*d*e+a^3*f-3*a^2*f*(a+b*(d*x
+c)^(1/3))+3*a*f*(a+b*(d*x+c)^(1/3))^2-f*(a+b*(d*x+c)^(1/3))^3)-2/9*a*b^3/f
*sum(_R1/(c*f-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1
)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*
_Z^2*a*f+3*_Z*a^2*f-a^3*f))+1/9*b^3/f^2*sum((b^3*c*f-b^3*d*e+2*_RR1^2*a*f-3
*_RR1*a^2*f+a^3*f)/(c*f-d*e)/(_RR1^2-2*_RR1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_RR
1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b
^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.43, size = 760, normalized size = 1.37

$$-\frac{\left(i\,dfx + i\,de - \sqrt{3}\,(dfx + de)\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}} + i\,a\right)} + \left(i\,dfx + i\,de + \sqrt{3}\,(dfx + de)\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(-i\sqrt{3}+1\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}} + i\,a\right)} + \left(-i\,dfx - i\,de + \sqrt{3}\,(dfx + de)\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}} - i\,a\right)} + \left(-i\,dfx - i\,de - \sqrt{3}\,(dfx + de)\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(-i\sqrt{3}+1\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}} - i\,a\right)} - 2\left(i\,dfx + i\,de\right)\left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b + \left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(i\,a - \left(\frac{i\,b^3cf - i\,b^3de}{f}\right)^{\frac{1}{3}}\right)} - 2\left(-i\,dfx - i\,de\right)\left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b + \left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\right)e^{\left(-i\,a - \left(\frac{-i\,b^3cf + i\,b^3de}{f}\right)^{\frac{1}{3}}\right)} + 12\,(cf - de)\sin\left((dx+c)^{\frac{1}{3}}b + a\right)}{12\,\left(cf^3x - dfe^2 - (df^2x - cf^2)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/12*((I*d*f*x + I*d*e - sqrt(3)*(d*f*x + d*e))*((I*b^3*c*f - I*b^3*d*e)/f
)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*c*f - I*b^3*d
*e)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*c*f - I*b^3*d*e)/f)^(1/3) + I*
a) + (I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*((I*b^3*c*f - I*b^3*d*e)/f)^
(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*c*f - I*b^3*d*e)
/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*c*f - I*b^3*d*e)/f)^(1/3) + I*a)
 + (-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e))*((-I*b^3*c*f + I*b^3*d*e)/f)^
(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*c*f + I*b^3*d
*e)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3) - I
*a) + (-I*d*f*x - I*d*e - sqrt(3)*(d*f*x + d*e))*((-I*b^3*c*f + I*b^3*d*e)/
f)^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*c*f + I*b^3
*d*e)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3)
- I*a) - 2*(I*d*f*x + I*d*e)*((I*b^3*c*f - I*b^3*d*e)/f)^(1/3)*Ei(I*(d*x +
c)^(1/3)*b + ((I*b^3*c*f - I*b^3*d*e)/f)^(1/3))*e^(I*a - ((I*b^3*c*f - I*b^
3*d*e)/f)^(1/3)) - 2*(-I*d*f*x - I*d*e)*((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3)*
Ei(-I*(d*x + c)^(1/3)*b + ((-I*b^3*c*f + I*b^3*d*e)/f)^(1/3))*e^(-I*a - ((-
I*b^3*c*f + I*b^3*d*e)/f)^(1/3)) + 12*(c*f - d*e)*sin((d*x + c)^(1/3)*b + a
))/(c*f^3*x - d*f*e^2 - (d*f^2*x - c*f^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{(e + fx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,(c+d\,x)^{1/3}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(e + f*x)^2, x)
```

3.212 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=513

$$\frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3}-\frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}+\frac{105f^2(c+dx)\cos\left(a+b\right.}{8b^3d^3}$$

```
[Out] 6*f*(-c*f+d*e)*cos(a+b*(d*x+c)^(2/3))/b^3/d^3-3/2*(-c*f+d*e)^2*(d*x+c)^(1/3
)*cos(a+b*(d*x+c)^(2/3))/b/d^3+105/8*f^2*(d*x+c)*cos(a+b*(d*x+c)^(2/3))/b^3
/d^3-3*f*(-c*f+d*e)*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))/b/d^3-3/2*f^2*(d*x
+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))/b/d^3-315/16*f^2*(d*x+c)^(1/3)*sin(a+b*(d*
x+c)^(2/3))/b^4/d^3+6*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2
/d^3+21/4*f^2*(d*x+c)^(5/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d^3+3/4*(-c*f+d*e)^2
*cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b
^(3/2)/d^3+315/32*f^2*cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2
))*2^(1/2)*Pi^(1/2)/b^(9/2)/d^3+315/32*f^2*FresnelC((d*x+c)^(1/3)*b^(1/2)*2
^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(9/2)/d^3-3/4*(-c*f+d*e)^2*Fresn
elS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
/d^3
```

Rubi [A]
time = 0.39, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3514, 3466, 3435, 3433, 3432, 3460, 3377, 2718, 3467, 3434}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}+\frac{315\sqrt{\frac{\pi}{2}}f^2\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3}+\frac{315\sqrt{\frac{\pi}{2}}f^2\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{16b^{9/2}d^3}-\frac{315f^2\sqrt[3]{c+dx}\,\sin\left(a+b(c+dx)^{2/3}\right)}{16b^4d^3}+\frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3}+\frac{105f^2(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d^3}+\frac{6f(c+dx)^{2/3}(de-cf)\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^3}+\frac{21f^2(c+dx)^{5/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d^3}-\frac{3f(c+dx)^{4/3}(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{bd^3}-\frac{3\sqrt[3]{c+dx}\,(de-cf)^2\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}-\frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (6*f*(d*e - c*f)*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^3) - (3*(d*e - c*f)^2*(
c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^3) + (105*f^2*(c + d*x)*C
os[a + b*(c + d*x)^(2/3)])/(8*b^3*d^3) - (3*f*(d*e - c*f)*(c + d*x)^(4/3)*C
os[a + b*(c + d*x)^(2/3)])/(b*d^3) - (3*f^2*(c + d*x)^(7/3)*Cos[a + b*(c +
d*x)^(2/3)])/(2*b*d^3) + (3*(d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^3) + (315*f^2*Sqrt[Pi/2]*Cos[a]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(16*b^(9/2)*d^3) + (315*f^2*
Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(16*b^(9/2)
*d^3) - (3*(d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(
1/3)]*Sin[a])/(2*b^(3/2)*d^3) - (315*f^2*(c + d*x)^(1/3)*Sin[a + b*(c + d*x
)^(2/3)])/(16*b^4*d^3) + (6*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c + d*
x)^(2/3)])/(b^2*d^3) + (21*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(2/3)])/
(4*b^2*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\text{Subst}\left(\int \left((de-cf)^2x^2\sin\left(a+bx^2\right)-2f(-de+cf)x^5\sin\left(a+bx^2 \right.\right.}{d^3}$$

$$= \frac{(3f^2)\,\text{Subst}\left(\int x^8\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\text{Su}}{}$$

$$= -\frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f^2(c+dx)^{7/3}\cos\left(\right.}{2ba}$$

$$= -\frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f(de-cf)(c+dx}{}$$

$$= -\frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} + \frac{105f^2(c+dx)\cos\left(\right.}{8b^3c}$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\right.}{2bd^3}$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\right.}{2bd^3}$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\right.}{2bd^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.78, size = 510, normalized size = 0.99

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (((-3*I)/64)*((Cos[a] + I*Sin[a])*((1 + I)*((-105*I)*f^2 + 8*b^3*(d*e - c*f
)^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]] + 2*Sqrt[b]
*(-105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 + 4*
b^2*f*(c + d*x)^(2/3)*(8*d*e - c*f + 7*d*f*x) + (2*I)*b*f*(16*d*e + 19*c*f
+ 35*d*f*x))*(Cos[b*(c + d*x)^(2/3)] + I*Sin[b*(c + d*x)^(2/3)])) - (2*Sqrt
[b]*(-105*f^2*(c + d*x)^(1/3) + (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 +
 4*b^2*f*(c + d*x)^(2/3)*(8*d*e - c*f + 7*d*f*x) - (2*I)*b*f*(16*d*e + 19*c
*f + 35*d*f*x)) - (1 + I)*((105*I)*f^2 + 8*b^3*(d^2*e^2 + c^2*f^2))*Sqrt[Pi
```

```
/2]*Erf[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]]*(Cos[b*(c + d*x)^(2/3)]
+ I*Sin[b*(c + d*x)^(2/3)]) + (8 + 8*I)*b^3*c*d*e*f*Sqrt[2*Pi]*Erf[((1 + I)
*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]]*(Cos[b*(c + d*x)^(2/3)] + I*Sin[b*(c + d
*x)^(2/3)]))*(Cos[a + b*(c + d*x)^(2/3)] - I*Sin[a + b*(c + d*x)^(2/3)])))/
(b^(9/2)*d^3)
```

Maple [A]
time = 0.07, size = 395, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{3f^2(dx+c)^{\frac{7}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{21f^2\left(\frac{(dx+c)^{\frac{5}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}-\frac{5\left(-\frac{(dx+c)\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\frac{3(dx+c)^{\frac{1}{3}}\sin\left(a+b(d\cdots\right.}{4b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots}{\cdots}$
default	$\frac{-\frac{3f^2(dx+c)^{\frac{7}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{21f^2\left(\frac{(dx+c)^{\frac{5}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}-\frac{5\left(-\frac{(dx+c)\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\frac{3(dx+c)^{\frac{1}{3}}\sin\left(a+b(d\cdots\right.}{4b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3*(-1/2*f^2/b*(d*x+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))+7/2*f^2/b*(1/2/b*(d*
x+c)^(5/3)*sin(a+b*(d*x+c)^(2/3))-5/2/b*(-1/2/b*(d*x+c)*cos(a+b*(d*x+c)^(2/
3))+3/2/b*(1/2/b*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(2/3))-1/4/b^(3/2)*2^(1/2)*P
i^(1/2)*(cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*Fre
snelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))))-1/2*(-2*c*f^2+2*d*e*f)/b*
(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))+2*(-2*c*f^2+2*d*e*f)/b*(1/2/b*(d*x+c)^
(2/3)*sin(a+b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))-1/2*(c^2*f^2-2
*c*d*e*f+d^2*e^2)/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4*(c^2*f^2-2*c*d
```

```
*e*f+d^2*e^2)/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/
2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2)
)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 562, normalized size = 1.10

$$-\frac{3\left(\frac{8\left(\sqrt{2}\sqrt{\pi}\left(\left(\left(i-1\right)\cos\left(a\right)+\left(i+1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{ib}\right)+\left(-\left(i+1\right)\cos\left(a\right)-\left(i-1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}+8\left(dx+c\right)^{\frac{1}{3}}b^{2}\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)c^{2}f^{2}}{b^{3}d^{2}}-\frac{16\left(\sqrt{2}\sqrt{\pi}\left(\left(\left(i-1\right)\cos\left(a\right)+\left(i+1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{ib}\right)+\left(-\left(i+1\right)\cos\left(a\right)-\left(i-1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}+8\left(dx+c\right)^{\frac{1}{3}}b^{2}\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)cfe}{b^{3}d}+\frac{128\left(2\left(dx+c\right)^{\frac{2}{3}}b\sin\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)-\left(\left(dx+c\right)^{\frac{4}{3}}b^{2}-2\right)\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)cf^{2}}{b^{3}d^{2}}+\frac{8\left(\sqrt{2}\sqrt{\pi}\left(\left(\left(i-1\right)\cos\left(a\right)+\left(i+1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{ib}\right)+\left(-\left(i+1\right)\cos\left(a\right)-\left(i-1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}+8\left(dx+c\right)^{\frac{1}{3}}b^{2}\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)e^{2}}{b^{3}}-\frac{128\left(2\left(dx+c\right)^{\frac{2}{3}}b\sin\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)-\left(\left(dx+c\right)^{\frac{4}{3}}b^{2}-2\right)\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)fe}{b^{3}d}-\frac{\left(105\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right)\cos\left(a\right)-\left(i-1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{ib}\right)+\left(-\left(i-1\right)\cos\left(a\right)+\left(i+1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}-16\left(4\left(dx+c\right)^{\frac{7}{3}}b^{5}-35\left(dx+c\right)b^{3}\right)\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)+56\left(4\left(dx+c\right)^{\frac{5}{3}}b^{4}-15\left(dx+c\right)^{\frac{1}{3}}b^{2}\right)\sin\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)f^{2}}{b^{6}d^{2}}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/128*(8*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c
)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)
*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*c
^2*f^2/(b^3*d^2) - 16*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*
erf((d*x + c)^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*
x + c)^(1/3)*sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/
3)*b + a))*c*f*e/(b^3*d) + 128*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b +
 a) - ((d*x + c)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*c*f^2/(b^3*d^2)
 + 8*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1/
3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt
(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*e^2/b^
3 - 128*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c)^(4/3)*
b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*f*e/(b^3*d) - (105*sqrt(2)*sqrt(pi)*((
(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(I*b)) + (-(I - 1)
*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*b^(3/2) - 16*(4*
(d*x + c)^(7/3)*b^5 - 35*(d*x + c)*b^3)*cos((d*x + c)^(2/3)*b + a) + 56*(4*
(d*x + c)^(5/3)*b^4 - 15*(d*x + c)^(1/3)*b^2)*sin((d*x + c)^(2/3)*b + a))*f
^2/(b^6*d^2))/d
```

Fricas [A]
time = 0.40, size = 314, normalized size = 0.61

$$\frac{3\left(\sqrt{2}\left(105\,\pi f^{2}\sin\left(a\right)+8\left(\pi b^{3}c^{2}f^{2}-2\,\pi b^{3}cdfe+\pi b^{3}d^{2}e^{2}\right)\cos\left(a\right)\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\left(dx+c\right)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\left(105\,\pi f^{2}\cos\left(a\right)-8\left(\pi b^{3}c^{2}f^{2}-2\,\pi b^{3}cdfe+\pi b^{3}d^{2}e^{2}\right)\sin\left(a\right)\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\left(dx+c\right)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)+4\left(35\,b^{2}df^{2}x+19\,b^{2}cf^{2}+16\,b^{2}dfe-4\left(b^{4}d^{2}f^{2}x^{2}+2\,b^{4}d^{2}fxe+b^{4}d^{2}e^{2}\right)\left(dx+c\right)^{\frac{1}{3}}\right)\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)-2\left(105\left(dx+c\right)^{\frac{1}{3}}bf^{2}-4\left(7\,b^{3}df^{2}x-b^{3}cf^{2}+8\,b^{3}dfe\right)\left(dx+c\right)^{\frac{2}{3}}\right)\sin\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)}{32\,b^{5}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/32*(sqrt(2)*(105*pi*f^2*sin(a) + 8*(pi*b^3*c^2*f^2 - 2*pi*b^3*c*d*f*e + p
i*b^3*d^2*e^2)*cos(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(
b/pi)) + sqrt(2)*(105*pi*f^2*cos(a) - 8*(pi*b^3*c^2*f^2 - 2*pi*b^3*c*d*f*e
+ pi*b^3*d^2*e^2)*sin(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sq
rt(b/pi)) + 4*(35*b^2*d*f^2*x + 19*b^2*c*f^2 + 16*b^2*d*f*e - 4*(b^4*d^2*f^
2*x^2 + 2*b^4*d^2*f*x*e + b^4*d^2*e^2)*(d*x + c)^(1/3))*cos((d*x + c)^(2/3)
```

```
*b + a) - 2*(105*(d*x + c)^(1/3)*b*f^2 - 4*(7*b^3*d*f^2*x - b^3*c*f^2 + 8*b
^3*d*f*e)*(d*x + c)^(2/3))*sin((d*x + c)^(2/3)*b + a))/(b^5*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin \left(a+b(c+dx)^{\frac{2}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.48, size = 777, normalized size = 1.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/64*(f^2*((I*sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 - 105)*erf(-1/2*sqrt(2)*(d*x
+ c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^4*(-I*b/abs(b) + 1)*s
qrt(abs(b))) - 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^(4/3)*b^3*c +
8*I*(d*x + c)^(1/3)*b^3*c^2 - 28*(d*x + c)^(5/3)*b^2 + 32*(d*x + c)^(2/3)*b
^2*c + 70*(-I*d*x - I*c)*b + 32*I*b*c + 105*(d*x + c)^(1/3))*e^(I*(d*x + c)
^(2/3)*b + I*a)/b^4)/d^2 + (I*sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 + 105)*erf(-1/
2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^4*(I*b
/abs(b) + 1)*sqrt(abs(b))) - 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^
(4/3)*b^3*c + 8*I*(d*x + c)^(1/3)*b^3*c^2 + 28*(d*x + c)^(5/3)*b^2 - 32*(d*
x + c)^(2/3)*b^2*c + 70*(-I*d*x - I*c)*b + 32*I*b*c - 105*(d*x + c)^(1/3))*
e^(-I*(d*x + c)^(2/3)*b - I*a)/b^4)/d^2) + 8*(sqrt(2)*sqrt(pi)*erf(-1/2*sqr
t(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b
) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I
*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2
*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*
(d*x + c)^(2/3)*b - I*a)/b)*e^2 - 16*(sqrt(2)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(
d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)
*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/a
bs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*I*(
I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c - 2*(d*x + c)^(2/3)*b - 2*I
)*e^(I*(d*x + c)^(2/3)*b + I*a)/b^3 + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x +
 c)^(1/3)*b^2*c + 2*(d*x + c)^(2/3)*b - 2*I)*e^(-I*(d*x + c)^(2/3)*b - I*a)
/b^3)*f*e/d)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,(c+d\,x)^{2/3}\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x)^2, x)
```

3.213 $\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=243

$$\frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2}-\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}$$

```
[Out] 3*f*cos(a+b*(d*x+c)^(2/3))/b^3/d^2-3/2*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b*(d*
x+c)^(2/3))/b/d^2-3/2*f*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))/b/d^2+3*f*(d*x
+c)^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d^2+3/4*(-c*f+d*e)*cos(a)*FresnelC((d*
x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d^2-3/4*(-c*f
+d*e)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1
/2)/b^(3/2)/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3514, 3466, 3435, 3433, 3432, 3460, 3377, 2718}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}+\frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2}+\frac{3f(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{b^2d^2}-\frac{3\sqrt[3]{c+dx}(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (3*f*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^2) - (3*(d*e - c*f)*(c + d*x)^(1/3)
*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^2) - (3*f*(c + d*x)^(4/3)*Cos[a + b*(c
+ d*x)^(2/3)])/(2*b*d^2) + (3*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^2) - (3*(d*e - c*f)*Sqrt[Pi/2]*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d^2) + (3*f
*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)])/(b^2*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{3\text{Subst}\left(\int \left((de-cf)x^2\sin\left(a+bx^2\right)+fx^5\sin\left(a+bx^2\right)\right)\,dx, x, \sqrt[3]{c\cdot\right.}{d^2}$$

$$= \frac{(3f)\text{Subst}\left(\int x^5\sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2} + \frac{(3(de-cf))\text{Subs}}{}$$

$$= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} + \frac{(3f)\text{Subst}\left(\int x^2\sin(\right.}{}$$

$$= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} - \frac{3f(c+dx)^{4/3}\cos\left(a\right.}{2bd^2}$$

$$= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2} - \frac{3f(c+dx)^{4/3}\cos\left(a\right.}{2bd^2}$$

$$= \frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2} - \frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2}\right.}{2bd^2}$$

Mathematica [A]

time = 0.55, size = 213, normalized size = 0.88

$$\frac{3\left(4f\cos\left(a+b(c+dx)^{2/3}\right)-2b^2de\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)-2b^2dfx\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)+b^{3/2}(de-cf)\sqrt{2\pi}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)-b^{3/2}(de-cf)\sqrt{2\pi}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)+4bf(c+dx)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\right)}{4b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (3*(4*f*Cos[a + b*(c + d*x)^(2/3)] - 2*b^2*d*e*(c + d*x)^(1/3)*Cos[a + b*(c
 + d*x)^(2/3)] - 2*b^2*d*f*x*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] + b
^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)] - b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*
x)^(1/3)]*Sin[a] + 4*b*f*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)]))/(4*b^
3*d^2)
```

Maple [A]

time = 0.01, size = 175, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{3f(dx+c)^{\frac{4}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{6f\left(\frac{(dx+c)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b^2}\right)}{b}+\frac{3(cf-de)(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right.}{2b}}{d^2}$

default	$\frac{-\frac{3f(dx+c)^{\frac{4}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{6f\left(\frac{(dx+c)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b^2}\right)}{b}+\frac{3(cf-de)(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2*(-1/2*f/b*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))+2*f/b*(1/2/b*(d*x+c)^(
2/3)*sin(a+b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))+1/2*(c*f-d*e)/b
*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))-1/4*(c*f-d*e)/b^(3/2)*2^(1/2)*Pi^(1/2
)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(
(d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 249, normalized size = 1.02

$$\frac{3\left(\frac{\left(\sqrt{2}\sqrt{\pi}\left(((i-1)\cos(a)+(i+1)\sin(a))\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{ib}\right)+(-(i+1)\cos(a)-(i-1)\sin(a))\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}+8(dx+c)^{\frac{1}{3}}b^2\cos\left((dx+c)^{\frac{2}{3}}b+a\right)\right)cf}{b^3d}-\frac{\left(\sqrt{2}\sqrt{\pi}\left(((i-1)\cos(a)+(i+1)\sin(a))\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{ib}\right)+(-(i+1)\cos(a)-(i-1)\sin(a))\operatorname{erf}\left((dx+c)^{\frac{1}{3}}\sqrt{-ib}\right)\right)b^{\frac{3}{2}}+8(dx+c)^{\frac{1}{3}}b^2\cos\left((dx+c)^{\frac{2}{3}}b+a\right)\right)e}{b^3}+\frac{8\left(2(dx+c)^{\frac{2}{3}}b\sin\left((dx+c)^{\frac{2}{3}}b+a\right)-\left((dx+c)^{\frac{4}{3}}b^2-2\right)\cos\left((dx+c)^{\frac{2}{3}}b+a\right)\right)f}{b^3d}\right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 3/16*((sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1
/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqr
t(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*c*f/(
b^3*d) - (sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)
^(1/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*
sqrt(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))*e/
b^3 + 8*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c)^(4/3)*
b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*f/(b^3*d))/d
```

Fricas [A]
time = 0.37, size = 164, normalized size = 0.67

$$-\frac{3\left(\sqrt{2}(\pi bcf-\pi bde)\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}(\pi bcf-\pi bde)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin(a)-4(dx+c)^{\frac{2}{3}}bf\sin\left((dx+c)^{\frac{2}{3}}b+a\right)+2\left((b^2dfx+b^2de)(dx+c)^{\frac{1}{3}}-2f\right)\cos\left((dx+c)^{\frac{2}{3}}b+a\right)\right)}{4b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] -3/4*(sqrt(2)*(pi*b*c*f - pi*b*d*e)*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(
d*x + c)^(1/3)*sqrt(b/pi)) - sqrt(2)*(pi*b*c*f - pi*b*d*e)*sqrt(b/pi)*fresn
el_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi))*sin(a) - 4*(d*x + c)^(2/3)*b*f*s
in((d*x + c)^(2/3)*b + a) + 2*((b^2*d*f*x + b^2*d*e)*(d*x + c)^(1/3) - 2*f)
*cos((d*x + c)^(2/3)*b + a))/(b^3*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.38, size = 407, normalized size = 1.67

$$-\frac{3\left(\left(\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{2(dx+c)^{\frac{1}{3}}e^{\left(i(dx+c)^{\frac{2}{3}}b+ia\right)}}{b}+\frac{2(dx+c)^{\frac{1}{3}}e^{\left(-i(dx+c)^{\frac{2}{3}}b-ia\right)}}{b}\right)e-\frac{\left(\frac{\sqrt{2}\sqrt{\pi}c\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{\sqrt{2}\sqrt{\pi}c\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{2i\left(i(dx+c)^{\frac{4}{3}}b^2-i(dx+c)^{\frac{1}{3}}b^2c-2(dx+c)^{\frac{2}{3}}b-2i\right)e^{\left(i(dx+c)^{\frac{2}{3}}b+ia\right)}}{b^3}+\frac{2i\left(i(dx+c)^{\frac{4}{3}}b^2-i(dx+c)^{\frac{1}{3}}b^2c+2(dx+c)^{\frac{2}{3}}b-2i\right)e^{\left(-i(dx+c)^{\frac{2}{3}}b-ia\right)}}{b^3}\right)f}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/8*((sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*
sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)
*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(
b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3)*b
 + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)*e - (sqrt(2
)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)
))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*c*erf(-1/2
*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/ab
s(b) + 1)*sqrt(abs(b))) + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^
2*c - 2*(d*x + c)^(2/3)*b - 2*I)*e^(I*(d*x + c)^(2/3)*b + I*a)/b^3 + 2*I*(I
*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c + 2*(d*x + c)^(2/3)*b - 2*I)
*e^(-I*(d*x + c)^(2/3)*b - I*a)/b^3)*f/d)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,\left(e+f\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))*(e + f*x), x)
```

3.214 $\int \sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=130

$$-\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt{\frac{\pi}{2}}\,\cos(a)C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)\,\mathrm{s}}{2b^{3/2}d}$$

```
[Out] -3/2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+3/4*cos(a)*FresnelC((d*x+c)^(
1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d-3/4*FresnelS((d*x
+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3444, 3466, 3435, 3433, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\,\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt{\frac{\pi}{2}}\,\sin(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) + (3*Sqrt[Pi/2]*Cos
[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d) - (3*Sqrt[P
i/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
```

```
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+b(c+dx)^{2/3}\right)\,dx &= \frac{3\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d}\\ &= -\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\text{Subst}\left(\int\cos\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right.}{2bd}\\ &= -\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{(3\cos(a))\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\sqrt[3]{c}\right.}{2bd}\\ &= -\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt{\frac{\pi}{2}}\,\cos(a)C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{2b^{3/2}d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 114, normalized size = 0.88

$$-\frac{3\left(2\sqrt{b}\,\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)-\sqrt{2\pi}\,\cos(a)C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)+\sqrt{2\pi}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)\sin(a)\right)}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(2*Sqrt[b]*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] - Sqrt[2*Pi]*Cos[
a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)] + Sqrt[2*Pi]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(4*b^(3/2)*d)
```

Maple [A]
time = 0.01, size = 86, normalized size = 0.66

method	result

derivativedivides	$\dfrac{-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}}{d}$	8
default	$\dfrac{-\frac{3(dx+c)^{\frac{1}{3}}\cos\left(a+b(dx+c)^{\frac{2}{3}}\right)}{2b}+\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{(dx+c)^{\frac{1}{3}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}}{d}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*(-1/2/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4/b^(3/2)*2^(1/2)*Pi^(1/
2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS
((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 92, normalized size = 0.71

$$-\frac{3\left(\sqrt{2}\,\sqrt{\pi}\,\left(\left((i-1)\,\cos\left(a\right)+(i+1)\,\sin\left(a\right)\right)\mathrm{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{i\,b}\right)+\left(-(i+1)\,\cos\left(a\right)-(i-1)\,\sin\left(a\right)\right)\mathrm{erf}\left(\left(dx+c\right)^{\frac{1}{3}}\sqrt{-i\,b}\right)\right)b^{\frac{3}{2}}+8\left(dx+c\right)^{\frac{1}{3}}b^{2}\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)}{16\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*(sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf((d*x + c)^(1
/3)*sqrt(I*b)) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf((d*x + c)^(1/3)*sqr
t(-I*b)))*b^(3/2) + 8*(d*x + c)^(1/3)*b^2*cos((d*x + c)^(2/3)*b + a))/(b^3*
d)
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.75

$$\frac{3\left(\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(a\right)\mathrm{C}\left(\sqrt{2}\,\left(dx+c\right)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,\left(dx+c\right)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)-2\left(dx+c\right)^{\frac{1}{3}}b\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)}{4\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/4*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(
b/pi)) - sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/p
i))*sin(a) - 2*(d*x + c)^(1/3)*b*cos((d*x + c)^(2/3)*b + a))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3)),x)
[Out] Integral(sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.69, size = 170, normalized size = 1.31

$$-\frac{3\left(\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,(dx+c)^{\frac{1}{3}}\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{2\,(dx+c)^{\frac{1}{3}}e^{\left(i\,(dx+c)^{\frac{2}{3}}b+i\,a\right)}}{b}+\frac{2\,(dx+c)^{\frac{1}{3}}e^{\left(-i\,(dx+c)^{\frac{2}{3}}b-i\,a\right)}}{b}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
[Out] -3/8*(sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*s
qrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*
erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b
*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3)*b
+ I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3)),x)
[Out] int(sin(a + b*(c + d*x)^(2/3)), x)
```

3.215 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

Mathematica [A]
time = 14.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b(c + dx)^{\frac{2}{3}}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x), x)
```

3.216 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2},x\right)$$

[Out] Unintegrable(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
[Out] Defer[Int][Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

Mathematica [A]
time = 15.07, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
[Out] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b(c + dx)^{\frac{2}{3}}\right)}{(e + fx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin \left(a+b\,(c+d\,x)^{2/3}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x)^2,x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(e + f*x)^2, x)
```

3.217 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx$

Optimal. Leaf size=855

$$\frac{b^5 f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3}-\frac{b^7 f^2 (c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3}+\frac{b(de-cf)^2(c+dx)^{2/3}\,\mathrm{c}}{2d^3}$$

```
[Out] -1/120960*b^9*f^2*Ci(b/(d*x+c)^(1/3))*cos(a)/d^3+1/2*b^3*(-c*f+d*e)^2*Ci(b/
(d*x+c)^(1/3))*cos(a)/d^3+1/120*b^5*f*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a+b/(d*x
+c)^(1/3))/d^3-1/120960*b^7*f^2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^3+1/
2*b*(-c*f+d*e)^2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d^3-1/60*b^3*f*(-c*f+
d*e)*(d*x+c)*cos(a+b/(d*x+c)^(1/3))/d^3+1/20160*b^5*f^2*(d*x+c)^(4/3)*cos(a
+b/(d*x+c)^(1/3))/d^3+1/5*b*f*(-c*f+d*e)*(d*x+c)^(5/3)*cos(a+b/(d*x+c)^(1/3
))/d^3-1/1008*b^3*f^2*(d*x+c)^2*cos(a+b/(d*x+c)^(1/3))/d^3+1/24*b*f^2*(d*x+
c)^(8/3)*cos(a+b/(d*x+c)^(1/3))/d^3+1/120*b^6*f*(-c*f+d*e)*cos(a)*Si(b/(d*x
+c)^(1/3))/d^3+1/120*b^6*f*(-c*f+d*e)*Ci(b/(d*x+c)^(1/3))*sin(a)/d^3+1/1209
60*b^9*f^2*Si(b/(d*x+c)^(1/3))*sin(a)/d^3-1/2*b^3*(-c*f+d*e)^2*Si(b/(d*x+c)
^(1/3))*sin(a)/d^3+1/120960*b^8*f^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d^
3-1/2*b^2*(-c*f+d*e)^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/d^3+1/120*b^4*f
*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(1/3))/d^3-1/60480*b^6*f^2*(d*x+c
)*sin(a+b/(d*x+c)^(1/3))/d^3+(-c*f+d*e)^2*(d*x+c)*sin(a+b/(d*x+c)^(1/3))/d^
3-1/20*b^2*f*(-c*f+d*e)*(d*x+c)^(4/3)*sin(a+b/(d*x+c)^(1/3))/d^3+1/5040*b^4
*f^2*(d*x+c)^(5/3)*sin(a+b/(d*x+c)^(1/3))/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+
b/(d*x+c)^(1/3))/d^3-1/168*b^2*f^2*(d*x+c)^(7/3)*sin(a+b/(d*x+c)^(1/3))/d^3
+1/3*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(1/3))/d^3
```

Rubi [A]
time = 0.71, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3512, 3378, 3384, 3380, 3383}

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b^5*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(120*d^3) -
(b^7*f^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(120960*d^3) + (b*(d*e
 - c*f)^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^3) - (b^3*f*(d*e
 - c*f)*(c + d*x)*Cos[a + b/(c + d*x)^(1/3)])/(60*d^3) + (b^5*f^2*(c + d*x)
^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(20160*d^3) + (b*f*(d*e - c*f)*(c + d*x)
^(5/3)*Cos[a + b/(c + d*x)^(1/3)])/(5*d^3) - (b^3*f^2*(c + d*x)^2*Cos[a + b
```

```
/(c + d*x)^(1/3)])/(1008*d^3) + (b*f^2*(c + d*x)^(8/3)*Cos[a + b/(c + d*x)^
(1/3)])/(24*d^3) - (b^9*f^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(120960*
d^3) + (b^3*(d*e - c*f)^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(2*d^3) +
(b^6*f*(d*e - c*f)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(120*d^3) + (b^8*
f^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(120960*d^3) - (b^2*(d*e -
c*f)^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^3) + (b^4*f*(d*e -
c*f)*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(120*d^3) - (b^6*f^2*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/(60480*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin
[a + b/(c + d*x)^(1/3)])/d^3 - (b^2*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b
/(c + d*x)^(1/3)])/(20*d^3) + (b^4*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^
(1/3)])/(5040*d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])
/d^3 - (b^2*f^2*(c + d*x)^(7/3)*Sin[a + b/(c + d*x)^(1/3)])/(168*d^3) + (f^
2*(c + d*x)^3*Sin[a + b/(c + d*x)^(1/3)])/(3*d^3) + (b^6*f*(d*e - c*f)*Cos[
a]*SinIntegral[b/(c + d*x)^(1/3)])/(120*d^3) + (b^9*f^2*Sin[a]*SinIntegral[
b/(c + d*x)^(1/3)])/(120960*d^3) - (b^3*(d*e - c*f)^2*Sin[a]*SinIntegral[b/
(c + d*x)^(1/3)])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
```

```
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = -\frac{3\text{Subst}\left(\int \left(\frac{f^2\sin(a+bx)}{d^2x^{10}}+\frac{2f(de-cf)\sin(a+bx)}{d^2x^7}+\frac{(de-cf)^2\sin(a+bx)}{d^2x^4}\right) dx, x\right.}{d}$$

$$= -\frac{(3f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{10}}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(6f(de-cf))\text{Subst}\left(\right.}{}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin}{d^3}$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{bf(de-cf)(c+dx)}{}$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{bf(de-cf)(c+dx)}{}$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^3f(de-cf)(c+dx}{6}$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^3f(de-cf)(c+dx}{6}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{b(de-cf)^2(c+dx)}{}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{b(de-cf)^2(c+dx)}{}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(\right.}{12096}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(\right.}{12096}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(\right.}{12096}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(\right.}{12096}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.

time = 2.83, size = 929, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] ((-1/241920*I)*((Cos[a] + I*Sin[a])*((60480*I)*b^3*d^2*e^2*ExpIntegralEi[(I
*b)/(c + d*x)^(1/3)] + 1008*b^6*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)]
- (120960*I)*b^3*c*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - I*b^9*f^2*E
xpIntegralEi[(I*b)/(c + d*x)^(1/3)] - 1008*b^6*c*f^2*ExpIntegralEi[(I*b)/(c
 + d*x)^(1/3)] + (60480*I)*b^3*c^2*f^2*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)]
 + (c + d*x)^(1/3)*(b^8*f^2 - I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*
x)^(2/3) + (24*I)*b^3*f*(c + d*x)^(2/3)*(-84*d*e + 79*c*f - 5*d*f*x) + (6*I
)*b^5*f*(168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*e - 41
*c*f + d*f*x) + 40320*(c + d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3
*e^2 + 3*e*f*x + f^2*x^2)) + (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d
*f*(48*e + 7*f*x) + d^2*(60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2
*f^2 - 2*c*d*f*(399*e + 16*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2))))*(C
os[b/(c + d*x)^(1/3)] + I*Sin[b/(c + d*x)^(1/3)])) - ((c + d*x)^(1/3)*(b^8*
f^2 + I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*x)^(2/3) - (6*I)*b^5*f*(
168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*e - 41*c*f + d*
f*x) + (24*I)*b^3*f*(c + d*x)^(2/3)*(84*d*e - 79*c*f + 5*d*f*x) + 40320*(c
+ d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2)
) - (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d*f*(48*e + 7*f*x) + d^2*(
60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2*f^2 - 2*c*d*f*(399*e + 1
6*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2))) + I*b^3*(-60480*d^2*e^2 + 1
008*((-I)*b^3 + 120*c)*d*e*f + (b^6 + (1008*I)*b^3*c - 60480*c^2)*f^2)*ExpI
ntegralEi[((-I)*b)/(c + d*x)^(1/3)]*(Cos[b/(c + d*x)^(1/3)] + I*Sin[b/(c +
d*x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)] - I*Sin[a + b/(c + d*x)^(1/3)])))
/d^3
```

Maple [A]
time = 0.48, size = 936, normalized size = 1.09

method	result	size
derivativedivides	Expression too large to display	936
default	Expression too large to display	936

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^3*b^3*(c^2*f^2*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d
*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1
/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+b^6*f^2*(-1/9
```

```
*sin(a+b/(d*x+c)^(1/3))/b^9*(d*x+c)^3-1/72*cos(a+b/(d*x+c)^(1/3))/b^8*(d*x+
c)^(8/3)+1/504*sin(a+b/(d*x+c)^(1/3))/b^7*(d*x+c)^(7/3)+1/3024*cos(a+b/(d*x
+c)^(1/3))/b^6*(d*x+c)^2-1/15120*sin(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)-1
/60480*cos(a+b/(d*x+c)^(1/3))/b^4*(d*x+c)^(4/3)+1/181440*sin(a+b/(d*x+c)^(1
/3))/b^3*(d*x+c)+1/362880*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)-1/362880
*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/362880*Si(b/(d*x+c)^(1/3))*sin(a)
+1/362880*Ci(b/(d*x+c)^(1/3))*cos(a))-2*c*d*e*f*(-1/3*sin(a+b/(d*x+c)^(1/3)
)/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x
+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^
(1/3))*cos(a))-2*b^3*c*f^2*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/30*
cos(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/3))/b^4*(
d*x+c)^(4/3)+1/360*cos(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+b/(d*x+c)
^(1/3))/b^2*(d*x+c)^(2/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/72
0*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))+2*b^3*d*e*f*
(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c)^2-1/30*cos(a+b/(d*x+c)^(1/3))/b^5*
(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/3))/b^4*(d*x+c)^(4/3)+1/360*cos(a+b/
(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)-1
/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)-1/720*Si(b/(d*x+c)^(1/3))*cos(a
)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))+d^2*e^2*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^
3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^
(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3
))*cos(a)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.81, size = 1004, normalized size = 1.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/241920*(60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(
a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 +
 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x
 + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
)*c^2*f^2/d^2 - 120960*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)
))*cos(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a)
)*b^3 + 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) -
2*((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3)))*c*f*e/d - 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)
^(1/3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(
a))*b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*
cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(
d*x + c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3)))*c*f^2/d^2 + 60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(
```

```
1/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*si
n(a))*b^3 + 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
) - 2*((d*x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3)))*e^2 + 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)
^(1/3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(
a))*b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*
cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(
d*x + c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3)))*f*e/d - (((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos
(a) - (-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^9
 + 2*((d*x + c)^(2/3)*b^7 - 6*(d*x + c)^(4/3)*b^5 + 120*(d*x + c)^2*b^3 - 5
040*(d*x + c)^(8/3)*b)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d
*x + c)^(1/3)*b^8 - 2*(d*x + c)*b^6 + 24*(d*x + c)^(5/3)*b^4 - 720*(d*x + c
)^(7/3)*b^2 + 40320*(d*x + c)^3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
)))*f^2/d^2)/d
```

Fricas [A]
time = 0.40, size = 647, normalized size = 0.76

$$-\frac{2\left(120\,b^3d^2f^2x^2 - 1776\,b^3cdf^2x - 1896\,b^3c^2f^2 + 2016\,(b^3d^2fx + b^3cdf)e - (5040\,bd^2f^2x^2 - 14112\,bcdf^2x + 60480\,bd^2e^2 - (b^7 - 41328\,bc^2)f^2 + 24192\,(bd^2fx - 4\,bcdf)e)(dx+c)^{\frac{2}{3}} - 6\,(b^5df^2x - 167\,b^5cf^2 + 168\,b^5dfe)(dx+c)^{\frac{1}{3}}\right)\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right) + \left((120960\,b^3cdfe - 60480\,b^3d^2e^2 + (b^9 - 60480\,b^3c^2)f^2)\cos(a) + 1008\,(b^6cf^2 - b^6dfe)\sin(a)\right)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) + \left((120960\,b^3cdfe - 60480\,b^3d^2e^2 + (b^9 - 60480\,b^3c^2)f^2)\cos(a) + 1008\,(b^6cf^2 - b^6dfe)\sin(a)\right)\operatorname{Ci}\left(-\frac{b}{(dx+c)^{\frac{1}{3}}}\right) + 2\left(2\,b^6df^2x - 40320\,d^3f^2x^3 + 2\,(b^6c - 20160\,c^3)f^2 - 120960\,(d^3x + cd^2)e^2 - 120960\,(d^3fx^2 - c^2df)e - 24\,(b^4df^2x - 41\,b^4cf^2 + 42\,b^4dfe)(dx+c)^{\frac{2}{3}} + (720\,b^2d^2f^2x^2 - 4608\,b^2cdf^2x + 60480\,b^2d^2e^2 - (b^8 - 55152\,b^2c^2)f^2 + 6048\,(b^2d^2fx - 19\,b^2cdf)e)(dx+c)^{\frac{1}{3}}\right)\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right) + 2\left(1008\,(b^6cf^2 - b^6dfe)\cos(a) - (120960\,b^3cdfe - 60480\,b^3d^2e^2 + (b^9 - 60480\,b^3c^2)f^2)\sin(a)\right)\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{241920\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -1/241920*(2*(120*b^3*d^2*f^2*x^2 - 1776*b^3*c*d*f^2*x - 1896*b^3*c^2*f^2 +
 2016*(b^3*d^2*f*x + b^3*c*d*f)*e - (5040*b*d^2*f^2*x^2 - 14112*b*c*d*f^2*x
 + 60480*b*d^2*e^2 - (b^7 - 41328*b*c^2)*f^2 + 24192*(b*d^2*f*x - 4*b*c*d*f
)*e)*(d*x + c)^(2/3) - 6*(b^5*d*f^2*x - 167*b^5*c*f^2 + 168*b^5*d*f*e)*(d*x
 + c)^(1/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) + ((120960*b^
3*c*d*f*e - 60480*b^3*d^2*e^2 + (b^9 - 60480*b^3*c^2)*f^2)*cos(a) + 1008*(b
^6*c*f^2 - b^6*d*f*e)*sin(a))*cos_integral(b/(d*x + c)^(1/3)) + ((120960*b^
3*c*d*f*e - 60480*b^3*d^2*e^2 + (b^9 - 60480*b^3*c^2)*f^2)*cos(a) + 1008*(b
^6*c*f^2 - b^6*d*f*e)*sin(a))*cos_integral(-b/(d*x + c)^(1/3)) + 2*(2*b^6*d
*f^2*x - 40320*d^3*f^2*x^3 + 2*(b^6*c - 20160*c^3)*f^2 - 120960*(d^3*x + c*
d^2)*e^2 - 120960*(d^3*f*x^2 - c^2*d*f)*e - 24*(b^4*d*f^2*x - 41*b^4*c*f^2
+ 42*b^4*d*f*e)*(d*x + c)^(2/3) + (720*b^2*d^2*f^2*x^2 - 4608*b^2*c*d*f^2*x
 + 60480*b^2*d^2*e^2 - (b^8 - 55152*b^2*c^2)*f^2 + 6048*(b^2*d^2*f*x - 19*b
^2*c*d*f)*e)*(d*x + c)^(1/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x +
c)) + 2*(1008*(b^6*c*f^2 - b^6*d*f*e)*cos(a) - (120960*b^3*c*d*f*e - 60480*
b^3*d^2*e^2 + (b^9 - 60480*b^3*c^2)*f^2)*sin(a))*sin_integral(b/(d*x + c)^(
1/3)))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11930 vs. $2(753) = 1506$.
time = 6.99, size = 11930, normalized size = 13.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -1/120960*((a^9*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3)) + a^9*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3)) - 9*((d*x + c)^(1/3)*a + b)*a^8*b^10*cos(a)*cos_integral(-a +
((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 9*((d*x + c)^(1/
3)*a + b)*a^8*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))/(d*x + c)^(1/3) + 36*((d*x + c)^(1/3)*a + b)^2*a^7*b^10*cos(a)*cos
_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3
6*((d*x + c)^(1/3)*a + b)^2*a^7*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 84*((d*x + c)^(1/3)*a + b)^3*a
^6*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(
d*x + c) - 84*((d*x + c)^(1/3)*a + b)^3*a^6*b^10*sin(a)*sin_integral(a - ((
d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 126*((d*x + c)^(1/3)*a +
 b)^4*a^5*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3))/(d*x + c)^(4/3) + 126*((d*x + c)^(1/3)*a + b)^4*a^5*b^10*sin(a)*sin_i
ntegral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 126*
((d*x + c)^(1/3)*a + b)^5*a^4*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3
)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - 126*((d*x + c)^(1/3)*a + b)^5*a
^4*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d
*x + c)^(5/3) + 84*((d*x + c)^(1/3)*a + b)^6*a^3*b^10*cos(a)*cos_integral(-
a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 + 1008*a^9*b^7*c*c
os_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a) + a^8*b^10
*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 1008*a^9*b^7*c*cos(a)*sin_i
ntegral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 84*((d*x + c)^(1/3)*
a + b)^6*a^3*b^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3))/(d*x + c)^2 - 36*((d*x + c)^(1/3)*a + b)^7*a^2*b^10*cos(a)*cos_inte
gral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(7/3) - 9072*(
(d*x + c)^(1/3)*a + b)*a^8*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3))*sin(a)/(d*x + c)^(1/3) - 8*((d*x + c)^(1/3)*a + b)*a^7*b^1
0*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 9072*((d*x
 + c)^(1/3)*a + b)*a^8*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b
)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 36*((d*x + c)^(1/3)*a + b)^7*a^2*b^10*
```

```
sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^
(7/3) + 9*((d*x + c)^(1/3)*a + b)^8*a*b^10*cos(a)*cos_integral(-a + ((d*x +
 c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(8/3) + 36288*((d*x + c)^(1/3)*
a + b)^2*a^7*b^7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))*sin(a)/(d*x + c)^(2/3) + 28*((d*x + c)^(1/3)*a + b)^2*a^6*b^10*sin(((d*x
 + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 36288*((d*x + c)^(1/3
)*a + b)^2*a^7*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3))/(d*x + c)^(2/3) + 9*((d*x + c)^(1/3)*a + b)^8*a*b^10*sin(a)*sin_
integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(8/3) + a^7
*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b
)^9*b^10*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/
(d*x + c)^3 - 84672*((d*x + c)^(1/3)*a + b)^3*a^6*b^7*c*cos_integral(-a + (
(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c) - 56*((d*x + c)^(1
/3)*a + b)^3*a^5*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c
) + 84672*((d*x + c)^(1/3)*a + b)^3*a^6*b^7*c*cos(a)*sin_integral(a - ((d*x
 + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x + c)^(1/3)*a + b)^9*b
^10*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x +
 c)^3 - 7*((d*x + c)^(1/3)*a + b)*a^6*b^10*cos(((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3))/(d*x + c)^(1/3) + 127008*((d*x + c)^(1/3)*a + b)^4*a^5*b^7*c*c
os_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^
(4/3) + 70*((d*x + c)^(1/3)*a + b)^4*a^4*b^10*sin(((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3))/(d*x + c)^(4/3) - 127008*((d*x + c)^(1/3)*a + b)^4*a^5*b^7*
c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c
)^(4/3) + 21*((d*x + c)^(1/3)*a + b)^2*a^5*b^10*cos(((d*x + c)^(1/3)*a + b)
/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 127008*((d*x + c)^(1/3)*a + b)^5*a^4*b^
7*c*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x
+ c)^(5/3) - 56*((d*x + c)^(1/3)*a + b)^5*a^3*b^10*sin(((d*x + c)^(1/3)*a +
 b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 127008*((d*x + c)^(1/3)*a + b)^5*a^4
*b^7*c*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*
x + c)^(5/3) - 35*((d*x + c)^(1/3)*a + b)^3*a^4*b^10*cos(((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3))/(d*x + c) - 1008*a^8*b^7*c*cos(((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3)) - 60480*a^9*b^4*c^2*cos(a)*cos_integral(-a + ((d*x + c)
^(1/3)*a + b)/(d*x + c)^(1/3)) + 84672*((d*x + c)^(1/3)*a + b)^6*a^3*b^7*c*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)
^2 + 28*((d*x + c)^(1/3)*a + b)^6*a^2*b^10*sin(((d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3))/(d*x + c)^2 - 2*a^6*b^10*sin(((d*x...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)\,(e+f\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x)^2, x)
```

3.218 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal. Leaf size=419

$$\frac{b^5 f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}+\frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}-\frac{b^3 f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+}}\right.}{120d^2}$$

```
[Out] 1/2*b^3*(-c*f+d*e)*Ci(b/(d*x+c)^(1/3))*cos(a)/d^2+1/240*b^5*f*(d*x+c)^(1/3)
*cos(a+b/(d*x+c)^(1/3))/d^2+1/2*b*(-c*f+d*e)*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^
(1/3))/d^2-1/120*b^3*f*(d*x+c)*cos(a+b/(d*x+c)^(1/3))/d^2+1/10*b*f*(d*x+c)^
(5/3)*cos(a+b/(d*x+c)^(1/3))/d^2+1/240*b^6*f*cos(a)*Si(b/(d*x+c)^(1/3))/d^2
+1/240*b^6*f*Ci(b/(d*x+c)^(1/3))*sin(a)/d^2-1/2*b^3*(-c*f+d*e)*Si(b/(d*x+c)
^(1/3))*sin(a)/d^2-1/2*b^2*(-c*f+d*e)*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/
d^2+1/240*b^4*f*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(1/3))/d^2+(-c*f+d*e)*(d*x+c)
*sin(a+b/(d*x+c)^(1/3))/d^2-1/40*b^2*f*(d*x+c)^(4/3)*sin(a+b/(d*x+c)^(1/3))
/d^2+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^(1/3))/d^2
```

Rubi [A]
time = 0.34, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3512, 3378, 3384, 3380, 3383}

$$\frac{b^6 f\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}+\frac{b^6 f\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}+\frac{b^5 f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}+\frac{b^4 f(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}+\frac{b^3\cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}-\frac{b^3\sin(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}-\frac{b^3 f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2}-\frac{b^2\sqrt[3]{c+dx}(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}-\frac{b^2 f(c+dx)^{4/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{40d^2}+\frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{b(c+dx)^{2/3}(de-cf)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}+\frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b^5*f*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(240*d^2) + (b*(d*e - c*
f)*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^2) - (b^3*f*(c + d*x)*C
os[a + b/(c + d*x)^(1/3)])/(120*d^2) + (b*f*(c + d*x)^(5/3)*Cos[a + b/(c +
d*x)^(1/3)])/(10*d^2) + (b^3*(d*e - c*f)*Cos[a]*CosIntegral[b/(c + d*x)^(1/
3)])/(2*d^2) + (b^6*f*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(240*d^2) - (b
^2*(d*e - c*f)*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2) + (b^4*f
*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(240*d^2) + ((d*e - c*f)*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/d^2 - (b^2*f*(c + d*x)^(4/3)*Sin[a + b/(c
+ d*x)^(1/3)])/(40*d^2) + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2
) + (b^6*f*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(240*d^2) - (b^3*(d*e - c
*f)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{3\text{Subst}\left(\int\left(\frac{f\sin(a+bx)}{dx^7}+\frac{(de-cf)\sin(a+bx)}{dx^4}\right)dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{(3f)\text{Subst}\left(\int\frac{\sin(a+bx)}{x^7}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{(3(de-cf))\text{Subst}\left(\int \sin \ldots}{d^2}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+\ldots}}\right.}{2d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\ldots\right.}{10d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\ldots\right.}{10d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\ldots\right.}{120d^2}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\ldots\right.}{120d^2}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\ldots\right.}{2d^2}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\ldots\right.}{2d^2}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\ldots\right.}{2d^2}$$

Mathematica [A]
time = 0.56, size = 540, normalized size = 1.29

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (e*(c + d*x)^(1/3)*Cos[b/(c + d*x)^(1/3)]*(b*(c + d*x)^(1/3)*Cos[a] - b^2*S
in[a] + 2*(c + d*x)^(2/3)*Sin[a]))/(2*d) + (f*(c + d*x)^(1/3)*Cos[b/(c + d*
```

```
x)^(1/3)]*(b^5*Cos[a] - 120*b*c*(c + d*x)^(1/3)*Cos[a] - 2*b^3*(c + d*x)^(2
/3)*Cos[a] + 24*b*(c + d*x)^(4/3)*Cos[a] + 120*b^2*c*Sin[a] + b^4*(c + d*x)
^(1/3)*Sin[a] - 240*c*(c + d*x)^(2/3)*Sin[a] - 6*b^2*(c + d*x)*Sin[a] + 120
*(c + d*x)^(5/3)*Sin[a]))/(240*d^2) + (e*(c + d*x)^(1/3)*(-(b^2*Cos[a]) + 2
*(c + d*x)^(2/3)*Cos[a] - b*(c + d*x)^(1/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)])
/(2*d) + (f*(c + d*x)^(1/3)*(120*b^2*c*Cos[a] + b^4*(c + d*x)^(1/3)*Cos[a]
- 240*c*(c + d*x)^(2/3)*Cos[a] - 6*b^2*(c + d*x)*Cos[a] + 120*(c + d*x)^(5/
3)*Cos[a] - b^5*Sin[a] + 120*b*c*(c + d*x)^(1/3)*Sin[a] + 2*b^3*(c + d*x)^(
2/3)*Sin[a] - 24*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)])/(240*d^2
) + (b^3*e*(Cos[a]*CosIntegral[b/(c + d*x)^(1/3)] - Sin[a]*SinIntegral[b/(c
 + d*x)^(1/3)]))/(2*d) + (b^3*f*(-120*c*Cos[a]*CosIntegral[b/(c + d*x)^(1/3
)] + b^3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] + b^3*Cos[a]*SinIntegral[b/(
c + d*x)^(1/3)] + 120*c*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)]))/(240*d^2)
```

Maple [A]
time = 0.03, size = 391, normalized size = 0.93

method	result
derivativedivides	$-\frac{3b^3\left(-cf\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\text{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{6}\right.\right.}{}$
default	$-\frac{3b^3\left(-cf\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\text{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{6}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^2*b^3*(-c*f*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+
c)^(1/3))/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*
Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+d*e*(-1/3*sin(a+
b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)+1
/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/
6*Ci(b/(d*x+c)^(1/3))*cos(a))+f*b^3*(-1/6*sin(a+b/(d*x+c)^(1/3))/b^6*(d*x+c
)^2-1/30*cos(a+b/(d*x+c)^(1/3))/b^5*(d*x+c)^(5/3)+1/120*sin(a+b/(d*x+c)^(1/
3))/b^4*(d*x+c)^(4/3)+1/360*cos(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/720*sin(a+
b/(d*x+c)^(1/3))/b^2*(d*x+c)^(2/3)-1/720*cos(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(
1/3)-1/720*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 460, normalized size = 1.10

$$-\frac{\frac{120\left(\left(\left(\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^3+2\,(dx+c)^{\frac{2}{3}}b\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-2\left((dx+c)^{\frac{1}{3}}b^2-2\,dx-2\,c\right)\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)\right)cf}{d}-120\left(\left(\left(\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^3+2\,(dx+c)^{\frac{2}{3}}b\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-2\left((dx+c)^{\frac{1}{3}}b^2-2\,dx-2\,c\right)\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)\right)e-\frac{\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^6+2\left((dx+c)^{\frac{1}{3}}b^5-2\,(dx+c)b^3+24\,(dx+c)^{\frac{5}{3}}b\right)\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)+2\left((dx+c)^{\frac{2}{3}}b^4-6\,(dx+c)^{\frac{4}{3}}b^2+120\,(dx+c)^2\right)\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)\right)f}{d}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/480*(120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) +
 (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(
d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c
)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*c*
f/d - 120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (
I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*
x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^
(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*e -
(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (Ei(I*
b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^6 + 2*((d*x + c)^(
1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*cos(((d*x + c)^(1/3)*a +
 b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*x + c)^(4/3)*b^2 + 120
*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*f/d)/d
```

Fricas [A]
time = 0.36, size = 305, normalized size = 0.73

$$\frac{2\left(\left(dx+c\right)^{\frac{1}{3}}b^5f-2\,b^3dfx-2\,b^3cf+24\,(bdfx-4\,bcf+5\,bde)(dx+c)^{\frac{2}{3}}\right)\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)+\left(b^6f\sin\left(a\right)-120\,(b^3cf-b^3de)\cos\left(a\right)\right)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+\left(b^6f\sin\left(a\right)-120\,(b^3cf-b^3de)\cos\left(a\right)\right)\operatorname{Ci}\left(-\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+2\left(\left(dx+c\right)^{\frac{2}{3}}b^4f+120\,d^2fx^2-120\,c^2f+240\,(d^2x+cd)e-6\,(b^2dfx-19\,b^2cf+20\,b^2de)(dx+c)^{\frac{1}{3}}\right)\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)+2\left(b^6f\cos\left(a\right)+120\,(b^3cf-b^3de)\sin\left(a\right)\right)\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{480\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/480*(2*((d*x + c)^(1/3)*b^5*f - 2*b^3*d*f*x - 2*b^3*c*f + 24*(b*d*f*x - 4
*b*c*f + 5*b*d*e)*(d*x + c)^(2/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d
*x + c)) + (b^6*f*sin(a) - 120*(b^3*c*f - b^3*d*e)*cos(a))*cos_integral(b/(
d*x + c)^(1/3)) + (b^6*f*sin(a) - 120*(b^3*c*f - b^3*d*e)*cos(a))*cos_integ
ral(-b/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4*f + 120*d^2*f*x^2 - 120*c^
2*f + 240*(d^2*x + c*d)*e - 6*(b^2*d*f*x - 19*b^2*c*f + 20*b^2*d*e)*(d*x +
c)^(1/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) + 2*(b^6*f*cos(a
) + 120*(b^3*c*f - b^3*d*e)*sin(a))*sin_integral(b/(d*x + c)^(1/3)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3728 vs. $2(363) = 726$.
time = 4.93, size = 3728, normalized size = 8.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 1/240*(120*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a + ((d
*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(1/3)*
a + b)*a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_integra
l(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*((d*x +
 c)^(1/3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos(a)*cos_i
ntegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x + c
)^(1/3)*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
 - 2*((d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3))/(d*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*x + c)^(1/3)*a
 + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x + c)^(1/3)*a + b)/
(d*x + c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x +
 c)^(1/3)))*e/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x + c)^(1/3) + 3*((d
*x + c)^(1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3/(d*x +
 c))*b) + (a^6*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
))*sin(a) - a^6*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3)) - 6*((d*x + c)^(1/3)*a + b)*a^5*b^7*cos_integral(-a + ((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(1/3) + 6*((d*x + c)^(1/3)*a
 + b)*a^5*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c)^(1/3) + 15*((d*x + c)^(1/3)*a + b)^2*a^4*b^7*cos_integral(-a
+ ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)^(2/3) - 15*((d*
x + c)^(1/3)*a + b)^2*a^4*b^7*cos(a)*sin_integral(a - ((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 20*((d*x + c)^(1/3)*a + b)^3*a^3*b^7*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x + c)
 + 20*((d*x + c)^(1/3)*a + b)^3*a^3*b^7*cos(a)*sin_integral(a - ((d*x + c)^
(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + 15*((d*x + c)^(1/3)*a + b)^4*a^2*
b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)/(d*x
+ c)^(4/3) - 15*((d*x + c)^(1/3)*a + b)^4*a^2*b^7*cos(a)*sin_integral(a - (
(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) - 6*((d*x + c)^(1/3
)*a + b)^5*a*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
*sin(a)/(d*x + c)^(5/3) + 6*((d*x + c)^(1/3)*a + b)^5*a*b^7*cos(a)*sin_inte
```

```
gral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) - a^5*b^7
*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 120*a^6*b^4*c*cos(a)*cos_in
tegral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + ((d*x + c)^(1/3)*a +
 b)^6*b^7*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a)
/(d*x + c)^2 - ((d*x + c)^(1/3)*a + b)^6*b^7*cos(a)*sin_integral(a - ((d*x
+ c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^2 - 120*a^6*b^4*c*sin(a)*sin_i
ntegral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 5*((d*x + c)^(1/3)*a
 + b)*a^4*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3)
+ 720*((d*x + c)^(1/3)*a + b)*a^5*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)
^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) + 720*((d*x + c)^(1/3)*a + b
)*a^5*b^4*c*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)/(d*x + c)^(1/3) - 10*((d*x + c)^(1/3)*a + b)^2*a^3*b^7*cos(((d*x + c)^(1/
3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) - 1800*((d*x + c)^(1/3)*a + b)^2
*a^4*b^4*c*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
)/(d*x + c)^(2/3) - 1800*((d*x + c)^(1/3)*a + b)^2*a^4*b^4*c*sin(a)*sin_int
egral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 10*((d
*x + c)^(1/3)*a + b)^3*a^2*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
/(d*x + c) + 2400*((d*x + c)^(1/3)*a + b)^3*a^3*b^4*c*cos(a)*cos_integral(-
a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) + a^4*b^7*sin(((d*x
+ c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2400*((d*x + c)^(1/3)*a + b)^3*a^3*b^4
*c*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x +
c) - 5*((d*x + c)^(1/3)*a + b)^4*a*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c
)^(1/3))/(d*x + c)^(4/3) - 1800*((d*x + c)^(1/3)*a + b)^4*a^2*b^4*c*cos(a)*
cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3)
- 4*((d*x + c)^(1/3)*a + b)*a^3*b^7*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3))/(d*x + c)^(1/3) - 1800*((d*x + c)^(1/3)*a + b)^4*a^2*b^4*c*sin(a)*sin
_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(4/3) + ((
d*x + c)^(1/3)*a + b)^5*b^7*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d
*x + c)^(5/3) + 720*((d*x + c)^(1/3)*a + b)^5*a*b^4*c*cos(a)*cos_integral(-
a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(5/3) + 6*((d*x + c)
^(1/3)*a + b)^2*a^2*b^7*sin(((d*x + c)^(1/3)*a ...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)\,(e+f\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))*(e + f*x), x)
```

3.219 $\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal. Leaf size=136

$$\frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{b^3\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^2\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{(c+dx)\,\text{s}}{}$$

```
[Out] 1/2*b^3*Ci(b/(d*x+c)^(1/3))*cos(a)/d+1/2*b*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1
/3))/d-1/2*b^3*Si(b/(d*x+c)^(1/3))*sin(a)/d-1/2*b^2*(d*x+c)^(1/3)*sin(a+b/(
d*x+c)^(1/3))/d+(d*x+c)*sin(a+b/(d*x+c)^(1/3))/d
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3442, 3378, 3384, 3380, 3383}

$$\frac{b^3\cos(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^3\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^2\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d}+\frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d) + (b^3*Cos[a]*CosInteg
ral[b/(c + d*x)^(1/3)])/(2*d) - (b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1
/3)])/(2*d) + ((c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/d - (b^3*Sin[a]*SinInt
egral[b/(c + d*x)^(1/3)])/(2*d)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{b^2\text{Subst}\left(\right.}{}$$

$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+}{}$$

$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+}{}$$

$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{b^3\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\,\sin}{}$$

Mathematica [A]
time = 0.08, size = 133, normalized size = 0.98

$$\frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+2c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+2dx\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-b^2\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-b^3\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^3*Cos[a]*CosIntegral[b/(c
 + d*x)^(1/3)] + 2*c*Sin[a + b/(c + d*x)^(1/3)] + 2*d*x*Sin[a + b/(c + d*x)
^(1/3)] - b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] - b^3*Sin[a]*SinIn
tegral[b/(c + d*x)^(1/3)])/(2*d)
```

Maple [A]
time = 0.02, size = 108, normalized size = 0.79

method	result
derivativedivides	$-\frac{3b^3\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a}{6}\right.}{d}$
default	$-\frac{3b^3\left(-\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)}{3b^3}-\frac{\cos\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{2}{3}}}{6b^2}+\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)(dx+c)^{\frac{1}{3}}}{6b}+\frac{\mathrm{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)\sin(a}{6}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b^3*(-1/3*sin(a+b/(d*x+c)^(1/3))/b^3*(d*x+c)-1/6*cos(a+b/(d*x+c)^(1/3)
)/b^2*(d*x+c)^(2/3)+1/6*sin(a+b/(d*x+c)^(1/3))/b*(d*x+c)^(1/3)+1/6*Si(b/(d*
x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 138, normalized size = 1.01

$$\frac{\left(\left(\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^3+2\,(dx+c)^{\frac{2}{3}}b\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-2\left((dx+c)^{\frac{1}{3}}b^2-2\,dx-2\,c\right)\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/4*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*Ei(I
*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*x + c)
^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)*
b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/d
```

Fricas [A]
time = 0.38, size = 139, normalized size = 1.02

$$\frac{b^3\cos(a)\,\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+b^3\cos(a)\,\mathrm{Ci}\left(-\frac{b}{(dx+c)^{\frac{1}{3}}}\right)-2\,b^3\sin(a)\,\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+2\,(dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-2\left((dx+c)^{\frac{1}{3}}b^2-2\,dx-2\,c\right)\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*cos(a)*cos_integral(b/(d*x + c)^(1/3)) + b^3*cos(a)*cos_integral(-
b/(d*x + c)^(1/3)) - 2*b^3*sin(a)*sin_integral(b/(d*x + c)^(1/3)) + 2*(d*x
+ c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 2*((d*x + c
)^(1/3)*b^2 - 2*d*x - 2*c)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a + \frac{b}{\sqrt[3]{c+dx}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(114) = 228.
time = 5.08, size = 663, normalized size = 4.88

$$\frac{a^3b^4\cos(a)\operatorname{Ci}\left(-a+\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)+a^3b^4\sin(a)\operatorname{Si}\left(a-\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-\frac{3\left((dx+c)^{\frac{1}{3}}a+b\right)a^2b^4\cos(a)\operatorname{Ci}\left(-a+\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{1}{3}}}-\frac{3\left((dx+c)^{\frac{1}{3}}a+b\right)a^2b^4\sin(a)\operatorname{Si}\left(a-\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{1}{3}}}+\frac{3\left((dx+c)^{\frac{1}{3}}a+b\right)^2ab^4\cos(a)\operatorname{Ci}\left(-a+\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{2}{3}}}+\frac{3\left((dx+c)^{\frac{1}{3}}a+b\right)^2ab^4\sin(a)\operatorname{Si}\left(a-\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{2}{3}}}-\frac{\left((dx+c)^{\frac{1}{3}}a+b\right)^3b^4\cos(a)\operatorname{Ci}\left(-a+\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{dx+c}-\frac{\left((dx+c)^{\frac{1}{3}}a+b\right)^3b^4\sin(a)\operatorname{Si}\left(a-\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{dx+c}+a^2b^4\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-\frac{2\left((dx+c)^{\frac{1}{3}}a+b\right)ab^4\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{1}{3}}}+\frac{\left((dx+c)^{\frac{1}{3}}a+b\right)^2b^4\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{2}{3}}}+ab^4\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-\frac{\left((dx+c)^{\frac{1}{3}}a+b\right)b^4\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{(dx+c)^{\frac{1}{3}}}-2b^4\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{2\left(a^3-\frac{3\left((dx+c)^{\frac{1}{3}}a+b\right)a^2}{(dx+c)^{\frac{1}{3}}}+\frac{3\left((dx+c)^{\frac{1}{3}}a+b\right)^2a}{(dx+c)^{\frac{2}{3}}}-\frac{\left((dx+c)^{\frac{1}{3}}a+b\right)^3}{dx+c}\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 1/2*(a^3*b^4*cos(a)*cos_integral(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3)) + a^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3)) - 3*((d*x + c)^(1/3)*a + b)*a^2*b^4*cos(a)*cos_integral(-a + ((d*x + c)
^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(1/3) - 3*((d*x + c)^(1/3)*a + b)*
a^2*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d
*x + c)^(1/3) + 3*((d*x + c)^(1/3)*a + b)^2*a*b^4*cos(a)*cos_integral(-a +
((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c)^(2/3) + 3*((d*x + c)^(1/
3)*a + b)^2*a*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)
^(1/3))/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3*b^4*cos(a)*cos_integral
(-a + ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d*x + c) - ((d*x + c)^(1/3)
*a + b)^3*b^4*sin(a)*sin_integral(a - ((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3))/(d*x + c) + a^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((
d*x + c)^(1/3)*a + b)*a*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(d
*x + c)^(1/3) + ((d*x + c)^(1/3)*a + b)^2*b^4*sin(((d*x + c)^(1/3)*a + b)/(
d*x + c)^(1/3))/(d*x + c)^(2/3) + a*b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3)) - ((d*x + c)^(1/3)*a + b)*b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x +
c)^(1/3))/(d*x + c)^(1/3) - 2*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3)))/((a^3 - 3*((d*x + c)^(1/3)*a + b)*a^2/(d*x + c)^(1/3) + 3*((d*x + c)^(
1/3)*a + b)^2*a/(d*x + c)^(2/3) - ((d*x + c)^(1/3)*a + b)^3/(d*x + c))*b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3)),x)
[Out] int(sin(a + b/(c + d*x)^(1/3)), x)
```

3.220 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$

Optimal. Leaf size=434

$$-\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f}+\frac{\text{Ci}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f}+\frac{\text{Ci}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+d}}\right.}{}$$

[Out] -3*cos(a)*Si(b/(d*x+c)^(1/3))/f-cos(a+(-1)^(1/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)
)*Si((-1)^(1/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)-b/(d*x+c)^(1/3))/f+cos(a-b*f^(1/
3)/(-c*f+d*e)^(1/3))*Si(b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1/3))/f+cos(a
-(-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3))*Si((-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^
(1/3)+b/(d*x+c)^(1/3))/f-3*Ci(b/(d*x+c)^(1/3))*sin(a)/f+Ci(b*f^(1/3)/(-c*f+
d*e)^(1/3)+b/(d*x+c)^(1/3))*sin(a-b*f^(1/3)/(-c*f+d*e)^(1/3))/f+Ci((-1)^(1/
3)*b*f^(1/3)/(-c*f+d*e)^(1/3)-b/(d*x+c)^(1/3))*sin(a+(-1)^(1/3)*b*f^(1/3)/(
-c*f+d*e)^(1/3))/f+Ci((-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3)+b/(d*x+c)^(1/3)
)*sin(a-(-1)^(2/3)*b*f^(1/3)/(-c*f+d*e)^(1/3))/f

Rubi [A]
time = 1.32, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3512, 3384, 3380, 3383, 3426}

$$\frac{\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\sin\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}-\frac{3\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f}-\frac{\cos\left(a+\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{\sqrt[3]{f}\,b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\cos\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{Si}\left(\frac{(-1)^{2/3}\sqrt[3]{f}\,b}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}-\frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]

[Out] (-3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/f + (CosIntegral[(b*f^(1/3))/(d*
e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - (b*f^(1/3))/(d*e - c*f)^(1/3)])
/f + (CosIntegral[((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1
/3)]*Sin[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f + (CosIntegral[((
-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - ((-1)^(
2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f - (3*Cos[a]*SinIntegral[b/(c + d*x)^(
1/3)])/f - (Cos[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(
(-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1/3)])/f + (Cos[a -
(b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(d*e - c*f)^(1/3) +
b/(c + d*x)^(1/3)])/f + (Cos[a - ((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*
SinIntegral[((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)])/
f

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = -\frac{3\text{Subst}\left(\int \left(\frac{d\sin(a+bx)}{fx}+\frac{d(-de+cf)x^2\sin(a+bx)}{f(f+(de-cf)x^3)}\right)dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{x}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f} + \frac{(3(de-cf))\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{f+(de-cf)x^3}dx,\right.}{f}$$

$$= \frac{(3(de-cf))\text{Subst}\left(\int\left(\frac{\sin(a+bx)}{3(de-cf)^{2/3}\left(\sqrt[3]{f}+\sqrt[3]{de-cf}\,x\right)}+\frac{\sin(a+b}{3(de-cf)^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{\right.}}\right.\right.}{f}$$

$$= -\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\sqrt[3]{de-cf}\,\text{Subst}\left(\int \frac{}{\sqrt[3]{}}\right.}{}$$

$$= -\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\left(\sqrt[3]{de-cf}\,\cos\left(a-\right.\right.}{}$$

$$= -\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} + \frac{\text{Ci}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right.}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.48, size = 170, normalized size = 0.39

$$\frac{i\left(\left(-3\text{Ei}\left(-\frac{ib}{\sqrt[3]{c+dx}}\right)+\text{RootSum}\left[de-cf+f\#1^3\&,e^{-\frac{ib}{\#1}}\text{Ei}\left(-ib\left(\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{\#1}\right)\right)\&\right]\right)(\cos(a)-i\sin(a))+\left(3\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)-\text{RootSum}\left[de-cf+f\#1^3\&,e^{\frac{ib}{\#1}}\text{Ei}\left(ib\left(\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{\#1}\right)\right)\&\right]\right)(\cos(a)+i\sin(a))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] ((I/2)*((-3*ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] + RootSum[d*e - c*f + f
*#1^3 & , ExpIntegralEi[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1) &
 ])*(Cos[a] - I*Sin[a]) + (3*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - RootSum
[d*e - c*f + f*#1^3 & , E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(-1/3) -
#1^(-1))] & ])*(Cos[a] + I*Sin[a])))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 156, normalized size = 0.36

method	result
derivativedivides	$-3b^3\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left((cf-de)\_Z^3+(-3acf+3ade)\_Z^2+\left(3a^2cf-3a^2de\right)\_Z-a^3cf+a^3de-f\,b^3\right)}\left(-\mathrm{sinIntegral}\left(-\frac{b}{(dx+c}\right.\right.}{3f\,b^3}\right.$
default	$-3b^3\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left((cf-de)\_Z^3+(-3acf+3ade)\_Z^2+\left(3a^2cf-3a^2de\right)\_Z-a^3cf+a^3de-f\,b^3\right)}\left(-\mathrm{sinIntegral}\left(-\frac{b}{(dx+c}\right.\right.}{3f\,b^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -3*b^3*(-1/3/f/b^3*sum(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1
/3)-_R1+a)*sin(_R1),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^
2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3))+1/f/b^3*(Si(b/(d*x+c)^(1/3))*co
s(a)+Ci(b/(d*x+c)^(1/3))*sin(a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.42, size = 566, normalized size = 1.30

$$\frac{-i\,\mathrm{Ei}\left(\frac{2i(dx+c)^{\frac{2}{3}}b-\left(\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}\left(dx-\sqrt{3}(-idx-ic)+c\right)}{2(dx+c)}\right)e^{\left(\frac{1}{2}\left(\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}(i\sqrt{3}+1)+ia\right)}+i\,\mathrm{Ei}\left(\frac{-2i(dx+c)^{\frac{2}{3}}b-\left(-\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}\left(dx-\sqrt{3}(-idx-ic)+c\right)}{2(dx+c)}\right)e^{\left(\frac{1}{2}\left(-\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}(i\sqrt{3}+1)-ia\right)}-i\,\mathrm{Ei}\left(\frac{2i(dx+c)^{\frac{2}{3}}b-\left(\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}\left(dx-\sqrt{3}(idx+ic)+c\right)}{2(dx+c)}\right)e^{\left(\frac{1}{2}\left(\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}(-i\sqrt{3}+1)+ia\right)}+i\,\mathrm{Ei}\left(\frac{-2i(dx+c)^{\frac{2}{3}}b-\left(-\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}\left(dx-\sqrt{3}(idx+ic)+c\right)}{2(dx+c)}\right)e^{\left(\frac{1}{2}\left(-\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}(-i\sqrt{3}+1)-ia\right)}+3i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)e^{(ia)}-3i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)e^{(-ia)}-i\,\mathrm{Ei}\left(\frac{i(dx+c)^{\frac{2}{3}}b+\left(\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}(dx+c)}{dx+c}\right)e^{\left(ia-\left(\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}\right)}+i\,\mathrm{Ei}\left(\frac{-i(dx+c)^{\frac{2}{3}}b+\left(-\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}(dx+c)}{dx+c}\right)e^{\left(-ia-\left(-\frac{ib^3f}{cf-de}\right)^{\frac{1}{3}}\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(c*f - d*e))^(1/3)*(d*x -
sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(c*f - d*e))^(1/3)*
(I*sqrt(3) + 1) + I*a) + I*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(c*f
- d*e))^(1/3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3
*f/(c*f - d*e))^(1/3)*(I*sqrt(3) + 1) - I*a) - I*Ei(1/2*(2*I*(d*x + c)^(2/3
)*b - (I*b^3*f/(c*f - d*e))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x +
 c))*e^(1/2*(I*b^3*f/(c*f - d*e))^(1/3)*(-I*sqrt(3) + 1) + I*a) + I*Ei(1/2*
```

```
(-2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(c*f - d*e))^(1/3)*(d*x - sqrt(3)*(I*d*
x + I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(c*f - d*e))^(1/3)*(-I*sqrt(3) +
 1) - I*a) + 3*I*Ei(I*b/(d*x + c)^(1/3))*e^(I*a) - 3*I*Ei(-I*b/(d*x + c)^(1
/3))*e^(-I*a) - I*Ei((I*(d*x + c)^(2/3)*b + (I*b^3*f/(c*f - d*e))^(1/3)*(d*
x + c))/(d*x + c))*e^(I*a - (I*b^3*f/(c*f - d*e))^(1/3)) + I*Ei((-I*(d*x +
c)^(2/3)*b + (-I*b^3*f/(c*f - d*e))^(1/3)*(d*x + c))/(d*x + c))*e^(-I*a - (
-I*b^3*f/(c*f - d*e))^(1/3)))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{\sqrt[3]{c + dx}}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e),x)
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a + \frac{b}{(c+d\,x)^{1/3}}\right)}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x), x)
```

3.221 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=566

$$-\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{Ci}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}}-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\operatorname{Ci}\left(\frac{(-1)}{\sqrt[3]{-}}\right.}{3f^{2/3}(-de+cf)^{4/3}}$$

```
[Out] -1/3*b*d*Ci(b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+c)^(1/3))*cos(a+b*f^(1/3)/(c*f
-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)+1/3*(-1)^(1/3)*b*d*Ci((-1)^(1/3)*b*f^(
1/3)/(c*f-d*e)^(1/3)+b/(d*x+c)^(1/3))*cos(a-(-1)^(1/3)*b*f^(1/3)/(c*f-d*e)^
(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(2/3)*b*d*Ci((-1)^(2/3)*b*f^(1/3)/(
c*f-d*e)^(1/3)-b/(d*x+c)^(1/3))*cos(a+(-1)^(2/3)*b*f^(1/3)/(c*f-d*e)^(1/3))
/f^(2/3)/(c*f-d*e)^(4/3)-1/3*b*d*Si(b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+c)^(1/
3))*sin(a+b*f^(1/3)/(c*f-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(1/3)
*b*d*Si((-1)^(1/3)*b*f^(1/3)/(c*f-d*e)^(1/3)+b/(d*x+c)^(1/3))*sin(a-(-1)^(1
/3)*b*f^(1/3)/(c*f-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)-1/3*(-1)^(2/3)*b*d*S
i((-1)^(2/3)*b*f^(1/3)/(c*f-d*e)^(1/3)-b/(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*
f^(1/3)/(c*f-d*e)^(1/3))/f^(2/3)/(c*f-d*e)^(4/3)+(d*x+c)*sin(a+b/(d*x+c)^(1
/3))/(-c*f+d*e)/(f*x+e)
```

Rubi [A]
time = 1.66, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3422, 3415, 3384, 3380, 3383}

$$-\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}+\frac{\sqrt[3]{-1}\,bd\cos\left(a-\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{cf-de}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}-\frac{bd\sin\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{Si}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}-\frac{(-1)^{2/3}bd\sin\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{Si}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}-\frac{\sqrt[3]{-1}\,bd\sin\left(a-\frac{\sqrt[3]{-1}\,b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{f}\,b}{\sqrt[3]{cf-de}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] -1/3*(b*d*Cos[a + (b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[(b*f^(1/3))
/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(f^(2/3)*(-(d*e) + c*f)^(4/3))
- ((-1)^(2/3)*b*d*Cos[a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosI
ntegral[((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(
3*f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((-1)^(1/3)*b*d*Cos[a - ((-1)^(1/3)*b*f^(
1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*
f)^(1/3) + b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((c + d*x
)*Sin[a + b/(c + d*x)^(1/3)])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*f^(
1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(-(d*e) + c*f)^(1/3) -
b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) - ((-1)^(2/3)*b*d*Sin[
a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[((-1)^(2/3)*b*
```

```
f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*
f)^(4/3)) - ((-1)^(1/3)*b*d*Sin[a - ((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(
1/3)]*SinIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) + b/(c + d*x)
^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^3\right)^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \frac{\cos(a+bx)}{\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\text{Subst}\left(\int \left(\frac{d\cos(a+bx)}{3f^{2/3}\left(\sqrt[3]{f}-\sqrt[3]{-de+cf}\,x\right)} + \cdots\right.\right.}{}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt[3]{f}-\sqrt[3]{-de+cf}\,x}\,dx, x, \frac{}{\sqrt[3]{c}\cdots}\right.}{3f^{2/3}(de-cf)}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{\left(bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int \frac{\cos\cdots}{\sqrt[3]{\cdots}}\right.}{3f^{2/3}(de-\cdots}$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} - \frac{(-1)^{2/3}bd\cos\cdots}{}$$

<u>Mathematica</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.76, size = 313, normalized size = 0.55

$$\frac{(\cos(a)+i\sin(a))\left(bd(e+fx)\text{RootSum}\left[de-cf+f\#1^3\&,\frac{\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)-e^{\frac{ib}{\#1}}\text{Ei}\left(ib\left(\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{\#1}\right)\right)}{\#1}\&\right]+(c+dx)\left(3if\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)-3f\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)\right)+i\left(-3cf-3dfx+bd(e+fx)\text{RootSum}\left[de-cf+f\#1^3\&,\frac{\text{Ei}\left(-\frac{ib}{\sqrt[3]{c+dx}}\right)-e^{-\frac{ib}{\#1}}\text{Ei}\left(-ib\left(\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{\#1}\right)\right)}{\#1}\&\right]\left(-i\cos\left(\frac{b}{\sqrt[3]{c+dx}}\right)+\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)\right)\left(\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-i\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{6f(-de+cf)(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] ((Cos[a] + I*Sin[a])*(b*d*(e + f*x)*RootSum[d*e - c*f + f*#1^3 & , (ExpInte
gralEi[(I*b)/(c + d*x)^(1/3)] - E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(
-1/3) - #1^(-1))])/#1 & ] + (c + d*x)*((3*I)*f*Cos[b/(c + d*x)^(1/3)] - 3*f
*Sin[b/(c + d*x)^(1/3)])) + I*(-3*c*f - 3*d*f*x + b*d*(e + f*x)*RootSum[d*e
 - c*f + f*#1^3 & , (ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] - ExpIntegralE
i[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1))/#1 & ]*((-I)*Cos[b/(c
+ d*x)^(1/3)] + Sin[b/(c + d*x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)] - I*Si
n[a + b/(c + d*x)^(1/3)]))/(6*f*(-(d*e) + c*f)*(e + f*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 1554, normalized size = 2.75

method	result	size
derivativedivides	Expression too large to display	1554
default	Expression too large to display	1554

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3*d*b^3*(a^2*(sin(a+b/(d*x+c)^(1/3))*(1/3/f/b^3*(a+b/(d*x+c)^(1/3))-1/3*a/
f/b^3)/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a^2*d*e*(a+b/(d*x+c
)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+c)^(1/3))^2-c*f*(a
+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)-2/9/f/b^3*sum(1/(_R1^2
*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c)^(1/
3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*f-d*e)
*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^
3))-1/9/f/b^3*sum(1/(-_RR1*c*f+_RR1*d*e+a*c*f-a*d*e)*(Si(-b/(d*x+c)^(1/3)+_
RR1-a)*sin(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e
)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b
^3)))+sin(a+b/(d*x+c)^(1/3))*(-2/3*a/f/b^3*(a+b/(d*x+c)^(1/3))^2+2/3*a^2/f/
b^3*(a+b/(d*x+c)^(1/3)))/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a
^2*d*e*(a+b/(d*x+c)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+
c)^(1/3))^2-c*f*(a+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)+2/9*
a/f/b^3*sum((_R1+a)/(_R1^2*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^
2*d*e)*(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(
_R1)),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*
e)*_Z-a^3*c*f+a^3*d*e-f*b^3))+2/9*a/f/b^3*sum(_RR1/(-_RR1*c*f+_RR1*d*e+a*c*
f-a*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*sin(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*
cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*
a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3))+sin(a+b/(d*x+c)^(1/3))*(2/3*a/f/b^3*(a+
b/(d*x+c)^(1/3))^2-a^2/f/b^3*(a+b/(d*x+c)^(1/3))+1/3*(a^3*c*f-a^3*d*e+b^3*f
)/f/b^3/(c*f-d*e))/(a^3*c*f-a^3*d*e-3*a^2*c*f*(a+b/(d*x+c)^(1/3))+3*a^2*d*e
*(a+b/(d*x+c)^(1/3))+3*a*c*f*(a+b/(d*x+c)^(1/3))^2-3*a*d*e*(a+b/(d*x+c)^(1/
3))^2-c*f*(a+b/(d*x+c)^(1/3))^3+d*e*(a+b/(d*x+c)^(1/3))^3+f*b^3)-2/9*a/f/b^
3*sum(_R1/(_R1^2*c*f-_R1^2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-S
i(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=
RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*
c*f+a^3*d*e-f*b^3))+1/9/f/b^3*sum((2*_RR1^2*a*c*f-2*_RR1^2*a*d*e-3*_RR1*a^2
*c*f+3*_RR1*a^2*d*e+a^3*c*f-a^3*d*e+b^3*f)/(c*f-d*e)/(_RR1^2*c*f-_RR1^2*d*e
-2*_RR1*a*c*f+2*_RR1*a*d*e+a^2*c*f-a^2*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*si
n(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-
3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-f*b^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.44, size = 827, normalized size = 1.46

$$-\frac{\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(-i d f x-i d e+\sqrt{3}(d f x+d e)\right) \operatorname{Ei}\left(\frac{2 i (d x+c)^{\frac{2}{3}} b-\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(d x-\sqrt{3}(-i d x-i c)+c\right)}{2(d x+c)}\right) e^{\left(\frac{1}{2}\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(i \sqrt{3}+1)+i a\right)}+\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(i d f x+i d e-\sqrt{3}(d f x+d e)\right) \operatorname{Ei}\left(\frac{-2 i (d x+c)^{\frac{2}{3}} b-\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(d x-\sqrt{3}(-i d x-i c)+c\right)}{2(d x+c)}\right) e^{\left(\frac{1}{2}\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(i \sqrt{3}+1)-i a\right)}+\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(-i d f x-i d e-\sqrt{3}(d f x+d e)\right) \operatorname{Ei}\left(\frac{2 i (d x+c)^{\frac{2}{3}} b-\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(d x-\sqrt{3}(i d x+i c)+c\right)}{2(d x+c)}\right) e^{\left(\frac{1}{2}\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(-i \sqrt{3}+1)+i a\right)}+\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(i d f x+i d e+\sqrt{3}(d f x+d e)\right) \operatorname{Ei}\left(\frac{-2 i (d x+c)^{\frac{2}{3}} b-\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\left(d x-\sqrt{3}(i d x+i c)+c\right)}{2(d x+c)}\right) e^{\left(\frac{1}{2}\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(-i \sqrt{3}+1)-i a\right)}-2\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(-i d f x-i d e) \operatorname{Ei}\left(\frac{i (d x+c)^{\frac{2}{3}} b+\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(d x+c)}{d x+c}\right) e^{\left(i a-\left(\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\right)}-2\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(i d f x+i d e) \operatorname{Ei}\left(\frac{-i (d x+c)^{\frac{2}{3}} b+\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}(d x+c)}{d x+c}\right) e^{\left(-i a-\left(-\frac{i b^{3} f}{c f-d e}\right)^{\frac{1}{3}}\right)}+12(d f x+c f) \sin\left(\frac{a d x+a c+(d x+c)^{\frac{2}{3}} b}{d x+c}\right)}{12\left(c f^{3} x-d f e^{2}-\left(d f^{2} x-c f^{2}\right) e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/12*((I*b^3*f/(c*f - d*e))^(1/3)*(-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e
))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(c*f - d*e))^(1/3)*(d*x - sqrt(
3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(c*f - d*e))^(1/3)*(I*sq
rt(3) + 1) + I*a) + (-I*b^3*f/(c*f - d*e))^(1/3)*(I*d*f*x + I*d*e - sqrt(3)
*(d*f*x + d*e))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(c*f - d*e))^(1/
3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(c*f - d
*e))^(1/3)*(I*sqrt(3) + 1) - I*a) + (I*b^3*f/(c*f - d*e))^(1/3)*(-I*d*f*x -
 I*d*e - sqrt(3)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(c
*f - d*e))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c))*e^(1/2*(I*b^
3*f/(c*f - d*e))^(1/3)*(-I*sqrt(3) + 1) + I*a) + (-I*b^3*f/(c*f - d*e))^(1/
3)*(I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b
 - (-I*b^3*f/(c*f - d*e))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c
))*e^(1/2*(-I*b^3*f/(c*f - d*e))^(1/3)*(-I*sqrt(3) + 1) - I*a) - 2*(I*b^3*f
/(c*f - d*e))^(1/3)*(-I*d*f*x - I*d*e)*Ei((I*(d*x + c)^(2/3)*b + (I*b^3*f/(
c*f - d*e))^(1/3)*(d*x + c))/(d*x + c))*e^(I*a - (I*b^3*f/(c*f - d*e))^(1/3
)) - 2*(-I*b^3*f/(c*f - d*e))^(1/3)*(I*d*f*x + I*d*e)*Ei((-I*(d*x + c)^(2/3
)*b + (-I*b^3*f/(c*f - d*e))^(1/3)*(d*x + c))/(d*x + c))*e^(-I*a - (-I*b^3*
f/(c*f - d*e))^(1/3)) + 12*(d*f*x + c*f)*sin((a*d*x + a*c + (d*x + c)^(2/3)
*b)/(d*x + c)))/(c*f^3*x - d*f*e^2 - (d*f^2*x - c*f^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\left(e+fx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)}{{\left(e+f\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(e + f*x)^2, x)
```

3.222 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal. Leaf size=630

$$\frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}+\frac{bf(de-cf)(c+dx)^{4/3}\cos\left(\right.}{2d^3}$$

```
[Out] 1/2*b^3*f*(-c*f+d*e)*Ci(b/(d*x+c)^(2/3))*cos(a)/d^3+2*b*(-c*f+d*e)^2*(d*x+c
)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d^3-8/315*b^3*f^2*(d*x+c)*cos(a+b/(d*x+c)^(2
/3))/d^3+1/2*b*f*(-c*f+d*e)*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))/d^3+2/21*b
*f^2*(d*x+c)^(7/3)*cos(a+b/(d*x+c)^(2/3))/d^3-1/2*b^3*f*(-c*f+d*e)*Si(b/(d*
x+c)^(2/3))*sin(a)/d^3+16/315*b^4*f^2*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/
d^3-1/2*b^2*f*(-c*f+d*e)*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(2/3))/d^3+(-c*f+d*e
)^2*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d^3-4/105*b^2*f^2*(d*x+c)^(5/3)*sin(a+b/
(d*x+c)^(2/3))/d^3+f*(-c*f+d*e)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))/d^3+1/3*f^
2*(d*x+c)^3*sin(a+b/(d*x+c)^(2/3))/d^3-16/315*b^(9/2)*f^2*cos(a)*FresnelC(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d^3+2*b^(3/2)*(-c*f
+d*e)^2*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^
(1/2)/d^3+2*b^(3/2)*(-c*f+d*e)^2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^
(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d^3+16/315*b^(9/2)*f^2*FresnelS(b^(1/2)*2^(1
/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d^3
```

Rubi [A]
time = 0.52, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {3514, 3490, 3468, 3469, 3434, 3433, 3432, 3460, 3378, 3384, 3380, 3383, 3435}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(d*e - c*f)^2*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^3 - (8*b^3
*f^2*(c + d*x)*Cos[a + b/(c + d*x)^(2/3)])/(315*d^3) + (b*f*(d*e - c*f)*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*d^3) + (2*b*f^2*(c + d*x)^(7/3)
*Cos[a + b/(c + d*x)^(2/3)])/(21*d^3) + (b^3*f*(d*e - c*f)*Cos[a]*CosIntegr
al[b/(c + d*x)^(2/3)])/(2*d^3) - (16*b^(9/2)*f^2*Sqrt[2*Pi]*Cos[a]*FresnelC
[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/(315*d^3) + (2*b^(3/2)*(d*e - c*f)^
2*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^3 + (
2*b^(3/2)*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^
(1/3)]*Sin[a])/d^3 + (16*b^(9/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
```

```
])/(c + d*x)^(1/3)]*Sin[a])/(315*d^3) + (16*b^4*f^2*(c + d*x)^(1/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(315*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(2*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)
^(2/3)])/d^3 - (4*b^2*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^(2/3)])/(105*
d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(2/3)])/d^3 + (f^2*(c
 + d*x)^3*Sin[a + b/(c + d*x)^(2/3)])/(3*d^3) - (b^3*f*(d*e - c*f)*Sin[a]*S
inIntegral[b/(c + d*x)^(2/3)])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \frac{3\text{Subst}\left(\int \left((de-cf)^2x^2 \sin\left(a+\frac{b}{x^2}\right)-2f(-de+cf)x^5 \sin\left(a+\frac{b}{x^2}\right.\right.\right.}{d^3}$$

$$= \frac{(3f^2)\,\text{Subst}\left(\int x^8 \sin\left(a+\frac{b}{x^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\text{Su}}{}$$

$$= -\frac{(3f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^{10}}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(3f(de-cf))\text{Subst}}{}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2 \sin\left(a\right.}{d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{4/3}}{2d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{4/3}}{2d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\right.}{315d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\right.}{315d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\right.}{315d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3} - \frac{8b^3f^2(c+dx)\cos\left(a+\right.}{315d^3}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.78, size = 613, normalized size = 0.97

$$\frac{ie^{-ia}\left(e^{-\frac{ib}{(c+dx)^{2/3}}}\sqrt[3]{c+dx}\left(32b^4f^2+16ib^3f^2(c+dx)^{2/3}+3b^2f\sqrt[3]{c+dx}\,(-105de+97cf-8dfx)-15ib(84d^2e^2+21def(-7c+dx)+f^2(67c^2-13cdx+4d^2x^2))+210(c+dx)^{2/3}\,(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2))\right)-e^{i\left(2a+\frac{b}{(c+dx)^{2/3}}\right)}\sqrt[3]{c+dx}\left(32b^4f^2-16ib^3f^2(c+dx)^{2/3}+3b^2f\sqrt[3]{c+dx}\,(-105de+97cf-8dfx)+15ib(84d^2e^2+21def(-7c+dx)+f^2(67c^2-13cdx+4d^2x^2))+210(c+dx)^{2/3}\,(c^2f^2-cdf(3e+fx)+d^2(3e^2+3efx+f^2x^2))\right)+4\sqrt[4]{-1}\,b^{3/2}e^{2ia}(315id^2e^2-630icdef+(8b^3+315ic^2)\,f^2)\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}\,\sqrt{b}}{\sqrt[3]{c+dx}}\right)-4\sqrt[4]{-1}\,b^{3/2}(315d^2e^2-630cdef+(8ib^3+315c^2)\,f^2)\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(-1)^{3/4}\sqrt{b}}{\sqrt[3]{c+dx}}\right)+315ib^3f(-de+cf)\mathrm{Ei}\left(-\frac{ib}{(c+dx)^{2/3}}\right)+315ib^3e^{2ia}f(-de+cf)\mathrm{Ei}\left(\frac{ib}{(c+dx)^{2/3}}\right)\right)}{1260d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((I/1260)*(((c + d*x)^(1/3)*(32*b^4*f^2 + (16*I)*b^3*f^2*(c + d*x)^(2/3) +
3*b^2*f*(c + d*x)^(1/3)*(-105*d*e + 97*c*f - 8*d*f*x) - (15*I)*b*(84*d^2*e^
2 + 21*d*e*f*(-7*c + d*x) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c +
 d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))
))/E^((I*b)/(c + d*x)^(2/3)) - E^(I*(2*a + b/(c + d*x)^(2/3)))*(c + d*x)^(1
/3)*(32*b^4*f^2 - (16*I)*b^3*f^2*(c + d*x)^(2/3) + 3*b^2*f*(c + d*x)^(1/3)*
(-105*d*e + 97*c*f - 8*d*f*x) + (15*I)*b*(84*d^2*e^2 + 21*d*e*f*(-7*c + d*x
) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c + d*x)^(2/3)*(c^2*f^2 - c
*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))) + 4*(-1)^(1/4)*b^(3/2)
*E^((2*I)*a)*((315*I)*d^2*e^2 - (630*I)*c*d*e*f + (8*b^3 + (315*I)*c^2)*f^2
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b])/(c + d*x)^(1/3)] - 4*(-1)^(1/4)*b^(3/2
)*(315*d^2*e^2 - 630*c*d*e*f + ((8*I)*b^3 + 315*c^2)*f^2)*Sqrt[Pi]*Erfi[((-
1)^(3/4)*Sqrt[b])/(c + d*x)^(1/3)] + (315*I)*b^3*f*(-(d*e) + c*f)*ExpIntegr
alEi[((-I)*b)/(c + d*x)^(2/3)] + (315*I)*b^3*E^((2*I)*a)*f*(-(d*e) + c*f)*E
xpIntegralEi[(I*b)/(c + d*x)^(2/3)]))/(d^3*E^(I*a))
```

Maple [A]
time = 0.06, size = 452, normalized size = 0.72

method	result
derivativedivides	$\dfrac{(c^2f^2-2cdef+d^2e^2)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2(c^2f^2-2cdef+d^2e^2)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\right.}{}$

default	$\frac{(c^2f^2-2cdef+d^2e^2)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2(c^2f^2-2cdef+d^2e^2)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\ }\right.}{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3*(1/3*(c^2*f^2-2*c*d*e*f+d^2*e^2)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*(
c^2*f^2-2*c*d*e*f+d^2*e^2)*b*(-(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)
*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+
sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))))+1/6*(-2*c*f^2+2*d
*e*f)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))-1/3*(-2*c*f^2+2*d*e*f)*b*(-1/4*(d*x+
c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(
2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*sin(a)*Si(b/(d*x+c)^(2/3)))))+1
/9*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(2/3))-2/9*f^2*b*(-1/7*(d*x+c)^(7/3)*cos(a
+b/(d*x+c)^(2/3))-2/7*b*(-1/5*(d*x+c)^(5/3)*sin(a+b/(d*x+c)^(2/3))+2/5*b*(-
1/3*(d*x+c)*cos(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2
/3))+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*
x+c)^(1/3))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.80, size = 1261, normalized size = 2.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/1260*(630*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*c
os(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt(
```

```
(d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1
)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqr
t(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d
*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) -
1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*c^2*f^2/
((d*x + c)^(1/3)*b*d^2) - 1260*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x
 + c)^(-4/3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d
*x + c)^(4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c
)^(2/3)) + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1
)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(p
i)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(
d*x + c)^(2/3))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x +
 c)^(4/3))*c*f*e/((d*x + c)^(1/3)*b*d) - 315*(((Ei(I*b/(d*x + c)^(2/3)) + E
i(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*Ei(-I*b/(d
*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^(2/3)*a +
b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin(((d*x + c
)^(2/3)*a + b)/(d*x + c)^(2/3)))*c*f^2/d^2 + 315*(((Ei(I*b/(d*x + c)^(2/3))
 + Ei(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*Ei(-I*
b/(d*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^(2/3)*
a + b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin(((d*x
 + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*f*e/d + 630*sqrt(2)*(2*sqrt(2)*(d*x +
c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(
2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/
3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/
3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a)
+ (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi
)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^
(1/4))*sqrt((d*x + c)^(4/3))*e^2/((d*x + c)^(1/3)*b) - 2*sqrt(2)*(8*((-(I -
 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(s
qrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I + 1)*sqrt(pi)*(erf(sqrt(I*b/
(d*x + c)^(2/3))) - 1) + (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3)))
- 1))*sin(a))*b^5*(b^2/(d*x + c)^(4/3))^(1/4) + 2*(4*sqrt(2)*(d*x + c)^(4/3
)*sqrt((d*x + c)^(-4/3))*b^4 - 15*sqrt(2)*(d*x + c)^(8/3)*sqrt((d*x + c)^(-
4/3))*b^2)*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - (16*sqrt(2)*(d*x
+ c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^5 - 12*sqrt(2)*(d*x + c)^2*sqrt((d*x +
c)^(-4/3))*b^3 + 105*sqrt(2)*(d*x + c)^(10/3)*sqrt((d*x + c)^(-4/3))*b)*sin
(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*sqrt((d*x + c)^(4/3))*f^2/((d*x
+ c)^(1/3)*b*d^2))/d
```

Fricas [A]
time = 0.42, size = 501, normalized size = 0.80

$$-\frac{315\left(b^{3}cf^{2}-b^{3}dfe\right)\cos\left(a\right)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+315\left(b^{3}cf^{2}-b^{3}dfe\right)\cos\left(a\right)\operatorname{Ci}\left(-\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+8\sqrt{2}\left(8\pi b^{4}f^{2}\cos\left(a\right)-315\left(\pi bc^{2}f^{2}-2\pi bcdfe+\pi bd^{2}e^{2}\right)\sin\left(a\right)\right)\sqrt{\frac{b}{\pi}}\,\operatorname{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)-8\sqrt{2}\left(8\pi b^{4}f^{2}\sin\left(a\right)+315\left(\pi bc^{2}f^{2}-2\pi bcdfe+\pi bd^{2}e^{2}\right)\cos\left(a\right)\right)\sqrt{\frac{b}{\pi}}\,\operatorname{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)-630\left(b^{3}cf^{2}-b^{3}dfe\right)\sin\left(a\right)\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2\left(16b^{3}df^{2}x+16b^{3}cf^{2}-15\left(4bd^{2}f^{2}x^{2}-13bcdf^{2}x+67bc^{2}f^{2}+84bd^{2}e^{2}+21\left(bd^{2}fx-7bcdf\right)e\right)(dx+c)^{\frac{1}{3}}\right)\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)-2\left(210d^{3}f^{2}x^{3}+32(dx+c)^{\frac{1}{3}}b^{4}f^{2}+210c^{3}f^{2}+630\left(d^{3}x+cd^{2}\right)e^{2}+630\left(d^{3}fx^{2}-c^{2}df\right)e-3\left(8b^{2}df^{2}x-97b^{2}cf^{2}+105b^{2}dfe\right)(dx+c)^{\frac{2}{3}}\right)\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{1260d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] -1/1260*(315*(b^3*c*f^2 - b^3*d*f*e)*cos(a)*cos_integral(b/(d*x + c)^(2/3))
 + 315*(b^3*c*f^2 - b^3*d*f*e)*cos(a)*cos_integral(-b/(d*x + c)^(2/3)) + 8*
sqrt(2)*(8*pi*b^4*f^2*cos(a) - 315*(pi*b*c^2*f^2 - 2*pi*b*c*d*f*e + pi*b*d^
2*e^2)*sin(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) -
 8*sqrt(2)*(8*pi*b^4*f^2*sin(a) + 315*(pi*b*c^2*f^2 - 2*pi*b*c*d*f*e + pi*b
*d^2*e^2)*cos(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)
) - 630*(b^3*c*f^2 - b^3*d*f*e)*sin(a)*sin_integral(b/(d*x + c)^(2/3)) + 2*
(16*b^3*d*f^2*x + 16*b^3*c*f^2 - 15*(4*b*d^2*f^2*x^2 - 13*b*c*d*f^2*x + 67*
b*c^2*f^2 + 84*b*d^2*e^2 + 21*(b*d^2*f*x - 7*b*c*d*f)*e)*(d*x + c)^(1/3))*c
os((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)) - 2*(210*d^3*f^2*x^3 + 32*(
d*x + c)^(1/3)*b^4*f^2 + 210*c^3*f^2 + 630*(d^3*x + c*d^2)*e^2 + 630*(d^3*f
*x^2 - c^2*d*f)*e - 3*(8*b^2*d*f^2*x - 97*b^2*c*f^2 + 105*b^2*d*f*e)*(d*x +
 c)^(2/3))*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{{(c+d\,x)}^{2/3}}\right)\,{(e+f\,x)}^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(e + f*x)^2,x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))*(e + f*x)^2, x)
```

3.223 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal. Leaf size=318

$$\frac{2b(de-cf)\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{b^3f\cos(a)\operatorname{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{2b^{3/2}\cdots}{\cdots}$$

```
[Out] 1/4*b^3*f*Ci(b/(d*x+c)^(2/3))*cos(a)/d^2+2*b*(-c*f+d*e)*(d*x+c)^(1/3)*cos(a
+b/(d*x+c)^(2/3))/d^2+1/4*b*f*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))/d^2-1/4*
b^3*f*Si(b/(d*x+c)^(2/3))*sin(a)/d^2-1/4*b^2*f*(d*x+c)^(2/3)*sin(a+b/(d*x+c
)^(2/3))/d^2+(-c*f+d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d^2+1/2*f*(d*x+c)^2*
sin(a+b/(d*x+c)^(2/3))/d^2+2*b^(3/2)*(-c*f+d*e)*cos(a)*FresnelS(b^(1/2)*2^(
1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d^2+2*b^(3/2)*(-c*f+d*e)*Fres
nelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d^2
```

Rubi [A]

time = 0.26, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3514, 3490, 3468, 3469, 3434, 3433, 3432, 3460, 3378, 3384, 3380, 3383}

$$\frac{2\sqrt{2\pi}\,b^{3/2}\sin(a)(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{2\sqrt{2\pi}\,b^{3/2}\cos(a)(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{b^3f\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}-\frac{b^3f\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}-\frac{b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{(c+dx)(de-cf)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{2b\sqrt[3]{c+dx}\,(de-cf)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^2 + (b*f*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d^2) + (b^3*f*Cos[a]*CosIntegra
l[b/(c + d*x)^(2/3)])/(4*d^2) + (2*b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fr
esnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^2 + (2*b^(3/2)*(d*e - c*f)*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d^2 - (b^
2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])/(4*d^2) + ((d*e - c*f)*(c +
 d*x)*Sin[a + b/(c + d*x)^(2/3)])/d^2 + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^
(2/3)])/(2*d^2) - (b^3*f*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{3\text{Subst}\left(\int \left((de-cf)x^2\sin\left(a+\frac{b}{x^2}\right)+fx^5\sin\left(a+\frac{b}{x^2}\right)\right)\,dx,x,\sqrt[3]{c+\right.}{d^2}$$

$$= \frac{(3f)\text{Subst}\left(\int x^5\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}+\frac{(3(de-cf))\text{Subst}\left(\right.}{}$$

$$= -\frac{(3f)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d^2}-\frac{(3(de-cf))\text{Subst}\left(\int \frac{\sin(\right.}{d^2}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right.}{2d^2}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{}{(c}\right.}{4d^2}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{}{(c}\right.}{4d^2}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{}{(c}\right.}{4d^2}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2}+\frac{bf(c+dx)^{4/3}\cos\left(a+\frac{}{(c}\right.}{4d^2}$$

Mathematica [A]
time = 0.72, size = 378, normalized size = 1.19

$$\frac{8bde\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)-7bcf\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+bdfx\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b^3f\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)+8b^{3/2}(de-cf)\sqrt{2\pi}\,\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+8b^{3/2}de\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)-8b^{3/2}cf\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)+4cde\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)-2c^2f\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+4d^2ex\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2d^2fx^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)-b^2f(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)-b^3f\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (8*b*d*e*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] - 7*b*c*f*(c + d*x)^(1/
3)*Cos[a + b/(c + d*x)^(2/3)] + b*d*f*x*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)
^(2/3)] + b^3*f*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)] + 8*b^(3/2)*(d*e - c*
f)*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 8*b^(
```

```
3/2)*d*e*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] -
 8*b^(3/2)*c*f*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Si
n[a] + 4*c*d*e*Sin[a + b/(c + d*x)^(2/3)] - 2*c^2*f*Sin[a + b/(c + d*x)^(2/
3)] + 4*d^2*e*x*Sin[a + b/(c + d*x)^(2/3)] + 2*d^2*f*x^2*Sin[a + b/(c + d*x
)^(2/3)] - b^2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] - b^3*f*Sin[a]*
SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Maple [A]
time = 0.02, size = 225, normalized size = 0.71

method	result
derivativedivides	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2(cf-de)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}}{\sqrt{\pi}}\right.\right.\right.}{}$
default	$\frac{-(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+2(cf-de)b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}}{\sqrt{\pi}}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2*(-1/3*(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))+2/3*(c*f-d*e)*b*(-(d*x
+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(
b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^
(1/2)/(d*x+c)^(1/3))))+1/6*f*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))-1/3*f*b*(-1/4
*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d*x+c)^(2/3)*sin(a+b/(d*
x+c)^(2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*sin(a)*Si(b/(d*x+c)^(2/3)
)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 586, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos((
(d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x
 + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sq
rt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I
*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x +
 c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*
sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*c*f/((d*x +
c)^(1/3)*b*d) - 4*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))
*b^2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)
*sqrt((d*x + c)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + ((
(I + 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(e
rf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(
I*b/(d*x + c)^(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3
))) - 1))*sin(a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))*e/
((d*x + c)^(1/3)*b) - (((Ei(I*b/(d*x + c)^(2/3)) + Ei(-I*b/(d*x + c)^(2/3))
)*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I*Ei(-I*b/(d*x + c)^(2/3)))*sin(a))
*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - 2
*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*sin(((d*x + c)^(2/3)*a + b)/(d*x + c
)^(2/3)))*f/d)/d
```

Fricas [A]
time = 0.39, size = 273, normalized size = 0.86

$$\frac{b^3 f \cos(a)\,\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) + b^3 f \cos(a)\,\mathrm{Ci}\left(-\frac{b}{(dx+c)^{\frac{2}{3}}}\right) - 2\,b^3 f \sin(a)\,\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) - 16\,\sqrt{2}\,(\pi bcf - \pi bde)\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right) - 16\,\sqrt{2}\,(\pi bcf - \pi bde)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)\sin(a) + 2\,(bdfx - 7\,bcf + 8\,bde)(dx+c)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right) + 2\left(2\,d^2 f x^2 - (dx+c)^{\frac{2}{3}}b^2 f - 2\,c^2 f + 4\,(d^2 x + cd)e\right)\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*f*cos(a)*cos_integral(b/(d*x + c)^(2/3)) + b^3*f*cos(a)*cos_integr
al(-b/(d*x + c)^(2/3)) - 2*b^3*f*sin(a)*sin_integral(b/(d*x + c)^(2/3)) - 1
6*sqrt(2)*(pi*b*c*f - pi*b*d*e)*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(
b/pi)/(d*x + c)^(1/3)) - 16*sqrt(2)*(pi*b*c*f - pi*b*d*e)*sqrt(b/pi)*fresne
l_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3))*sin(a) + 2*(b*d*f*x - 7*b*c*f + 8
*b*d*e)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)) +
2*(2*d^2*f*x^2 - (d*x + c)^(2/3)*b^2*f - 2*c^2*f + 4*(d^2*x + c*d)*e)*sin((
a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)\sin\left(a + \frac{b}{(c + dx)^{\frac{2}{3}}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)\,\left(e+f\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))*(e + f*x), x)
```

3.224 $\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal. Leaf size=141

$$\frac{2b\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{2b^{3/2}\sqrt{2\pi}\,\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{2b^{3/2}\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d}+\frac{(c+d}{}$$

```
[Out] 2*b*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d
+2*b^(3/2)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*
Pi^(1/2)/d+2*b^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a
)*2^(1/2)*Pi^(1/2)/d
```

Rubi [A]

time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3444, 3490, 3468, 3469, 3434, 3433, 3432}

$$\frac{2\sqrt{2\pi}\,b^{3/2}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{2\sqrt{2\pi}\,b^{3/2}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{2b\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d + (2*b^(3/2)*Sqrt[2*Pi]*
Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d + (2*b^(3/2)*Sqrt[
2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d + ((c + d*x)
*Sin[a + b/(c + d*x)^(2/3)])/d
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3444

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{3\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} - \frac{(2b)\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{(4b^2)\text{Subst}\left(\right.}{}$$

$$= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{(4b^2\cos(a))}{}$$

$$= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d} + \frac{2b^{3/2}\sqrt{2\pi}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d} + \frac{2b^{3/2}\surd}{}$$

Mathematica [A]
time = 0.11, size = 146, normalized size = 1.04

$$\frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2b^{3/2}\sqrt{2\pi}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+2b^{3/2}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)+c\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+dx\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Cos[
a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)
^(2/3)] + d*x*Sin[a + b/(c + d*x)^(2/3)])/d
```

Maple [A]
time = 0.01, size = 105, normalized size = 0.74

method	result

derivativedivides	$\frac{(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)+\sin(a)\right.\right.}{d}$
default	$\frac{(dx+c)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2b\left(-(dx+c)^{\frac{1}{3}}\cos\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,(dx+c)^{\frac{1}{3}}}\right)+\sin(a)\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*(1/3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*x+c)^(1/3)*cos(a+b/(d*x+
c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2
)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.41, size = 219, normalized size = 1.55

$$\frac{\sqrt{2}\left(2\sqrt{2}\,(dx+c)^{\frac{2}{3}}\sqrt{\frac{1}{(dx+c)^{\frac{4}{3}}}}\,b^2\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)+\sqrt{2}\,(dx+c)^{\frac{4}{3}}\sqrt{\frac{1}{(dx+c)^{\frac{4}{3}}}}\,b\sin\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)+\left(\left((i+1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{(dx+c)^{\frac{2}{3}}}}\right)-1\right)-(i-1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{(dx+c)^{\frac{2}{3}}}}\right)-1\right)\right)\cos(a)+\left(-(i-1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{(dx+c)^{\frac{2}{3}}}}\right)-1\right)+(i+1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{(dx+c)^{\frac{2}{3}}}}\right)-1\right)\right)\sin(a)\right)b^2\left(\frac{b^2}{(dx+c)^{\frac{4}{3}}}\right)^{\frac{1}{4}}\sqrt{(dx+c)^{\frac{4}{3}}}\right)}{2\,(dx+c)^{\frac{1}{3}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*(d*x + c)^(2/3)*sqrt((d*x + c)^(-4/3))*b^2*cos(((d*x
 + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + sqrt(2)*(d*x + c)^(4/3)*sqrt((d*x + c
)^(-4/3))*b*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I + 1)*sqrt(p
i)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/(
d*x + c)^(2/3))) - 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^
(2/3))) - 1) + (I + 1)*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(
a))*b^2*(b^2/(d*x + c)^(4/3))^(1/4))*sqrt((d*x + c)^(4/3))/((d*x + c)^(1/3)
*b*d)
```

Fricas [A]
time = 0.38, size = 143, normalized size = 1.01

$$\frac{2\sqrt{2}\,\pi b\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)+2\sqrt{2}\,\pi b\sqrt{\frac{b}{\pi}}\;\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)\sin(a)+2\,(dx+c)^{\frac{1}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)+(dx+c)\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] (2*sqrt(2)*pi*b*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^
(1/3)) + 2*sqrt(2)*pi*b*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)
^(1/3))*sin(a) + 2*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/
(d*x + c)) + (d*x + c)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3)),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3)), x)
```

3.225 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Defer[Int][Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Mathematica [A]
time = 18.73, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
[Out] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x), x)
```

3.226 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2},x\right)$$

[Out] Unintegrable(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx$$

Verification is not applicable to the result.

[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]

[Out] Defer[Int][Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2, x]

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2} dx$$

Mathematica [F]
time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]

[Out] $Aborted

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f^2*x^2 + 2*f*x*
e + e^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e + fx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x)^2,x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(e + f*x)^2, x)
```

3.227 $\int (ce+dex)^{4/3} \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=289

$$\frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}}-\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d}+\frac{90e(c+dx)\sqrt[3]{}}{}$$

```
[Out] 2160*e*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^7/d/(d*x+c)^(1/3)-1080*e*
(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^5/d+90*e*(d*x+c)*(
e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*e*(d*x+c)^(5/3)*(e*(d*x+c))
^(1/3)*cos(a+b*(d*x+c)^(1/3))/b/d+2160*e*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^
(1/3))/b^6/d-360*e*(d*x+c)^(2/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1/3))/b
^4/d+18*e*(d*x+c)^(4/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^2/d
```

Rubi [A]

time = 0.18, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2718}

$$\frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}}+\frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^6d}-\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d}-\frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d}+\frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (2160*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^7*d*(c + d*x)^(1
/3)) - (1080*e*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3
)])/(b^5*d) + (90*e*(c + d*x)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)
])/(b^3*d) - (3*e*(c + d*x)^(5/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(
1/3)])/(b*d) + (2160*e*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^6
*d) - (360*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)]
)/(b^4*d) + (18*e*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(
1/3)])/(b^2*d)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\text{Subst}\left(\int x^2(ex^3)^{4/3}\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^6\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{\left(18e\sqrt[3]{e(c+\right.}{}$$

$$= -\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{18e(c+dx)^{4}}{}$$

$$= \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{}}{}$$

$$= \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{}}{}$$

$$= -\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx}{}$$

$$= -\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx}{}$$

$$= \frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}} - \frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c-}}{}$$

Mathematica [**A**]

time = 0.38, size = 226, normalized size = 0.78

$$\frac{3(e(c+dx))^{4/3}\left(-\cos\left(b\sqrt[3]{c+dx}\right)\left(\left(-720+360b^2(c+dx)^{2/3}-30b^4(c+dx)^{4/3}+b^6(c+dx)^2\right)\cos(a)-6b\left(120\sqrt[3]{c+dx}-20b^2(c+dx)+b^4(c+dx)^{5/3}\right)\sin(a)\right)+\left(6b\left(120\sqrt[3]{c+dx}-20b^2(c+dx)+b^4(c+dx)^{5/3}\right)\cos(a)+\left(-720+360b^2(c+dx)^{2/3}-30b^4(c+dx)^{4/3}+b^6(c+dx)^2\right)\sin(a)\right)\sin\left(b\sqrt[3]{c+dx}\right)\right)}{b^7d(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(e*(c + d*x))^(4/3)*(-(Cos[b*(c + d*x)^(1/3)]*((-720 + 360*b^2*(c + d*x)
^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Cos[a] - 6*b*(120*(c + d
*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Sin[a])) + (6*b*(120*(c
 + d*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Cos[a] + (-720 + 36
0*b^2*(c + d*x)^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Sin[a])*S
in[b*(c + d*x)^(1/3)]))/(b^7*d*(c + d*x)^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+b(dx+c)^{\frac{1}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 195, normalized size = 0.67

$$-\frac{3\left(2\left(b^6d^2x^2e+2\,b^6cdxe+b^6c^2e\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-3\left(e\Gamma\left(6,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+e\Gamma\left(6,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+e\Gamma\left(6,i\,(dx+c)^{\frac{1}{3}}b\right)+e\Gamma\left(6,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\cos(a)-3\left(-i\,e\Gamma\left(6,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,e\Gamma\left(6,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,e\Gamma\left(6,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,e\Gamma\left(6,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\sin(a)\right)e^{\frac{1}{3}}}{2\,b^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/2*(2*(b^6*d^2*x^2*e + 2*b^6*c*d*x*e + b^6*c^2*e)*cos((d*x + c)^(1/3)*b +
 a) - 3*(e*gamma(6, I*b*conjugate((d*x + c)^(1/3))) + e*gamma(6, -I*b*conju
gate((d*x + c)^(1/3))) + e*gamma(6, I*(d*x + c)^(1/3)*b) + e*gamma(6, -I*(d
*x + c)^(1/3)*b))*cos(a) - 3*(-I*e*gamma(6, I*b*conjugate((d*x + c)^(1/3)))
 + I*e*gamma(6, -I*b*conjugate((d*x + c)^(1/3))) - I*e*gamma(6, I*(d*x + c)
^(1/3)*b) + I*e*gamma(6, -I*(d*x + c)^(1/3)*b))*sin(a))*e^(1/3)/(b^7*d)
```

Fricas [A]
time = 0.76, size = 229, normalized size = 0.79

$$-\frac{3\left(\left(\left(b^6d^2x^2+2\,b^6cdx+b^6c^2-720\right)(dx+c)^{\frac{2}{3}}e+360\left(b^2dx+b^2c\right)(dx+c)^{\frac{1}{3}}e-30\left(b^4d^2x^2+2\,b^4cdx+b^4c^2\right)e\right)(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)e^{\frac{1}{3}}+6\left(20\left(b^3dx+b^3c\right)(dx+c)^{\frac{2}{3}}e-\left(b^5d^2x^2+2\,b^5cdx+b^5c^2\right)(dx+c)^{\frac{1}{3}}e-120\left(bdx+bc\right)e\right)(dx+c)^{\frac{1}{3}}e^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^7d^2x+b^7cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")

[Out] -3*(((b^6*d^2*x^2 + 2*b^6*c*d*x + b^6*c^2 - 720)*(d*x + c)^(2/3)*e + 360*(b^2*d*x + b^2*c)*(d*x + c)^(1/3)*e - 30*(b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2)*e)*(d*x + c)^(1/3)*cos((d*x + c)^(1/3)*b + a)*e^(1/3) + 6*(20*(b^3*d*x + b^3*c)*(d*x + c)^(2/3)*e - (b^5*d^2*x^2 + 2*b^5*c*d*x + b^5*c^2)*(d*x + c)^(1/3)*e - 120*(b*d*x + b*c)*e)*(d*x + c)^(1/3)*e^(1/3)*sin((d*x + c)^(1/3)*b + a))/(b^7*d^2*x + b^7*c*d)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(1/3)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(195) = 390.
time = 4.27, size = 566, normalized size = 1.96

$$\frac{3\left(\left(2\left(\frac{\left((dxe+ce)b^3ce^3-6(dxe+ce)^{\frac{1}{3}}bce^{\frac{11}{3}}\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{8}{3}\right)}}{b^4d^2}-\frac{3\left((dxe+ce)^{\frac{2}{3}}b^2ce^{\frac{10}{3}}-2ce^4\right)e^{\left(-\frac{8}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^4d^2}\right)e^{(-1)}-\left(\frac{\left((dxe+ce)^2b^6e^5-30(dxe+ce)^{\frac{4}{3}}b^4e^{\frac{17}{3}}+360(dxe+ce)^{\frac{2}{3}}b^2e^{\frac{19}{3}}-720e^7\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{14}{3}\right)}}{b^7d^2}-\frac{6\left((dxe+ce)^{\frac{5}{3}}b^5e^{\frac{16}{3}}-20(dxe+ce)b^3e^6+120(dxe+ce)^{\frac{1}{3}}be^{\frac{20}{3}}\right)e^{\left(-\frac{14}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^7d^2}\right)e^{(-2)}-\frac{c^2\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{1}{3}}}{bd^2}\right)d^2e-\frac{c^2\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{4}{3}}}{b}+2\left(\frac{c\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{4}{3}}}{b}-\frac{\left((dxe+ce)b^3e^3-6(dxe+ce)^{\frac{1}{3}}be^{\frac{11}{3}}\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{8}{3}\right)}}{b^4}+\frac{3\left((dxe+ce)^{\frac{2}{3}}b^2e^{\frac{10}{3}}-2e^4\right)e^{\left(-\frac{8}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^4}\right)c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")

[Out] 3*((2*(((d*x*e + c*e)*b^3*c*e^3 - 6*(d*x*e + c*e)^(1/3)*b*c*e^(11/3))*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-8/3)/(b^4*d^2) - 3*((d*x*e + c*e)^(2/3)*b^2*c*e^(10/3) - 2*c*e^4)*e^(-8/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/(b^4*d^2))*e^(-1) - (((d*x*e + c*e)^2*b^6*e^5 - 30*(d*x*e + c*e)^(4/3)*b^4*e^(17/3) + 360*(d*x*e + c*e)^(2/3)*b^2*e^(19/3) - 720*e^7)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-14/3)/(b^7*d^2) - 6*((d*x*e + c*e)^(5/3)*b^5*e^(16/3) - 20*(d*x*e + c*e)*b^3*e^6 + 120*(d*x*e + c*e)^(1/3)*b*e^(20/3))*e^(-14/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/(b^7*d^2))*e^(-2) - c^2*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(1/3)/(b*d^2))*d^2*e - c^2*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(4/3)/b + 2*(c*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(4/3)/b - ((d*x*e + c*e)*b^3*e^3 - 6*(d*x*e + c*e)^(1/3)*b*e^(11/3))*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-8/3)/b^4 + 3*((d*x*e + c*e)^(2/3)*b^2*e^(10/3) - 2*e^4)*e^(-8/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^4)*c)/d

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(4/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(4/3), x)
```

3.228 $\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=202

$$\frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}-\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos}{bd}$$

```
[Out] 36*(e*(d*x+c))^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-72*(e*(d*x+c))^(2/3)*cos(
a+b*(d*x+c)^(1/3))/b^5/d/(d*x+c)^(2/3)-3*(d*x+c)^(2/3)*(e*(d*x+c))^(2/3)*co
s(a+b*(d*x+c)^(1/3))/b/d-72*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^4/d/
(d*x+c)^(1/3)+12*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*sin(a+b*(d*x+c)^(1/3))/b^2
/d
```

Rubi [A]
time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2718}

$$-\frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}-\frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{12\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (36*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (72*(e*(c + d
*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d*(c + d*x)^(2/3)) - (3*(c + d*
x)^(2/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (72*(e*(c
+ d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (12*(c
+ d*x)^(1/3)*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\text{Subst}\left(\int x^2\left(ex^3\right)^{2/3}\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$
$$= -\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{(12(e(c+ }{}$$
$$= -\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{12\sqrt[3]{c+dx}}{}$$
$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}(e(c+dx))}{}$$
$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}(e(c+dx))}{}$$
$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{72(e(c+dx))^{2/3}\cos\left(a\right.}{b^5d(c+dx)}$$

Mathematica [A]
time = 0.20, size = 111, normalized size = 0.55

$$-\frac{3(e(c+dx))^{2/3}\left(\left(24-12b^2(c+dx)^{2/3}+b^4(c+dx)^{4/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-4b\left(-6\sqrt[3]{c+dx}+b^2(c+dx)\right)\sin\left(a+b\sqrt[3]{c+dx}\right)\right)}{b^5d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(2/3)*((24 - 12*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(4/3)
)*Cos[a + b*(c + d*x)^(1/3)] - 4*b*(-6*(c + d*x)^(1/3) + b^2*(c + d*x))*Sin
[a + b*(c + d*x)^(1/3)]))/(b^5*d*(c + d*x)^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{2}{3}} \sin\left(a + b(dx + c)^{\frac{1}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.48, size = 191, normalized size = 0.95

$$-\frac{3\left((b^4dx + b^4c)(dx + c)^{\frac{1}{3}}\cos\left((dx + c)^{\frac{1}{3}}b + a\right)e^{\frac{2}{3}} + \left(3\left(\Gamma\left(3, ib\overline{(dx + c)^{\frac{1}{3}}}\right) + \Gamma\left(3, -ib\overline{(dx + c)^{\frac{1}{3}}}\right) + \Gamma\left(3, i(dx + c)^{\frac{1}{3}}b\right) + \Gamma\left(3, -i(dx + c)^{\frac{1}{3}}b\right)\right)\cos(a) - 4(b^3dx + b^3c)\sin\left((dx + c)^{\frac{1}{3}}b + a\right) - 3\left(i\Gamma\left(3, ib\overline{(dx + c)^{\frac{1}{3}}}\right) - i\Gamma\left(3, -ib\overline{(dx + c)^{\frac{1}{3}}}\right) + i\Gamma\left(3, i(dx + c)^{\frac{1}{3}}b\right) - i\Gamma\left(3, -i(dx + c)^{\frac{1}{3}}b\right)\right)\sin(a)\right)e^{\frac{2}{3}}\right)}{b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*((b^4*d*x + b^4*c)*(d*x + c)^(1/3)*cos((d*x + c)^(1/3)*b + a)*e^(2/3) +
(3*(gamma(3, I*b*conjugate((d*x + c)^(1/3))) + gamma(3, -I*b*conjugate((d*x
 + c)^(1/3))) + gamma(3, I*(d*x + c)^(1/3)*b) + gamma(3, -I*(d*x + c)^(1/3)
*b))*cos(a) - 4*(b^3*d*x + b^3*c)*sin((d*x + c)^(1/3)*b + a) - 3*(I*gamma(3
, I*b*conjugate((d*x + c)^(1/3))) - I*gamma(3, -I*b*conjugate((d*x + c)^(1/
3))) + I*gamma(3, I*(d*x + c)^(1/3)*b) - I*gamma(3, -I*(d*x + c)^(1/3)*b))*
sin(a))*e^(2/3))/(b^5*d)
```

Fricas [A]
time = 0.80, size = 141, normalized size = 0.70

$$\frac{3\left(\left(12\,b^2dx + 12\,b^2c - (b^4dx + b^4c)(dx + c)^{\frac{2}{3}} - 24\,(dx + c)^{\frac{1}{3}}\right)(dx + c)^{\frac{2}{3}}\cos\left((dx + c)^{\frac{1}{3}}b + a\right)e^{\frac{2}{3}} - 4\,(dx + c)^{\frac{2}{3}}\left(6\,(dx + c)^{\frac{2}{3}}b - (b^3dx + b^3c)(dx + c)^{\frac{1}{3}}\right)e^{\frac{2}{3}}\sin\left((dx + c)^{\frac{1}{3}}b + a\right)\right)}{b^5d^2x + b^5cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((12*b^2*d*x + 12*b^2*c - (b^4*d*x + b^4*c)*(d*x + c)^(2/3) - 24*(d*x + c
)^(1/3))*(d*x + c)^(2/3)*cos((d*x + c)^(1/3)*b + a)*e^(2/3) - 4*(d*x + c)^(
2/3)*(6*(d*x + c)^(2/3)*b - (b^3*d*x + b^3*c)*(d*x + c)^(1/3))*e^(2/3)*sin(
(d*x + c)^(1/3)*b + a))/(b^5*d^2*x + b^5*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c + dx))^{\frac{2}{3}} \sin\left(a + b\sqrt[3]{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(137) = 274.
time = 3.50, size = 310, normalized size = 1.53

$$-\frac{3\left(\left(\frac{(dxe+ce)^{\frac{1}{3}}\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{1}{3}}}{b}-\frac{e^{\frac{2}{3}}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^{2}}\right)c-\left(\left(\frac{(dxe+ce)^{\frac{1}{3}}c\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{1}{3}}}{b}-\frac{ce^{\frac{2}{3}}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^{2}}\right)e-\frac{\left((dxe+ce)^{\frac{4}{3}}b^{4}e^{\frac{11}{3}}-12(dxe+ce)^{\frac{2}{3}}b^{2}e^{\frac{13}{3}}+24e^{5}\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{10}{3}\right)}}{b^{5}}+\frac{4\left((dxe+ce)b^{3}e^{4}-6(dxe+ce)^{\frac{1}{3}}be^{\frac{14}{3}}\right)e^{\left(-\frac{10}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^{5}}\right)e^{(-1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(((d*x*e + c*e)^(1/3)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*
e^(1/3)/b - e^(2/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^2)*
c - (((d*x*e + c*e)^(1/3)*c*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1
))*e^(1/3)/b - c*e^(2/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/
b^2)*e - ((d*x*e + c*e)^(4/3)*b^4*e^(11/3) - 12*(d*x*e + c*e)^(2/3)*b^2*e^(
13/3) + 24*e^5)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-10/3)
/b^5 + 4*((d*x*e + c*e)*b^3*e^4 - 6*(d*x*e + c*e)^(1/3)*b*e^(14/3))*e^(-10/
3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^5)*e^(-1))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(2/3), x)
```

3.229 $\int \sqrt[3]{ce+dex}\,\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=160

$$\frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{18\sqrt[3]{e(c+dx)}\sin\left(a\right.}{b^4d\sqrt[3]{c+}}$$

```
[Out] 18*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^3/d-3*(d*x+c)^(2/3)*(e*(d*x+c
))^(1/3)*cos(a+b*(d*x+c)^(1/3))/b/d-18*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^(1
/3))/b^4/d/(d*x+c)^(1/3)+9*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*sin(a+b*(d*x+c)^
(1/3))/b^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2717}

$$-\frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (18*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (18*(e*(c + d
*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (9*(c + d*
x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{ce+dex}\,\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{3\mathrm{Subst}\left(\int x^2\sqrt[3]{ex^3}\,\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d}\\ &= \frac{\left(3\sqrt[3]{e(c+dx)}\right)\mathrm{Subst}\left(\int x^3\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}\\ &= -\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{\left(9\sqrt[3]{e(c+dx\right.}}{}\\ &= -\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{9\sqrt[3]{c+dx}\,\sqrt[3]{}}{}\\ &= \frac{18\sqrt[3]{e(c+dx)}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}{bd}\\ &= \frac{18\sqrt[3]{e(c+dx)}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}{bd}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 97, normalized size = 0.61

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\left(-6b\sqrt[3]{c+dx}+b^3(c+dx)\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-3\left(-2+b^2(c+dx)^{2/3}\right)\sin\left(a+b\sqrt[3]{c+dx}\right)\right)}{b^4d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(1/3)*((-6*b*(c + d*x)^(1/3) + b^3*(c + d*x))*Cos[a + b*(
c + d*x)^(1/3)] - 3*(-2 + b^2*(c + d*x)^(2/3))*Sin[a + b*(c + d*x)^(1/3)]))
/(b^4*d*(c + d*x)^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 155, normalized size = 0.97

$$-\frac{3\left(4\left(b^3dx+b^3c\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-3\left(-i\Gamma\left(3,ib\overline{(dx+c)^{\frac{1}{3}}}\right)+i\Gamma\left(3,-ib\overline{(dx+c)^{\frac{1}{3}}}\right)-i\Gamma\left(3,i(dx+c)^{\frac{1}{3}}b\right)+i\Gamma\left(3,-i(dx+c)^{\frac{1}{3}}b\right)\right)\cos(a)+3\left(\Gamma\left(3,ib\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(3,-ib\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(3,i(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(3,-i(dx+c)^{\frac{1}{3}}b\right)\right)\sin(a)\right)e^{\frac{1}{3}}}{4\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/4*(4*(b^3*d*x + b^3*c)*cos((d*x + c)^(1/3)*b + a) - 3*(-I*gamma(3, I*b*c
onjugate((d*x + c)^(1/3))) + I*gamma(3, -I*b*conjugate((d*x + c)^(1/3))) -
I*gamma(3, I*(d*x + c)^(1/3)*b) + I*gamma(3, -I*(d*x + c)^(1/3)*b))*cos(a)
+ 3*(gamma(3, I*b*conjugate((d*x + c)^(1/3))) + gamma(3, -I*b*conjugate((d*
x + c)^(1/3))) + gamma(3, I*(d*x + c)^(1/3)*b) + gamma(3, -I*(d*x + c)^(1/3
)*b))*sin(a))*e^(1/3)/(b^4*d)
```

Fricas [A]
time = 0.74, size = 126, normalized size = 0.79

$$\frac{3\left(\left(6\,bdx+6\,bc-\left(b^3dx+b^3c\right)(dx+c)^{\frac{2}{3}}\right)(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)e^{\frac{1}{3}}+3\,(dx+c)^{\frac{1}{3}}\left(\left(b^2dx+b^2c\right)(dx+c)^{\frac{1}{3}}-2\,(dx+c)^{\frac{2}{3}}\right)e^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^4d^2x+b^4cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((6*b*d*x + 6*b*c - (b^3*d*x + b^3*c)*(d*x + c)^(2/3))*(d*x + c)^(1/3)*co
s((d*x + c)^(1/3)*b + a)*e^(1/3) + 3*(d*x + c)^(1/3)*((b^2*d*x + b^2*c)*(d*
x + c)^(1/3) - 2*(d*x + c)^(2/3))*e^(1/3)*sin((d*x + c)^(1/3)*b + a))/(b^4*
d^2*x + b^4*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[3]{e\,(c+dx)}\,\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [A]
time = 3.70, size = 196, normalized size = 1.22

$$-\frac{3\left(\frac{c\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{1}{3}}}{b}-\left(\frac{c\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{4}{3}}}{b}-\frac{\left((dxe+ce)b^3e^3-6\,(dxe+ce)^{\frac{1}{3}}be^{\frac{11}{3}}\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{8}{3}\right)}}{b^4}+\frac{3\left((dxe+ce)^{\frac{2}{3}}b^2e^{\frac{10}{3}}-2\,e^4\right)e^{\left(-\frac{8}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^4}\right)e^{(-1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(c*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(1/3)/b - (c*cos(
((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(4/3)/b - ((d*x*e + c*e)*b^
3*e^3 - 6*(d*x*e + c*e)^(1/3)*b*e^(11/3))*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3
) + a*e)*e^(-1))*e^(-8/3)/b^4 + 3*((d*x*e + c*e)^(2/3)*b^2*e^(10/3) - 2*e^4
)*e^(-8/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^4)*e^(-1))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3), x)
```

3.230 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=85

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}+\frac{3\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

```
[Out] -3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d/(e*(d*x+c))^(1/3)+3*(d*x+c)^(1/
3)*sin(a+b*(d*x+c)^(1/3))/b^2/d/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2717}

$$\frac{3\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(1/3)) +
 (3*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx &= \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\sqrt[3]{ex^3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\ &= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{e(c+dx)}}\\ &= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}+\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int\cos(a+bx)\,dx,\right.}{bd\sqrt[3]{e(c+dx)}}\\ &= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}+\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 0.82

$$\frac{-3b(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)+3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(c + d*x)^(1/3)*Sin[a
+ b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 128, normalized size = 1.51

$$-\frac{3\left(\left(-i\,\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(2,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(2,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\cos\left(a\right)-\left(\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(2,i\,(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(2,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\sin\left(a\right)\right)e^{\left(-\frac{1}{3}\right)}}{4\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/4*((-I*gamma(2, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(2, -I*b*conjug
ate((d*x + c)^(1/3))) - I*gamma(2, I*(d*x + c)^(1/3)*b) + I*gamma(2, -I*(d*
x + c)^(1/3)*b))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(1/3))) + gamma
(2, -I*b*conjugate((d*x + c)^(1/3))) + gamma(2, I*(d*x + c)^(1/3)*b) + gamm
a(2, -I*(d*x + c)^(1/3)*b))*sin(a))*e^(-1/3)/(b^2*d)
```

Fricas [A]
time = 0.75, size = 66, normalized size = 0.78

$$-\frac{3\left((dx+c)^{\frac{4}{3}}b\cos\left((dx+c)^{\frac{1}{3}}b+a\right)e^{\frac{2}{3}}-(dx+c)e^{\frac{2}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)e^{(-1)}}{b^2d^2x+b^2cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] -3*((d*x + c)^(4/3)*b*cos((d*x + c)^(1/3)*b + a)*e^(2/3) - (d*x + c)*e^(2/3
)*sin((d*x + c)^(1/3)*b + a))*e^(-1)/(b^2*d^2*x + b^2*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{e\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Giac [A]
time = 5.92, size = 83, normalized size = 0.98

$$-\frac{3\left(\frac{(dxe+ce)^{\frac{1}{3}}\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{1}{3}}}{b}-\frac{e^{\frac{2}{3}}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^2}\right)e^{(-1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] -3*((d*x*e + c*e)^(1/3)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e
^(1/3)/b - e^(2/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^2)*e
^(-1)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3), x)
```

3.231 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=42

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

```
[Out] -3*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b/d/(e*(d*x+c))^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3512, 15, 2718}

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx = \frac{3\mathrm{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{2/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$
$$= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

Mathematica [**A**]
time = 0.05, size = 42, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [**A**]
time = 0.32, size = 22, normalized size = 0.52

$$-\frac{3\cos\left((dx+c)^{\frac{1}{3}}b+a\right)e^{\left(-\frac{2}{3}\right)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
[Out] -3*cos((d*x + c)^(1/3)*b + a)*e^(-2/3)/(b*d)
```

Fricas [A]
time = 0.35, size = 34, normalized size = 0.81

$$-\frac{3\,(dx+c)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)e^{\left(-\frac{2}{3}\right)}}{bd^2x+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
[Out] -3*(d*x + c)*cos((d*x + c)^(1/3)*b + a)*e^(-2/3)/(b*d^2*x + b*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(e\,(c+dx)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Giac [A]
time = 6.51, size = 35, normalized size = 0.83

$$-\frac{3\,\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{2}{3}\right)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
[Out] -3*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-2/3)/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sin\left(a+b\,{(c+d\,x)}^{1/3}\right)}{{(c\,e+d\,e\,x)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3),x)
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3), x)
```

3.232 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=120

$$\frac{3b\sqrt[3]{c+dx}\cos(a)\text{Ci}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3b\sqrt[3]{c+dx}\sin(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[Out] 3*b*(d*x+c)^(1/3)*Ci(b*(d*x+c)^(1/3))*cos(a)/d/e/(e*(d*x+c))^(1/3)-3*b*(d*x
+c)^(1/3)*Si(b*(d*x+c)^(1/3))*sin(a)/d/e/(e*(d*x+c))^(1/3)-3*sin(a+b*(d*x+c
)^(1/3))/d/e/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\frac{3b\cos(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3b\sin(a)\sqrt[3]{c+dx}\,\text{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*
b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))
^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx &= \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{4/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}\\
&= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}+\frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}\\
&= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}+\frac{\left(3b\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}\\
&= \frac{3b\sqrt[3]{c+dx}\cos(a)\text{Ci}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3b\sqrt[3]{c+dx}\sin
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 85, normalized size = 0.71

$$-\frac{3\left(-b\sqrt[3]{c+dx}\cos(a)\text{Ci}\left(b\sqrt[3]{c+dx}\right)+\sin\left(a+b\sqrt[3]{c+dx}\right)+b\sqrt[3]{c+dx}\sin(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)]) + Sin[a + b
*(c + d*x)^(1/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)]
))/(d*e*(e*(c + d*x))^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 125, normalized size = 1.04

$$\frac{3\left(\left(\Gamma\left(-1, i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1, -i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1, i\,(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(-1, -i\,(dx+c)^{\frac{1}{3}}b\right)\right)\cos\left(a\right)+\left(-i\,\Gamma\left(-1, i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-1, -i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-1, i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(-1, -i\,(dx+c)^{\frac{1}{3}}b\right)\right)\sin\left(a\right)\right)be^{\left(-\frac{4}{3}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/4*((gamma(-1, I*b*conjugate((d*x + c)^(1/3))) + gamma(-1, -I*b*conjugate(
(d*x + c)^(1/3))) + gamma(-1, I*(d*x + c)^(1/3)*b) + gamma(-1, -I*(d*x + c)
^(1/3)*b))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(1/3))) + I*gamma
(-1, -I*b*conjugate((d*x + c)^(1/3))) - I*gamma(-1, I*(d*x + c)^(1/3)*b) +
I*gamma(-1, -I*(d*x + c)^(1/3)*b))*sin(a))*b*e^(-4/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(2/3)*e^(-4/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*x^2 + 2*c
*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\left(e\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*x*e + c*e)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3), x)
```

3.233 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=175

$$-\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b^2(c+dx)^{2/3}\text{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b^2(c+}{}$$

```
[Out] -3/2*b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b^2*(
d*x+c)^(2/3)*cos(a)*Si(b*(d*x+c)^(1/3))/d/e/(e*(d*x+c))^(2/3)-3/2*b^2*(d*x+
c)^(2/3)*Ci(b*(d*x+c)^(1/3))*sin(a)/d/e/(e*(d*x+c))^(2/3)-3/2*sin(a+b*(d*x+
c)^(1/3))/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A]
time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$-\frac{3b^2\sin(a)(c+dx)^{2/3}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b^2\cos(a)(c+dx)^{2/3}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(2/3
)) - (3*b^2*(c + d*x)^(2/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(2*d*e*(
e*(c + d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(
2/3)) - (3*b^2*(c + d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/(2*d*
e*(e*(c + d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{5/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \sqrt[3]{c+dx}\right)}{de(e(c+dx))^{2/3}}$$

$$= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3b\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{\left(3b^2(c+dx)^{2/\cdots\right.}{}$$

$$= -\frac{3b\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{\left(3b^2(c+dx)^{2/\cdots\right.}{}$$

$$= -\frac{3b\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2(c+dx)^{2/3}\text{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{2de(e(c+dx))^{2/3}} - \cdots$$

Mathematica [A]
time = 0.12, size = 115, normalized size = 0.66

$$-\frac{3\left(b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)+b^2(c+dx)^{2/3}\text{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)+\sin\left(a+b\sqrt[3]{c+dx}\right)+b^2(c+dx)^{2/3}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)\right)}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*(b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x)^(2/3)*Cos
Integral[b*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(1/3)] + b^2*(c +
d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)]))/(2*d*e*(e*(c + d*x))^(2/
3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 128, normalized size = 0.73

$$-\frac{3\left(\left(-i\Gamma\left(-2,ib\overline{(dx+c)^{\frac{1}{3}}}\right)+i\Gamma\left(-2,-ib\overline{(dx+c)^{\frac{1}{3}}}\right)-i\Gamma\left(-2,i(dx+c)^{\frac{1}{3}}b\right)+i\Gamma\left(-2,-i(dx+c)^{\frac{1}{3}}b\right)\right)\cos(a)-\left(\Gamma\left(-2,ib\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-2,-ib\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-2,i(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(-2,-i(dx+c)^{\frac{1}{3}}b\right)\right)\sin(a)\right)b^2e^{\left(-\frac{5}{3}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] -3/4*((-I*gamma(-2, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(-2, -I*b*conj
ugate((d*x + c)^(1/3))) - I*gamma(-2, I*(d*x + c)^(1/3)*b) + I*gamma(-2, -I
*(d*x + c)^(1/3)*b))*cos(a) - (gamma(-2, I*b*conjugate((d*x + c)^(1/3))) +
gamma(-2, -I*b*conjugate((d*x + c)^(1/3))) + gamma(-2, I*(d*x + c)^(1/3)*b)
 + gamma(-2, -I*(d*x + c)^(1/3)*b))*sin(a))*b^2*e^(-5/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)*e^(-5/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*x^2 + 2*c
*d*x + c^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{\left(e\left(c + dx\right)\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(5/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*x*e + c*e)^(5/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3), x)
```

3.234 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal. Leaf size=267

$$\frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{b^4\sqrt[3]{c+dx}\,\mathrm{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{8de^2\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{\right.}}{4de^2(c+dx)\sqrt[3]{}}$$

```
[Out] 1/8*b^3*cos(a+b*(d*x+c)^(1/3))/d/e^2/(e*(d*x+c))^(1/3)-1/4*b*cos(a+b*(d*x+c
)^(1/3))/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)+1/8*b^4*(d*x+c)^(1/3)*cos(a)
*Si(b*(d*x+c)^(1/3))/d/e^2/(e*(d*x+c))^(1/3)+1/8*b^4*(d*x+c)^(1/3)*Ci(b*(d*
x+c)^(1/3))*sin(a)/d/e^2/(e*(d*x+c))^(1/3)-3/4*sin(a+b*(d*x+c)^(1/3))/d/e^2
/(d*x+c)/(e*(d*x+c))^(1/3)+1/8*b^2*sin(a+b*(d*x+c)^(1/3))/d/e^2/(d*x+c)^(1/
3)/(e*(d*x+c))^(1/3)
```

Rubi [A]

time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\frac{b^4\sin(a)\sqrt[3]{c+dx}\,\mathrm{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^4\cos(a)\sqrt[3]{c+dx}\,\mathrm{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}}-\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (b^3*Cos[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(e*(c + d*x))^(1/3)) - (b*Cos[a +
 b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) + (b^4*(
c + d*x)^(1/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(8*d*e^2*(e*(c + d*x)
)^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)*(e*(c + d*x))^
(1/3)) + (b^2*Sin[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(c + d*x)^(1/3)*(e*(c +
d*x))^(1/3)) + (b^4*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/
(8*d*e^2*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{7/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx, x, \sqrt[3]{c+dx}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^4}\,dx, x, \sqrt[3]{c+d\right.}{4de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} - \frac{\left(b^2\sqrt[3]{c+dx}\right)}{}$$

$$= -\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{b^2\sin\left(a+b\right.}{8de^2\sqrt[3]{c+dx}}$$

$$= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+\right.}{4de^2(c+dx)\sqrt[3]{e(c}}$$

$$= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+\right.}{4de^2(c+dx)\sqrt[3]{e(c}}$$

$$= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{b^4\sqrt[3]{c+dx}\,\text{Ci}\left(b\sqrt[3]{\right.}{8de^2\sqrt[3]{e(c}}$$

Mathematica [A]
time = 0.24, size = 184, normalized size = 0.69

$$\frac{b^3c\cos\left(a+b\sqrt[3]{c+dx}\right)+b^3dx\cos\left(a+b\sqrt[3]{c+dx}\right)-2b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)+b^4(c+dx)^{4/3}\text{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)-6\sin\left(a+b\sqrt[3]{c+dx}\right)+b^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)+b^4(c+dx)^{4/3}\cos(a)\text{Si}\left(b\sqrt[3]{c+dx}\right)}{8de(e(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (b^3*c*Cos[a + b*(c + d*x)^(1/3)] + b^3*d*x*Cos[a + b*(c + d*x)^(1/3)] - 2*
b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*CosInteg
ral[b*(c + d*x)^(1/3)]*Sin[a] - 6*Sin[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x
)^(2/3)*Sin[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*Cos[a]*SinIntegral
[b*(c + d*x)^(1/3)])/(8*d*e*(e*(c + d*x))^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 128, normalized size = 0.48

$$\frac{3\left(\left(-i\,\Gamma\left(-4,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-4,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)-i\,\Gamma\left(-4,i\,(dx+c)^{\frac{1}{3}}b\right)+i\,\Gamma\left(-4,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\cos(a)-\left(\Gamma\left(-4,i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-4,-i\,b\overline{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-4,i\,(dx+c)^{\frac{1}{3}}b\right)+\Gamma\left(-4,-i\,(dx+c)^{\frac{1}{3}}b\right)\right)\sin(a)\right)b^4e^{\left(-\frac{7}{3}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] 3/4*((-I*gamma(-4, I*b*conjugate((d*x + c)^(1/3))) + I*gamma(-4, -I*b*conju
gate((d*x + c)^(1/3))) - I*gamma(-4, I*(d*x + c)^(1/3)*b) + I*gamma(-4, -I*
(d*x + c)^(1/3)*b))*cos(a) - (gamma(-4, I*b*conjugate((d*x + c)^(1/3))) + g
amma(-4, -I*b*conjugate((d*x + c)^(1/3))) + gamma(-4, I*(d*x + c)^(1/3)*b)
+ gamma(-4, -I*(d*x + c)^(1/3)*b))*sin(a))*b^4*e^(-7/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(2/3)*e^(-7/3)*sin((d*x + c)^(1/3)*b + a)/(d^3*x^3 + 3*c
*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*x*e + c*e)^(7/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3), x)
```

3.235 $\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=267

$$\frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d}-\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}-\frac{45e\sqrt{\pi}\sqrt[3]{e(c+d}}{8}$$

```
[Out] 45/8*e*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b^3/d-3/2*e*(d*x+c)^(4/3)*(
e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+15/4*e*(d*x+c)^(2/3)*(e*(d*x+c)
)^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d-45/16*e*(e*(d*x+c))^(1/3)*cos(a)*Fresn
elC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)/d/(d*x+c)^(1/3
)*2^(1/2)+45/16*e*(e*(d*x+c))^(1/3)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/
Pi^(1/2))*sin(a)*Pi^(1/2)/b^(7/2)/d/(d*x+c)^(1/3)*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3516, 3498, 3496, 3466, 3467, 3435, 3433, 3432}

$$-\frac{45\sqrt{\pi}\,e\cos(a)\sqrt[3]{e(c+dx)}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{8\sqrt{2}\,b^{7/2}d\sqrt[3]{c+dx}}+\frac{45\sqrt{\pi}\,e\sin(a)\sqrt[3]{e(c+dx)}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}\,b^{7/2}d\sqrt[3]{c+dx}}+\frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d}+\frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d}-\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (45*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(8*b^3*d) - (3*e*(c +
 d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) - (45*e
*Sqrt[Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(8*Sqrt[2]*b^(7/2)*d*(c + d*x)^(1/3)) + (45*e*Sqrt[Pi]*(e*(c + d*x)
)^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2]*b^(
7/2)*d*(c + d*x)^(1/3)) + (15*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a +
 b*(c + d*x)^(2/3)])/(4*b^2*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{\operatorname{Subst}\left(\int (ex)^{4/3}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{\left(e\sqrt[3]{e(c+dx)}\right)\operatorname{Subst}\left(\int x^{4/3}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\operatorname{Subst}\left(\int x^{6}\sin\left(a+bx^{2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{\left(15e\sqrt[3]{e(c\right.}{}$$

$$= -\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{15e(c+dx}{}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+d}{}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+d}{}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+d}{}$$

Mathematica [A]
time = 0.51, size = 175, normalized size = 0.66

$$-\frac{3(e(c+dx))^{4/3}\left(15\sqrt{2\pi}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)-15\sqrt{2\pi}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)+2\sqrt{b}\left(\sqrt[3]{c+dx}\left(-15+4b^2(c+dx)^{4/3}\right)\cos\left(a+b(c+dx)^{2/3}\right)-10b(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)\right)\right)}{16b^{7/2}d(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(4/3)*(15*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(
c + d*x)^(1/3)] - 15*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)
]*Sin[a] + 2*Sqrt[b]*((c + d*x)^(1/3)*(-15 + 4*b^2*(c + d*x)^(4/3))*Cos[a +
 b*(c + d*x)^(2/3)] - 10*b*(c + d*x)*Sin[a + b*(c + d*x)^(2/3)])))/(16*b^(7
/2)*d*(c + d*x)^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{4}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 425, normalized size = 1.59

$$\frac{3\left(\left(\left(ie\Gamma\left(\frac{7}{2},-ib\overline{(dx+c)^{\frac{2}{3}}}\right)-ie\Gamma\left(\frac{7}{2},i(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(-ie\Gamma\left(\frac{7}{2},ib\overline{(dx+c)^{\frac{2}{3}}}\right)+ie\Gamma\left(\frac{7}{2},-i(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)-\left(e\Gamma\left(\frac{7}{2},-ib\overline{(dx+c)^{\frac{2}{3}}}\right)+e\Gamma\left(\frac{7}{2},i(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(e\Gamma\left(\frac{7}{2},ib\overline{(dx+c)^{\frac{2}{3}}}\right)+e\Gamma\left(\frac{7}{2},-i(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)\right)\cos(a)-\left(\left(e\Gamma\left(\frac{7}{2},-ib\overline{(dx+c)^{\frac{2}{3}}}\right)+e\Gamma\left(\frac{7}{2},i(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(e\Gamma\left(\frac{7}{2},ib\overline{(dx+c)^{\frac{2}{3}}}\right)+e\Gamma\left(\frac{7}{2},-i(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)-\left(-ie\Gamma\left(\frac{7}{2},-ib\overline{(dx+c)^{\frac{2}{3}}}\right)+ie\Gamma\left(\frac{7}{2},i(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)-\left(-ie\Gamma\left(\frac{7}{2},ib\overline{(dx+c)^{\frac{2}{3}}}\right)+ie\Gamma\left(\frac{7}{2},-i(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)\right)\sin(a)\right)\sqrt{(dx+c)^{\frac{2}{3}}b}\,e^{\frac{1}{3}}}{8\,(dx+c)^{\frac{1}{3}}b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 3/8*(((I*e*gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) - I*e*gamma(7/2, I*(
d*x + c)^(2/3)*b))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*e*gamma(7/2,
 I*b*conjugate((d*x + c)^(2/3))) + I*e*gamma(7/2, -I*(d*x + c)^(2/3)*b))*co
s(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (e*gamma(7/2, -I*b*conjugate((d*x +
c)^(2/3))) + e*gamma(7/2, I*(d*x + c)^(2/3)*b))*sin(7/4*pi + 7/3*arctan2(0,
 d*x + c)) + (e*gamma(7/2, I*b*conjugate((d*x + c)^(2/3))) + e*gamma(7/2, -
I*(d*x + c)^(2/3)*b))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((e*
gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) + e*gamma(7/2, I*(d*x + c)^(2/3
)*b))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (e*gamma(7/2, I*b*conjugate((
d*x + c)^(2/3))) + e*gamma(7/2, -I*(d*x + c)^(2/3)*b))*cos(-7/4*pi + 7/3*ar
ctan2(0, d*x + c)) - (-I*e*gamma(7/2, -I*b*conjugate((d*x + c)^(2/3))) + I*
e*gamma(7/2, I*(d*x + c)^(2/3)*b))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) -
(-I*e*gamma(7/2, I*b*conjugate((d*x + c)^(2/3))) + I*e*gamma(7/2, -I*(d*x +
 c)^(2/3)*b))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c
)^(2/3)*b)*e^(1/3)/((d*x + c)^(1/3)*b^4*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(4/3)*e^(4/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [C] Result contains complex when optimal does not.
time = 3.49, size = 713, normalized size = 2.67

$$-\frac{3\left(8\left(-\frac{i\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(ia)}}{\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}}+\frac{i\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(-ia)}}{\sqrt{ibe^{\left(-\frac{2}{3}\right)}}}\right)c^{2}e+\left(-\frac{8i\sqrt{\pi}c^{2}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(ia)}}{\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}d^{2}}+\frac{8i\sqrt{\pi}c^{2}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(-ia)}}{\sqrt{ibe^{\left(-\frac{2}{3}\right)}}d^{2}}+\frac{-\frac{2i\left(4i(dxe+ce)^{\frac{5}{3}}b^{2}e^{\left(-\frac{4}{3}\right)}-10(dxe+ce)be^{\left(-\frac{2}{3}\right)}-15i(dxe+ce)^{\frac{1}{3}}\right)e^{\left(i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+ia\right)}}{b^{3}}-\frac{15\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(ia)}}{\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}b^{3}}}{d^{2}}+\frac{-\frac{2i\left(4i(dxe+ce)^{\frac{5}{3}}b^{2}e^{\left(-\frac{4}{3}\right)}+10(dxe+ce)be^{\left(-\frac{2}{3}\right)}-15i(dxe+ce)^{\frac{1}{3}}\right)e^{\left(-i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-ia\right)}}{b^{3}}-\frac{15\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(-ia)}}{\sqrt{ibe^{\left(-\frac{2}{3}\right)}}b^{3}}}{d^{2}}-\frac{16i\left(-i(dxe+ce)^{\frac{2}{3}}bce^{\left(-\frac{2}{3}\right)}+c\right)e^{\left(i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+ia+\frac{1}{3}\right)}}{b^{2}d^{2}}-\frac{16i\left(-i(dxe+ce)^{\frac{2}{3}}bce^{\left(-\frac{2}{3}\right)}-c\right)e^{\left(-i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-ia+\frac{1}{3}\right)}}{b^{2}d^{2}}\right)d^{2}e+16\left(\frac{i\sqrt{\pi}c\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(ia+1)}}{\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}}-\frac{i\sqrt{\pi}c\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(-ia+1)}}{\sqrt{ibe^{\left(-\frac{2}{3}\right)}}}-\frac{i\left(i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-1\right)e^{\left(i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+ia+\frac{4}{3}\right)}}{b^{2}}-\frac{i\left(i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+1\right)e^{\left(-i(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-ia+\frac{4}{3}\right)}}{b^{2}}\right)c\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/32*(8*(-I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a)
/sqrt(-I*b*e^(-2/3)) + I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3
)))*e^(-I*a)/sqrt(I*b*e^(-2/3)))*c^2*e + (-8*I*sqrt(pi)*c^2*erf(-(d*x*e + c
*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a)/(sqrt(-I*b*e^(-2/3))*d^2) + 8*I*sqrt
(pi)*c^2*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))*e^(-I*a)/(sqrt(I*b*e^
(-2/3))*d^2) + (-2*I*(4*I*(d*x*e + c*e)^(5/3)*b^2*e^(-4/3) - 10*(d*x*e + c*
e)*b*e^(-2/3) - 15*I*(d*x*e + c*e)^(1/3))*e^(I*(d*x*e + c*e)^(2/3)*b*e^(-2/
3) + I*a)/b^3 - 15*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e
^(I*a)/(sqrt(-I*b*e^(-2/3))*b^3))/d^2 + (-2*I*(4*I*(d*x*e + c*e)^(5/3)*b^2*
e^(-4/3) + 10*(d*x*e + c*e)*b*e^(-2/3) - 15*I*(d*x*e + c*e)^(1/3))*e^(-I*(d
*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a)/b^3 - 15*sqrt(pi)*erf(-(d*x*e + c*e)^(1
/3)*sqrt(I*b*e^(-2/3)))*e^(-I*a)/(sqrt(I*b*e^(-2/3))*b^3))/d^2 - 16*I*(-I*(
d*x*e + c*e)^(2/3)*b*c*e^(-2/3) + c)*e^(I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) +
I*a + 1/3)/(b^2*d^2) - 16*I*(-I*(d*x*e + c*e)^(2/3)*b*c*e^(-2/3) - c)*e^(-I
*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a + 1/3)/(b^2*d^2))*d^2*e + 16*(I*sqrt(
pi)*c*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a + 1)/sqrt(-I*b*e
^(-2/3)) - I*sqrt(pi)*c*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))*e^(-I*
a + 1)/sqrt(I*b*e^(-2/3)) - I*(I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - 1)*e^(I*(
d*x*e + c*e)^(2/3)*b*e^(-2/3) + I*a + 4/3)/b^2 - I*(I*(d*x*e + c*e)^(2/3)*b
*e^(-2/3) + 1)*e^(-I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a + 4/3)/b^2)*c)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3), x)
```

3.236 $\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=227

$$-\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}-\frac{9\sqrt{\pi}\,(e(c+dx))^{2/3}\cos(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{4\sqrt{2}\,b^{5/2}d(c+dx)^{2/3}}-\frac{9\sqrt{}}{}$$

```
[Out] -3/2*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*cos(a+b*(d*x+c)^(2/3))/b/d+9/4*(e*(d*x
+c))^(2/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d/(d*x+c)^(1/3)-9/8*(e*(d*x+c))^(2/3)
*cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)/d
/(d*x+c)^(2/3)*2^(1/2)-9/8*(e*(d*x+c))^(2/3)*FresnelC((d*x+c)^(1/3)*b^(1/2)
*2^(1/2)/Pi^(1/2))*sin(a)*Pi^(1/2)/b^(5/2)/d/(d*x+c)^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.296$, Rules used = {3516, 3498, 3496, 3466, 3467, 3434, 3433, 3432}

$$-\frac{9\sqrt{\pi}\,\sin(a)(e(c+dx))^{2/3}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt[3]{c+dx}\right)}{4\sqrt{2}\,b^{5/2}d(c+dx)^{2/3}}-\frac{9\sqrt{\pi}\,\cos(a)(e(c+dx))^{2/3}S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{4\sqrt{2}\,b^{5/2}d(c+dx)^{2/3}}+\frac{9(e(c+dx))^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)}{4b^2d\sqrt[3]{c+dx}}-\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 - (9*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)^(1/3)])/(4*Sqrt[2]*b^(5/2)*d*(c + d*x)^(2/3)) - (9*Sqrt[Pi]*(e*(c + d*
x))^(2/3)*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(4*Sqrt[2]*b
^(5/2)*d*(c + d*x)^(2/3)) + (9*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(2/3
)])/(4*b^2*d*(c + d*x)^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx = \frac{\operatorname{Subst}\left(\int (ex)^{2/3}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{2/3}\operatorname{Subst}\left(\int x^{2/3}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d(c+dx)^{2/3}}$$

$$= \frac{\left(3(e(c+dx))^{2/3}\right)\operatorname{Subst}\left(\int x^4\sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{(9(e(c+d \ldots}{}$$

$$= -\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{9(e(c+dx \ldots}{}$$

$$= -\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{9(e(c+dx \ldots}{}$$

$$= -\frac{3\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} - \frac{9\sqrt{\pi}\,(e(c- \ldots}{}$$

Mathematica [A]
time = 0.31, size = 160, normalized size = 0.70

$$-\frac{3(e(c+dx))^{2/3}\left(3\sqrt{2\pi}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+3\sqrt{2\pi}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)+2\sqrt{b}\left(2b(c+dx)\cos\left(a+b(c+dx)^{2/3}\right)-3\sqrt[3]{c+dx}\sin\left(a+b(c+dx)^{2/3}\right)\right)\right)}{8b^{5/2}d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(2/3)*(3*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)] + 3*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*
Sin[a] + 2*Sqrt[b]*(2*b*(c + d*x)*Cos[a + b*(c + d*x)^(2/3)] - 3*(c + d*x)^
(1/3)*Sin[a + b*(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(c + d*x)^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{2}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 422, normalized size = 1.86

$$-\frac{3\left(3\,(dx+c)^{\frac{2}{3}}\left(\left(\left(\Gamma\left(\frac{3}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(\frac{3}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(\frac{3}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(\frac{3}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(-\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)+\left(i\,\Gamma\left(\frac{3}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(\frac{3}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)+\left(i\,\Gamma\left(\frac{3}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(\frac{3}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(-\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)\right)\cos\left(a\right)+\left(\left(i\,\Gamma\left(\frac{3}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(\frac{3}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)+\left(-i\,\Gamma\left(\frac{3}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(\frac{3}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(-\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)-\left(\Gamma\left(\frac{3}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(\frac{3}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(\frac{3}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(\frac{3}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(-\frac{3}{4}\pi+\arctan\left(0,dx+c\right)\right)\right)\sin\left(a\right)\right)\sqrt{(dx+c)^{\frac{2}{3}}b}\,e^{\frac{2}{3}}+8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left((dx+c)^{\frac{2}{3}}b+a\right)e^{\frac{2}{3}}\right)}{16\left(b^3d^2x+b^3cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*(3*(d*x + c)^(2/3)*(((gamma(3/2, -I*b*conjugate((d*x + c)^(2/3))) + g
amma(3/2, I*(d*x + c)^(2/3)*b))*cos(3/4*pi + arctan2(0, d*x + c)) + (gamma(
3/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(3/2, -I*(d*x + c)^(2/3)*b))*co
s(-3/4*pi + arctan2(0, d*x + c)) + (I*gamma(3/2, -I*b*conjugate((d*x + c)^(
2/3))) - I*gamma(3/2, I*(d*x + c)^(2/3)*b))*sin(3/4*pi + arctan2(0, d*x + c
)) + (I*gamma(3/2, I*b*conjugate((d*x + c)^(2/3))) - I*gamma(3/2, -I*(d*x +
 c)^(2/3)*b))*sin(-3/4*pi + arctan2(0, d*x + c)))*cos(a) + ((I*gamma(3/2, -
I*b*conjugate((d*x + c)^(2/3))) - I*gamma(3/2, I*(d*x + c)^(2/3)*b))*cos(3/
4*pi + arctan2(0, d*x + c)) + (-I*gamma(3/2, I*b*conjugate((d*x + c)^(2/3))
) + I*gamma(3/2, -I*(d*x + c)^(2/3)*b))*cos(-3/4*pi + arctan2(0, d*x + c))
- (gamma(3/2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(3/2, I*(d*x + c)^(2/
3)*b))*sin(3/4*pi + arctan2(0, d*x + c)) + (gamma(3/2, I*b*conjugate((d*x +
 c)^(2/3))) + gamma(3/2, -I*(d*x + c)^(2/3)*b))*sin(-3/4*pi + arctan2(0, d*
x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(2/3) + 8*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos((d*x + c)^(2/3)*b + a)*e^(2/3))/(b^3*d^2*x + b^3*c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(2/3)*e^(2/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{2}{3}}\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 5.87, size = 321, normalized size = 1.41

$$-\frac{3\left(\frac{4\,c\left(\cos\left(\left(dxe+ce\right)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+a\right)+\cos\left(-\left(dxe+ce\right)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-a\right)\right)e^{\frac{2}{3}}}{b}-\left(\frac{4\,ce^{\left(i\left(dxe+ce\right)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+i\,a+\frac{5}{3}\right)}}{b}+\frac{4\,ce^{\left(-i\left(dxe+ce\right)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-i\,a+\frac{5}{3}\right)}}{b}+\frac{2i\left(2i\left(dxe+ce\right)be^{\left(-\frac{2}{3}\right)}-3\left(dxe+ce\right)^{\frac{1}{3}}\right)e^{\left(i\left(dxe+ce\right)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+i\,a+\frac{4}{3}\right)}}{b^{2}}+\frac{2i\left(2i\left(dxe+ce\right)be^{\left(-\frac{2}{3}\right)}+3\left(dxe+ce\right)^{\frac{1}{3}}\right)e^{\left(-i\left(dxe+ce\right)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-i\,a+\frac{4}{3}\right)}}{b^{2}}-\frac{3i\sqrt{\pi}\operatorname{erf}\left(-\left(dxe+ce\right)^{\frac{1}{3}}\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{\left(i\,a+\frac{4}{3}\right)}}{\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}\,b^{2}}+\frac{3i\sqrt{\pi}\operatorname{erf}\left(-\left(dxe+ce\right)^{\frac{1}{3}}\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{\left(-i\,a+\frac{4}{3}\right)}}{\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}\,b^{2}}\right)e^{\left(-1\right)}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/16*(4*c*(cos((d*x*e + c*e)^(2/3)*b*e^(-2/3) + a) + cos(-(d*x*e + c*e)^(2
/3)*b*e^(-2/3) - a))*e^(2/3)/b - (4*c*e^(I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) +
 I*a + 5/3)/b + 4*c*e^(-I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a + 5/3)/b + 2
*I*(2*I*(d*x*e + c*e)*b*e^(-2/3) - 3*(d*x*e + c*e)^(1/3))*e^(I*(d*x*e + c*e
)^(2/3)*b*e^(-2/3) + I*a + 4/3)/b^2 + 2*I*(2*I*(d*x*e + c*e)*b*e^(-2/3) + 3
*(d*x*e + c*e)^(1/3))*e^(-I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a + 4/3)/b^2
 - 3*I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a + 4/3)
/(sqrt(-I*b*e^(-2/3))*b^2) + 3*I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b
*e^(-2/3)))*e^(-I*a + 4/3)/(sqrt(I*b*e^(-2/3))*b^2))*e^(-1))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3), x)
```

3.237 $\int \sqrt[3]{ce+dex} \sin\left(a+b(c+dx)^{2/3}\right) dx$

Optimal. Leaf size=89

$$-\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d+3/2*(e*(d*x
+c))^(1/3)*sin(a+b*(d*x+c)^(2/3))/b^2/d/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3516, 3462, 3460, 3377, 2717}

$$\frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}-\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 + (3*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(2/3)])/(2*b^2*d*(c + d*x)^(1
/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \sqrt[3]{ce+dex}\,\sin\left(a+b(c+dx)^{2/3}\right)\,dx=\frac{\text{Subst}\left(\int \sqrt[3]{ex}\,\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{\sqrt[3]{e(c+dx)}\,\text{Subst}\left(\int \sqrt[3]{x}\,\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$=\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,(c+dx)^{2/3}\right)}{2d\sqrt[3]{c+dx}}$$

$$=-\frac{3\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{\left(3\sqrt[3]{e(c+dx\right.}}{}$$

$$=-\frac{3\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt[3]{e(c+dx)}}{2b}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.81

$$-\frac{3\sqrt[3]{e(c+dx)}\left(b(c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)-\sin\left(a+b(c+dx)^{2/3}\right)\right)}{2b^2d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(2/3)] - Sin
[a + b*(c + d*x)^(2/3)]))/(2*b^2*d*(c + d*x)^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 128, normalized size = 1.44

$$-\frac{3\left(\left(-i\,\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(2,i\,(dx+c)^{\frac{2}{3}}b\right)+i\,\Gamma\left(2,-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(a\right)-\left(\Gamma\left(2,i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(2,-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(2,i\,(dx+c)^{\frac{2}{3}}b\right)+\Gamma\left(2,-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(a\right)\right)e^{\frac{1}{3}}}{8\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/8*((-I*gamma(2, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(2, -I*b*conjug
ate((d*x + c)^(2/3))) - I*gamma(2, I*(d*x + c)^(2/3)*b) + I*gamma(2, -I*(d*
x + c)^(2/3)*b))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(2/3))) + gamma
(2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(2, I*(d*x + c)^(2/3)*b) + gamm
a(2, -I*(d*x + c)^(2/3)*b))*sin(a))*e^(1/3)/(b^2*d)
```

Fricas [A]
time = 0.75, size = 71, normalized size = 0.80

$$-\frac{3\left((bdx+bc)(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)e^{\frac{1}{3}}-(dx+c)e^{\frac{1}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)\right)}{2\left(b^2d^2x+b^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] -3/2*((b*d*x + b*c)*(d*x + c)^(2/3)*cos((d*x + c)^(2/3)*b + a)*e^(1/3) - (d
*x + c)*e^(1/3)*sin((d*x + c)^(2/3)*b + a))/(b^2*d^2*x + b^2*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[3]{e\left(c+dx\right)}\,\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 6.94, size = 265, normalized size = 2.98

$$-\frac{3\left(\left(\left(-\frac{i\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{(i\,a)}}{\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}}+\frac{i\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{(-i\,a)}}{\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}}\right)c+\left(\frac{i\sqrt{\pi}\,c\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{(i\,a+1)}}{\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}}-\frac{i\sqrt{\pi}\,c\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{(-i\,a+1)}}{\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}}-\frac{i\left(i\,(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-1\right)e^{\left(i\,(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+i\,a+\frac{4}{3}\right)}}{b^2}-\frac{i\left(i\,(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+1\right)e^{\left(-i\,(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-i\,a+\frac{4}{3}\right)}}{b^2}\right)e^{(-1)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/4*((-I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a)/sq
rt(-I*b*e^(-2/3)) + I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))
*e^(-I*a)/sqrt(I*b*e^(-2/3)))*c + (I*sqrt(pi)*c*erf(-(d*x*e + c*e)^(1/3)*sq
rt(-I*b*e^(-2/3)))*e^(I*a + 1)/sqrt(-I*b*e^(-2/3)) - I*sqrt(pi)*c*erf(-(d*x
*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))*e^(-I*a + 1)/sqrt(I*b*e^(-2/3)) - I*(I*
(d*x*e + c*e)^(2/3)*b*e^(-2/3) - 1)*e^(I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) + I
*a + 4/3)/b^2 - I*(I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) + 1)*e^(-I*(d*x*e + c*e
)^(2/3)*b*e^(-2/3) - I*a + 4/3)/b^2)*e^(-1))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin \left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)\,{\left(c\,e+d\,e\,x\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3), x)
```

3.238 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=44

$$-\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))/b/d/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3516, 3462, 3460, 2718}

$$-\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{\sqrt[3]{ex}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\operatorname{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{\sqrt[3]{x}}\,dx, x, c+dx\right)}{d\sqrt[3]{e(c+dx)}}\\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\operatorname{Subst}\left(\int \sin(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&= -\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 1.00

$$-\frac{3\sqrt[3]{c+dx}\,\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [A]
time = 0.30, size = 22, normalized size = 0.50

$$-\frac{3\cos\left((dx+c)^{\frac{2}{3}}b+a\right)e^{\left(-\frac{1}{3}\right)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/2*cos((d*x + c)^(2/3)*b + a)*e^(-1/3)/(b*d)
```

Fricas [A]
time = 0.36, size = 34, normalized size = 0.77

$$-\frac{3\,(dx+c)\cos\left((dx+c)^{\frac{2}{3}}b+a\right)e^{\left(-\frac{1}{3}\right)}}{2\,(bd^2x+bcd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(d*x + c)*cos((d*x + c)^(2/3)*b + a)*e^(-1/3)/(b*d^2*x + b*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(22) = 44.
time = 6.31, size = 52, normalized size = 1.18

$$-\frac{3\left(\cos\left((dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+a\right)+\cos\left(-(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-a\right)\right)e^{\left(-\frac{1}{3}\right)}}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] -3/4*(cos((d*x*e + c*e)^(2/3)*b*e^(-2/3) + a) + cos(-(d*x*e + c*e)^(2/3)*b*
e^(-2/3) - a))*e^(-1/3)/(b*d)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3), x)
```

3.239 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=133

$$\frac{3\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}\cos(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{\sqrt{b}\,d(e(c+dx))^{2/3}}+\frac{3\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)\sin(a)}{\sqrt{b}\,d(e(c+dx))^{2/3}}$$

```
[Out] 3/2*(d*x+c)^(2/3)*cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*2
^(1/2)*Pi^(1/2)/d/(e*(d*x+c))^(2/3)/b^(1/2)+3/2*(d*x+c)^(2/3)*FresnelC((d*x
+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/d/(e*(d*x+c))^(
2/3)/b^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3516, 3498, 3464, 3434, 3433, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\,\sin(a)(c+dx)^{2/3}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt[3]{c+dx}\right)}{\sqrt{b}\,d(e(c+dx))^{2/3}}+\frac{3\sqrt{\frac{\pi}{2}}\,\cos(a)(c+dx)^{2/3}S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{\sqrt{b}\,d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(Sqrt[b]*d*(e*(c + d*x))^(2/3)) + (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(Sqrt[b]*d*(e*(c + d*x))^
(2/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3464

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{(ex)^{2/3}}\,dx,x,c+dx\right)}{d} \\
&= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{x^{2/3}}\,dx,x,c+dx\right)}{d(e(c+dx))^{2/3}} \\
&= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}} \\
&= \frac{\left(3(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}} + \frac{\left(3(c+dx)^{2/3}\sin\right.}{} \\
&= \frac{3\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}\cos(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{\sqrt{b}\,d(e(c+dx))^{2/3}} + \frac{3\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}C\left(\sqrt{b}\right.}{\sqrt{b}\,d(e(c}
\end{aligned}
$$

Mathematica [A]

time = 0.10, size = 96, normalized size = 0.72

$$\frac{3\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}\left(\cos(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)+C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)\sin(a)\right)}{\sqrt{b}\,d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)
^(1/3)] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[b]*d*
(e*(c + d*x))^(2/3))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 486, normalized size = 3.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(((-I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1) + I*sqr
t(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*cos(1/4*pi + 1/3*arctan2(0, d*x
 + c)) + (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) - I*sq
rt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*cos(-1/4*pi + 1/3*arctan2(0,
d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1) + sq
rt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*sin(1/4*pi + 1/3*arctan2(0, d*
x + c)) - (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) + sqrt(
pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*sin(-1/4*pi + 1/3*arctan2(0, d*x
 + c)))*cos(a) + ((sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1
) + sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*cos(1/4*pi + 1/3*arctan2
(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1) +
 sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*cos(-1/4*pi + 1/3*arctan2(
```

```
0, d*x + c)) + (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(2/3)))) - 1)
 - I*sqrt(pi)*(erf(sqrt(I*(d*x + c)^(2/3)*b)) - 1))*sin(1/4*pi + 1/3*arctan
2(0, d*x + c)) + (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(2/3)))) - 1
) - I*sqrt(pi)*(erf(sqrt(-I*(d*x + c)^(2/3)*b)) - 1))*sin(-1/4*pi + 1/3*arc
tan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(-2/3)/((d*x + c)^(1/3
)*b*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
[Out] integral(e^(-2/3)*sin((d*x + c)^(2/3)*b + a)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b(c + dx)^{\frac{2}{3}}\right)}{\left(e\left(c + dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Giac [C] Result contains complex when optimal does not.
time = 6.30, size = 84, normalized size = 0.63

$$-\frac{3\left(-\frac{i\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{(i\,a)}}{\sqrt{-i\,be^{\left(-\frac{2}{3}\right)}}} + \frac{i\sqrt{\pi}\operatorname{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}\right)e^{(-i\,a)}}{\sqrt{i\,be^{\left(-\frac{2}{3}\right)}}}\right)e^{(-1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
[Out] -3/4*(-I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a)/sqr
t(-I*b*e^(-2/3)) + I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))*
e^(-I*a)/sqrt(I*b*e^(-2/3)))*e^(-1)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3), x)
```

3.240 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=168

$$\frac{3\sqrt{b}\,\sqrt{2\pi}\,\sqrt[3]{c+dx}\,\cos(a)C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{b}\,\sqrt{2\pi}\,\sqrt[3]{c+dx}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)\sin(a)}{de\sqrt[3]{e(c+dx)}}-$$

```
[Out] -3*sin(a+b*(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(1/3)+3*(d*x+c)^(1/3)*cos(a)*Fres
nelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/e/(
e*(d*x+c))^(1/3)-3*(d*x+c)^(1/3)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^
(1/2))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d/e/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3516, 3498, 3496, 3468, 3435, 3433, 3432}

$$\frac{3\sqrt{2\pi}\,\sqrt{b}\,\cos(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{2\pi}\,\sqrt{b}\,\sin(a)\sqrt[3]{c+dx}\,S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)
^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(d*e*(e*(c + d*x))^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{(ex)^{4/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{x^{4/3}}\,dx, x, c+dx\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(6b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \sqrt[3]{c\right.}}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(6b\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[3]{\right.}}{de\sqrt[3]{e(c+dx)}}$$

$$= \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\,S}{de\sqrt[3]{}}$$

Mathematica [A]
time = 0.17, size = 133, normalized size = 0.79

$$-\frac{3\left(-\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)+\sin\left(a+b(c+dx)^{2/3}\right)\right)}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi
]*(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*FresnelS[Sqrt[b]*S
qrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(2/3)]))/(d*e*(e*(c
 + d*x))^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 379, normalized size = 2.26

$$-\frac{3\left(\left(\left(-i\,\Gamma\left(-\frac{1}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-\frac{1}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(i\,\Gamma\left(-\frac{1}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(-\frac{1}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)-\left(\Gamma\left(-\frac{1}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-\frac{1}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(-\frac{1}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-\frac{1}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)\right)\cos\left(a\right)+\left(\left(\Gamma\left(-\frac{1}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-\frac{1}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(-\frac{1}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-\frac{1}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(-i\,\Gamma\left(-\frac{1}{2},-i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-\frac{1}{2},i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(-i\,\Gamma\left(-\frac{1}{2},i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-\frac{1}{2},-i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)\right)\sin\left(a\right)\right)\sqrt{(dx+c)^{\frac{2}{3}}b}\,e^{\left(-\frac{4}{3}\right)}}{8\,(dx+c)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/8*(((-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2, I*(
d*x + c)^(2/3)*b))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (I*gamma(-1/2, I
*b*conjugate((d*x + c)^(2/3))) - I*gamma(-1/2, -I*(d*x + c)^(2/3)*b))*cos(-
1/4*pi + 1/3*arctan2(0, d*x + c)) - (gamma(-1/2, -I*b*conjugate((d*x + c)^(
2/3))) + gamma(-1/2, I*(d*x + c)^(2/3)*b))*sin(1/4*pi + 1/3*arctan2(0, d*x
+ c)) + (gamma(-1/2, I*b*conjugate((d*x + c)^(2/3))) + gamma(-1/2, -I*(d*x
+ c)^(2/3)*b))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*cos(a) + ((gamma(-1/
2, -I*b*conjugate((d*x + c)^(2/3))) + gamma(-1/2, I*(d*x + c)^(2/3)*b))*cos
(1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, I*b*conjugate((d*x + c)^(
2/3))) + gamma(-1/2, -I*(d*x + c)^(2/3)*b))*cos(-1/4*pi + 1/3*arctan2(0, d*
x + c)) + (-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2,
I*(d*x + c)^(2/3)*b))*sin(1/4*pi + 1/3*arctan2(0, d*x + c)) + (-I*gamma(-1/
2, I*b*conjugate((d*x + c)^(2/3))) + I*gamma(-1/2, -I*(d*x + c)^(2/3)*b))*s
in(-1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))*sqrt((d*x + c)^(2/3)*b)*e^(-
4/3)/((d*x + c)^(1/3)*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(2/3)*e^(-4/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*x^2 + 2*c
*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{\left(e\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(d*x*e + c*e)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,{\left(c+d\,x\right)}^{2/3}\right)}{{\left(c\,e+d\,e\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3), x)
```

3.241 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=126

$$\frac{3b(c+dx)^{2/3}\cos(a)\text{Ci}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

```
[Out] 3/2*b*(d*x+c)^(2/3)*Ci(b*(d*x+c)^(2/3))*cos(a)/d/e/(e*(d*x+c))^(2/3)-3/2*b*
(d*x+c)^(2/3)*Si(b*(d*x+c)^(2/3))*sin(a)/d/e/(e*(d*x+c))^(2/3)-3/2*sin(a+b*
(d*x+c)^(2/3))/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3516, 3462, 3460, 3378, 3384, 3380, 3383}

$$\frac{3b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(2*d*e*(e*(c + d*x))^(2/3)) -
 (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
 d*x))^(2/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{(ex)^{5/3}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin(a+bx^{2/3})}{x^{5/3}}\,dx,x,c+dx\right)}{de(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,(c+dx)^{2/}\right.}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,(c+\right.}{2de(e(c+dx))^{2/3}}$$

$$= \frac{3b(c+dx)^{2/3}\cos(a)\text{Ci}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b(c+dx)^{2}}{2c}$$

Mathematica [A]
time = 0.11, size = 87, normalized size = 0.69

$$-\frac{3\left(-b(c+dx)^{2/3}\cos(a)\text{Ci}\left(b(c+dx)^{2/3}\right)+\sin\left(a+b(c+dx)^{2/3}\right)+b(c+dx)^{2/3}\sin(a)\text{Si}\left(b(c+dx)^{2/3}\right)\right)}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)]) + Sin[a + b
*(c + d*x)^(2/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)]
))/(2*d*e*(e*(c + d*x))^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^{\frac{2}{3}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 125, normalized size = 0.99

$$\frac{3\left(\left(\Gamma\left(-1, i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1, -i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1, i\,(dx+c)^{\frac{2}{3}}b\right)+\Gamma\left(-1, -i\,(dx+c)^{\frac{2}{3}}b\right)\right)\cos(a)+\left(-i\,\Gamma\left(-1, i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-1, -i\,b\overline{(dx+c)^{\frac{2}{3}}}\right)-i\,\Gamma\left(-1, i\,(dx+c)^{\frac{2}{3}}b\right)+i\,\Gamma\left(-1, -i\,(dx+c)^{\frac{2}{3}}b\right)\right)\sin(a)\right)be^{\left(-\frac{5}{3}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] 3/8*((gamma(-1, I*b*conjugate((d*x + c)^(2/3))) + gamma(-1, -I*b*conjugate(
(d*x + c)^(2/3))) + gamma(-1, I*(d*x + c)^(2/3)*b) + gamma(-1, -I*(d*x + c)
^(2/3)*b))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(2/3))) + I*gamma
(-1, -I*b*conjugate((d*x + c)^(2/3))) - I*gamma(-1, I*(d*x + c)^(2/3)*b) +
I*gamma(-1, -I*(d*x + c)^(2/3)*b))*sin(a))*b*e^(-5/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)*e^(-5/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*x^2 + 2*c
*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b(c + dx)^{\frac{2}{3}}\right)}{\left(e\left(c + dx\right)\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(d*x*e + c*e)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin \left(a+b\,(c+d\,x)^{2/3}\right)}{(c\,e+d\,e\,x)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3),x)
[Out] int(sin(a + b*(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3), x)
```

3.242 $\int \sqrt[3]{ce+dex} \sin \left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx$

Optimal. Leaf size=247

$$-\frac{b^3 \sqrt[3]{e(c+dx)} \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}+\frac{b(c+dx)^{2/3} \sqrt[3]{e(c+dx)} \cos \left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d}-\frac{b^4 \sqrt[3]{e(c+dx)} \operatorname{Ci}(}{8d\sqrt[3]{c}}$$

```
[Out] -1/8*b^3*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(1/3))/d+1/4*b*(d*x+c)^(2/3)*(e*
(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(1/3))/d-1/8*b^4*(e*(d*x+c))^(1/3)*cos(a)*Si
(b/(d*x+c)^(1/3))/d/(d*x+c)^(1/3)-1/8*b^4*(e*(d*x+c))^(1/3)*Ci(b/(d*x+c)^(1
/3))*sin(a)/d/(d*x+c)^(1/3)-1/8*b^2*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*sin(a+b
/(d*x+c)^(1/3))/d+3/4*(d*x+c)*(e*(d*x+c))^(1/3)*sin(a+b/(d*x+c)^(1/3))/d
```

Rubi [A]
time = 0.16, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$-\frac{b^4 \sin(a) \sqrt[3]{e(c+dx)} \operatorname{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}-\frac{b^4 \cos(a) \sqrt[3]{e(c+dx)} \operatorname{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}-\frac{b^3 \sqrt[3]{e(c+dx)} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}-\frac{b^2 \sqrt[3]{c+dx} \sqrt[3]{e(c+dx)} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}+\frac{3(c+dx) \sqrt[3]{e(c+dx)} \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d}+\frac{b(c+dx)^{2/3} \sqrt[3]{e(c+dx)} \cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] -1/8*(b^3*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/d + (b*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4*(e*(c +
d*x))^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(8*d*(c + d*x)^(1/3)) -
(b^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(8*d)
+ (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4
*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(8*d*(c + d*x)^
(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt[3]{ce+dex}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{3\text{Subst}\left(\int \frac{\sqrt[3]{\frac{e}{x^3}}\sin(a+bx)}{x^4}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{3(c+dx)\sqrt[3]{e(c+dx)}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{\left(3b\sqrt[3]{e(c+dx)}\right)\cdots}{\cdots}$$

$$= \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c\cdots}}{\cdots}$$

$$= \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c+dx}\,\sqrt[3]{e\cdots}}{\cdots}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)\cdots}}{4\cdots}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)\cdots}}{4\cdots}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)\cdots}}{4\cdots}$$

<u>Mathematica</u> [A]

time = 0.23, size = 208, normalized size = 0.84

$$-\frac{\sqrt[3]{e(c+dx)}\left(-2bc\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-2bdx\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^4\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)-6c\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)-6dx\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^2(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^4\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{8d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] -1/8*((e*(c + d*x))^(1/3)*(-2*b*c*Cos[a + b/(c + d*x)^(1/3)] - 2*b*d*x*Cos[
a + b/(c + d*x)^(1/3)] + b^3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)] + b
^4*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] - 6*c*(c + d*x)^(1/3)*Sin[a + b/(c
 + d*x)^(1/3)] - 6*d*x*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] + b^2*(c
```

```
+ d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)] + b^4*Cos[a]*SinIntegral[b/(c + d*x
)^(1/3)]))/(d*(c + d*x)^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{1}{3}} \sin \left(a + \frac{b}{(dx + c)^{\frac{1}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 128, normalized size = 0.52

$$-\frac{3 \left(\left(-i \Gamma\left(-4, i b \overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right) + i \Gamma\left(-4, -i b \overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right) - i \Gamma\left(-4, \frac{i b}{(dx+c)^{\frac{1}{3}}}\right) + i \Gamma\left(-4, -\frac{i b}{(dx+c)^{\frac{1}{3}}}\right)\right) \cos(a) - \left(\Gamma\left(-4, i b \overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right) + \Gamma\left(-4, -i b \overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right) + \Gamma\left(-4, \frac{i b}{(dx+c)^{\frac{1}{3}}}\right) + \Gamma\left(-4, -\frac{i b}{(dx+c)^{\frac{1}{3}}}\right)\right) \sin(a)\right) b^4 e^{\frac{1}{3}}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/4*((-I*gamma(-4, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(-4, -I*b*con
jugate((d*x + c)^(-1/3))) - I*gamma(-4, I*b/(d*x + c)^(1/3)) + I*gamma(-4,
-I*b/(d*x + c)^(1/3)))*cos(a) - (gamma(-4, I*b*conjugate((d*x + c)^(-1/3)))
 + gamma(-4, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(-4, I*b/(d*x + c)^(1
/3)) + gamma(-4, -I*b/(d*x + c)^(1/3)))*sin(a))*b^4*e^(1/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)*e^(1/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x
 + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{e (c + dx)} \sin \left(a + \frac{b}{\sqrt[3]{c + dx}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(1/3)),x)
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
[Out] integrate((d*x*e + c*e)^(1/3)*sin(a + b/(d*x + c)^(1/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)\,{\left(c\,e+d\,e\,x\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))*(c*e + d*e*x)^(1/3), x)
```

3.243 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=168

$$\frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b^2\sqrt[3]{c+dx}\,\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b}{}$$

```
[Out] 3/2*b*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)+3/2*b^2*(d*x
+c)^(1/3)*cos(a)*Si(b/(d*x+c)^(1/3))/d/(e*(d*x+c))^(1/3)+3/2*b^2*(d*x+c)^(1
/3)*Ci(b/(d*x+c)^(1/3))*sin(a)/d/(e*(d*x+c))^(1/3)+3/2*(d*x+c)*sin(a+b/(d*x
+c)^(1/3))/d/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$\frac{3b^2\sin(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b^2\cos(a)\sqrt[3]{c+dx}\,\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x))^(1/3))
+ (3*b^2*(c + d*x)^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(2*d*(e*(c
+ d*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x)
)^(1/3)) + (3*b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2
*d*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{\frac{e}{x^3}}\,x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+}}\right.}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{\right.}}{}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{\right.}}{}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\sqrt[3]{c+dx}\,\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(}{}$$

Mathematica [A]
time = 0.12, size = 131, normalized size = 0.78

$$\frac{3\left(b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^2\sqrt[3]{c+dx}\,\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)+c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+dx\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^2\sqrt[3]{c+dx}\,\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*CosI
ntegral[b/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)^(1/3)] + d*x*Sin[
a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)
^(1/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 171, normalized size = 1.02

$$-\frac{3\left((dx+c)^{\frac{1}{3}}\left(\left(-i\,\Gamma\left(-1,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\Gamma\left(-1,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\Gamma\left(-1,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(-1,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)-\left(\Gamma\left(-1,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\Gamma\left(-1,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\Gamma\left(-1,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(-1,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^2-4\,(dx+c)\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)\right)e^{\left(-\frac{1}{3}\right)}}{8\,(dx+c)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*((d*x + c)^(1/3)*((-I*gamma(-1, I*b*conjugate((d*x + c)^(-1/3))) + I*g
amma(-1, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(-1, I*b/(d*x + c)^(1/3
)) + I*gamma(-1, -I*b/(d*x + c)^(1/3)))*cos(a) - (gamma(-1, I*b*conjugate((
d*x + c)^(-1/3))) + gamma(-1, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(-1,
 I*b/(d*x + c)^(1/3)) + gamma(-1, -I*b/(d*x + c)^(1/3)))*sin(a))*b^2 - 4*(d
*x + c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*e^(-1/3)/((d*x + c)^(
1/3)*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(e^(-1/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*x + c
)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*x*e + c*e)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(1/3), x)
```

3.244 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=116

$$-\frac{3b(c+dx)^{2/3}\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

```
[Out] -3*b*(d*x+c)^(2/3)*Ci(b/(d*x+c)^(1/3))*cos(a)/d/(e*(d*x+c))^(2/3)+3*b*(d*x+
c)^(2/3)*Si(b/(d*x+c)^(1/3))*sin(a)/d/(e*(d*x+c))^(2/3)+3*(d*x+c)*sin(a+b/(
d*x+c)^(1/3))/d/(e*(d*x+c))^(2/3)
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3512, 15, 3378, 3384, 3380, 3383}

$$-\frac{3b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c + d*x
))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)
) + (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c +
d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{2/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{\sqrt[3]{c+}}\right.}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x,\right.}{d(e(c+dx))^{2/3}}$$

$$= -\frac{3b(c+dx)^{2/3}\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3b(}{}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.76

$$\frac{3\left(-b(c+dx)^{2/3}\cos(a)\mathrm{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b(c+dx)^{2/3}\sin(a)\mathrm{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)]) + (c + d*x)*
Sin[a + b/(c + d*x)^(1/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*
x)^(1/3)]))/(d*(e*(c + d*x))^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 152, normalized size = 1.31

$$-\frac{3\left(\left(\left(\mathrm{Ei}\left(ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\mathrm{Ei}\left(-ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\mathrm{Ei}\left(-ib\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)be^{\frac{1}{3}}-4(dx+c)^{\frac{1}{3}}e^{\frac{1}{3}}\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)\right)e^{(-1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] -3/4*(((Ei(I*b*conjugate((d*x + c)^(-1/3))) + Ei(-I*b*conjugate((d*x + c)^(
-1/3))) + Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*E
i(I*b*conjugate((d*x + c)^(-1/3))) - I*Ei(-I*b*conjugate((d*x + c)^(-1/3)))
 + I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b*e^(1/3
) - 4*(d*x + c)^(1/3)*e^(1/3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))
*e^(-1)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(e^(-2/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*x + c
)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{\sqrt[3]{c + dx}}\right)}{\left(e\left(c + dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*x*e + c*e)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(2/3), x)
```

3.245 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=45

$$\frac{3\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

```
[Out] 3*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/b/d/e/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3512, 15, 2718}

$$\frac{3\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{4/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 0.93

$$\frac{3(c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bd(e(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
[Out] (3*(c + d*x)^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [A]
time = 0.36, size = 30, normalized size = 0.67

$$\frac{3\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)e^{\left(-\frac{4}{3}\right)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] 3*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*e^(-4/3)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.
time = 0.35, size = 48, normalized size = 1.07

$$\frac{3\,(dx+c)\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)e^{\left(-\frac{4}{3}\right)}}{bd^2x+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] 3*(d*x + c)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))*e^(-4/3)/(b*d^
2*x + b*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\left(e\,(c+dx)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*x*e + c*e)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{1/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(4/3), x)
```

3.246 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=91

$$\frac{3\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}-\frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

```
[Out] 3*(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(1/3))/b/d/e/(e*(d*x+c))^(2/3)-3*(d*x+c)^(2
/3)*sin(a+b/(d*x+c)^(1/3))/b^2/d/e/(e*(d*x+c))^(2/3)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2717}

$$\frac{3\sqrt[3]{c+dx}\,\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}-\frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(2/3))
- (3*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e*(e*(c + d*x))^(2/
3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{5/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$
$$= -\frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de(e(c+dx))^{2/3}}$$
$$= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int \cos(a+bx)\,dx, x\right.}{bde(e(c+dx))^{2/3}}$$
$$= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 0.79

$$\frac{3(c+dx)^{5/3}\left(\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}} - \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}\right)}{d(e(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(5/3)*(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(1/3)) - Sin[a
+ b/(c + d*x)^(1/3)]/b^2))/(d*(e*(c + d*x))^(5/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.50, size = 170, normalized size = 1.87

$$-\frac{3\left(4\,b^2\sin\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-(dx+c)^{\frac{2}{3}}\left(\left(-i\,\Gamma\left(3,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+i\,\Gamma\left(3,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)-i\,\Gamma\left(3,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+i\,\Gamma\left(3,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)-\left(\Gamma\left(3,i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\Gamma\left(3,-i\,b\overline{\frac{1}{(dx+c)^{\frac{1}{3}}}}\right)+\Gamma\left(3,\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)+\Gamma\left(3,-\frac{i\,b}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)\right)e^{\left(-\frac{5}{3}\right)}}{8\,(dx+c)^{\frac{2}{3}}b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] -3/8*(4*b^2*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - (d*x + c)^(2/3)*
((-I*gamma(3, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(3, -I*b*conjugate(
(d*x + c)^(-1/3))) - I*gamma(3, I*b/(d*x + c)^(1/3)) + I*gamma(3, -I*b/(d*x
 + c)^(1/3)))*cos(a) - (gamma(3, I*b*conjugate((d*x + c)^(-1/3))) + gamma(3
, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(3, I*b/(d*x + c)^(1/3)) + gamma
(3, -I*b/(d*x + c)^(1/3)))*sin(a)))*e^(-5/3)/((d*x + c)^(2/3)*b^2*d)
```

Fricas [A]
time = 0.80, size = 94, normalized size = 1.03

$$\frac{3\left((dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)e^{\frac{1}{3}}-(dx+c)e^{\frac{1}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)\right)e^{(-2)}}{b^2d^2x+b^2cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] 3*((d*x + c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))*e^(1/
3) - (d*x + c)*e^(1/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)))*e^
(-2)/(b^2*d^2*x + b^2*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e\,(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*x*e + c*e)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{{(c\,e+d\,e\,x)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(5/3), x)
```

3.247 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal. Leaf size=172

$$-\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}-\frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}}$$

```
[Out] -18*cos(a+b/(d*x+c)^(1/3))/b^3/d/e^2/(e*(d*x+c))^(1/3)+3*cos(a+b/(d*x+c)^(1
/3))/b/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)-9*sin(a+b/(d*x+c)^(1/3))/b^2/d
/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(1/3)+18*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3)
)/b^4/d/e^2/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2717}

$$\frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}}-\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}-\frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-18*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(1/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) - (9*S
in[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3))
+ (18*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c + d*x))^
(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx &= -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{7/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\ &= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2\sqrt[3]{e(c+dx)}}\\ &= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{\left(9\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \frac{}{\sqrt[]{}}\right.}{bde^2\sqrt[3]{e(c+dx)}}\\ &= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}} + \frac{\left(18\sqrt[3]{c+dx}\right)}{}\\ &= -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{}{\sqrt[3]{}}\right.}{b^2de^2\sqrt[3]{c+dx}\,\sqrt[3]{}}\\ &= -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{}{\sqrt[3]{}}\right.}{b^2de^2\sqrt[3]{c+dx}\,\sqrt[3]{}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 107, normalized size = 0.62

$$-\frac{3\left(\left(-b^3\sqrt[3]{c+dx}+6b(c+dx)\right)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+3\sqrt[3]{c+dx}\left(-2c-2dx+b^2\sqrt[3]{c+dx}\right)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{b^4de(e(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-3*((-(b^3*(c + d*x)^(1/3)) + 6*b*(c + d*x))*Cos[a + b/(c + d*x)^(1/3)] +
3*(c + d*x)^(1/3)*(-2*c - 2*d*x + b^2*(c + d*x)^(1/3))*Sin[a + b/(c + d*x)^
(1/3)]))/(b^4*d*e*(e*(c + d*x))^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.39, size = 1400, normalized size = 8.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] -3/16*(2*(cos(a)^2 + sin(a)^2)*b^4*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1
/3)) - 2*(b^4*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2*sin(a) + b^4*s
in(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*cos((2*(d*x + c)^(1/3
)*a + b)/(d*x + c)^(1/3)) + 2*(b^4*cos(a)*cos(((d*x + c)^(1/3)*a + b)/(d*x
+ c)^(1/3))^2 + b^4*cos(a)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*
sin((2*(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + ((((-I*gamma(5, I*b*conjug
ate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*g
amma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^3 -
 (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x
+ c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/
3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*ga
mma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3))
+ I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 - (gamma(5, I*b*conjuga
te((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(
5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*d*x + (
(-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((
d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x
+ c)^(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(
5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamm
a(5, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d
```

```
*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5
, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2
- (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x
 + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1
/3)))*sin(a)^3)*c)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2 + (((-I*g
amma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x +
 c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^
(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I
*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5,
-I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5, I*b*conjugate((d*x +
c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b
/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 - (ga
mma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)
^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))
*sin(a)^3)*d*x + ((-I*gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5
, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*g
amma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^3 - (gamma(5, I*b*conjugate((d*x + c)
^(-1/3))) + gamma(5, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x
+ c)^(1/3)) + gamma(5, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (-I*gamma(5
, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(5, -I*b*conjugate((d*x + c)^(-
1/3))) - I*gamma(5, I*b/(d*x + c)^(1/3)) + I*gamma(5, -I*b/(d*x + c)^(1/3))
)*cos(a)*sin(a)^2 - (gamma(5, I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, -
I*b*conjugate((d*x + c)^(-1/3))) + gamma(5, I*b/(d*x + c)^(1/3)) + gamma(5,
 -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^
(1/3))^2)*(d*x + c)^(1/3))*e^(-1/3)/((((cos(a)^2*e^2 + e^2*sin(a)^2)*b^4*d^
2*x + (cos(a)^2*e^2 + e^2*sin(a)^2)*b^4*c*d)*cos(((d*x + c)^(1/3)*a + b)/(d
*x + c)^(1/3))^2 + ((cos(a)^2*e^2 + e^2*sin(a)^2)*b^4*d^2*x + (cos(a)^2*e^2
 + e^2*sin(a)^2)*b^4*c*d)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))^2)*(
d*x + c)^(1/3))
```

Fricas [A]

time = 0.86, size = 151, normalized size = 0.88

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b^3-6\,bdx-6\,bc\right)(dx+c)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)e^{\frac{2}{3}}-3\left((dx+c)^{\frac{2}{3}}b^2-2\,(dx+c)^{\frac{4}{3}}\right)(dx+c)^{\frac{2}{3}}e^{\frac{2}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)\right)e^{(-3)}}{b^4d^3x^2+2\,b^4cd^2x+b^4c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(1/3)*b^3 - 6*b*d*x - 6*b*c)*(d*x + c)^(2/3)*cos((a*d*x + a*c
 + (d*x + c)^(2/3)*b)/(d*x + c))*e^(2/3) - 3*((d*x + c)^(2/3)*b^2 - 2*(d*x
+ c)^(4/3))*(d*x + c)^(2/3)*e^(2/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(
d*x + c)))*e^(-3)/(b^4*d^3*x^2 + 2*b^4*c*d^2*x + b^4*c^2*d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*x*e + c*e)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(7/3), x)
```

3.248 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal. Leaf size=217

$$-\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}}+\frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}}-\frac{12\sin\left(\right.}{b^2de^2\sqrt[3]{c+}}$$

```
[Out] -36*cos(a+b/(d*x+c)^(1/3))/b^3/d/e^2/(e*(d*x+c))^(2/3)+3*cos(a+b/(d*x+c)^(1
/3))/b/d/e^2/(d*x+c)^(2/3)/(e*(d*x+c))^(2/3)+72*(d*x+c)^(2/3)*cos(a+b/(d*x+
c)^(1/3))/b^5/d/e^2/(e*(d*x+c))^(2/3)-12*sin(a+b/(d*x+c)^(1/3))/b^2/d/e^2/(
d*x+c)^(1/3)/(e*(d*x+c))^(2/3)+72*(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(1/3))/b^4/
d/e^2/(e*(d*x+c))^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3512, 15, 3377, 2718}

$$\frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}}+\frac{72\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}-\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}}-\frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] (-36*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(2/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(2/3)) + (72*
(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(b^5*d*e^2*(e*(c + d*x))^(2/3))
 - (12*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))
^(2/3)) + (72*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c
+ d*x))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = -\frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{8/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{\left(12(c+dx)^{2/3}\right)\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x,\right.}{bde^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}} + \frac{\left(36(c+dx)^{2/3}\right.}{}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{}{\sqrt[3]{}}\right.}{b^2de^2\sqrt[3]{c+dx}\,(e}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{}{\sqrt[3]{}}\right.}{b^2de^2\sqrt[3]{c+dx}\,(e}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} + \frac{72(c+dx)^{2/3}\cos}{b^5de^2(e(c}$$

Mathematica [A]

time = 0.19, size = 112, normalized size = 0.52

$$\frac{(c+dx)^{4/3}\left(3\left(b^4-12b^2(c+dx)^{2/3}+24(c+dx)^{4/3}\right)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+12b\left(6c+6dx-b^2\sqrt[3]{c+dx}\right)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{b^5d(e(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] ((c + d*x)^(4/3)*(3*(b^4 - 12*b^2*(c + d*x)^(2/3) + 24*(c + d*x)^(4/3))*Cos
[a + b/(c + d*x)^(1/3)] + 12*b*(6*c + 6*d*x - b^2*(c + d*x)^(1/3))*Sin[a +
b/(c + d*x)^(1/3)]))/(b^5*d*(e*(c + d*x))^(8/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.08, size = 935, normalized size = 4.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

```
[Out] 3/20*(2*(b^5*cos((2*(d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))*sin(a) - b^5*co
s(b/(d*x + c)^(1/3))*sin(a) - b^5*cos(a)*sin((2*(d*x + c)^(1/3)*a + b)/(d*x
 + c)^(1/3)) - b^5*cos(a)*sin(b/(d*x + c)^(1/3)))*(d*x + c)^(1/3)*e^(1/3) +
 (((gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*
x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(
1/3)))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6
, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*g
amma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a) + (gamma(6, I*b*conjugate((d
*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*
b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 + (-I*
gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x
+ c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)
^(1/3)))*sin(a)^3)*d^2*x^2 + 2*((gamma(6, I*b*conjugate((d*x + c)^(-1/3)))
```

```
+ gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)
) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I*gamma(6, I*b*conjugate((
d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(
6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^2*sin(a)
 + (gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*
x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(
1/3)))*cos(a)*sin(a)^2 + (-I*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*
gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)
) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c*d*x + ((gamma(6, I*b*conj
ugate((d*x + c)^(-1/3))) + gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gam
ma(6, I*b/(d*x + c)^(1/3)) + gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)^3 + (-I
*gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x
 + c)^(-1/3))) - I*gamma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c
)^(1/3)))*cos(a)^2*sin(a) + (gamma(6, I*b*conjugate((d*x + c)^(-1/3))) + ga
mma(6, -I*b*conjugate((d*x + c)^(-1/3))) + gamma(6, I*b/(d*x + c)^(1/3)) +
gamma(6, -I*b/(d*x + c)^(1/3)))*cos(a)*sin(a)^2 + (-I*gamma(6, I*b*conjugat
e((d*x + c)^(-1/3))) + I*gamma(6, -I*b*conjugate((d*x + c)^(-1/3))) - I*gam
ma(6, I*b/(d*x + c)^(1/3)) + I*gamma(6, -I*b/(d*x + c)^(1/3)))*sin(a)^3)*c^
2)*e^(1/3))/((cos(a)^2*e^3 + e^3*sin(a)^2)*b^5*d^3*x^2 + 2*(cos(a)^2*e^3 +
e^3*sin(a)^2)*b^5*c*d^2*x + (cos(a)^2*e^3 + e^3*sin(a)^2)*b^5*c^2*d)
```

Fricas [A]
time = 0.90, size = 172, normalized size = 0.79

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b^{4}-12\,b^{2}dx-12\,b^{2}c+24\,(dx+c)^{\frac{5}{3}}\right)(dx+c)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)e^{\frac{1}{3}}-4\left((dx+c)^{\frac{2}{3}}b^{3}-6\,(bdx+bc)(dx+c)^{\frac{1}{3}}\right)(dx+c)^{\frac{1}{3}}e^{\frac{1}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)\right)e^{(-3)}}{b^{5}d^{3}x^{2}+2\,b^{5}cd^{2}x+b^{5}c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(1/3)*b^4 - 12*b^2*d*x - 12*b^2*c + 24*(d*x + c)^(5/3))*(d*x
+ c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))*e^(1/3) - 4*((d
*x + c)^(2/3)*b^3 - 6*(b*d*x + b*c)*(d*x + c)^(1/3))*(d*x + c)^(1/3)*e^(1/3
)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)))*e^(-3)/(b^5*d^3*x^2 + 2
*b^5*c*d^2*x + b^5*c^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(8/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*x*e + c*e)^(8/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{1/3}}\right)}{{(c\,e+d\,e\,x)}^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(8/3),x)
[Out] int(sin(a + b/(c + d*x)^(1/3))/(c*e + d*e*x)^(8/3), x)
```

3.249 $\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx$

Optimal. Leaf size=299

$$-\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d}+\frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d}-\frac{8b^{7/2}e\sqrt{2\pi}\sqrt[3]{e(c+}}{35d}$$

```
[Out] -8/35*b^3*e*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+6/35*b*e*(d*x+c)^(4/
3)*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d-4/35*b^2*e*(d*x+c)^(2/3)*(e*(
d*x+c))^(1/3)*sin(a+b/(d*x+c)^(2/3))/d+3/7*e*(d*x+c)^2*(e*(d*x+c))^(1/3)*si
n(a+b/(d*x+c)^(2/3))/d-8/35*b^(7/2)*e*(e*(d*x+c))^(1/3)*cos(a)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(1/3)-8/35*
b^(7/2)*e*(e*(d*x+c))^(1/3)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)
)*sin(a)*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.22, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3516, 3498, 3496, 3490, 3468, 3469, 3434, 3433, 3432}

$$-\frac{8\sqrt{2\pi}\,b^{7/2}e\sin(a)\sqrt[3]{e(c+dx)}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}-\frac{8\sqrt{2\pi}\,b^{7/2}e\cos(a)\sqrt[3]{e(c+dx)}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}-\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d}-\frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d}+\frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d}+\frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (-8*b^3*e*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) + (6*b*e*(
c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) - (8*
b^(7/2)*e*Sqrt[2*Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)])/(35*d*(c + d*x)^(1/3)) - (8*b^(7/2)*e*Sqrt[2*Pi]*(e*(c
 + d*x))^(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(35*d
*(c + d*x)^(1/3)) - (4*b^2*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b/
(c + d*x)^(2/3)])/(35*d) + (3*e*(c + d*x)^2*(e*(c + d*x))^(1/3)*Sin[a + b/(
c + d*x)^(2/3)])/(7*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
```

```
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{\text{Subst}\left(\int (ex)^{4/3}\sin\left(a+\frac{b}{x^{2/3}}\right)dx,x,c+dx\right)}{d}$$

$$= \frac{\left(e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{4/3}\sin\left(a+\frac{b}{x^{2/3}}\right)dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{6}\sin\left(a+\frac{b}{x^{2}}\right)dx,x,\sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^8}dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} - \frac{\left(6be\sqrt[3]{e(c+dx)}\right.}{}$$

$$= \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{}}{}$$

$$= \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} - \frac{4b^2e(c+dx)^{2/}}{}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+}}{3}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+}}{3}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+}}{3}$$

Mathematica [A]

time = 0.61, size = 237, normalized size = 0.79

$$\frac{(e(c+dx))^{4/3}\left(\frac{\cos\left(\frac{b}{(c+dx)^{2/3}}\right)\left(-8b^3\cos(a)+6b(c+dx)^{4/3}\cos(a)-4b^2(c+dx)^{2/3}\sin(a)+15(c+dx)^2\sin(a)\right)}{c+dx}-\frac{8b^{7/2}\sqrt{2\pi}\left(\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)\right)}{(c+dx)^{4/3}}+\frac{\left(-4b^2(c+dx)^{2/3}\cos(a)+15(c+dx)^2\cos(a)+8b^3\sin(a)-6b(c+dx)^{4/3}\sin(a)\right)\sin\left(\frac{b}{(c+dx)^{2/3}}\right)}{c+dx}\right)}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((e*(c + d*x))^(4/3)*((Cos[b/(c + d*x)^(2/3)]*(-8*b^3*Cos[a] + 6*b*(c + d*x
)^(4/3)*Cos[a] - 4*b^2*(c + d*x)^(2/3)*Sin[a] + 15*(c + d*x)^2*Sin[a]))/(c
+ d*x) - (8*b^(7/2)*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(c + d
*x)^(4/3) + ((-4*b^2*(c + d*x)^(2/3)*Cos[a] + 15*(c + d*x)^2*Cos[a] + 8*b^3
*Sin[a] - 6*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(2/3)])/(c + d*x)))/(
35*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{4}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.08, size = 1231, normalized size = 4.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/8*((((-I*e^(4/3)*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*e^(4/3
)*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c))
+ (I*e^(4/3)*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*e^(4/3)*gamm
a(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (e^(
4/3)*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, -I*
b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (e^(4/3)*gamma(
-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, I*b/(d*x + c)
^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((e^(4/3)*gamma(-
```

```
7/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, -I*b/(d*x + c)^
(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (e^(4/3)*gamma(-7/2, -I*b*c
onjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos
(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*e^(4/3)*gamma(-7/2, I*b*conjugate
((d*x + c)^(-2/3))) + I*e^(4/3)*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*
pi + 7/3*arctan2(0, d*x + c)) - (-I*e^(4/3)*gamma(-7/2, -I*b*conjugate((d*x
 + c)^(-2/3))) + I*e^(4/3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi +
7/3*arctan2(0, d*x + c)))*sin(a))*d^2*x^2 + 2*(((-I*e^(4/3)*gamma(-7/2, I*b
*conjugate((d*x + c)^(-2/3))) + I*e^(4/3)*gamma(-7/2, -I*b/(d*x + c)^(2/3))
)*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (I*e^(4/3)*gamma(-7/2, -I*b*conju
gate((d*x + c)^(-2/3))) - I*e^(4/3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-
7/4*pi + 7/3*arctan2(0, d*x + c)) + (e^(4/3)*gamma(-7/2, I*b*conjugate((d*x
 + c)^(-2/3))) + e^(4/3)*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/
3*arctan2(0, d*x + c)) - (e^(4/3)*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/
3))) + e^(4/3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(
0, d*x + c)))*cos(a) - ((e^(4/3)*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3)
)) + e^(4/3)*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0,
 d*x + c)) + (e^(4/3)*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + e^(4/
3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c))
 - (-I*e^(4/3)*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*e^(4/3)*gam
ma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I
*e^(4/3)*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*e^(4/3)*gamma(-7
/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*sin(a))*c
*d*x + (((-I*e^(4/3)*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*e^(4/
3)*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c))
 + (I*e^(4/3)*gamma(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*e^(4/3)*gam
ma(-7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c)) + (e^
(4/3)*gamma(-7/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, -I
*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) - (e^(4/3)*gamma
(-7/2, -I*b*conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, I*b/(d*x + c
)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) - ((e^(4/3)*gamma(
-7/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, -I*b/(d*x + c)
^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (e^(4/3)*gamma(-7/2, -I*b*
conjugate((d*x + c)^(-2/3))) + e^(4/3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*co
s(-7/4*pi + 7/3*arctan2(0, d*x + c)) - (-I*e^(4/3)*gamma(-7/2, I*b*conjugat
e((d*x + c)^(-2/3))) + I*e^(4/3)*gamma(-7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4
*pi + 7/3*arctan2(0, d*x + c)) - (-I*e^(4/3)*gamma(-7/2, -I*b*conjugate((d*
x + c)^(-2/3))) + I*e^(4/3)*gamma(-7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*pi +
 7/3*arctan2(0, d*x + c)))*sin(a))*c^2)*(d*x + c)^(1/3)*(b/(d*x + c)^(2/3))
^(7/2)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(4/3)*e^(4/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x
 + c)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,6,1,0,0,0]%%%}+%%%{-2,[0,3,1,1,1,0]%%%}+%%%{1,[0,0,1,2,2
,0]%%%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)\,(c\,e+d\,e\,x)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(4/3), x)
```

3.250 $\int (ce+dex)^{2/3} \sin \left(a+\frac{b}{(c+dx)^{2/3}}\right) \, dx$

Optimal. Leaf size=262

$$\frac{2b\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}+\frac{4\sqrt{2}\,b^{5/2}\sqrt{\pi}\,(e(c+dx))^{2/3}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4\sqrt{2}\,b^{5/2}}{}$$

```
[Out] 2/5*b*(d*x+c)^(1/3)*(e*(d*x+c))^(2/3)*cos(a+b/(d*x+c)^(2/3))/d-4/5*b^2*(e*(
d*x+c))^(2/3)*sin(a+b/(d*x+c)^(2/3))/d/(d*x+c)^(1/3)+3/5*(d*x+c)*(e*(d*x+c)
)^(2/3)*sin(a+b/(d*x+c)^(2/3))/d+4/5*b^(5/2)*(e*(d*x+c))^(2/3)*cos(a)*Fresn
elC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(2/3
)-4/5*b^(5/2)*(e*(d*x+c))^(2/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(
1/3))*sin(a)*2^(1/2)*Pi^(1/2)/d/(d*x+c)^(2/3)
```

Rubi [A]

time = 0.17, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3516, 3498, 3496, 3490, 3468, 3469, 3435, 3433, 3432}

$$\frac{4\sqrt{2}\sqrt{\pi}\,b^{5/2}\cos(a)(e(c+dx))^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4\sqrt{2}\sqrt{\pi}\,b^{5/2}\sin(a)(e(c+dx))^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}}+\frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}+\frac{2b\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(5*d)
+ (4*Sqrt[2]*b^(5/2)*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/(c + d*x)^(1/3)])/(5*d*(c + d*x)^(2/3)) - (4*Sqrt[2]*b^(5/2)*Sq
rt[Pi]*(e*(c + d*x))^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*S
in[a])/(5*d*(c + d*x)^(2/3)) - (4*b^2*(e*(c + d*x))^(2/3)*Sin[a + b/(c + d*
x)^(2/3)])/(5*d*(c + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(5*d)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\,dx = \frac{\text{Subst}\left(\int (ex)^{2/3}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{2/3}\text{Subst}\left(\int x^{2/3}\sin\left(a+\frac{b}{x^{2/3}}\right)\,dx,x,c+dx\right)}{d(c+dx)^{2/3}}$$

$$= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+\frac{b}{x^2}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$

$$= -\frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^6}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d(c+dx)^{2/3}}$$

$$= \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{\left(6b(e(c+dx))^{2/3}\right)}{}$$

$$= \frac{2b\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{3(c+dx)(e(c+}{}$$

$$= \frac{2b\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{4b^2(e(c+dx))^{2/}}{5d\sqrt[3]{}}$$

$$= \frac{2b\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{4b^2(e(c+dx))^{2/}}{5d\sqrt[3]{}}$$

$$= \frac{2b\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{4\sqrt{2}\,b^{5/2}\sqrt{\pi}\,(e}{}$$

Mathematica [A]
time = 0.23, size = 228, normalized size = 0.87

$$\frac{(e(c+dx))^{2/3}\left(2bc\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2bdx\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+4b^{5/2}\sqrt{2\pi}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)-4b^{5/2}\sqrt{2\pi}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)-4b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+3c(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+3dx(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{5d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((e*(c + d*x))^(2/3)*(2*b*c*Cos[a + b/(c + d*x)^(2/3)] + 2*b*d*x*Cos[a + b/
(c + d*x)^(2/3)] + 4*b^(5/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]
```

```
)/(c + d*x)^(1/3)] - 4*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
+ d*x)^(1/3)]*Sin[a] - 4*b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)] + 3
*c*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] + 3*d*x*(c + d*x)^(2/3)*Sin[a
 + b/(c + d*x)^(2/3)]))/(5*d*(c + d*x)^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{2}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 823, normalized size = 3.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/8*(((((-I*e^(2/3)*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*e^(2/3
)*gamma(-5/2, -I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c))
+ (I*e^(2/3)*gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*e^(2/3)*gamm
a(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (e^(
2/3)*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(2/3)*gamma(-5/2, -I*
b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) - (e^(2/3)*gamma(
-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + e^(2/3)*gamma(-5/2, I*b/(d*x + c)
^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*cos(a) - ((e^(2/3)*gamma(-
5/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(2/3)*gamma(-5/2, -I*b/(d*x + c)^
(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (e^(2/3)*gamma(-5/2, -I*b*c
onjugate((d*x + c)^(-2/3))) + e^(2/3)*gamma(-5/2, I*b/(d*x + c)^(2/3)))*cos
(-5/4*pi + 5/3*arctan2(0, d*x + c)) - (-I*e^(2/3)*gamma(-5/2, I*b*conjugate
((d*x + c)^(-2/3))) + I*e^(2/3)*gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*
pi + 5/3*arctan2(0, d*x + c)) - (-I*e^(2/3)*gamma(-5/2, -I*b*conjugate((d*x
 + c)^(-2/3))) + I*e^(2/3)*gamma(-5/2, I*b/(d*x + c)^(2/3)))*sin(-5/4*pi +
5/3*arctan2(0, d*x + c)))*sin(a))*d*x + (((-I*e^(2/3)*gamma(-5/2, I*b*conju
gate((d*x + c)^(-2/3))) + I*e^(2/3)*gamma(-5/2, -I*b/(d*x + c)^(2/3)))*cos(
5/4*pi + 5/3*arctan2(0, d*x + c)) + (I*e^(2/3)*gamma(-5/2, -I*b*conjugate((
d*x + c)^(-2/3))) - I*e^(2/3)*gamma(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi
 + 5/3*arctan2(0, d*x + c)) + (e^(2/3)*gamma(-5/2, I*b*conjugate((d*x + c)^
(-2/3))) + e^(2/3)*gamma(-5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arct
```

```
an2(0, d*x + c)) - (e^(2/3)*gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) +
 e^(2/3)*gamma(-5/2, I*b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x
 + c)))*cos(a) - ((e^(2/3)*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + e
^(2/3)*gamma(-5/2, -I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x +
 c)) + (e^(2/3)*gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + e^(2/3)*gam
ma(-5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi + 5/3*arctan2(0, d*x + c)) - (-I
*e^(2/3)*gamma(-5/2, I*b*conjugate((d*x + c)^(-2/3))) + I*e^(2/3)*gamma(-5/
2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) - (-I*e^(2/
3)*gamma(-5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*e^(2/3)*gamma(-5/2, I*
b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*sin(a))*c)*(d*x
 + c)^(2/3)*(b/(d*x + c)^(2/3))^(5/2)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
[Out] integral((d*x + c)^(2/3)*e^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x
 + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{2}{3}}\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b/(d*x+c)**(2/3)),x)
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
[Out] integrate((d*x*e + c*e)^(2/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)\,{\left(c\,e+d\,e\,x\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(2/3), x)
```

3.251 $\int \sqrt[3]{ce+dex}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$

Optimal. Leaf size=168

$$\frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}+\frac{3b^2\sqrt[3]{e(c+dx)}\,\mathrm{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)}{4d\sqrt[3]{c+dx}}+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\,\sin}{4d}$$

```
[Out] 3/4*b*(d*x+c)^(1/3)*(e*(d*x+c))^(1/3)*cos(a+b/(d*x+c)^(2/3))/d+3/4*b^2*(e*(
d*x+c))^(1/3)*cos(a)*Si(b/(d*x+c)^(2/3))/d/(d*x+c)^(1/3)+3/4*b^2*(e*(d*x+c)
)^(1/3)*Ci(b/(d*x+c)^(2/3))*sin(a)/d/(d*x+c)^(1/3)+3/4*(d*x+c)*(e*(d*x+c))^
(1/3)*sin(a+b/(d*x+c)^(2/3))/d
```

Rubi [A]
time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3516, 3462, 3460, 3378, 3384, 3380, 3383}

$$\frac{3b^2\sin(a)\sqrt[3]{e(c+dx)}\,\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}+\frac{3b^2\cos(a)\sqrt[3]{e(c+dx)}\,\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}+\frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (3*b*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d)
+ (3*b^2*(e*(c + d*x))^(1/3)*CosIntegral[b/(c + d*x)^(2/3)]*Sin[a])/(4*d*(c
 + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*d) + (3*b^2*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]
)/(4*d*(c + d*x)^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \sqrt[3]{ce+dex}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{\text{Subst}\left(\int \sqrt[3]{ex}\,\sin\left(a+\frac{b}{x^{2/3}}\right)dx,x,c+dx\right)}{d}$$

$$= \frac{\sqrt[3]{e(c+dx)}\,\text{Subst}\left(\int \sqrt[3]{x}\,\sin\left(a+\frac{b}{x^{2/3}}\right)dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{c+dx}}$$

$$= \frac{3(c+dx)\sqrt[3]{e(c+dx)}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} - \frac{\left(3b\sqrt[3]{e(c+dx)}\right)\text{Su}}{4}$$

$$= \frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+}}{}$$

$$= \frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+}}{}$$

$$= \frac{3b\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}\,\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3b^2\sqrt[3]{e(c+dx)}\,\text{Ci}}{4d\sqrt[3]{c}}$$

__Mathematica__ [A]
time = 0.20, size = 113, normalized size = 0.67

$$\frac{3\sqrt[3]{e(c+dx)}\left(b(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b^2\text{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)+(c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b^2\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)\right)}{4d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)] + b^2*
CosIntegral[b/(c + d*x)^(2/3)]*Sin[a] + (c + d*x)^(4/3)*Sin[a + b/(c + d*x)
^(2/3)] + b^2*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]))/(4*d*(c + d*x)^(1/3))
```

__Maple__ [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{1}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 128, normalized size = 0.76

$$\frac{3\left(\left(-i\,\Gamma\left(-2,i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(-2,-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\,\Gamma\left(-2,\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)+i\,\Gamma\left(-2,-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(a\right)-\left(\Gamma\left(-2,i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-2,-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-2,\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-2,-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(a\right)\right)b^2e^{\frac{1}{3}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 3/8*((-I*gamma(-2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-2, -I*b*conj
ugate((d*x + c)^(-2/3))) - I*gamma(-2, I*b/(d*x + c)^(2/3)) + I*gamma(-2, -
I*b/(d*x + c)^(2/3)))*cos(a) - (gamma(-2, I*b*conjugate((d*x + c)^(-2/3)))
+ gamma(-2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(-2, I*b/(d*x + c)^(2/
3)) + gamma(-2, -I*b/(d*x + c)^(2/3)))*sin(a))*b^2*e^(1/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)*e^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x
 + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{e\left(c+dx\right)}\,\sin\left(a+\frac{b}{\left(c+dx\right)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((d*x*e + c*e)^(1/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)\,{\left(c\,e+d\,e\,x\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))*(c*e + d*e*x)^(1/3), x)
```

3.252 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=122

$$-\frac{3b\sqrt[3]{c+dx}\,\cos(a)\mathrm{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b\sqrt[3]{c+dx}\,\sin(a)\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

```
[Out] -3/2*b*(d*x+c)^(1/3)*Ci(b/(d*x+c)^(2/3))*cos(a)/d/(e*(d*x+c))^(1/3)+3/2*b*(
d*x+c)^(1/3)*Si(b/(d*x+c)^(2/3))*sin(a)/d/(e*(d*x+c))^(1/3)+3/2*(d*x+c)*sin
(a+b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3516, 3462, 3460, 3378, 3384, 3380, 3383}

$$-\frac{3b\cos(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b\sin(a)\sqrt[3]{c+dx}\,\mathrm{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)])/(2*d*(e*(c + d
*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(2*d*(e*(c + d*x))^(
1/3)) + (3*b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(2*d*(e
*(c + d*x))^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{\sqrt[3]{ex}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{\sqrt[3]{x}}\,dx, x, c+dx\right)}{d\sqrt[3]{e(c+dx)}} \\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} \\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} \\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\,\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{(c+dx}\right.}{2d\sqrt[3]{e(c+dx)}} \\
&= -\frac{3b\sqrt[3]{c+dx}\,\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b\sqrt[3]{c+dx}}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 90, normalized size = 0.74

$$
\frac{3\left(-b\sqrt[3]{c+dx}\,\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b\sqrt[3]{c+dx}\,\sin(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{e(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)]) + (c + d*x)*
Sin[a + b/(c + d*x)^(2/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*
x)^(2/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{1}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 125, normalized size = 1.02

$$-\frac{3\left(\left(\Gamma\left(-1,ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-1,-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-1,\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(-1,-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos(a)+\left(-i\Gamma\left(-1,ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\Gamma\left(-1,-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\Gamma\left(-1,\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)+i\Gamma\left(-1,-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin(a)\right)be^{\left(-\frac{1}{3}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*((gamma(-1, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1, -I*b*conjugat
e((d*x + c)^(-2/3))) + gamma(-1, I*b/(d*x + c)^(2/3)) + gamma(-1, -I*b/(d*x
 + c)^(2/3)))*cos(a) + (-I*gamma(-1, I*b*conjugate((d*x + c)^(-2/3))) + I*g
amma(-1, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-1, I*b/(d*x + c)^(2/3
)) + I*gamma(-1, -I*b/(d*x + c)^(2/3)))*sin(a))*b*e^(-1/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(e^(-1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*x + c
)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{\sqrt[3]{e(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(1/3), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3),x)
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(1/3), x)
```

3.253 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=164

$$-\frac{3\sqrt{b}\,\sqrt{2\pi}\,(c+dx)^{2/3}\cos(a)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3\sqrt{b}\,\sqrt{2\pi}\,(c+dx)^{2/3}S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(\right.}{d(e(c+}$$

```
[Out] 3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))/d/(e*(d*x+c))^(2/3)-3*(d*x+c)^(2/3)*cos(a)
*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*b^(1/2)*2^(1/2)*Pi^(1/2)/
d/(e*(d*x+c))^(2/3)+3*(d*x+c)^(2/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+
c)^(1/3))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/2)/d/(e*(d*x+c))^(2/3)
```

Rubi [A]

time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3516, 3498, 3496, 3440, 3468, 3435, 3433, 3432}

$$-\frac{3\sqrt{2\pi}\,\sqrt{b}\,\cos(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3\sqrt{2\pi}\,\sqrt{b}\,\sin(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])
/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)) + (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x
)^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(d*(e*(c + d
*x))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(d*(e*(c + d*x))^(2/
3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3440

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{2/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{2/3}}\,dx, x, c+dx\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a+\frac{b}{x^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(6b(c+dx)^{2/3}\right)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{}{\sqrt[3]{c}}\right.}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(6b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \right.}{d(e(c+dx))^{2/3}}$$

$$= -\frac{3\sqrt{b}\,\sqrt{2\pi}\,(c+dx)^{2/3}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3\sqrt{b}\,\sqrt{2\pi}\,(c+dx)^{2/3}S\left(\frac{\sqrt{b}}{\sqrt[3]{c+}}\right.}{d(e(c+dx))^{2/3}}$$

Mathematica [A]
time = 0.21, size = 136, normalized size = 0.83

$$\frac{3\left(-\sqrt{b}\,\sqrt{2\pi}\,(c+dx)^{2/3}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+\sqrt{b}\,\sqrt{2\pi}\,(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + (c + d*x)*Sin[a + b/(c + d*x)^(2/3)]
))/(d*(e*(c + d*x))^(2/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 382, normalized size = 2.33

$$-\frac{3\,(dx+c)^{\frac{1}{3}}\left(\left(\left(-i\,\Gamma\left(-\frac{1}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(-\frac{1}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(i\,\Gamma\left(-\frac{1}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\,\Gamma\left(-\frac{1}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(-\frac{1}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-\frac{1}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)-\left(\Gamma\left(-\frac{1}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-\frac{1}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)\right)\cos\left(a\right)-\left(\left(\Gamma\left(-\frac{1}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-\frac{1}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)+\left(\Gamma\left(-\frac{1}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(-\frac{1}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)-\left(-i\,\Gamma\left(-\frac{1}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(-\frac{1}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)-\left(-i\,\Gamma\left(-\frac{1}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,\Gamma\left(-\frac{1}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{3}\arctan\left(0,dx+c\right)\right)\right)\sin\left(a\right)\right)\sqrt{\frac{b}{(dx+c)^{\frac{2}{3}}}}\,e^{\left(-\frac{2}{3}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] -3/8*(d*x + c)^(1/3)*(((-I*gamma(-1/2, I*b*conjugate((d*x + c)^(-2/3))) + I
*gamma(-1/2, -I*b/(d*x + c)^(2/3)))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) +
 (I*gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(-1/2, I*b/(d*x
+ c)^(2/3)))*cos(-1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, I*b*conj
ugate((d*x + c)^(-2/3))) + gamma(-1/2, -I*b/(d*x + c)^(2/3)))*sin(1/4*pi +
1/3*arctan2(0, d*x + c)) - (gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) +
 gamma(-1/2, I*b/(d*x + c)^(2/3)))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*
cos(a) - ((gamma(-1/2, I*b*conjugate((d*x + c)^(-2/3))) + gamma(-1/2, -I*b/
(d*x + c)^(2/3)))*cos(1/4*pi + 1/3*arctan2(0, d*x + c)) + (gamma(-1/2, -I*b
*conjugate((d*x + c)^(-2/3))) + gamma(-1/2, I*b/(d*x + c)^(2/3)))*cos(-1/4*
pi + 1/3*arctan2(0, d*x + c)) - (-I*gamma(-1/2, I*b*conjugate((d*x + c)^(-2
/3))) + I*gamma(-1/2, -I*b/(d*x + c)^(2/3)))*sin(1/4*pi + 1/3*arctan2(0, d*
x + c)) - (-I*gamma(-1/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(-1/2,
 I*b/(d*x + c)^(2/3)))*sin(-1/4*pi + 1/3*arctan2(0, d*x + c)))*sin(a))*sqrt
(b/(d*x + c)^(2/3))*e^(-2/3)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(e^(-2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*x + c
)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(2/3), x)
```

3.254 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=141

$$-\frac{3\sqrt{\pi}\,\sqrt[3]{c+dx}\,\cos(a)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\,\sqrt{b}\,de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{\pi}\,\sqrt[3]{c+dx}\,C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{\sqrt{2}\,\sqrt{b}\,de\sqrt[3]{e(c+dx)}}$$

```
[Out] -3/2*(d*x+c)^(1/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*
Pi^(1/2)/d/e/(e*(d*x+c))^(1/3)*2^(1/2)/b^(1/2)-3/2*(d*x+c)^(1/3)*FresnelC(b
^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*Pi^(1/2)/d/e/(e*(d*x+c))^(1/3
)*2^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3516, 3498, 3464, 3434, 3433, 3432}

$$-\frac{3\sqrt{\pi}\,\sin(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\,\sqrt{b}\,de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{\pi}\,\cos(a)\sqrt[3]{c+dx}\,S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\,\sqrt{b}\,de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(Sqrt[2]*Sqrt[b]*d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[Pi]*(c + d*x)^
(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(Sqrt[2]*Sqrt[
b]*d*e*(e*(c + d*x))^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3464

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{4/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{4/3}}\,dx, x, c+dx\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\,\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{\left(3\sqrt[3]{c+dx}\,\sin(a\right.}{}$$

$$= -\frac{3\sqrt{\pi}\,\sqrt[3]{c+dx}\,\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\,\sqrt{b}\,de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{\pi}\,\sqrt[3]{c+dx}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{\sqrt{2}\,\sqrt{b}\,de\sqrt[3]{e(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 96, normalized size = 0.68

$$-\frac{3\sqrt{\frac{\pi}{2}}\,(c+dx)^{4/3}\left(\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)\right)}{\sqrt{b}\,d(e(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*Sqrt[Pi/2]*(c + d*x)^(4/3)*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[
b]*d*(e*(c + d*x))^(4/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 486, normalized size = 3.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(((-I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))) - 1) + I*sqr
t(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/3*arctan2(0, d*
x + c)) + (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3)))) - 1) - I
*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/3*arctan2(0
, d*x + c)) - (sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))) - 1) +
sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/3*arctan2(0,
 d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3)))) - 1) +
sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi + 1/3*arctan2(0,
 d*x + c)))*cos(a) - ((sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3))))
 - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/3*ar
ctan2(0, d*x + c)) + (sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/3))))
 - 1) + sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/3*ar
ctan2(0, d*x + c)) - (I*sqrt(pi)*(erf(sqrt(I*b*conjugate((d*x + c)^(-2/3)))
) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/3
*arctan2(0, d*x + c)) - (I*sqrt(pi)*(erf(sqrt(-I*b*conjugate((d*x + c)^(-2/
3)))) - 1) - I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi +
 1/3*arctan2(0, d*x + c)))*sin(a))*e^(-4/3)/((d*x + c)^(1/3)*d*sqrt(b/(d*x
+ c)^(2/3)))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(2/3)*e^(-4/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*
x + c))/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(4/3), x)
```

3.255 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=47

$$\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

[Out] 3/2*(d*x+c)^(2/3)*cos(a+b/(d*x+c)^(2/3))/b/d/e/(e*(d*x+c))^(2/3)

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3516, 3462, 3460, 2718}

$$\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]

[Out] (3*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e*(e*(c + d*x))^(2/3)
)

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{5/3}}\,dx,x,c+dx\right)}{d}\\ &= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{5/3}}\,dx,x,c+dx\right)}{de(e(c+dx))^{2/3}}\\ &= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin(a+bx)\,dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}\\ &= \frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.94

$$\frac{3(c+dx)^{5/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bd(e(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
[Out] (3*(c + d*x)^(5/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(5/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [A]
time = 0.30, size = 30, normalized size = 0.64

$$\frac{3\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)e^{\left(-\frac{5}{3}\right)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] 3/2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3))*e^(-5/3)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.
time = 0.37, size = 48, normalized size = 1.02

$$\frac{3\,(dx+c)\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)e^{\left(-\frac{5}{3}\right)}}{2\,(bd^2x+bcd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] 3/2*(d*x + c)*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))*e^(-5/3)/(b*
d^2*x + b*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e\,(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(5/3), x)
```

3.256 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}}-\frac{3\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

```
[Out] 3/2*cos(a+b/(d*x+c)^(2/3))/b/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(1/3)-3/2*(d*x
+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^2/(e*(d*x+c))^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3516, 3462, 3460, 3377, 2717}

$$\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}}-\frac{3\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/
3)) - (3*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)])/(2*b^2*d*e^2*(e*(c + d
*x))^(1/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3462

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{7/3}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{7/3}}\,dx, x, c+dx\right)}{de^2\sqrt[3]{e(c+dx)}}\\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2de^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}} - \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,\sqrt[3]{e(c+dx)}} - \frac{3\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 72, normalized size = 0.76

$$-\frac{3(c+dx)^{5/3}\left(-b\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2b^2d(e(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(5/3)*(-(b*Cos[a + b/(c + d*x)^(2/3)]) + (c + d*x)^(2/3)*Sin[
a + b/(c + d*x)^(2/3)]))/(2*b^2*d*(e*(c + d*x))^(7/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 128, normalized size = 1.35

$$\frac{3\left(\left(-i\Gamma\left(2,ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\Gamma\left(2,-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\Gamma\left(2,\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)+i\Gamma\left(2,-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos(a)-\left(\Gamma\left(2,ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(2,-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(2,\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)+\Gamma\left(2,-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin(a)\right)e^{\left(-\frac{7}{3}\right)}}{8\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] 3/8*((-I*gamma(2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(2, -I*b*conjug
ate((d*x + c)^(-2/3))) - I*gamma(2, I*b/(d*x + c)^(2/3)) + I*gamma(2, -I*b/
(d*x + c)^(2/3)))*cos(a) - (gamma(2, I*b*conjugate((d*x + c)^(-2/3))) + gam
ma(2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(2, I*b/(d*x + c)^(2/3)) + g
amma(2, -I*b/(d*x + c)^(2/3)))*sin(a))*e^(-7/3)/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(52) = 104.
time = 0.81, size = 110, normalized size = 1.16

$$\frac{3\left((dx+c)^{\frac{4}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)e^{\frac{2}{3}}-(dx+c)^2e^{\frac{2}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)\right)e^{(-3)}}{2\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] 3/2*((d*x + c)^(4/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))*e^(
2/3) - (d*x + c)^2*e^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))
)*e^(-3)/(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{b}{(c+d\,x)^{2/3}}\right)}{(c\,e+d\,e\,x)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(7/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(7/3), x)
```

3.257 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal. Leaf size=237

$$\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}}+\frac{9\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}\cos(a)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}+\frac{9\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\text{si}}{4b^{5/2}de^2(e(c+dx))^{2/3}}$$

```
[Out] 3/2*cos(a+b/(d*x+c)^(2/3))/b/d/e^2/(d*x+c)^(1/3)/(e*(d*x+c))^(2/3)-9/4*(d*x
+c)^(1/3)*sin(a+b/(d*x+c)^(2/3))/b^2/d/e^2/(e*(d*x+c))^(2/3)+9/8*(d*x+c)^(2
/3)*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*2^(1/2)*Pi^(1/2
)/b^(5/2)/d/e^2/(e*(d*x+c))^(2/3)+9/8*(d*x+c)^(2/3)*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)/(d*x+c)^(1/3))*sin(a)*2^(1/2)*Pi^(1/2)/b^(5/2)/d/e^2/(e*(d*x+c))
^(2/3)
```

Rubi [A]

time = 0.17, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3516, 3498, 3496, 3490, 3466, 3467, 3434, 3433, 3432}

$$\frac{9\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}+\frac{9\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}-\frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}}+\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(2/
3)) + (9*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
 + d*x)^(1/3)])/(4*b^(5/2)*d*e^2*(e*(c + d*x))^(2/3)) + (9*Sqrt[Pi/2]*(c +
d*x)^(2/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(4*b^(5/2
)*d*e^2*(e*(c + d*x))^(2/3)) - (9*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*b^2*d*e^2*(e*(c + d*x))^(2/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
```

```
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{8/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{8/3}}\,dx, x, c+dx\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^6}\,dx, x, \sqrt[3]{c+dx}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}} - \frac{\left(9(c+dx)^{2/3}\right)\text{Subst}\left(\int x^2\cos\left(a+bx^2\right)\,dx, x, \frac{}{\sqrt[3]{}}\right.}{2bde^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\right)\text{S}}{}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\,\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\text{co}\right.}{}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\,(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}\,(c+dx)^{2/3}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.62, size = 165, normalized size = 0.70

$$\frac{(c+dx)^{5/3}\left(9\sqrt{2\pi}\,(c+dx)\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+9\sqrt{2\pi}\,(c+dx)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)+6\sqrt{b}\left(2b\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)-3(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)\right)}{8b^{5/2}d(e(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] ((c + d*x)^(5/3)*(9*Sqrt[2*Pi]*(c + d*x)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)] + 9*Sqrt[2*Pi]*(c + d*x)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/
(c + d*x)^(1/3)]*Sin[a] + 6*Sqrt[b]*(2*b*Cos[a + b/(c + d*x)^(2/3)] - 3*(c
+ d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(e*(c + d*x))^(8/3)
)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 442, normalized size = 1.86

$$\frac{3\,(dx+c)^{\frac{1}{3}}\left(\left(\left(i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2}, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)+\left(-i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)+\left(e^{\frac{1}{3}}\Gamma\left(\frac{5}{2}, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)-\left(e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)\right)\cos(a)+\left(\left(e^{\frac{1}{3}}\Gamma\left(\frac{5}{2}, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)+\left(e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)+\left(-i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2}, i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)+\left(-i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},-i\,b\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\,e^{\frac{1}{3}}\Gamma\left(\frac{5}{2},\frac{i\,b}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{3}\arctan(0,dx+c)\right)\right)\sin(a)\right)}{8\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

```
[Out] 3/8*(d*x + c)^(1/3)*(((I*e^(1/3)*gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))
) - I*e^(1/3)*gamma(5/2, -I*b/(d*x + c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0,
 d*x + c)) + (-I*e^(1/3)*gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + I*e
^(1/3)*gamma(5/2, I*b/(d*x + c)^(2/3)))*cos(-5/4*pi + 5/3*arctan2(0, d*x +
c)) + (e^(1/3)*gamma(5/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(1/3)*gamma(
5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*pi + 5/3*arctan2(0, d*x + c)) - (e^(1/3
)*gamma(5/2, -I*b*conjugate((d*x + c)^(-2/3))) + e^(1/3)*gamma(5/2, I*b/(d*
x + c)^(2/3)))*sin(-5/4*pi + 5/3*arctan2(0, d*x + c)))*cos(a) + ((e^(1/3)*g
amma(5/2, I*b*conjugate((d*x + c)^(-2/3))) + e^(1/3)*gamma(5/2, -I*b/(d*x +
 c)^(2/3)))*cos(5/4*pi + 5/3*arctan2(0, d*x + c)) + (e^(1/3)*gamma(5/2, -I*
b*conjugate((d*x + c)^(-2/3))) + e^(1/3)*gamma(5/2, I*b/(d*x + c)^(2/3)))*c
os(-5/4*pi + 5/3*arctan2(0, d*x + c)) + (-I*e^(1/3)*gamma(5/2, I*b*conjugat
e((d*x + c)^(-2/3))) + I*e^(1/3)*gamma(5/2, -I*b/(d*x + c)^(2/3)))*sin(5/4*
pi + 5/3*arctan2(0, d*x + c)) + (-I*e^(1/3)*gamma(5/2, -I*b*conjugate((d*x
+ c)^(-2/3))) + I*e^(1/3)*gamma(5/2, I*b/(d*x + c)^(2/3)))*sin(-5/4*pi + 5/
3*arctan2(0, d*x + c)))*sin(a))/((d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)*
(b/(d*x + c)^(2/3))^(5/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)*e^(-8/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*
x + c))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(8/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+\frac{b}{{(c+d\,x)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{8/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(8/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(8/3), x)
```

3.258 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx$

Optimal. Leaf size=277

$$-\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}+\frac{45\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{45\sqrt{\pi}\sqrt[3]{\ }\cdots}{8\cdots}$$

```
[Out] -45/8*cos(a+b/(d*x+c)^(2/3))/b^3/d/e^3/(e*(d*x+c))^(1/3)+3/2*cos(a+b/(d*x+c
)^(2/3))/b/d/e^3/(d*x+c)^(4/3)/(e*(d*x+c))^(1/3)-15/4*sin(a+b/(d*x+c)^(2/3)
)/b^2/d/e^3/(d*x+c)^(2/3)/(e*(d*x+c))^(1/3)+45/16*(d*x+c)^(1/3)*cos(a)*Fres
nelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))*Pi^(1/2)/b^(7/2)/d/e^3/(e*(d*x
+c))^(1/3)*2^(1/2)-45/16*(d*x+c)^(1/3)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d
*x+c)^(1/3))*sin(a)*Pi^(1/2)/b^(7/2)/d/e^3/(e*(d*x+c))^(1/3)*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3516, 3498, 3496, 3490, 3466, 3467, 3435, 3433, 3432}

$$\frac{45\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{45\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}}-\frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

```
[Out] (-45*Cos[a + b/(c + d*x)^(2/3)])/(8*b^3*d*e^3*(e*(c + d*x))^(1/3)) + (3*Cos
[a + b/(c + d*x)^(2/3)])/(2*b*d*e^3*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)) +
(45*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(8*Sqrt[2]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (45*Sqrt[Pi]*(c +
d*x)^(1/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2
]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (15*Sin[a + b/(c + d*x)^(2/3)])/(4*b
^2*d*e^3*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3490

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3496

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3498

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3516

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(h*(x/f))^m*(a + b
```

```
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{10/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{10/3}}\,dx, x, c+dx\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^8}\,dx, x, \sqrt[3]{c+dx}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^6\sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^3\sqrt[3]{e(c+dx)}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{\left(15\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^4\cos\left(a+bx^2\right)\,dx, x,\right.}{2bde^3\sqrt[3]{e(c+dx)}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{\left(45\sqrt[3]{c+dx}\right.}{}$$

$$= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{}{(c+d}\right.}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{}}$$

$$= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{}{(c+d}\right.}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{}}$$

$$= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} + \frac{45\sqrt{\pi}\,\sqrt[3]{c+dx}\,\cos}{8\sqrt{2}\,b^{7/2}de^3}$$

Mathematica [A]

time = 0.85, size = 192, normalized size = 0.69

$$\frac{(e(c+dx))^{2/3}\left(45\sqrt{2\pi}\,(c+dx)^{5/3}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)-45\sqrt{2\pi}\,(c+dx)^{5/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)-6\sqrt{b}\left((-4b^2+15(c+dx)^{4/3})\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+10b(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)\right)}{16b^{7/2}de^4(c+dx)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

```
[Out] ((e*(c + d*x))^(2/3)*(45*Sqrt[2*Pi]*(c + d*x)^(5/3)*Cos[a]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)^(1/3)] - 45*Sqrt[2*Pi]*(c + d*x)^(5/3)*FresnelS[(Sq
rt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] - 6*Sqrt[b]*((-4*b^2 + 15*(c + d*
x)^(4/3))*Cos[a + b/(c + d*x)^(2/3)] + 10*b*(c + d*x)^(2/3)*Sin[a + b/(c +
d*x)^(2/3)])))/(16*b^(7/2)*d*e^4*(c + d*x)^(7/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 402, normalized size = 1.45

$$\frac{3\left(\left(\left(i\Gamma\left(\frac{7}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)-i\Gamma\left(\frac{7}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(-i\Gamma\left(\frac{7}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\Gamma\left(\frac{7}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(\Gamma\left(\frac{7}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(\frac{7}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)-\left(\Gamma\left(\frac{7}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(\frac{7}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)\right)\cos(a)+\left(\left(\Gamma\left(\frac{7}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(\frac{7}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(\Gamma\left(\frac{7}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+\Gamma\left(\frac{7}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(-i\Gamma\left(\frac{7}{2},ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\Gamma\left(\frac{7}{2},-\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)+\left(-i\Gamma\left(\frac{7}{2},-ib\overline{\frac{1}{(dx+c)^{\frac{2}{3}}}}\right)+i\Gamma\left(\frac{7}{2},\frac{ib}{(dx+c)^{\frac{2}{3}}}\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{3}\arctan(0,dx+c)\right)\right)\sin(a)\right)}{8\left(d^3x^2e^{\frac{10}{3}}+2cd^2xe^{\frac{10}{3}}+c^2de^{\frac{10}{3}}\right)(dx+c)^{\frac{1}{3}}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="maxima")
```

```
[Out] 3/8*(((I*gamma(7/2, I*b*conjugate((d*x + c)^(-2/3))) - I*gamma(7/2, -I*b/(d
*x + c)^(2/3)))*cos(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, -I*b
*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4
*pi + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, I*b*conjugate((d*x + c)^(-2/3)
)) + gamma(7/2, -I*b/(d*x + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)
) - (gamma(7/2, -I*b*conjugate((d*x + c)^(-2/3))) + gamma(7/2, I*b/(d*x + c
)^(2/3)))*sin(-7/4*pi + 7/3*arctan2(0, d*x + c)))*cos(a) + ((gamma(7/2, I*b
*conjugate((d*x + c)^(-2/3))) + gamma(7/2, -I*b/(d*x + c)^(2/3)))*cos(7/4*p
i + 7/3*arctan2(0, d*x + c)) + (gamma(7/2, -I*b*conjugate((d*x + c)^(-2/3))
) + gamma(7/2, I*b/(d*x + c)^(2/3)))*cos(-7/4*pi + 7/3*arctan2(0, d*x + c))
 + (-I*gamma(7/2, I*b*conjugate((d*x + c)^(-2/3))) + I*gamma(7/2, -I*b/(d*x
```

```
 + c)^(2/3)))*sin(7/4*pi + 7/3*arctan2(0, d*x + c)) + (-I*gamma(7/2, -I*b*c
onjugate((d*x + c)^(-2/3))) + I*gamma(7/2, I*b/(d*x + c)^(2/3)))*sin(-7/4*p
i + 7/3*arctan2(0, d*x + c)))*sin(a))/((d^3*x^2*e^(10/3) + 2*c*d^2*x*e^(10/
3) + c^2*d*e^(10/3))*(d*x + c)^(1/3)*(b/(d*x + c)^(2/3))^(7/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(2/3)*e^(-10/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d
*x + c))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*x*e + c*e)^(10/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+\frac{b}{{\left(c+d\,x\right)}^{2/3}}\right)}{{\left(c\,e+d\,e\,x\right)}^{10/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(10/3),x)
```

```
[Out] int(sin(a + b/(c + d*x)^(2/3))/(c*e + d*e*x)^(10/3), x)
```

3.259 $\int (ex)^m \sin\left(a+b(c+dx)^n\right)\,dx$

Optimal. Leaf size=21

$$\text{Int}\left((ex)^m \sin\left(a+b(c+dx)^n\right),x\right)$$

```
[Out] Unintegrable((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m \sin\left(a+b(c+dx)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
[Out] Defer[Int][(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Rubi steps

$$\int (ex)^m \sin\left(a+b(c+dx)^n\right)\,dx = \int (ex)^m \sin\left(a+b(c+dx)^n\right)\,dx$$

Mathematica [A]
time = 5.75, size = 0, normalized size = 0.00

$$\int (ex)^m \sin\left(a+b(c+dx)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
[Out] Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex)^m \sin\left(a+b(dx+c)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate((x*e)^m*sin((d*x + c)^n*b + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral((x*e)^m*sin((d*x + c)^n*b + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sin(a + b(c + dx)^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(a+b*(d*x+c)**n),x)
[Out] Integral((e*x)**m*sin(a + b*(c + d*x)**n), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="giac")
[Out] integrate((x*e)^m*sin((d*x + c)^n*b + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sin\left(a + b\,(c + d\,x)^n\right)\,(e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^n)*(e*x)^m,x)
[Out] int(sin(a + b*(c + d*x)^n)*(e*x)^m, x)
```

3.260 $\int x^3 \sin\left(a + b(c + dx)^n\right) \, dx$

Optimal. Leaf size=503

$$-\frac{ic^3e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n}+\frac{ic^3e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^4n}$$

```
[Out] -1/2*I*c^3*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^
n)^(1/n))+1/2*I*c^3*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(
d*x+c)^n)^(1/n))+3/2*I*c^2*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^4
/n/((-I*b*(d*x+c)^n)^(2/n))-3/2*I*c^2*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^
4/exp(I*a)/n/((I*b*(d*x+c)^n)^(2/n))-3/2*I*c*exp(I*a)*(d*x+c)^3*GAMMA(3/n,-
I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^n)^(3/n))+3/2*I*c*(d*x+c)^3*GAMMA(3/n,I
*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(3/n))+1/2*I*exp(I*a)*(d*x+c)
^4*GAMMA(4/n,-I*b*(d*x+c)^n)/d^4/n/((-I*b*(d*x+c)^n)^(4/n))-1/2*I*(d*x+c)^4
*GAMMA(4/n,I*b*(d*x+c)^n)/d^4/exp(I*a)/n/((I*b*(d*x+c)^n)^(4/n))
```

Rubi [A]

time = 0.27, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3514, 3446, 2239, 3504, 2250}

$$-\frac{ie^{ia}c^3(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n}+\frac{ie^{-ia}c^3(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^4n}+\frac{3ie^{ia}c^2(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^4n}-\frac{3ie^{-ia}c^2(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^4n}+\frac{ie^{ia}(c+dx)^4\left(-ib(c+dx)^n\right)^{-4/n}\text{Gamma}\left(\frac{4}{n},-ib(c+dx)^n\right)}{2d^4n}-\frac{3ie^{ia}c(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^4n}+\frac{3ie^{-ia}c(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^4n}-\frac{ie^{-ia}(c+dx)^4\left(ib(c+dx)^n\right)^{-4/n}\text{Gamma}\left(\frac{4}{n},ib(c+dx)^n\right)}{2d^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-1/2*I)*c^3*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^4*n*(
(-I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c^3*(c + d*x)*Gamma[n^(-1), I*b*(c + d
*x)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + (((3*I)/2)*c^2*E^(I*a)*(
c + d*x)^2*Gamma[2/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(2/n
)) - (((3*I)/2)*c^2*(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n
*(I*b*(c + d*x)^n)^(2/n)) - (((3*I)/2)*c*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I
)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(3/n)) + (((3*I)/2)*c*(c + d*
x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n)) +
 ((I/2)*E^(I*a)*(c + d*x)^4*Gamma[4/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*
(c + d*x)^n)^(4/n)) - ((I/2)*(c + d*x)^4*Gamma[4/n, I*b*(c + d*x)^n])/(d^4*
E^(I*a)*n*(I*b*(c + d*x)^n)^(4/n))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\sin\left(a+b(c+dx)^n\right)\,dx &= \frac{\mathrm{Subst}\left(\int\left(-c^3\sin\left(a+bx^n\right)+3c^2x\sin\left(a+bx^n\right)-3cx^2\sin\left(a+bx^n\right)+x^3\sin\right.\right.}{d^4}\\
&= \frac{\mathrm{Subst}\left(\int x^3\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^4}-\frac{(3c)\mathrm{Subst}\left(\int x^2\sin\left(a+bx^n\right)\,dx,\right.}{d^4}\\
&= \frac{i\mathrm{Subst}\left(\int e^{-ia-ibx^n}x^3\,dx,x,c+dx\right)}{2d^4}-\frac{i\mathrm{Subst}\left(\int e^{ia+ibx^n}x^3\,dx,x,c+dx\right)}{2d^4}-\\
&= -\frac{ic^3e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n}+\frac{ic^3e^{-ia}(c+dx)\left(i\right.}{}\end{aligned}$$

Mathematica [F]
time = 9.18, size = 0, normalized size = 0.00

$$\int x^3\sin\left(a+b(c+dx)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3*Sin[a + b*(c + d*x)^n],x]
[Out] Integrate[x^3*Sin[a + b*(c + d*x)^n], x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3 \sin\left(a + b(dx + c)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(a+b*(d*x+c)^n),x)
[Out] int(x^3*sin(a+b*(d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
[Out] integral(x^3*sin((d*x + c)^n*b + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sin\left(a + b(c + dx)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(a+b*(d*x+c)**n),x)
[Out] Integral(x**3*sin(a + b*(c + d*x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sin\left(a+b\left(c+d\,x\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(a + b*(c + d*x)^n),x)
```

```
[Out] int(x^3*sin(a + b*(c + d*x)^n), x)
```

3.261 $\int x^2 \sin \left(a+b(c+dx)^n\right) \, dx$

Optimal. Leaf size=369

$$\frac{ic^2e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n}-\frac{ic^2e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n}-$$

```
[Out] 1/2*I*c^2*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^3/n/((-I*b*(d*x+c)^n
)^(1/n))-1/2*I*c^2*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^3/exp(I*a)/n/((I*b*(d
*x+c)^n)^(1/n))-I*c*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^3/n/((-I
*b*(d*x+c)^n)^(2/n))+I*c*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^3/exp(I*a)/n/
((I*b*(d*x+c)^n)^(2/n))+1/2*I*exp(I*a)*(d*x+c)^3*GAMMA(3/n,-I*b*(d*x+c)^n)/
d^3/n/((-I*b*(d*x+c)^n)^(3/n))-1/2*I*(d*x+c)^3*GAMMA(3/n,I*b*(d*x+c)^n)/d^3
/exp(I*a)/n/((I*b*(d*x+c)^n)^(3/n))
```

Rubi [A]
time = 0.17, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3514, 3446, 2239, 3504, 2250}

$$\frac{ie^{ia}c^2(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n}-\frac{ie^{-ia}c^2(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n}+\frac{ie^{ia}(c+dx)^3\left(-ib(c+dx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},-ib(c+dx)^n\right)}{2d^3n}-\frac{ie^{ia}c(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{d^3n}+\frac{ie^{-ia}c(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},ib(c+dx)^n\right)}{d^3n}-\frac{ie^{-ia}(c+dx)^3\left(ib(c+dx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},ib(c+dx)^n\right)}{2d^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((I/2)*c^2*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^n^(-1)) - ((I/2)*c^2*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)
^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) - (I*c*E^(I*a)*(c + d*x)^2*Ga
mma[2/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I)*b*(c + d*x)^n)^(2/n)) + (I*c*(c
+ d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(2/n
)) + ((I/2)*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^(3/n)) - ((I/2)*(c + d*x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(
d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \sin\left(a+b(c+dx)^n\right)\,dx &= \frac{\text{Subst}\left(\int \left(c^2 \sin\left(a+bx^n\right)-2cx\sin\left(a+bx^n\right)+x^2\sin\left(a+bx^n\right)\right)\,dx,x,c+d\right.}{d^3}\\ &= \frac{\text{Subst}\left(\int x^2\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^3}-\frac{(2c)\text{Subst}\left(\int x\sin\left(a+bx^n\right)\,dx,x\right.}{d^3}\\ &= \frac{i\text{Subst}\left(\int e^{-ia-ibx^n}x^2\,dx,x,c+dx\right)}{2d^3}-\frac{i\text{Subst}\left(\int e^{ia+ibx^n}x^2\,dx,x,c+dx\right)}{2d^3}-\\ &= \frac{ic^2e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n}-\frac{ic^2e^{-ia}(c+dx)\left(ib(\right.}{}\end{aligned}$$

Mathematica [F]
time = 5.20, size = 0, normalized size = 0.00

$$\int x^2\sin\left(a+b(c+dx)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sin[a + b*(c + d*x)^n],x]
[Out] Integrate[x^2*Sin[a + b*(c + d*x)^n], x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \sin\left(a+b(dx+c)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(x^2*sin(a+b*(d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(x^2*sin((d*x + c)^n*b + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sin\left(a+b(c+dx)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(x**2*sin(a + b*(c + d*x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sin\left(a + b\left(c + d\,x\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*(c + d*x)^n),x)
```

```
[Out] int(x^2*sin(a + b*(c + d*x)^n), x)
```

3.262 $\int x \sin\left(a + b(c + dx)^n\right)\, dx$

Optimal. Leaf size=243

$$-\frac{ice^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}+\frac{ice^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^2n}+$$

```
[Out] -1/2*I*c*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d^2/n/((-I*b*(d*x+c)^n)
^(1/n))+1/2*I*c*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d^2/exp(I*a)/n/((I*b*(d*x+
c)^n)^(1/n))+1/2*I*exp(I*a)*(d*x+c)^2*GAMMA(2/n,-I*b*(d*x+c)^n)/d^2/n/((-I*
b*(d*x+c)^n)^(2/n))-1/2*I*(d*x+c)^2*GAMMA(2/n,I*b*(d*x+c)^n)/d^2/exp(I*a)/n
/((I*b*(d*x+c)^n)^(2/n))
```

Rubi [A]

time = 0.10, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3514, 3446, 2239, 3504, 2250}

$$\frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^2n}-\frac{ie^{ia}c(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}+\frac{ie^{-ia}c(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^2n}-\frac{ie^{-ia}(c+dx)^2\left(ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},ib(c+dx)^n\right)}{2d^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-1/2*I)*c*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^2*n*((-
I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^
n])/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*E^(I*a)*(c + d*x)^2*G
amma[2/n, (-I)*b*(c + d*x)^n])/(d^2*n*((-I)*b*(c + d*x)^n)^(2/n)) - ((I/2)*
(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^(
2/n))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
```

```
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin\left(a+b(c+dx)^n\right)\,dx &= \frac{\text{Subst}\left(\int\left(-c\sin\left(a+bx^n\right)+x\sin\left(a+bx^n\right)\right)\,dx,x,c+dx\right)}{d^2}\\ &= \frac{\text{Subst}\left(\int x\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^2}-\frac{c\text{Subst}\left(\int\sin\left(a+bx^n\right)\,dx,x,c+d\right.}{d^2}\\ &= \frac{i\text{Subst}\left(\int e^{-ia-ibx^n}x\,dx,x,c+dx\right)}{2d^2}-\frac{i\text{Subst}\left(\int e^{ia+ibx^n}x\,dx,x,c+dx\right)}{2d^2}-\frac{(ic)}{}\\ &= -\frac{ice^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}+\frac{ice^{-ia}(c+dx)\,(ib(c}{}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 192, normalized size = 0.79

$$\frac{(c+dx)\left(\left(-ib(c+dx)^n\right)^{-2/n}\left(c\left(-ib(c+dx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)-(c+dx)\Gamma\left(\frac{2}{n},-ib(c+dx)^n\right)\right)(-i\cos(a)+\sin(a))+\left(ib(c+dx)^n\right)^{-2/n}\left(c\left(ib(c+dx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)-(c+dx)\Gamma\left(\frac{2}{n},ib(c+dx)^n\right)\right)(i\cos(a)+\sin(a))\right)}{2d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((c + d*x)*(((c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*b*(c + d*x)^
n] - (c + d*x)*Gamma[2/n, (-I)*b*(c + d*x)^n])*((-I)*Cos[a] + Sin[a]))/((-I
)*b*(c + d*x)^n)^(2/n) + ((c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), I*b*(c
+ d*x)^n] - (c + d*x)*Gamma[2/n, I*b*(c + d*x)^n])*(I*Cos[a] + Sin[a]))/(I*
b*(c + d*x)^n)^(2/n)))/(2*d^2*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\sin\left(a+b(dx+c)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(x*sin(a+b*(d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(x*sin((d*x + c)^n*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(x*sin((d*x + c)^n*b + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(a+b(c+dx)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(x*sin(a + b*(c + d*x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(x*sin((d*x + c)^n*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \sin \left(a + b \left(c + d x\right)^n\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*(c + d*x)^n),x)
```

```
[Out] int(x*sin(a + b*(c + d*x)^n), x)
```

3.263 $\int \sin \left(a+b(c+dx)^n\right) \, dx$

Optimal. Leaf size=117

$$\frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

```
[Out] 1/2*I*exp(I*a)*(d*x+c)*GAMMA(1/n,-I*b*(d*x+c)^n)/d/n/((-I*b*(d*x+c)^n)^(1/n
))-1/2*I*(d*x+c)*GAMMA(1/n,I*b*(d*x+c)^n)/d/exp(I*a)/n/((I*b*(d*x+c)^n)^(1/
n))
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3446, 2239}

$$\frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((I/2)*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d*n*((-I)*b*(c
 + d*x)^n)^n^(-1)) - ((I/2)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n])/(d*E^
(I*a)*n*(I*b*(c + d*x)^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sin \left(a+b(c+dx)^n\right) \, dx &= \frac{1}{2}i\int e^{-ia-ib(c+dx)^n} \, dx-\frac{1}{2}i\int e^{ia+ib(c+dx)^n} \, dx\\ &= \frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 121, normalized size = 1.03

$$-\frac{i(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)(\cos(a)-i\sin(a))}{2dn}+\frac{i(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)(\cos(a)+i\sin(a))}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-1/2*I)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n]*(Cos[a] - I*Sin[a]))/(d*
n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*
x)^n]*(Cos[a] + I*Sin[a]))/(d*n*((-I)*b*(c + d*x)^n)^n^(-1))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(a+b(dx+c)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(sin(a+b*(d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^n*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^n*b + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+b(c+dx)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^n*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin \left(a+b\,{\left(c+d\,x\right)}^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^n),x)
```

```
[Out] int(sin(a + b*(c + d*x)^n), x)
```

3.264 $\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\sin\left(a+b(c+dx)^n\right)}{x},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^n)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^n]/x,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^n]/x, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x}\,dx = \int \frac{\sin\left(a+b(c+dx)^n\right)}{x}\,dx$$

Mathematica [A]
time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^n]/x,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^n]/x, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^n)/x,x)
[Out] int(sin(a+b*(d*x+c)^n)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^n*b + a)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="fricas")
[Out] integral(sin((d*x + c)^n*b + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(a + b(c + dx)^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**n)/x,x)
[Out] Integral(sin(a + b*(c + d*x)**n)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^n*b + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sin \left(a + b\,{\left(c + d\,x\right)}^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^n)/x,x)
[Out] int(sin(a + b*(c + d*x)^n)/x, x)
```

3.265 $\int \frac{\sin(a+b(c+dx)^n)}{x^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\sin\left(a+b(c+dx)^n\right)}{x^2},x\right)$$

```
[Out] Unintegrable(sin(a+b*(d*x+c)^n)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^n]/x^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^n]/x^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

Mathematica [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^n]/x^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^n]/x^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b(dx+c)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^n)/x^2,x)
[Out] int(sin(a+b*(d*x+c)^n)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="maxima")
[Out] integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="fricas")
[Out] integral(sin((d*x + c)^n*b + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + b(c + dx)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**n)/x**2,x)
[Out] Integral(sin(a + b*(c + d*x)**n)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="giac")
[Out] integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sin\left(a + b\,{\left(c + d\,x\right)}^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*(c + d*x)^n)/x^2,x)
```

```
[Out] int(sin(a + b*(c + d*x)^n)/x^2, x)
```

3.266 $\int x^3(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal. Leaf size=519

$$\frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{ibe^{-ic}f^3(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id\right.}{2g^4n}$$

```
[Out] 1/4*a*x^4-1/2*I*b*exp(I*c)*f^3*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^4/n/((-I
*d*(g*x+f)^n)^(1/n))+1/2*I*b*f^3*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/
g^4/n/((I*d*(g*x+f)^n)^(1/n))+3/2*I*b*exp(I*c)*f^2*(g*x+f)^2*GAMMA(2/n,-I*d
*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)^(2/n))-3/2*I*b*f^2*(g*x+f)^2*GAMMA(2/n,
I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*x+f)^n)^(2/n))-3/2*I*b*exp(I*c)*f*(g
*x+f)^3*GAMMA(3/n,-I*d*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)^(3/n))+3/2*I*b*f*
(g*x+f)^3*GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*x+f)^n)^(3/n))+1
/2*I*b*exp(I*c)*(g*x+f)^4*GAMMA(4/n,-I*d*(g*x+f)^n)/g^4/n/((-I*d*(g*x+f)^n)
^(4/n))-1/2*I*b*(g*x+f)^4*GAMMA(4/n,I*d*(g*x+f)^n)/exp(I*c)/g^4/n/((I*d*(g*
x+f)^n)^(4/n))
```

Rubi [A]
time = 0.36, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {14, 3514, 3446, 2239, 3504, 2250}

$$-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{ibe^{-ic}f^3(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2g^4n}+\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n}-\frac{3ibe^{-ic}f^2(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},id(f+gx)^n\right)}{2g^4n}+\frac{ibe^{ic}(f+gx)^4\left(-id(f+gx)^n\right)^{-4/n}\text{Gamma}\left(\frac{4}{n},-id(f+gx)^n\right)}{2g^4n}-\frac{3ibe^{ic}f(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{3ibe^{-ic}f(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},id(f+gx)^n\right)}{2g^4n}-\frac{ibe^{-ic}(f+gx)^4\left(id(f+gx)^n\right)^{-4/n}\text{Gamma}\left(\frac{4}{n},id(f+gx)^n\right)}{2g^4n}+\frac{ax^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^4)/4 - ((I/2)*b*E^(I*c)*f^3*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^4*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f^3*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^n^(-1)) + (((3*I)/2)*
b*E^(I*c)*f^2*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f
 + g*x)^n)^(2/n)) - (((3*I)/2)*b*f^2*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n
])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(2/n)) - (((3*I)/2)*b*E^(I*c)*f*(f + g*
x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(3/n)) + (
((3*I)/2)*b*f*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*
(f + g*x)^n)^(3/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^4*Gamma[4/n, (-I)*d*(f + g
*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) - ((I/2)*b*(f + g*x)^4*Gamma[4/n
, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3(a+b\sin(c+d(f+gx)^n))\,dx = \int \left(ax^3+bx^3\sin(c+d(f+gx)^n)\right)\,dx$$
$$= \frac{ax^4}{4}+b\int x^3\sin(c+d(f+gx)^n)\,dx$$
$$= \frac{ax^4}{4}+\frac{b\mathrm{Subst}\left(\int\left(-f^3\sin(c+dx^n)+3f^2x\sin(c+dx^n)-3fx^2\sin\right.\right.}{g^4}$$
$$= \frac{ax^4}{4}+\frac{b\mathrm{Subst}\left(\int x^3\sin(c+dx^n)\,dx,x,f+gx\right)}{g^4}-\frac{(3bf)\mathrm{Subst}\left(\int x^2\right.}{}$$
$$= \frac{ax^4}{4}+\frac{(ib)\mathrm{Subst}\left(\int e^{-ic-idx^n}x^3\,dx,x,f+gx\right)}{2g^4}-\frac{(ib)\mathrm{Subst}\left(\int e^{ic+idx^n}\right.}{2g^4}$$
$$= \frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{ib}{}$$

Mathematica [A]
time = 10.91, size = 539, normalized size = 1.04

$$\frac{1}{4}\left(ax^4-\frac{2ib(f+gx)\left(d^2(f+gx)^{2n}\right)^{-4/n}\left(-f^3(-id(f+gx)^n)^{4/n}(id(f+gx)^n)^{3/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\left(-3f^2(-id(f+gx)^n)^{4/n}(id(f+gx)^n)^{2/n}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)-(f+gx)\left(-3f(-id(f+gx)^n)^{4/n}(id(f+gx)^n)^{\frac{1}{n}}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)-(f+gx)\left(-(-id(f+gx)^n)^{4/n}\Gamma\left(\frac{4}{n},id(f+gx)^n\right)+(id(f+gx)^n)^{4/n}\Gamma\left(\frac{4}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)+3f(-id(f+gx)^n)^{\frac{1}{n}}(id(f+gx)^n)^{4/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)+3f^2(-id(f+gx)^n)^{2/n}(id(f+gx)^n)^{4/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)+f^3(-id(f+gx)^n)^{3/n}(id(f+gx)^n)^{4/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)(\cos(c)-i\sin(c))}{g^4n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^4 - ((2*I)*b*(f + g*x)*(-(f^3*((-I)*d*(f + g*x)^n)^(4/n)*(I*d*(f + g*x
)^n)^(3/n)*Gamma[n^(-1), I*d*(f + g*x)^n]) - (f + g*x)*(-3*f^2*((-I)*d*(f +
 g*x)^n)^(4/n)*(I*d*(f + g*x)^n)^(2/n)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g
*x)*(-3*f*((-I)*d*(f + g*x)^n)^(4/n)*(I*d*(f + g*x)^n)^n^(-1)*Gamma[3/n, I*
d*(f + g*x)^n] - (f + g*x)*(-(((-I)*d*(f + g*x)^n)^(4/n)*Gamma[4/n, I*d*(f
+ g*x)^n]) + (I*d*(f + g*x)^n)^(4/n)*Gamma[4/n, (-I)*d*(f + g*x)^n]*(Cos[c]
 + I*Sin[c])^2) + 3*f*((-I)*d*(f + g*x)^n)^n^(-1)*(I*d*(f + g*x)^n)^(4/n)*G
amma[3/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2) + 3*f^2*((-I)*d*(f + g
*x)^n)^(2/n)*(I*d*(f + g*x)^n)^(4/n)*Gamma[2/n, (-I)*d*(f + g*x)^n]*(Cos[c]
 + I*Sin[c])^2) + f^3*((-I)*d*(f + g*x)^n)^(3/n)*(I*d*(f + g*x)^n)^(4/n)*Ga
mma[n^(-1), (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2)*(Cos[c] - I*Sin[c]))
/(g^4*n*(d^2*(f + g*x)^(2*n))^(4/n)))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3(a+b\sin(c+d(gx+f)^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + b*integrate(x^3*sin((g*x + f)^n*d + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(b*x^3*sin((g*x + f)^n*d + c) + a*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\sin{(c + d(f + gx)^n)})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x**3*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x^3*(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.267 $\int x^2(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal. Leaf size=383

$$\frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f\right.}{2g^3n}$$

```
[Out] 1/3*a*x^3+1/2*I*b*exp(I*c)*f^2*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^3/n/((-I
*d*(g*x+f)^n)^(1/n))-1/2*I*b*f^2*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/
g^3/n/((I*d*(g*x+f)^n)^(1/n))-I*b*exp(I*c)*f*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+
f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(2/n))+I*b*f*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^
n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(2/n))+1/2*I*b*exp(I*c)*(g*x+f)^3*GAMMA(
3/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(3/n))-1/2*I*b*(g*x+f)^3*GAMMA(
3/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))
```

Rubi [A]
time = 0.24, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {14, 3514, 3446, 2239, 3504, 2250}

$$\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2g^3n}+\frac{ibe^{ic}(f+gx)^3\left(-id(f+gx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{ic}f(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{g^3n}+\frac{ibe^{-ic}f(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},id(f+gx)^n\right)}{g^3n}-\frac{ibe^{-ic}(f+gx)^3\left(id(f+gx)^n\right)^{-3/n}\text{Gamma}\left(\frac{3}{n},id(f+gx)^n\right)}{2g^3n}+\frac{ax^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^3)/3 + ((I/2)*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*f^2*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^(-1)) - (I*b*E^(I*c
)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)
^(2/n)) + (I*b*f*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I
*d*(f + g*x)^n)^(2/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f
+ g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)^3*Gamma[
3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2(a+b\sin(c+d(f+gx)^n))\,dx &= \int \left(ax^2+bx^2\sin(c+d(f+gx)^n)\right)\,dx\\ &= \frac{ax^3}{3}+b\int x^2\sin(c+d(f+gx)^n)\,dx\\ &= \frac{ax^3}{3}+\frac{b\text{Subst}\left(\int\left(f^2\sin(c+dx^n)-2fx\sin(c+dx^n)+x^2\sin(c+d\right.\right.}{g^3}\\ &= \frac{ax^3}{3}+\frac{b\text{Subst}\left(\int x^2\sin(c+dx^n)\,dx,x,f+gx\right)}{g^3}-\frac{(2bf)\text{Subst}\left(\int x\,\text{si}\right.}{}\\ &= \frac{ax^3}{3}+\frac{(ib)\text{Subst}\left(\int e^{-ic-idx^n}x^2\,dx,x,f+gx\right)}{2g^3}-\frac{(ib)\text{Subst}\left(\int e^{ic+idx^n}\right.}{2g^3}\\ &= \frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ib}{}\end{aligned}$$

Mathematica [A]
time = 5.69, size = 403, normalized size = 1.05

$$\frac{ax^3}{3}+\frac{ib(f+gx)\left(d^2(f+gx)^{2n}\right)^{-3/n}\left(-f^2(-id(f+gx)^n)^{3/n}(id(f+gx)^n)^{2/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\left(-2f(-id(f+gx)^n)^{3/n}(id(f+gx)^n)^{\frac{1}{n}}\Gamma\left(\frac{2}{n},id(f+gx)^n\right)-(f+gx)\left(-(-id(f+gx)^n)^{3/n}\Gamma\left(\frac{3}{n},id(f+gx)^n\right)+(id(f+gx)^n)^{3/n}\Gamma\left(\frac{3}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)+2f(-id(f+gx)^n)^{\frac{1}{n}}(id(f+gx)^n)^{3/n}\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)+f^2(-id(f+gx)^n)^{2/n}(id(f+gx)^n)^{3/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))^2\right)(\cos(c)-i\sin(c))}{2g^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^3)/3 + ((I/2)*b*(f + g*x)*(-(f^2*((-I)*d*(f + g*x)^n)^(3/n)*(I*d*(f +
g*x)^n)^(2/n)*Gamma[n^(-1), I*d*(f + g*x)^n]) - (f + g*x)*(-2*f*((-I)*d*(f
+ g*x)^n)^(3/n)*(I*d*(f + g*x)^n)^n^(-1)*Gamma[2/n, I*d*(f + g*x)^n] - (f +
 g*x)*(-(((-I)*d*(f + g*x)^n)^(3/n)*Gamma[3/n, I*d*(f + g*x)^n]) + (I*d*(f
+ g*x)^n)^(3/n)*Gamma[3/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2) + 2*f
*((-I)*d*(f + g*x)^n)^n^(-1)*(I*d*(f + g*x)^n)^(3/n)*Gamma[2/n, (-I)*d*(f +
 g*x)^n]*(Cos[c] + I*Sin[c])^2) + f^2*((-I)*d*(f + g*x)^n)^(2/n)*(I*d*(f +
g*x)^n)^(3/n)*Gamma[n^(-1), (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2)*(Cos
[c] - I*Sin[c]))/(g^3*n*(d^2*(f + g*x)^(2*n))^(3/n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(a+b\sin(c+d(gx+f)^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*sin((g*x + f)^n*d + c) + a*x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\sin{(c + d(f + gx)^n)})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x**2*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a + b\,\sin\left(c + d\,{\left(f + g\,x\right)}^n\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x^2*(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.268 $\int x(a+b\sin(c+d(f+gx)^n))\,dx$

Optimal. Leaf size=255

$$\frac{ax^2}{2}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n}+\frac{ibe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f-\right.}{2g^2n}$$

```
[Out] 1/2*a*x^2-1/2*I*b*exp(I*c)*f*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^2/n/((-I*d
*(g*x+f)^n)^(1/n))+1/2*I*b*f*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g^2/
n/((I*d*(g*x+f)^n)^(1/n))+1/2*I*b*exp(I*c)*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)
^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))-1/2*I*b*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^
n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n))
```

Rubi [A]

time = 0.13, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {14, 3514, 3446, 2239, 3504, 2250}

$$\frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n}+\frac{ibe^{-ic}f(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2g^2n}-\frac{ibe^{-ic}(f+gx)^2\left(id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},id(f+gx)^n\right)}{2g^2n}+\frac{ax^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^2)/2 - ((I/2)*b*E^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])
/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*E^(I*c
)*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(
2/n)) - ((I/2)*b*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I
*d*(f + g*x)^n)^(2/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
```

```
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a+b\sin(c+d(f+gx)^n))\,dx &= \int (ax+bx\sin(c+d(f+gx)^n))\,dx\\
&= \frac{ax^2}{2}+b\int x\sin(c+d(f+gx)^n)\,dx\\
&= \frac{ax^2}{2}+\frac{b\text{Subst}\left(\int(-f\sin(c+dx^n)+x\sin(c+dx^n))\,dx,x,f+gx\right)}{g^2}\\
&= \frac{ax^2}{2}+\frac{b\text{Subst}\left(\int x\sin(c+dx^n)\,dx,x,f+gx\right)}{g^2}-\frac{(bf)\text{Subst}\left(\int\sin(c\cdots\right.}{\cdots}\\
&= \frac{ax^2}{2}+\frac{(ib)\text{Subst}\left(\int e^{-ic-idx^n}x\,dx,x,f+gx\right)}{2g^2}-\frac{(ib)\text{Subst}\left(\int e^{ic+idx^n}x\cdots\right.}{2g^2}\\
&= \frac{ax^2}{2}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n}+\frac{ibe^{\cdots}}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 215, normalized size = 0.84

$$\frac{ax^2}{2}+\frac{b(f+gx)\left(-id(f+gx)^n\right)^{-2/n}\left(f\left(-id(f+gx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)-(f+gx)\Gamma\left(\frac{2}{n},-id(f+gx)^n\right)\right)(-i\cos(c)+\sin(c))}{2g^2n}+\frac{b(f+gx)\left(id(f+gx)^n\right)^{-2/n}\left(f\left(id(f+gx)^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)-(f+gx)\Gamma\left(\frac{2}{n},id(f+gx)^n\right)\right)(i\cos(c)+\sin(c))}{2g^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^2)/2 + (b*(f + g*x)*(f*((-I)*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*
d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n])*((-I)*Cos[c] + S
in[c]))/(2*g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (b*(f + g*x)*(f*(I*d*(f + g*
x)^n)^n^(-1)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, I*d*(f +
 g*x)^n])*(I*Cos[c] + Sin[c]))/(2*g^2*n*(I*d*(f + g*x)^n)^(2/n))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int x(a+b\sin\left(c+d(gx+f)^{n}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x*sin((g*x + f)^n*d + c) + a*x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\sin\left(c+d(f+gx)^{n}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Integral(x*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*sin(c + d*(f + g*x)^n)),x)
```

```
[Out] int(x*(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.269 $\int (a+b\sin (c+d(f+gx)^n))\,dx$

Optimal. Leaf size=122

$$ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx\right.}{2gn}$$

```
[Out] a*x+1/2*I*b*exp(I*c)*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g/n/((-I*d*(g*x+f)^n
)^(1/n))-1/2*I*b*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g/n/((I*d*(g*x+f
)^n)^(1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3446, 2239}

$$\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}+ax$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*(f + g*x)^n],x]
```

```
[Out] a*x + ((I/2)*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g*n*((
-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)
^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin (c+d(f+gx)^n))\,dx &= ax+b\int \sin (c+d(f+gx)^n)\,dx\\ &= ax+\frac{1}{2}(ib)\int e^{-ic-id(f+gx)^n}\,dx-\frac{1}{2}(ib)\int e^{ic+id(f+gx)^n}\,dx\\ &= ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f\ldots}{}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 126, normalized size = 1.03

$$ax-\frac{ib(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)(\cos(c)-i\sin(c))}{2gn}+\frac{ib(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))}{2gn}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*(f + g*x)^n],x]
```

```
[Out] a*x - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])
)/(g*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*(f + g*x)*Gamma[n^(-1), (-I)*d*
(f + g*x)^n]*(Cos[c] + I*Sin[c]))/(g*n*((-I)*d*(f + g*x)^n)^n^(-1))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int a+b\sin\left(c+d(gx+f)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(c+d*(g*x+f)^n),x)
```

```
[Out] int(a+b*sin(c+d*(g*x+f)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="maxima")
```

```
[Out] a*x + b*integrate(sin((g*x + f)^n*d + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="fricas")
```

```
[Out] integral(b*sin((g*x + f)^n*d + c) + a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+d(f+gx)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)**n),x)
```

```
[Out] Integral(a + b*sin(c + d*(f + g*x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="giac")
```

```
[Out] integrate(b*sin((g*x + f)^n*d + c) + a, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(c + d*(f + g*x)^n),x)
```

```
[Out] int(a + b*sin(c + d*(f + g*x)^n), x)
```

3.270 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx$

Optimal. Leaf size=26

$$a\log(x)+b\text{Int}\left(\frac{\sin\left(c+d(f+gx)^n\right)}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(sin(c+d*(g*x+f)^n)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
[Out] a*Log[x] + b*Defer[Int][Sin[c + d*(f + g*x)^n]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx &= \int \left(\frac{a}{x}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x}\right)dx\\ &= a\log(x)+b\int \frac{\sin\left(c+d(f+gx)^n\right)}{x}\,dx\end{aligned}$$

Mathematica [A]
time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+d(gx+f)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(sin((g*x + f)^n*d + c)/x, x) + a*log(x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sin \left(c + d(f + gx)^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))/x,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\sin\left(c+d\left(f+g\,x\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*(f + g*x)^n))/x,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))/x, x)
```

3.271 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{a}{x}+b\text{Int}\left(\frac{\sin\left(c+d(f+gx)^n\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sin(c+d*(g*x+f)^n)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sin[c + d*(f + g*x)^n]/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx &= \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x^2}\right)dx\\ &= -\frac{a}{x}+b\int \frac{\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx\end{aligned}$$

Mathematica [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+d(gx+f)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="maxima")
```

```
[Out] b*integrate(sin((g*x + f)^n*d + c)/x^2, x) - a/x
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sin \left(c + d(f + gx)^{n}\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\sin\left(c + d\left(f + g\,x\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*(f + g*x)^n))/x^2,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))/x^2, x)
```

3.272 $\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal. Leaf size=856

$$\frac{(2a^2+b^2)f^2x}{2g^2}-\frac{(2a^2+b^2)f(f+gx)^2}{2g^3}+\frac{(2a^2+b^2)(f+gx)^3}{6g^3}+\frac{iabe^{ic}f^2(f+gx)(-id(f+gx)^n)^{-1/n}\Gamma(\frac{1}{n},-}{g^3n}$$

```
[Out] 1/2*(2*a^2+b^2)*f^2*x/g^2-1/2*(2*a^2+b^2)*f*(g*x+f)^2/g^3+1/6*(2*a^2+b^2)*(
g*x+f)^3/g^3+I*a*b*exp(I*c)*f^2*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^3/n/((-
I*d*(g*x+f)^n)^(1/n))-I*a*b*f^2*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g
^3/n/((I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*exp(2*I*c)*f^2*(g*x+f)*GAMMA(1/
n,-2*I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*f^2*(g*x+
f)*GAMMA(1/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(1/n))-2*I*
a*b*exp(I*c)*f*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^
(2/n))+2*I*a*b*f*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g
*x+f)^n)^(2/n))-2^(-1-2/n)*b^2*exp(2*I*c)*f*(g*x+f)^2*GAMMA(2/n,-2*I*d*(g*x
+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(2/n))-2^(-1-2/n)*b^2*f*(g*x+f)^2*GAMMA(2/n,
2*I*d*(g*x+f)^n)/exp(2*I*c)/g^3/n/((I*d*(g*x+f)^n)^(2/n))+I*a*b*exp(I*c)*(g
*x+f)^3*GAMMA(3/n,-I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*x+f)^n)^(3/n))-I*a*b*(g*x
+f)^3*GAMMA(3/n,I*d*(g*x+f)^n)/exp(I*c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))+2^(-2
-3/n)*b^2*exp(2*I*c)*(g*x+f)^3*GAMMA(3/n,-2*I*d*(g*x+f)^n)/g^3/n/((-I*d*(g*
x+f)^n)^(3/n))+2^(-2-3/n)*b^2*(g*x+f)^3*GAMMA(3/n,2*I*d*(g*x+f)^n)/exp(2*I*
c)/g^3/n/((I*d*(g*x+f)^n)^(3/n))
```

Rubi [A]
time = 0.65, antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3514, 3448, 3447, 2239, 3446, 3506, 6, 3505, 2250, 3504}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*f^2*x)/(2*g^2) - ((2*a^2 + b^2)*f*(f + g*x)^2)/(2*g^3) + ((2
*a^2 + b^2)*(f + g*x)^3)/(6*g^3) + (I*a*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1
), (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*f^2*(f
 + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^
(-1)) + (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f^2*(f + g*x)*Gamma[n^(-1), (-2*I)
*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2
*f^2*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*
(f + g*x)^n)^n^(-1)) - ((2*I)*a*b*E^(I*c)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(
f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) + ((2*I)*a*b*f*(f + g*x)^2*
Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) - (2^(
-1 - 2/n)*b^2*E^((2*I)*c)*f*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n])/(
```

```
g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) - (2^(-1 - 2/n)*b^2*f*(f + g*x)^2*Gamma[2
/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) + (I*
a*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f +
 g*x)^n)^(3/n)) - (I*a*b*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*
g^3*n*(I*d*(f + g*x)^n)^(3/n)) + (2^(-2 - 3/n)*b^2*E^((2*I)*c)*(f + g*x)^3*
Gamma[3/n, (-2*I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) + (2^(
-2 - 3/n)*b^2*(f + g*x)^3*Gamma[3/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3
*n*(I*d*(f + g*x)^n)^(3/n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2(a+b\sin(c+d(f+gx)^n))^2\,dx = \frac{\text{Subst}\left(\int \left(f^2(a+b\sin(c+dx^n))^2-2fx(a+b\sin(c+dx^n))^2+x^2(\right.\right.}{g^3}$$

$$= \frac{\text{Subst}\left(\int x^2(a+b\sin(c+dx^n))^2\,dx,x,f+gx\right)}{g^3} - \frac{(2f)\text{Subst}\left(\int x(a\right.}{}$$

$$= \frac{\text{Subst}\left(\int \left(a^2x^2+\frac{b^2x^2}{2}-\frac{1}{2}b^2x^2\cos(2c+2dx^n)+2abx^2\sin(c+dx^n\right.\right.}{g^3}$$

$$= \frac{(2a^2+b^2)f^2x}{2g^2} + \frac{\text{Subst}\left(\int \left(\left(a^2+\frac{b^2}{2}\right)x^2-\frac{1}{2}b^2x^2\cos(2c+2dx^n)+\right.\right.}{g^3}$$

$$= \frac{(2a^2+b^2)f^2x}{2g^2} - \frac{(2a^2+b^2)f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)(f+gx)^3}{6g^3} + \frac{(}{}$$

$$= \frac{(2a^2+b^2)f^2x}{2g^2} - \frac{(2a^2+b^2)f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)(f+gx)^3}{6g^3} + \frac{i}{}$$

$$= \frac{(2a^2+b^2)f^2x}{2g^2} - \frac{(2a^2+b^2)f(f+gx)^2}{2g^3} + \frac{(2a^2+b^2)(f+gx)^3}{6g^3} + \frac{i}{}$$

Mathematica [**A**]
time = 16.95, size = 786, normalized size = 0.92

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (4*a^2*x^3 + 2*b^2*x^3 - (3*b*(f + g*x)*(((-8*I)*a*f*(f + g*x)*Gamma[2/n, I
*d*(f + g*x)^n]*(Cos[c] - I*Sin[c]))/(I*d*(f + g*x)^n)^(2/n) + ((8*I)*a*f*(
f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g
*x)^n)^(2/n) + (4*a*f^2*Gamma[n^(-1), (-I)*d*(f + g*x)^n]*((-I)*Cos[c] + Si
n[c]))/((-I)*d*(f + g*x)^n)^n^(-1) + (4*a*(f + g*x)^2*Gamma[3/n, (-I)*d*(f
+ g*x)^n]*((-I)*Cos[c] + Sin[c]))/((-I)*d*(f + g*x)^n)^(3/n) + (4*a*f^2*Gam
ma[n^(-1), I*d*(f + g*x)^n]*(I*Cos[c] + Sin[c]))/(I*d*(f + g*x)^n)^n^(-1) +
 (4*a*(f + g*x)^2*Gamma[3/n, I*d*(f + g*x)^n]*(I*Cos[c] + Sin[c]))/(I*d*(f
+ g*x)^n)^(3/n) - (b*f^2*Gamma[n^(-1), (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin
[c])^2*(Cosh[Log[2]/n] - Sinh[Log[2]/n]))/(I*d*(f + g*x)^n)^n^(-1) - (b*f^2
*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Log[2]/n]
- Sinh[Log[2]/n]))/((-I)*d*(f + g*x)^n)^n^(-1) + (2*b*f*(f + g*x)*Gamma[2/n
, (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/
n]))/(I*d*(f + g*x)^n)^(2/n) + (2*b*f*(f + g*x)*Gamma[2/n, (-2*I)*d*(f + g*
x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/n]))/((-I)*d*(f +
 g*x)^n)^(2/n) - (b*(f + g*x)^2*Gamma[3/n, (2*I)*d*(f + g*x)^n]*(Cos[c] - I
*Sin[c])^2*(Cosh[Log[8]/n] - Sinh[Log[8]/n]))/(I*d*(f + g*x)^n)^(3/n) - (b*
(f + g*x)^2*Gamma[3/n, (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Lo
g[8]/n] - Sinh[Log[8]/n]))/((-I)*d*(f + g*x)^n)^(3/n)))/(g^3*n))/12
```

Maple [**F**]
time = 0.15, size = 0, normalized size = 0.00

$$\int x^2(a + b\sin(c + d(gx + f)^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*b^2*x^3 - 1/2*b^2*integrate(x^2*cos(2*(g*x + f)^n*d + 2*c
), x) + 2*a*b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*x^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*x^2*sin((g*x + f)^n*d +
c) + (a^2 + b^2)*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\sin(c + d(f + gx)^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

```
[Out] int(x^2*(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.273 $\int x(a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal. Leaf size=556

$$-\frac{(2a^2+b^2)\,fx}{2g}+\frac{(2a^2+b^2)\,(f+gx)^2}{4g^2}-\frac{iabe^{ic}f(f+gx)\,(-id(f+gx)^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{g^2n}+\frac{iabe^{-ic}f(f$$

```
[Out] -1/2*(2*a^2+b^2)*f*x/g+1/4*(2*a^2+b^2)*(g*x+f)^2/g^2-I*a*b*exp(I*c)*f*(g*x+
f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(1/n))+I*a*b*f*(g*x+f)
*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(1/n))-2^(-2-1/n)
*b^2*exp(2*I*c)*f*(g*x+f)*GAMMA(1/n,-2*I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^
n)^(1/n))-2^(-2-1/n)*b^2*f*(g*x+f)*GAMMA(1/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^
2/n/((I*d*(g*x+f)^n)^(1/n))+I*a*b*exp(I*c)*(g*x+f)^2*GAMMA(2/n,-I*d*(g*x+f)
^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))-I*a*b*(g*x+f)^2*GAMMA(2/n,I*d*(g*x+f)^n)
/exp(I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n))+4^(-1-1/n)*b^2*exp(2*I*c)*(g*x+f)^2
*GAMMA(2/n,-2*I*d*(g*x+f)^n)/g^2/n/((-I*d*(g*x+f)^n)^(2/n))+4^(-1-1/n)*b^2*
(g*x+f)^2*GAMMA(2/n,2*I*d*(g*x+f)^n)/exp(2*I*c)/g^2/n/((I*d*(g*x+f)^n)^(2/n
))
```

Rubi [A]

time = 0.30, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3514, 3448, 3447, 2239, 3446, 3506, 6, 3505, 2250, 3504}

$$\frac{iabe^{ic}(f+gx)^2(-id(f+gx)^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n}-\frac{iabe^{ic}f(f+gx)(-id(f+gx)^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{g^2n}+\frac{iabe^{-ic}f(f+gx)(id(f+gx)^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{g^2n}-\frac{iabe^{-ic}(f+gx)^2(id(f+gx)^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},id(f+gx)^n\right)}{g^2n}+\frac{b^2e^{2ic}4^{-\frac{1}{n}-1}(f+gx)^2(-id(f+gx)^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-2id(f+gx)^n\right)}{g^2n}-\frac{b^2e^{2ic}f2^{-\frac{1}{n}-2}(f+gx)(-id(f+gx)^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-2id(f+gx)^n\right)}{g^2n}-\frac{b^2e^{-2ic}f2^{-\frac{1}{n}-2}(f+gx)(id(f+gx)^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},2id(f+gx)^n\right)}{g^2n}+\frac{b^2e^{-2ic}4^{-\frac{1}{n}-1}(f+gx)^2(id(f+gx)^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},2id(f+gx)^n\right)}{g^2n}+\frac{(2a^2+b^2)(f+gx)^2}{4g^2}-\frac{fx(2a^2+b^2)}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] -1/2*((2*a^2 + b^2)*f*x)/g + ((2*a^2 + b^2)*(f + g*x)^2)/(4*g^2) - (I*a*b*E
^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g
*x)^n)^n^(-1)) + (I*a*b*f*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c
)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) - (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f*(f +
 g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-
1)) - (2^(-2 - n^(-1))*b^2*f*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/
(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + (I*a*b*E^(I*c)*(f + g*x)^2*G
amma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) - (I*a*b*
(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^(
2/n)) + (4^(-1 - n^(-1))*b^2*E^((2*I)*c)*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f
 + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (4^(-1 - n^(-1))*b^2*(f +
g*x)^2*Gamma[2/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n
)^(2/n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x(a+b\sin(c+d(f+gx)^n))^2\,dx = \frac{\text{Subst}\left(\int\left(-f(a+b\sin(c+dx^n))^2+x(a+b\sin(c+dx^n))^2\right)\,dx,x,f\ \ldots}{g^2}$$

$$= \frac{\text{Subst}\left(\int x(a+b\sin(c+dx^n))^2\,dx,x,f+gx\right)}{g^2} - \frac{f\text{Subst}\left(\int (a+b\sin\ \ldots\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int\left(a^2x+\frac{b^2x}{2}-\frac{1}{2}b^2x\cos(2c+2dx^n)+2abx\sin(c+dx^n)\right)\,dx\ \ldots\right.}{g^2}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{\text{Subst}\left(\int\left(\left(a^2+\frac{b^2}{2}\right)x-\frac{1}{2}b^2x\cos(2c+2dx^n)+2a\ \ldots\right.\right.}{g^2}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{(2a^2+b^2)(f+gx)^2}{4g^2} + \frac{(2ab)\text{Subst}\left(\int x\sin(c+dx^n\ \ldots\right.}{g^2}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{(2a^2+b^2)(f+gx)^2}{4g^2} - \frac{iabe^{ic}f(f+gx)\left(-id(f+g\ \ldots\right.}{g\ \ldots}$$

$$= -\frac{(2a^2+b^2)\,fx}{2g} + \frac{(2a^2+b^2)(f+gx)^2}{4g^2} - \frac{iabe^{ic}f(f+gx)\left(-id(f+g\ \ldots\right.}{g\ \ldots}$$

Mathematica [A]
time = 27.06, size = 552, normalized size = 0.99

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (2*a^2*g^2*n*x^2 + b^2*g^2*n*x^2 - ((4*I)*a*b*(f + g*x)^2*Gamma[2/n, I*d*(f
 + g*x)^n]*(Cos[c] - I*Sin[c]))/(I*d*(f + g*x)^n)^(2/n) + ((4*I)*a*b*(f + g
*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g*x)
^n)^(2/n) + (4*a*b*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n]*((-I)*Cos[
c] + Sin[c]))/((-I)*d*(f + g*x)^n)^n^(-1) + (4*a*b*f*(f + g*x)*Gamma[n^(-1)
, I*d*(f + g*x)^n]*(I*Cos[c] + Sin[c]))/(I*d*(f + g*x)^n)^n^(-1) - (b^2*f*(
f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])^2*(Cosh[Log
[2]/n] - Sinh[Log[2]/n]))/(I*d*(f + g*x)^n)^n^(-1) - (b^2*f*(f + g*x)*Gamma
[n^(-1), (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Log[2]/n] - Sinh
[Log[2]/n]))/((-I)*d*(f + g*x)^n)^n^(-1) + (b^2*(f + g*x)^2*Gamma[2/n, (2*I
)*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/n]))/(
I*d*(f + g*x)^n)^(2/n) + (b^2*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n]*
(Cos[c] + I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/n]))/((-I)*d*(f + g*x)^
n)^(2/n))/(4*g^2*n)
```

Maple [**F**]
time = 0.20, size = 0, normalized size = 0.00

$$\int x(a+b\sin(c+d(gx+f)^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/4*b^2*x^2 - 1/2*b^2*integrate(x*cos(2*(g*x + f)^n*d + 2*c),
 x) + 2*a*b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*x*cos((g*x + f)^n*d + c)^2 + 2*a*b*x*sin((g*x + f)^n*d + c) +
 (a^2 + b^2)*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\sin\left(c+d(f+gx)^n\right))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral(x*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

```
[Out] int(x*(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.274 $\int (a+b\sin(c+d(f+gx)^n))^2\,dx$

Optimal. Leaf size=261

$$\frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}}{gn}$$

```
[Out] 1/2*(2*a^2+b^2)*x+I*a*b*exp(I*c)*(g*x+f)*GAMMA(1/n,-I*d*(g*x+f)^n)/g/n/((-I
*d*(g*x+f)^n)^(1/n))-I*a*b*(g*x+f)*GAMMA(1/n,I*d*(g*x+f)^n)/exp(I*c)/g/n/((
I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*exp(2*I*c)*(g*x+f)*GAMMA(1/n,-2*I*d*(g
*x+f)^n)/g/n/((-I*d*(g*x+f)^n)^(1/n))+2^(-2-1/n)*b^2*(g*x+f)*GAMMA(1/n,2*I*
d*(g*x+f)^n)/exp(2*I*c)/g/n/((I*d*(g*x+f)^n)^(1/n))
```

Rubi [A]
time = 0.10, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3448, 3447, 2239, 3446}

$$\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{gn}+\frac{b^2e^{2ic}2^{-\frac{1}{n}-2}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2id(f+gx)^n\right)}{gn}+\frac{b^2e^{-2ic}2^{-\frac{1}{n}-2}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2id(f+gx)^n\right)}{gn}+\frac{1}{2}x\left(2a^2+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (I*a*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*
x)^n])/(g*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*
b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g*n*((-I)*d
*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2*(f + g*x)*Gamma[n^(-1), (2*I)*
d*(f + g*x)^n])/(E^((2*I)*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
```

```
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(c+d(f+gx)^n))^2\,dx &= \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos(2c+2d(f+gx)^n)+2ab\sin(c+d(f+gx)^n)\right)\\ &= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin(c+d(f+gx)^n)\,dx-\frac{1}{2}b^2\int \cos(2c+2d(\\ &= \frac{1}{2}\left(2a^2+b^2\right)x+(iab)\int e^{-ic-id(f+gx)^n}\,dx-(iab)\int e^{ic+id(f+gx)^n}\,dx-\frac{1}{4}\\ &= \frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}\end{aligned}$$

Mathematica [A]
time = 0.53, size = 247, normalized size = 0.95

$$\frac{2(2a^2+b^2)gnx+2^{-1/n}b^2e^{2ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2id(f+gx)^n\right)+2^{-1/n}b^2e^{-2ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2id(f+gx)^n\right)-4iab(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)(\cos(c)-i\sin(c))+4iab(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)(\cos(c)+i\sin(c))}{4gn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*g*n*x + (b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*
(f + g*x)^n])/(2^n^(-1)*((-I)*d*(f + g*x)^n)^n^(-1)) + (b^2*(f + g*x)*Gamma
[n^(-1), (2*I)*d*(f + g*x)^n])/(2^n^(-1)*E^((2*I)*c)*(I*d*(f + g*x)^n)^n^(-
1)) - ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*Sin[c
]))/(I*d*(f + g*x)^n)^n^(-1) + ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), (-I)*d*(f
 + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g*x)^n)^n^(-1))/(4*g*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+d(gx+f)^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/2*b^2*x - 1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c), x) + 2*a*
b*integrate(sin((g*x + f)^n*d + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*sin((g*x + f)^n*d + c) + a^2
 + b^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin (c + d(f + gx)^n))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \sin (c + d (f + g x)^n))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*(f + g*x)^n))^2,x)
```

```
[Out] int((a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.275 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{(a+b\sin\left(c+d(f+gx)^n\right))^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin\left(c+d(f+gx)^n\right))^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Rubi steps

$$\int \frac{(a+b\sin\left(c+d(f+gx)^n\right))^2}{x}\,dx=\int \frac{(a+b\sin\left(c+d(f+gx)^n\right))^2}{x}\,dx$$

Mathematica [A]
time = 2.46, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin\left(c+d(f+gx)^n\right))^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin\left(c+d(gx+f)^n\right))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c)/x, x) + 2*a*b*integrate(sin((
g*x + f)^n*d + c)/x, x) + a^2*log(x) + 1/2*b^2*log(x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a^
2 - b^2)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin{(c + d(f + gx)^n)})^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*(f + g*x)^n))^2/x,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))^2/x, x)
```

3.276 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2},x\right)$$

[Out] Unintegrable((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]

[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]

Rubi steps

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Mathematica [A]
time = 2.09, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]

[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+d(gx+f)^n))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -a^2/x - 1/2*(b^2*x*integrate(cos(2*(g*x + f)^n*d + 2*c)/x^2, x) - 4*a*b*x*
integrate(sin((g*x + f)^n*d + c)/x^2, x) + b^2)/x
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a^
2 - b^2)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(c + d(f + gx)^n))^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*(f + g*x)^n))^2/x^2,x)
[Out] int((a + b*sin(c + d*(f + g*x)^n))^2/x^2, x)
```

3.277 $\int \frac{x^2}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Defer[Int][x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi steps

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx = \int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\sin\left(c+d\left(gx+f\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{a + b\,\sin\left(c + d\,{\left(f + g\,x\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x^2/(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.278 $\int \frac{x}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Defer[Int][x/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi steps

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx = \int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]),x]
[Out] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\sin\left(c + d\left(f + gx\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(x/(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{a + b\,\sin\left(c + d\left(f + g\,x\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(x/(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.279 $\int \frac{1}{a+b\sin(c+d(f+gx)^n)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

`[Out] Unintegrable(1/(a+b*sin(c+d*(g*x+f)^n)),x)`

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]`

`[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]`

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]`

`[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]`

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+d\left(f+gx\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*(f + g*x)^n)),x)
[Out] int(1/(a + b*sin(c + d*(f + g*x)^n)), x)
```

3.280 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

[Out] Unintegrable(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]

[Out] Defer[Int][1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]

[Out] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(1/(b*x*sin((g*x + f)^n*d + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(f+gx\right)^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Integral(1/(x*(a + b*sin(c + d*(f + g*x)**n))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))),x)
[Out] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))), x)
```

3.281 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

`[Out] Unintegrable(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]`

`[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]`

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]`

`[Out] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]`

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
[Out] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*sin((g*x + f)^n*d + c) + a*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))),x)
[Out] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))), x)
```

3.282 $\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2},x\right)$$

[Out] Unintegrable(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

Verification is not applicable to the result.

[In] Int[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]

[Out] Defer[Int][x^2/(a + b*Sin[c + d*(f + g*x)^n])^2, x]

Rubi steps

$$\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx = \int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$$

Mathematica [F]
time = 180.09, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]

[Out] $Aborted

Maple [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\sin(c+d(gx+f)^n))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a*b*g*x^3 + a*b*f*x^2)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c
) + 2*(a*b*g*x^3 + a*b*f*x^2)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*
x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n
*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((
g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*
d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*si
n((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^
n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x +
f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(
2*(g*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*g*n*x^2*cos((g*x
+ f)^n*d + c)^2 + 2*(g*x + f)^n*a^2*d*g*n*x^2*sin((g*x + f)^n*d + c)^2 + (g
*x + f)^n*a*b*d*g*n*x^2*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*g*n*x^2
*sin((g*x + f)^n*d + c) + (2*a*b*f*x - (a*b*g*n - 3*a*b*g)*x^2)*cos((g*x +
f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) - (2*a*b*f*x - (a*b*g*n - 3*a*b*g)*
x^2)*cos((g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*g*n*x^2*cos((g*x + f)^n*d
+ c) - 2*b^2*f*x + (b^2*g*n - 3*b^2*g)*x^2 - (2*a*b*f*x - (a*b*g*n - 3*a*b*
g)*x^2)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4
)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x +
 f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*
cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g
*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x
+ f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g
*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)
*cos(2*(g*x + f)^n*d + 2*c)), x) + 2*(b^2*g*x^3 + b^2*f*x^2 + (a*b*g*x^3 +
a*b*f*x^2)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 -
b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*
x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g
*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x
 + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*
d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g
*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)
```

```
*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g
*n)*cos(2*(g*x + f)^n*d + 2*c))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-x^2/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c)
- a^2 - b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sin(c + d*(f + g*x)^n))^2,x)
```

```
[Out] int(x^2/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.283 $\int \frac{x}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x}{(a+b\sin\left(c+d(f+gx)^n\right))^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
[Out] Defer[Int][x/(a + b*Sin[c + d*(f + g*x)^n])^2, x]
```

Rubi steps

$$\int \frac{x}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx = \int \frac{x}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx$$

Mathematica [F]
time = 180.07, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
[Out] $Aborted
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\sin\left(c+d\left(gx+f\right)^n\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a*b*g*x^2 + a*b*f*x)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
+ 2*(a*b*g*x^2 + a*b*f*x)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g
*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x
+ f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*
n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g
*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d
+ c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n
*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g
*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*g*n*x*cos((g*x + f)^n
*d + c)^2 + 2*(g*x + f)^n*a^2*d*g*n*x*sin((g*x + f)^n*d + c)^2 + (g*x + f)^
n*a*b*d*g*n*x*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*g*n*x*sin((g*x +
f)^n*d + c) + (a*b*f - (a*b*g*n - 2*a*b*g)*x)*cos((g*x + f)^n*d + c))*cos(2
*(g*x + f)^n*d + 2*c) - (a*b*f - (a*b*g*n - 2*a*b*g)*x)*cos((g*x + f)^n*d +
 c) + ((g*x + f)^n*a*b*d*g*n*x*cos((g*x + f)^n*d + c) - b^2*f + (b^2*g*n -
2*b^2*g)*x - (a*b*f - (a*b*g*n - 2*a*b*g)*x)*sin((g*x + f)^n*d + c))*sin(2*
(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*
d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 +
 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)
^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^
2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b
 - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x +
 f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)
 + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^n*d + 2*c)), x) + 2*(
b^2*g*x^2 + b^2*f*x + (a*b*g*x^2 + a*b*f*x)*sin((g*x + f)^n*d + c))*sin(2*(
g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d
 + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 +
4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^
n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2
 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b
- a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x +
f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)
+ (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^n*d + 2*c))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
[Out] integral(-x/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
a^2 - b^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int(x/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.284 $\int \frac{1}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(a+b\sin\left(c+d(f+gx)^n\right))^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx = \int \frac{1}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx$$

Mathematica [A]
time = 7.47, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin\left(c+d(f+gx)^n\right))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin\left(c+d\left(gx+f\right)^{n}\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)

[Out] int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")

[Out] (2*(a*b*g*x + a*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c) + 2*(a*b*g*x + a*b*f)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*n*cos((g*x + f)^n*d + c)^2 + 2*(g*x + f)^n*a^2*d*n*sin((g*x + f)^n*d + c)^2 + (g*x + f)^n*a*b*d*n*sin((g*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*n*sin((g*x + f)^n*d + c) - (a*b*n - a*b)*cos((g*x + f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) + (a*b*n - a*b)*cos((g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*n*cos((g*x + f)^n*d + c) + b^2*n - b^2 + (a*b*n - a*b)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n)*cos(2*(g*x + f)^n*d + 2*c)), x) + 2*(b^2*g*x + b^2*f + (a*b*g*x + a*b*f)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^n*d + 2*c))

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
[Out] integral(-1/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
a^2 - b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^(-2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*(f + g*x)^n))^2,x)
[Out] int(1/(a + b*sin(c + d*(f + g*x)^n))^2, x)
```

3.285 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 137.98, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
[Out] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a^3*b*g*x + a^3*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
- 2*(b^4*g*x*sin(2*c) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^3*b -
a*b^3)*g*x + (a^3*b - a*b^3)*f + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))*co
s(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*sin(
c))*cos((g*x + f)^n*d) - (a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*sin(2*(g*x
 + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*sin(
(g*x + f)^n*d))*cos((g*x + f)^n*d + c) + 4*((a^3*b - a*b^3)*g*x*cos(c) + (a
^3*b - a*b^3)*f*cos(c))*cos((g*x + f)^n*d) + ((g*x + f)^n*a^4*b^2*d*g*n*x*c
os(2*(g*x + f)^n*d + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x*sin(2*(g*x + f)^n
*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x*cos(2*(
g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^
2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)^2 + (b^6*cos(
2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x*sin(2*(g*x + f)^n*d)^2 + 4*(a^
5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x + f)^n*d) + 4*
((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^
2)*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)
*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6
)*(g*x + f)^n*d*g*n*x - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3
- a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d) - (a^2*b^4
 - b^6)*(g*x + f)^n*d*g*n*x*cos(2*c) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c)
 + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d
))*cos(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x*cos(2*(g*x + f)^n*
d)*cos(2*c) - (g*x + f)^n*a^2*b^4*d*g*n*x*sin(2*(g*x + f)^n*d)*sin(2*c) + 2
*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos(c)*sin((g*x + f)^n*d) + 2*(a^5*b
 - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - a^2*
b^4)*(g*x + f)^n*d*g*n*x)*cos(2*(g*x + f)^n*d + 2*c) - 2*(2*((a^3*b^3 - a*b
^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g*n*
x*cos((g*x + f)^n*d) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*
b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x*sin((g*x + f)^n*d) + (a^2*b^4 - b
^6)*(g*x + f)^n*d*g*n*x*sin(2*c))*sin(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2
*b^4*d*g*n*x*cos(2*c)*sin(2*(g*x + f)^n*d) + (g*x + f)^n*a^2*b^4*d*g*n*x*co
s(2*(g*x + f)^n*d)*sin(2*c) - 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*cos((
```

```
g*x + f)^n*d)*cos(c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x*sin((g*x + f
)^n*d)*sin(c))*sin(2*(g*x + f)^n*d + 2*c))*integrate(-2*((b^4*g*n*x*sin(2*c
) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) + ((g*x + f)^n*a^3*b*d*g*n*x*sin((
g*x + f)^n*d + c) - (a^3*b*g*n*x + a^3*b*f)*cos((g*x + f)^n*d + c))*cos(2*(
g*x + f)^n*d + 2*c) + ((a^3*b - a*b^3)*g*n*x + (a^3*b - a*b^3)*f + ((g*x +
f)^n*a*b^3*d*g*n*x*sin(2*c) + a*b^3*g*n*x*cos(2*c) + a*b^3*f*cos(2*c))*cos(
2*(g*x + f)^n*d) - 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - (a^4 - a
^2*b^2)*g*n*x*sin(c) - (a^4 - a^2*b^2)*f*sin(c))*cos((g*x + f)^n*d) + ((g*x
 + f)^n*a*b^3*d*g*n*x*cos(2*c) - a*b^3*g*n*x*sin(2*c) - a*b^3*f*sin(2*c))*s
in(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*sin(c) + (a^4
- a^2*b^2)*g*n*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*sin((g*x + f)^n*d))*cos
((g*x + f)^n*d + c) - 2*((a^3*b - a*b^3)*g*n*x*cos(c) + (a^3*b - a*b^3)*f*c
os(c))*cos((g*x + f)^n*d) + (b^4*g*n*x*cos(2*c) + b^4*f*cos(2*c))*sin(2*(g*
x + f)^n*d) - ((g*x + f)^n*a^3*b*d*g*n*x*cos((g*x + f)^n*d + c) + a^2*b^2*g
*n*x + a^2*b^2*f + (a^3*b*g*n*x + a^3*b*f)*sin((g*x + f)^n*d + c))*sin(2*(g
*x + f)^n*d + 2*c) - ((a^3*b - a*b^3)*(g*x + f)^n*d*g*n*x + ((g*x + f)^n*a*
b^3*d*g*n*x*cos(2*c) - a*b^3*g*n*x*sin(2*c) - a*b^3*f*sin(2*c))*cos(2*(g*x
+ f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*sin(c) + (a^4 - a^2*b^2)
*g*n*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*cos((g*x + f)^n*d) - ((g*x + f)^n
*a*b^3*d*g*n*x*sin(2*c) + a*b^3*g*n*x*cos(2*c) + a*b^3*f*cos(2*c))*sin(2*(g
*x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - (a^4 - a^2*b
^2)*g*n*x*sin(c) - (a^4 - a^2*b^2)*f*sin(c))*sin((g*x + f)^n*d))*sin((g*x +
 f)^n*d + c) + 2*((a^3*b - a*b^3)*g*n*x*sin(c) + (a^3*b - a*b^3)*f*sin(c))*
sin((g*x + f)^n*d))/((g*x + f)^n*a^4*b^2*d*g*n*x^2*cos(2*(g*x + f)^n*d + 2*
c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x^2*sin(2*(g*x + f)^n*d + 2*c)^2 + (b^6*co
s(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*cos(2*(g*x + f)^n*d)^2 + 4
*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)
^2)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(
2*c)^2)*(g*x + f)^n*d*g*n*x^2*sin(2*(g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3
 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x + f)^n*d) + 4*((a^6 - 2*a^4
*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^
n*d*g*n*x^2*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^
n*d*g*n*x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 - 2*a^2*b^4 + b^6)*(g*x +
f)^n*d*g*n*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5
)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x*cos((g*x + f)^n*d + c)^2 - 2*a*b*x*sin((g*x + f)^n*d + c
) - (a^2 + b^2)*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))^2),x)
[Out] int(1/(x*(a + b*sin(c + d*(f + g*x)^n))^2), x)
```

3.286 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

[Out] Unintegrable(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]

[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2), x]

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [F]
time = 180.05, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]

[Out] $Aborted

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
[Out] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
[Out] (2*(a^3*b*g*x + a^3*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c)
- 2*(b^4*g*x*sin(2*c) + b^4*f*sin(2*c))*cos(2*(g*x + f)^n*d) - 2*((a^3*b -
a*b^3)*g*x + (a^3*b - a*b^3)*f + (a*b^3*g*x*cos(2*c) + a*b^3*f*cos(2*c))*co
s(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*sin(c) + (a^4 - a^2*b^2)*f*sin(
c))*cos((g*x + f)^n*d) - (a*b^3*g*x*sin(2*c) + a*b^3*f*sin(2*c))*sin(2*(g*x
 + f)^n*d) + 2*((a^4 - a^2*b^2)*g*x*cos(c) + (a^4 - a^2*b^2)*f*cos(c))*sin(
(g*x + f)^n*d))*cos((g*x + f)^n*d + c) + 4*((a^3*b - a*b^3)*g*x*cos(c) + (a
^3*b - a*b^3)*f*cos(c))*cos((g*x + f)^n*d) + ((g*x + f)^n*a^4*b^2*d*g*n*x^2
*cos(2*(g*x + f)^n*d + 2*c)^2 + (g*x + f)^n*a^4*b^2*d*g*n*x^2*sin(2*(g*x +
f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*c
os(2*(g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*
a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)^2 + (
b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^2*sin(2*(g*x + f)^n*d)
^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos(c)*sin((g*x +
f)^n*d) + 4*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*
b^4)*sin(c)^2)*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d)^2 + 4*(a^5*b - 2*a^
3*b^3 + a*b^5)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*sin(c) + (a^4*b^2 -
 2*a^2*b^4 + b^6)*(g*x + f)^n*d*g*n*x^2 - 2*(2*((a^3*b^3 - a*b^5)*cos(c)*si
n(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g*x
+ f)^n*d) - (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x^2*cos(2*c) - 2*((a^3*b^3 -
a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*(g*x + f)^n*d*g
*n*x^2*sin((g*x + f)^n*d))*cos(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*
g*n*x^2*cos(2*(g*x + f)^n*d)*cos(2*c) - (g*x + f)^n*a^2*b^4*d*g*n*x^2*sin(2
*(g*x + f)^n*d)*sin(2*c) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos(c)
*sin((g*x + f)^n*d) + 2*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos((g*x +
f)^n*d)*sin(c) + (a^4*b^2 - a^2*b^4)*(g*x + f)^n*d*g*n*x^2)*cos(2*(g*x + f)
^n*d + 2*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*s
in(2*c)*sin(c))*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d) + 2*((a^3*b^3 - a*
b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*(g*x + f)^n*d*g*n
*x^2*sin((g*x + f)^n*d) + (a^2*b^4 - b^6)*(g*x + f)^n*d*g*n*x^2*sin(2*c))*s
in(2*(g*x + f)^n*d) - 2*((g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*c)*sin(2*(g*x
+ f)^n*d) + (g*x + f)^n*a^2*b^4*d*g*n*x^2*cos(2*(g*x + f)^n*d)*sin(2*c) - 2
*(a^5*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*cos((g*x + f)^n*d)*cos(c) + 2*(a^5
*b - a^3*b^3)*(g*x + f)^n*d*g*n*x^2*sin((g*x + f)^n*d)*sin(c))*sin(2*(g*x +
```

```
f)^n*d + 2*c))*integrate(-2*((2*b^4*f*sin(2*c) + (b^4*g*n*sin(2*c) + b^4*g
*sin(2*c))*x)*cos(2*(g*x + f)^n*d) + ((g*x + f)^n*a^3*b*d*g*n*x*sin((g*x +
f)^n*d + c) - (2*a^3*b*f + (a^3*b*g*n + a^3*b*g)*x)*cos((g*x + f)^n*d + c))
*cos(2*(g*x + f)^n*d + 2*c) + (2*(a^3*b - a*b^3)*f + ((a^3*b - a*b^3)*g*n +
 (a^3*b - a*b^3)*g)*x + ((g*x + f)^n*a*b^3*d*g*n*x*sin(2*c) + 2*a*b^3*f*cos
(2*c) + (a*b^3*g*n*cos(2*c) + a*b^3*g*cos(2*c))*x)*cos(2*(g*x + f)^n*d) - 2
*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - 2*(a^4 - a^2*b^2)*f*sin(c) -
 ((a^4 - a^2*b^2)*g*n*sin(c) + (a^4 - a^2*b^2)*g*sin(c))*x)*cos((g*x + f)^n
*d) + ((g*x + f)^n*a*b^3*d*g*n*x*cos(2*c) - 2*a*b^3*f*sin(2*c) - (a*b^3*g*n
*sin(2*c) + a*b^3*g*sin(2*c))*x)*sin(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*
(g*x + f)^n*d*g*n*x*sin(c) + 2*(a^4 - a^2*b^2)*f*cos(c) + ((a^4 - a^2*b^2)*
g*n*cos(c) + (a^4 - a^2*b^2)*g*cos(c))*x)*sin((g*x + f)^n*d))*cos((g*x + f)
^n*d + c) - 2*(2*(a^3*b - a*b^3)*f*cos(c) + ((a^3*b - a*b^3)*g*n*cos(c) + (
a^3*b - a*b^3)*g*cos(c))*x)*cos((g*x + f)^n*d) + (2*b^4*f*cos(2*c) + (b^4*g
*n*cos(2*c) + b^4*g*cos(2*c))*x)*sin(2*(g*x + f)^n*d) - ((g*x + f)^n*a^3*b*
d*g*n*x*cos((g*x + f)^n*d + c) + 2*a^2*b^2*f + (a^2*b^2*g*n + a^2*b^2*g)*x
+ (2*a^3*b*f + (a^3*b*g*n + a^3*b*g)*x)*sin((g*x + f)^n*d + c))*sin(2*(g*x
+ f)^n*d + 2*c) - ((a^3*b - a*b^3)*(g*x + f)^n*d*g*n*x + ((g*x + f)^n*a*b^3
*d*g*n*x*cos(2*c) - 2*a*b^3*f*sin(2*c) - (a*b^3*g*n*sin(2*c) + a*b^3*g*sin(
2*c))*x)*cos(2*(g*x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*sin(
c) + 2*(a^4 - a^2*b^2)*f*cos(c) + ((a^4 - a^2*b^2)*g*n*cos(c) + (a^4 - a^2*
b^2)*g*cos(c))*x)*cos((g*x + f)^n*d) - ((g*x + f)^n*a*b^3*d*g*n*x*sin(2*c)
+ 2*a*b^3*f*cos(2*c) + (a*b^3*g*n*cos(2*c) + a*b^3*g*cos(2*c))*x)*sin(2*(g*
x + f)^n*d) + 2*((a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*x*cos(c) - 2*(a^4 - a^2*
b^2)*f*sin(c) - ((a^4 - a^2*b^2)*g*n*sin(c) + (a^4 - a^2*b^2)*g*sin(c))*x)*
sin((g*x + f)^n*d))*sin((g*x + f)^n*d + c) + 2*(2*(a^3*b - a*b^3)*f*sin(c)
+ ((a^3*b - a*b^3)*g*n*sin(c) + (a^3*b - a*b^3)*g*sin(c))*x)*sin((g*x + f)^
n*d))/((g*x + f)^n*a^4*b^2*d*g*n*x^3*cos(2*(g*x + f)^n*d + 2*c)^2 + (g*x +
f)^n*a^4*b^2*d*g*n*x^3*sin(2*(g*x + f)^n*d + 2*c)^2 + (b^6*cos(2*c)^2 + b^6
*sin(2*c)^2)*(g*x + f)^n*d*g*n*x^3*cos(2*(g*x + f)^n*d)^2 + 4*((a^6 - 2*a^4
*b^2 + a^2*b^4)*cos(c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(c)^2)*(g*x + f)^
n*d*g*n*x^3*cos((g*x + f)^n*d)^2 + (b^6*cos(2*c)^2 + b^6*sin(2*c)^2)*(g*x +
 f)^n*d*g*n*x^3*sin(2*(g*x + f)^n*d)^2 + 4*(a^5...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*x^2*sin((g*x + f)^n*d
 + c) - (a^2 + b^2)*x^2), x)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,{\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))^2),x)
```

```
[Out] int(1/(x^2*(a + b*sin(c + d*(f + g*x)^n))^2), x)
```

3.287 $\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p\,dx$

Optimal. Leaf size=27

$$\text{Int}\left((ex)^m (a+b\sin(c+d(f+gx)^n))^p,x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p\,dx = \int (ex)^m (a+b\sin(c+d(f+gx)^n))^p\,dx$$

Mathematica [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int (ex)^m (a+b\sin(c+d(f+gx)^n))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m (a+b\sin(c+d(gx+f)^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
[Out] int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(c+d*(g*x+f)**n))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="giac")
[Out] integrate((x*e)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,x)^m\,\left(a+b\,\sin\left(c+d\,{\left(f+g\,x\right)}^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sin(c + d*(f + g*x)^n))^p,x)
[Out] int((e*x)^m*(a + b*sin(c + d*(f + g*x)^n))^p, x)
```

3.288 $\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) dx$

Optimal. Leaf size=224

$$ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)-bde^2\cos(c)\text{Ci}\left(\frac{d}{x}\right)+\frac{1}{6}bd^3f^2\cos(c)\text{Ci}\left(\frac{d}{x}\right)$$

```
[Out] a*e^2*x+a*e*f*x^2+1/3*a*f^2*x^3-b*d*e^2*Ci(d/x)*cos(c)+1/6*b*d^3*f^2*Ci(d/x
)*cos(c)+b*d*e*f*x*cos(c+d/x)+1/6*b*d*f^2*x^2*cos(c+d/x)+b*d^2*e*f*cos(c)*S
i(d/x)+b*d^2*e*f*Ci(d/x)*sin(c)+b*d*e^2*Si(d/x)*sin(c)-1/6*b*d^3*f^2*Si(d/x
)*sin(c)+b*e^2*x*sin(c+d/x)-1/6*b*d^2*f^2*x*sin(c+d/x)+b*e*f*x^2*sin(c+d/x)
+1/3*b*f^2*x^3*sin(c+d/x)
```

Rubi [A]
time = 0.30, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3512, 14, 3378, 3384, 3380, 3383}

$$ae^2x+aefx^2+\frac{1}{3}af^2x^3+\frac{1}{6}bd^3f^2\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+bd^2ef\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-bde^2\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)-\frac{1}{6}bd^3f^2\sin(c)\text{Si}\left(\frac{d}{x}\right)+bd^2ef\cos(c)\text{Si}\left(\frac{d}{x}\right)-\frac{1}{6}bd^2f^2x\sin\left(c+\frac{d}{x}\right)+bde^2\sin(c)\text{Si}\left(\frac{d}{x}\right)+be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

```
[Out] a*e^2*x + a*e*f*x^2 + (a*f^2*x^3)/3 + b*d*e*f*x*Cos[c + d/x] + (b*d*f^2*x^2
*Cos[c + d/x])/6 - b*d*e^2*Cos[c]*CosIntegral[d/x] + (b*d^3*f^2*Cos[c]*CosI
ntegral[d/x])/6 + b*d^2*e*f*CosIntegral[d/x]*Sin[c] + b*e^2*x*Sin[c + d/x]
- (b*d^2*f^2*x*Sin[c + d/x])/6 + b*e*f*x^2*Sin[c + d/x] + (b*f^2*x^3*Sin[c
+ d/x])/3 + b*d^2*e*f*Cos[c]*SinIntegral[d/x] + b*d*e^2*Sin[c]*SinIntegral[
d/x] - (b*d^3*f^2*Sin[c]*SinIntegral[d/x])/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) dx = -\text{Subst}\left(\int \left(\frac{f^2(a+b\sin(c+dx))}{x^4}+\frac{2ef(a+b\sin(c+dx))}{x^3}+\frac{e^2(\right.\right.$$

$$= -\left(e^2\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)\right)-(2ef)\text{Subst}\left(\int \frac{a-}{}\right.$$

$$= -\left(e^2\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b\sin(c+dx)}{x^2}\right)dx,x,\frac{1}{x}\right)\right)-(2ef)\text{Subst}\left(\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3-\left(be^2\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)-$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\right.$$

Mathematica [A]
time = 0.38, size = 150, normalized size = 0.67

$$\frac{1}{6}\left(bd\,\mathrm{Ci}\left(\frac{d}{x}\right)\left(\left(-6e^2+d^2f^2\right)\cos(c)+6def\sin(c)\right)+x\left(2a\left(3e^2+3efx+f^2x^2\right)+bdf(6e+fx)\cos\left(c+\frac{d}{x}\right)+b\left(6e^2+6efx-f^2\left(d^2-2x^2\right)\right)\sin\left(c+\frac{d}{x}\right)\right)-bd\left(-6def\cos(c)+\left(-6e^2+d^2f^2\right)\sin(c)\right)\mathrm{Si}\left(\frac{d}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

```
[Out] (b*d*CosIntegral[d/x]*((-6*e^2 + d^2*f^2)*Cos[c] + 6*d*e*f*Sin[c]) + x*(2*a
*(3*e^2 + 3*e*f*x + f^2*x^2) + b*d*f*(6*e + f*x)*Cos[c + d/x] + b*(6*e^2 +
6*e*f*x - f^2*(d^2 - 2*x^2))*Sin[c + d/x]) - b*d*(-6*d*e*f*Cos[c] + (-6*e^2
 + d^2*f^2)*Sin[c])*SinIntegral[d/x])/6
```

Maple [A]
time = 0.17, size = 209, normalized size = 0.93

method	result
derivativedivides	$-d\left(-\frac{a f^2 x^3}{3d}-\frac{aef x^2}{d}-\frac{a e^2 x}{d}+b d^2 f^2\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^3}{3d^3}-\frac{\cos\left(c+\frac{d}{x}\right)x^2}{6d^2}+\frac{\sin\left(c+\frac{d}{x}\right)x}{6d}+\frac{\text{sinIntegral}\left(\frac{d}{x}\right.}{6}\right.\right.$
default	$-d\left(-\frac{a f^2 x^3}{3d}-\frac{aef x^2}{d}-\frac{a e^2 x}{d}+b d^2 f^2\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^3}{3d^3}-\frac{\cos\left(c+\frac{d}{x}\right)x^2}{6d^2}+\frac{\sin\left(c+\frac{d}{x}\right)x}{6d}+\frac{\text{sinIntegral}\left(\frac{d}{x}\right.}{6}\right.\right.$
risch	$a e^2 x+\frac{a f^2 x^3}{3}+aef x^2+\frac{bd e^2 \mathrm{e}^{-ic}\,\text{expIntegral}\left(1,\frac{id}{x}\right)}{2}-\frac{b d^3 f^2 \mathrm{e}^{-ic}\,\text{expIntegral}\left(1,\frac{id}{x}\right)}{12}-\frac{ib d^2 ef\,\mathrm{e}^{-ic}\,\text{expIn}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*sin(c+d/x)),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-1/3*a/d*f^2*x^3-a/d*e*f*x^2-a*e^2/d*x+b*d^2*f^2*(-1/3*sin(c+d/x)/d^3*x
^3-1/6*cos(c+d/x)/d^2*x^2+1/6*sin(c+d/x)/d*x+1/6*Si(d/x)*sin(c)-1/6*Ci(d/x)
*cos(c))+2*b*d*e*f*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c+d/x)/d*x-1/2*Si(d/x)*
cos(c)-1/2*Ci(d/x)*sin(c))+b*e^2*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*co
s(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 258, normalized size = 1.15

$$\frac{1}{3}af^2x^3+afx^2e+\frac{1}{12}\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)+\left(i\,\mathrm{Ei}\left(\frac{id}{x}\right)-i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d^3+2dx^2\cos\left(\frac{cx+d}{x}\right)-2\left(d^2x-2x^3\right)\sin\left(\frac{cx+d}{x}\right)\right)bf^2+\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)+\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d^2+2dx\cos\left(\frac{cx+d}{x}\right)+2x^2\sin\left(\frac{cx+d}{x}\right)\right)bfe-\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d-2x\sin\left(\frac{cx+d}{x}\right)\right)be^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*f*x^2*e + 1/12*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) + (I*Ei(
I*d/x) - I*Ei(-I*d/x))*sin(c))*d^3 + 2*d*x^2*cos((c*x + d)/x) - 2*(d^2*x -
2*x^3)*sin((c*x + d)/x))*b*f^2 + 1/2*(((-I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c)
```

```
 + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x + d)/x) + 2*x^2*si
n((c*x + d)/x))*b*f*e - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/
x) + I*Ei(-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e^2 + a*x*e^2
```

Fricas [**A**]
time = 0.36, size = 223, normalized size = 1.00

$$\frac{1}{3}\,af^2x^3+afx^2e+axe^2+\frac{1}{12}\left(12\,bd^2fe\,\mathrm{Si}\left(\frac{d}{x}\right)+\left(bd^3f^2-6\,bde^2\right)\mathrm{Ci}\left(\frac{d}{x}\right)+\left(bd^3f^2-6\,bde^2\right)\mathrm{Ci}\left(-\frac{d}{x}\right)\right)\cos\left(c\right)+\frac{1}{6}\left(bdf^2x^2+6\,bdfxe\right)\cos\left(\frac{cx+d}{x}\right)+\frac{1}{6}\left(3\,bd^2f\,\mathrm{Ci}\left(\frac{d}{x}\right)e+3\,bd^2f\,\mathrm{Ci}\left(-\frac{d}{x}\right)e-\left(bd^3f^2-6\,bde^2\right)\mathrm{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right)-\frac{1}{6}\left(bd^2f^2x-2\,bf^2x^3-6\,bfx^2e-6\,bxe^2\right)\sin\left(\frac{cx+d}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] 1/3*a*f^2*x^3 + a*f*x^2*e + a*x*e^2 + 1/12*(12*b*d^2*f*e*sin_integral(d/x)
+ (b*d^3*f^2 - 6*b*d*e^2)*cos_integral(d/x) + (b*d^3*f^2 - 6*b*d*e^2)*cos_i
ntegral(-d/x))*cos(c) + 1/6*(b*d*f^2*x^2 + 6*b*d*f*x*e)*cos((c*x + d)/x) +
1/6*(3*b*d^2*f*cos_integral(d/x)*e + 3*b*d^2*f*cos_integral(-d/x)*e - (b*d^
3*f^2 - 6*b*d*e^2)*sin_integral(d/x))*sin(c) - 1/6*(b*d^2*f^2*x - 2*b*f^2*x
^3 - 6*b*f*x^2*e - 6*b*x*e^2)*sin((c*x + d)/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\left(e+fx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((a + b*sin(c + d/x))*(e + f*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. 2(213) = 426.
time = 7.74, size = 1264, normalized size = 5.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] 1/6*(b*c^3*d^4*f^2*cos(c)*cos_integral(-c + (c*x + d)/x) + b*c^3*d^4*f^2*si
n(c)*sin_integral(c - (c*x + d)/x) - 3*(c*x + d)*b*c^2*d^4*f^2*cos(c)*cos_i
ntegral(-c + (c*x + d)/x)/x + 6*b*c^3*d^3*f*cos_integral(-c + (c*x + d)/x)*
e*sin(c) - 6*b*c^3*d^3*f*cos(c)*e*sin_integral(c - (c*x + d)/x) - 3*(c*x +
d)*b*c^2*d^4*f^2*sin(c)*sin_integral(c - (c*x + d)/x)/x + 3*(c*x + d)^2*b*c
*d^4*f^2*cos(c)*cos_integral(-c + (c*x + d)/x)/x^2 - 18*(c*x + d)*b*c^2*d^3
*f*cos_integral(-c + (c*x + d)/x)*e*sin(c)/x + b*c^2*d^4*f^2*sin((c*x + d)/
```

```
x) + 18*(c*x + d)*b*c^2*d^3*f*cos(c)*e*sin_integral(c - (c*x + d)/x)/x + 3*
(c*x + d)^2*b*c*d^4*f^2*sin(c)*sin_integral(c - (c*x + d)/x)/x^2 + b*c*d^4*
f^2*cos((c*x + d)/x) - (c*x + d)^3*b*d^4*f^2*cos(c)*cos_integral(-c + (c*x
+ d)/x)/x^3 - 6*b*c^3*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e^2 - 6*b*c
^2*d^3*f*cos((c*x + d)/x)*e + 18*(c*x + d)^2*b*c*d^3*f*cos_integral(-c + (c
*x + d)/x)*e*sin(c)/x^2 - 2*(c*x + d)*b*c*d^4*f^2*sin((c*x + d)/x)/x - 18*(
c*x + d)^2*b*c*d^3*f*cos(c)*e*sin_integral(c - (c*x + d)/x)/x^2 - (c*x + d)
^3*b*d^4*f^2*sin(c)*sin_integral(c - (c*x + d)/x)/x^3 - 6*b*c^3*d^2*e^2*sin
(c)*sin_integral(c - (c*x + d)/x) - (c*x + d)*b*d^4*f^2*cos((c*x + d)/x)/x
+ 18*(c*x + d)*b*c^2*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e^2/x + 12*(
c*x + d)*b*c*d^3*f*cos((c*x + d)/x)*e/x - 6*(c*x + d)^3*b*d^3*f*cos_integra
l(-c + (c*x + d)/x)*e*sin(c)/x^3 - 2*b*d^4*f^2*sin((c*x + d)/x) + (c*x + d)
^2*b*d^4*f^2*sin((c*x + d)/x)/x^2 + 6*b*c*d^3*f*e*sin((c*x + d)/x) + 6*(c*x
 + d)^3*b*d^3*f*cos(c)*e*sin_integral(c - (c*x + d)/x)/x^3 + 18*(c*x + d)*b
*c^2*d^2*e^2*sin(c)*sin_integral(c - (c*x + d)/x)/x - 2*a*d^4*f^2 - 18*(c*x
 + d)^2*b*c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e^2/x^2 + 6*a*c*d^3*f
*e - 6*(c*x + d)^2*b*d^3*f*cos((c*x + d)/x)*e/x^2 - 6*b*c^2*d^2*e^2*sin((c*
x + d)/x) - 6*(c*x + d)*b*d^3*f*e*sin((c*x + d)/x)/x - 18*(c*x + d)^2*b*c*d
^2*e^2*sin(c)*sin_integral(c - (c*x + d)/x)/x^2 - 6*a*c^2*d^2*e^2 + 6*(c*x
+ d)^3*b*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e^2/x^3 - 6*(c*x + d)*a*
d^3*f*e/x + 12*(c*x + d)*b*c*d^2*e^2*sin((c*x + d)/x)/x + 6*(c*x + d)^3*b*d
^2*e^2*sin(c)*sin_integral(c - (c*x + d)/x)/x^3 + 12*(c*x + d)*a*c*d^2*e^2/
x - 6*(c*x + d)^2*b*d^2*e^2*sin((c*x + d)/x)/x^2 - 6*(c*x + d)^2*a*d^2*e^2/
x^2)/((c^3 - 3*(c*x + d)*c^2/x + 3*(c*x + d)^2*c/x^2 - (c*x + d)^3/x^3)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^2\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)^2*(a + b*sin(c + d/x)), x)
```

3.289 $\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal. Leaf size=118

$$aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)-bde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\text{Ci}\left(\frac{d}{x}\right)\sin(c)+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c\right.$$

```
[Out] a*e*x+1/2*a*f*x^2-b*d*e*Ci(d/x)*cos(c)+1/2*b*d*f*x*cos(c+d/x)+1/2*b*d^2*f*c
os(c)*Si(d/x)+1/2*b*d^2*f*Ci(d/x)*sin(c)+b*d*e*Si(d/x)*sin(c)+b*e*x*sin(c+d
/x)+1/2*b*f*x^2*sin(c+d/x)
```

Rubi [A]
time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3512, 14, 3378, 3384, 3380, 3383}

$$aex+\frac{1}{2}afx^2+\frac{1}{2}bd^2f\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-bde\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+bde\sin(c)\text{Si}\left(\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*d*f*x*Cos[c + d/x])/2 - b*d*e*Cos[c]*CosIntegral[d
/x] + (b*d^2*f*CosIntegral[d/x]*Sin[c])/2 + b*e*x*Sin[c + d/x] + (b*f*x^2*S
in[c + d/x])/2 + (b*d^2*f*Cos[c]*SinIntegral[d/x])/2 + b*d*e*Sin[c]*SinInte
gral[d/x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx = -\text{Subst}\left(\int \left(\frac{f(a+b\sin(c+dx))}{x^3}+\frac{e(a+b\sin(c+dx))}{x^2}\right)dx,x,\frac{1}{x}\right)$$

$$= -\left(e\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{x^2}dx,x,\frac{1}{x}\right)\right)-f\text{Subst}\left(\int \frac{a+b\sin(c}{x^3}\right.$$

$$= -\left(e\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b\sin(c+dx)}{x^2}\right)dx,x,\frac{1}{x}\right)\right)-f\text{Subst}\left(\int \left(\frac{a}{x^3}\right.\right.$$

$$= aex+\frac{1}{2}afx^2-(be)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}dx,x,\frac{1}{x}\right)-(bf)\text{Subst}\left(\int\right.$$

$$= aex+\frac{1}{2}afx^2+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)-(bde)\text{Subst}$$

$$= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(\right.$$

$$= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)-bde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+bex\sin\left(c\right.$$

$$= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)-bde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\text{Ci}\left(\right.$$

Mathematica [A]

time = 0.15, size = 79, normalized size = 0.67

$$\frac{1}{2}\left(bdfx\cos\left(c+\frac{d}{x}\right)+bd\text{Ci}\left(\frac{d}{x}\right)(-2e\cos(c)+df\sin(c))+x(2e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)+bd(df\cos(c)+2e\sin(c))\text{Si}\left(\frac{d}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

```
[Out] (b*d*f*x*Cos[c + d/x] + b*d*CosIntegral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) + x
*(2*e + f*x)*(a + b*Sin[c + d/x]) + b*d*(d*f*Cos[c] + 2*e*Sin[c])*SinIntegr
al[d/x])/2
```

Maple [A]
time = 0.06, size = 115, normalized size = 0.97

method	result
derivativedivides	$-d\left(-\frac{a f\,x^{2}}{2 d}-\frac{a e x}{d}+b f d\left(-\frac{\sin\left(c+\frac{d}{x}\right) x^{2}}{2 d^{2}}-\frac{\cos\left(c+\frac{d}{x}\right) x}{2 d}-\frac{\mathrm{sinIntegral}\left(\frac{d}{x}\right)\cos\left(c\right)}{2}-\frac{\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\mathrm{s}}{2}\right.\right.$
default	$-d\left(-\frac{a f\,x^{2}}{2 d}-\frac{a e x}{d}+b f d\left(-\frac{\sin\left(c+\frac{d}{x}\right) x^{2}}{2 d^{2}}-\frac{\cos\left(c+\frac{d}{x}\right) x}{2 d}-\frac{\mathrm{sinIntegral}\left(\frac{d}{x}\right)\cos\left(c\right)}{2}-\frac{\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\mathrm{s}}{2}\right.\right.$
risch	$a e x+\frac{a f\,x^{2}}{2}+\frac{b d e\,{\mathrm e}^{-i c}\,\mathrm{expIntegral}\left(1, \frac{i d}{x}\right)}{2}-\frac{i b\,d^{2} f\,{\mathrm e}^{-i c}\,\mathrm{expIntegral}\left(1, \frac{i d}{x}\right)}{4}+\frac{b d e\,{\mathrm e}^{i c}\,\mathrm{expIntegral}\left(1, -\frac{i d}{x}\right)}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*sin(c+d/x)),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-1/2*a*f/d*x^2-a*e/d*x+b*f*d*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c+d/x)/d*
x-1/2*Si(d/x)*cos(c)-1/2*Ci(d/x)*sin(c))+b*e*(-sin(c+d/x)/d*x-Si(d/x)*sin(c
)+Ci(d/x)*cos(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 155, normalized size = 1.31

$$\frac{1}{2}\,afx^{2}+\frac{1}{4}\,\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)+\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d^{2}+2\,dx\cos\left(\frac{cx+d}{x}\right)+2\,x^{2}\sin\left(\frac{cx+d}{x}\right)\right)bf-\frac{1}{2}\,\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right)be+axe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/4*(((-I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(
-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x + d)/x))*b*f
 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*si
n(c))*d - 2*x*sin((c*x + d)/x))*b*e + a*x*e
```

Fricas [A]
time = 0.37, size = 138, normalized size = 1.17

$$\frac{1}{2}\,bdfx\cos\left(\frac{cx+d}{x}\right)+\frac{1}{2}\,afx^{2}+axe+\frac{1}{2}\,\left(bd^{2}f\,\mathrm{Si}\left(\frac{d}{x}\right)-bd\,\mathrm{Ci}\left(\frac{d}{x}\right)e-bd\,\mathrm{Ci}\left(-\frac{d}{x}\right)e\right)\cos\left(c\right)+\frac{1}{4}\,\left(bd^{2}f\,\mathrm{Ci}\left(\frac{d}{x}\right)+bd^{2}f\,\mathrm{Ci}\left(-\frac{d}{x}\right)+4\,bde\,\mathrm{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right)+\frac{1}{2}\,\left(bfx^{2}+2\,bxe\right)\sin\left(\frac{cx+d}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*d*f*x*cos((c*x + d)/x) + 1/2*a*f*x^2 + a*x*e + 1/2*(b*d^2*f*sin_integ
ral(d/x) - b*d*cos_integral(d/x)*e - b*d*cos_integral(-d/x)*e)*cos(c) + 1/4
*(b*d^2*f*cos_integral(d/x) + b*d^2*f*cos_integral(-d/x) + 4*b*d*e*sin_inte
gral(d/x))*sin(c) + 1/2*(b*f*x^2 + 2*b*x*e)*sin((c*x + d)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((a + b*sin(c + d/x))*(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(112) = 224.
time = 5.75, size = 530, normalized size = 4.49

$$\frac{bc^2d^3f\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)\sin\left(c\right)-bc^2d^3f\cos\left(c\right)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)-2\,bc^2d^2\cos\left(c\right)\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)e-\frac{2\,(cx+d)bcd^3f\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)\sin(c)}{x}+\frac{2\,(cx+d)bcd^3f\cos(c)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)}{x}-2\,bc^2d^2e\sin\left(c\right)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)-bcd^3f\cos\left(\frac{cx+d}{x}\right)+\frac{4\,(cx+d)bcd^2\cos(c)\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)e}{x}+\frac{(cx+d)^2bd^3f\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)\sin(c)}{x^2}-\frac{(cx+d)^2bd^3f\cos(c)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)}{x^2}+\frac{4\,(cx+d)bcd^2e\sin(c)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)}{x}+\frac{(cx+d)bd^3f\cos\left(\frac{cx+d}{x}\right)}{x}-\frac{2\,(cx+d)^2bd^2\cos(c)\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)e}{x^2}+bd^3f\sin\left(\frac{cx+d}{x}\right)-2\,bcd^2e\sin\left(\frac{cx+d}{x}\right)-\frac{2\,(cx+d)^2bd^2e\sin(c)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)}{x^2}+ad^3f-2\,acd^2e+\frac{2\,(cx+d)bd^2e\sin\left(\frac{cx+d}{x}\right)}{x}+\frac{2\,(cx+d)ad^2e}{x}}{2\left(c^2-\frac{2\,(cx+d)c}{x}+\frac{(cx+d)^2}{x^2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c) - b*c^2*d^3*f*cos(c)
*sin_integral(c - (c*x + d)/x) - 2*b*c^2*d^2*cos(c)*cos_integral(-c + (c*x
+ d)/x)*e - 2*(c*x + d)*b*c*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x +
 2*(c*x + d)*b*c*d^3*f*cos(c)*sin_integral(c - (c*x + d)/x)/x - 2*b*c^2*d^2
*e*sin(c)*sin_integral(c - (c*x + d)/x) - b*c*d^3*f*cos((c*x + d)/x) + 4*(c
*x + d)*b*c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e/x + (c*x + d)^2*b*d
^3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x^2 - (c*x + d)^2*b*d^3*f*cos(c)
*sin_integral(c - (c*x + d)/x)/x^2 + 4*(c*x + d)*b*c*d^2*e*sin(c)*sin_integ
ral(c - (c*x + d)/x)/x + (c*x + d)*b*d^3*f*cos((c*x + d)/x)/x - 2*(c*x + d)
^2*b*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e/x^2 + b*d^3*f*sin((c*x + d
)/x) - 2*b*c*d^2*e*sin((c*x + d)/x) - 2*(c*x + d)^2*b*d^2*e*sin(c)*sin_inte
gral(c - (c*x + d)/x)/x^2 + a*d^3*f - 2*a*c*d^2*e + 2*(c*x + d)*b*d^2*e*sin
((c*x + d)/x)/x + 2*(c*x + d)*a*d^2*e/x)/((c^2 - 2*(c*x + d)*c/x + (c*x + d
)^2/x^2)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int(e+f\,x)\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)*(a + b*sin(c + d/x)), x)
```

3.290 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal. Leaf size=38

$$ax-bd\cos(c)\text{Ci}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)+bd\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

```
[Out] a*x-b*d*Ci(d/x)*cos(c)+b*d*Si(d/x)*sin(c)+b*x*sin(c+d/x)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {3442, 3378, 3384, 3380, 3383}

$$ax-bd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+bd\sin(c)\text{Si}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d/x],x]
```

```
[Out] a*x - b*d*Cos[c]*CosIntegral[d/x] + b*x*Sin[c + d/x] + b*d*Sin[c]*SinIntegr
al[d/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx&=ax+b\int\sin\left(c+\frac{d}{x}\right)dx\\&=ax-b\mathrm{Subst}\left(\int\frac{\sin(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)\\&=ax+bx\sin\left(c+\frac{d}{x}\right)-(bd)\mathrm{Subst}\left(\int\frac{\cos(c+dx)}{x}\,dx,x,\frac{1}{x}\right)\\&=ax+bx\sin\left(c+\frac{d}{x}\right)-(bd\cos(c))\mathrm{Subst}\left(\int\frac{\cos(dx)}{x}\,dx,x,\frac{1}{x}\right)+(bd\sin(c))\mathrm{S}\\&=ax-bd\cos(c)\mathrm{Ci}\left(\frac{d}{x}\right)+bx\sin\left(c+\frac{d}{x}\right)+bd\sin(c)\mathrm{Si}\left(\frac{d}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.32

$$ax+bx\cos\left(\frac{d}{x}\right)\sin(c)+bx\cos(c)\sin\left(\frac{d}{x}\right)-bd\left(\cos(c)\mathrm{Ci}\left(\frac{d}{x}\right)-\sin(c)\mathrm{Si}\left(\frac{d}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d/x],x]
[Out] a*x + b*x*Cos[d/x]*Sin[c] + b*x*Cos[c]*Sin[d/x] - b*d*(Cos[c]*CosIntegral[d
/x] - Sin[c]*SinIntegral[d/x])
```

Maple [A]
time = 0.06, size = 43, normalized size = 1.13

method	result
default	$ax-bd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\mathrm{sinIntegral}\left(\frac{d}{x}\right)\sin\left(c\right)+\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\cos\left(c\right)\right)$

derivativedivides	$-d\left(-\frac{ax}{d}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\mathrm{sinIntegral}\left(\frac{d}{x}\right)\sin\left(c\right)+\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\cos\left(c\right)\right)\right)$
risch	$ax-\frac{i\mathrm{e}^{-ic}\pi\,\mathrm{csgn}\left(\frac{d}{x}\right)bd}{2}+i\mathrm{e}^{-ic}\,\mathrm{sinIntegral}\left(\frac{d}{x}\right)bd+\frac{\mathrm{expIntegral}\left(1,-\frac{id}{x}\right)\mathrm{e}^{-ic}bd}{2}+\frac{\mathrm{e}^{ic}\,\mathrm{expIntegral}\left(1,-\frac{id}{x}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-b*d*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 65, normalized size = 1.71

$$-\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos\left(c\right)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin\left(c\right)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="maxima")
```

```
[Out] -1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(
c))*d - 2*x*sin((c*x + d)/x))*b + a*x
```

Fricas [A]
time = 0.39, size = 52, normalized size = 1.37

$$bd\sin\left(c\right)\mathrm{Si}\left(\frac{d}{x}\right)+bx\sin\left(\frac{cx+d}{x}\right)+ax-\frac{1}{2}\left(bd\,\mathrm{Ci}\left(\frac{d}{x}\right)+bd\,\mathrm{Ci}\left(-\frac{d}{x}\right)\right)\cos\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="fricas")
```

```
[Out] b*d*sin(c)*sin_integral(d/x) + b*x*sin((c*x + d)/x) + a*x - 1/2*(b*d*cos_in
tegral(d/x) + b*d*cos_integral(-d/x))*cos(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x)
```

```
[Out] Integral(a + b*sin(c + d/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(38) = 76.

time = 4.06, size = 137, normalized size = 3.61

$$ax - \frac{\left(cd^2\cos\left(c\right)\operatorname{Ci}\left(-c + \frac{cx+d}{x}\right) + cd^2\sin\left(c\right)\operatorname{Si}\left(c - \frac{cx+d}{x}\right) - \frac{(cx+d)d^2\cos(c)\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)}{x} - \frac{(cx+d)d^2\sin(c)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)}{x} + d^2\sin\left(\frac{cx+d}{x}\right)\right)b}{\left(c - \frac{cx+d}{x}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="giac")
```

```
[Out] a*x - (c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x) + c*d^2*sin(c)*sin_integ
ral(c - (c*x + d)/x) - (c*x + d)*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)/
x - (c*x + d)*d^2*sin(c)*sin_integral(c - (c*x + d)/x)/x + d^2*sin((c*x + d
)/x))*b/((c - (c*x + d)/x)*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int a + b\sin\left(c + \frac{d}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(c + d/x),x)
```

```
[Out] int(a + b*sin(c + d/x), x)
```

3.291 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$

Optimal. Leaf size=103

$$\frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\text{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{b\text{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)}{f}+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\cos(c)\text{Si}}{f}$$

```
[Out] a*ln(f+e/x)/f+a*ln(x)/f+b*cos(c-d*f/e)*Si(d*(f/e+1/x))/f-b*cos(c)*Si(d/x)/f
-b*Ci(d/x)*sin(c)/f+b*Ci(d*(f/e+1/x))*sin(c-d*f/e)/f
```

Rubi [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3512, 14, 3384, 3380, 3383, 3398}

$$\frac{a\log\left(\frac{e}{x}+f\right)}{f}+\frac{a\log(x)}{f}+\frac{b\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x),x]
```

```
[Out] (a*Log[f + e/x])/f + (a*Log[x])/f - (b*CosIntegral[d/x]*Sin[c])/f + (b*CosI
ntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (b*Cos[c - (d*f)/e]*SinInte
gral[d*(f/e + x^(-1))])/f - (b*Cos[c]*SinIntegral[d/x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx &= -\text{Subst}\left(\int\left(\frac{a+b\sin(c+dx)}{fx}-\frac{e(a+b\sin(c+dx))}{f(f+ex)}\right)dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int\frac{a+b\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{e\text{Subst}\left(\int\frac{a+b\sin(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{f}\\
&= -\frac{\text{Subst}\left(\int\left(\frac{a}{x}+\frac{b\sin(c+dx)}{x}\right)dx,x,\frac{1}{x}\right)}{f}+\frac{e\text{Subst}\left(\int\left(\frac{a}{f+ex}+\frac{b\sin(c+dx)}{f+ex}\right)dx,x,\frac{1}{x}\right)}{f}\\
&= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\text{Subst}\left(\int\frac{\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{(be)\text{Subst}\left(\int\frac{\sin(c+dx)}{f+ex}\,d\right.}{f}\\
&= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{(b\cos(c))\text{Subst}\left(\int\frac{\sin(dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{\left(be\cos\left(c-\frac{df}{e}\right)\right)}{}\\
&= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\text{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{b\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{f}+\frac{b\cos\left(c-\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 83, normalized size = 0.81

$$\frac{a\log(e+fx)-b\text{Ci}\left(\frac{d}{x}\right)\sin(c)+b\text{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)+b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)-b\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] - b*CosIntegral[d/x]*Sin[c] + b*CosIntegral[d*(f/e + x^(-1)
)]*Sin[c - (d*f)/e] + b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - b*
Cos[c]*SinIntegral[d/x])/f
```

Maple [A]
time = 0.09, size = 164, normalized size = 1.59

method	result
risch	$\frac{a\ln(fx+e)}{f}-\frac{ib\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}\left(1,\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2f}+\frac{ib\,\mathrm{expIntegral}\left(1,\frac{id}{x}\right)\mathrm{e}^{-ic}}{2f}+\frac{ib\,\mathrm{e}^{\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}}{2}$
derivativedivides	$-d\left(\frac{a\ln\left(\frac{d}{x}\right)}{fd}-\frac{a\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}+\frac{b\left(\mathrm{sinIntegral}\left(\frac{d}{x}\right)\cos(c)+\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\sin(c)\right)}{fd}-\frac{be\left(-\frac{\mathrm{sinIntegr}}{}\right.}{}\right.$
default	$-d\left(\frac{a\ln\left(\frac{d}{x}\right)}{fd}-\frac{a\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}+\frac{b\left(\mathrm{sinIntegral}\left(\frac{d}{x}\right)\cos(c)+\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\sin(c)\right)}{fd}-\frac{be\left(-\frac{\mathrm{sinIntegr}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(a/f/d*ln(d/x)-a/f/d*ln(-c*e+d*f+e*(c+d/x))+b/f/d*(Si(d/x)*cos(c)+Ci(d/x
)*sin(c))-b*e/f/d*(-Si(-d/x-c-(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-
c*e+d*f)/e)*sin((-c*e+d*f)/e)/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e)
*sin((c*x + d)/x)^2), x) + integrate(1/2*sin((c*x + d)/x)/(f*x + e), x)) +
a*log(f*x + e)/f
```

Fricas [A]
time = 0.36, size = 135, normalized size = 1.31

$$\frac{2\,b\cos\left(-(df-ce)e^{(-1)}\right)\mathrm{Si}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)-2\,b\cos\left(c\right)\mathrm{Si}\left(\frac{d}{x}\right)+2\,a\log\left(fx+e\right)+\left(b\,\mathrm{Ci}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+b\,\mathrm{Ci}\left(-\frac{(dfx+de)e^{(-1)}}{x}\right)\right)\sin\left(-(df-ce)e^{(-1)}\right)-\left(b\,\mathrm{Ci}\left(\frac{d}{x}\right)+b\,\mathrm{Ci}\left(-\frac{d}{x}\right)\right)\sin\left(c\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*cos(-(d*f - c*e)*e^(-1))*sin_integral((d*f*x + d*e)*e^(-1)/x) - 2*
b*cos(c)*sin_integral(d/x) + 2*a*log(f*x + e) + (b*cos_integral((d*f*x + d*
e)*e^(-1)/x) + b*cos_integral(-(d*f*x + d*e)*e^(-1)/x))*sin(-(d*f - c*e)*e^
(-1)) - (b*cos_integral(d/x) + b*cos_integral(-d/x))*sin(c))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sin\left(c + \frac{d}{x}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x), x)
```

Giac [A]
time = 3.21, size = 172, normalized size = 1.67

$$\frac{bd\operatorname{Ci}\left(\left(df - ce + \frac{(cx+d)e}{x}\right)e^{(-1)}\right)\sin\left(-(df - ce)e^{(-1)}\right) - bd\operatorname{Ci}\left(-c + \frac{cx+d}{x}\right)\sin\left(c\right) - bd\cos\left(-(df - ce)e^{(-1)}\right)\operatorname{Si}\left(-\left(df - ce + \frac{(cx+d)e}{x}\right)e^{(-1)}\right) + bd\cos\left(c\right)\operatorname{Si}\left(c - \frac{cx+d}{x}\right) + ad\log\left(-df + ce - \frac{(cx+d)e}{x}\right) - ad\log\left(c - \frac{cx+d}{x}\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="giac")
```

```
[Out] (b*d*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*sin(-(d*f - c*e)*e^(-
1)) - b*d*cos_integral(-c + (c*x + d)/x)*sin(c) - b*d*cos(-(d*f - c*e)*e^(-
1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + b*d*cos(c)*sin_inte
gral(c - (c*x + d)/x) + a*d*log(-d*f + c*e - (c*x + d)*e/x) - a*d*log(c - (
c*x + d)/x))/(d*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\sin\left(c + \frac{d}{x}\right)}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))/(e + f*x),x)
```

```
[Out] int((a + b*sin(c + d/x))/(e + f*x), x)
```

3.292 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=94

$$\frac{a}{e\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\mathrm{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}+\frac{bd\sin\left(c-\frac{df}{e}\right)\mathrm{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}$$

```
[Out] a/e/(f+e/x)-b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^2+b*d*Si(d*(f/e+1/x))*sin(c-
d*f/e)/e^2+b*sin(c+d/x)/e/(f+e/x)
```

Rubi [A]
time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383}

$$\frac{a}{e\left(\frac{e}{x}+f\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\mathrm{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{bd\sin\left(c-\frac{df}{e}\right)\mathrm{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

```
[Out] a/(e*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^2
+ (b*Sin[c + d/x])/(e*(f + e/x)) + (b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e
 + x^(-1))])/e^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx &= -\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a}{(f+ex)^2}+\frac{b\sin(c+dx)}{(f+ex)^2}\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} - b\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(bd)\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{\left(bd\cos\left(c-\frac{df}{e}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e} + \frac{(bd
\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(\frac{d(f+
\end{aligned}$$

Mathematica [A]

time = 0.48, size = 85, normalized size = 0.90

$$\frac{-bd\cos\left(c-\frac{df}{e}\right)\mathrm{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+\frac{e\left(-ae+bfx\sin\left(c+\frac{d}{x}\right)\right)}{f(e+fx)}+bd\sin\left(c-\frac{df}{e}\right)\mathrm{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

```
[Out] (-(b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]) + (e*(-(a*e) + b*f*x
*Sin[c + d/x]))/(f*(e + f*x)) + b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x
^(-1))])/e^2
```

Maple [A]
time = 0.10, size = 149, normalized size = 1.59

method	result
derivativedivides	$-d\left(-\frac{a}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{sinIntegral}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\mathrm{cosineI}}{e}\right.\right.$
default	$-d\left(-\frac{a}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\mathrm{sinIntegral}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\mathrm{cosineI}}{e}\right.\right.$
risch	$-\frac{a}{f(fx+e)}+\frac{bd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}\left(1,\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2e^2}+\frac{bd\,\mathrm{e}^{\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}\left(1,-\frac{id}{x}-ic-\frac{-ice+idf}{e}\right)}{2e^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-a/(-c*e+d*f+e*(c+d/x))/e+b*(-sin(c+d/x)/(-c*e+d*f+e*(c+d/x))/e+(-Si(-d
/x-c-(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f
)/e)/e)/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/(f^2*x^2 + 2*f*x*e + e^2), x) + integrate
(1/2*sin((c*x + d)/x)/((f^2*x^2 + 2*f*x*e + e^2)*cos((c*x + d)/x)^2 + (f^2*
x^2 + 2*f*x*e + e^2)*sin((c*x + d)/x)^2), x)) - a/(f^2*x + f*e)
```

Fricas [A]
time = 0.37, size = 165, normalized size = 1.76

$$\frac{2\,bfxe\sin\left(\frac{cx+d}{x}\right)+2\,(bdf^2x+bdfe)\sin\left(-(df-ce)e^{(-1)}\right)\operatorname{Si}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)-\left((bdf^2x+bdfe)\operatorname{Ci}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+(bdf^2x+bdfe)\operatorname{Ci}\left(-\frac{(dfx+de)e^{(-1)}}{x}\right)\right)\cos\left(-(df-ce)e^{(-1)}\right)-2\,ae^2}{2\,(f^2xe^2+fe^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*f*x*e*sin((c*x + d)/x) + 2*(b*d*f^2*x + b*d*f*e)*sin(-(d*f - c*e)*
e^(-1))*sin_integral((d*f*x + d*e)*e^(-1)/x) - ((b*d*f^2*x + b*d*f*e)*cos_i
ntegral((d*f*x + d*e)*e^(-1)/x) + (b*d*f^2*x + b*d*f*e)*cos_integral(-(d*f*
x + d*e)*e^(-1)/x))*cos(-(d*f - c*e)*e^(-1)) - 2*a*e^2)/(f^2*x*e^2 + f*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(88) = 176.
time = 5.63, size = 347, normalized size = 3.69

$$-\frac{bd^3f\cos\left(-(df-ce)e^{(-1)}\right)\operatorname{Ci}\left(\left(df-ce+\frac{(cx+d)e}{x}\right)e^{(-1)}\right)-bcd^2\cos\left(-(df-ce)e^{(-1)}\right)\operatorname{Ci}\left(\left(df-ce+\frac{(cx+d)e}{x}\right)e^{(-1)}\right)e+bd^3f\sin\left(-(df-ce)e^{(-1)}\right)\operatorname{Si}\left(-\left(df-ce+\frac{(cx+d)e}{x}\right)e^{(-1)}\right)-bcd^2e\sin\left(-(df-ce)e^{(-1)}\right)\operatorname{Si}\left(-\left(df-ce+\frac{(cx+d)e}{x}\right)e^{(-1)}\right)+\frac{(cx+d)bd^2\cos\left(-(df-ce)e^{(-1)}\right)\operatorname{Ci}\left(\left(df-ce+\frac{(cx+d)e}{x}\right)e^{(-1)}\right)e}{x}+\frac{(cx+d)bd^2e\sin\left(-(df-ce)e^{(-1)}\right)\operatorname{Si}\left(-\left(df-ce+\frac{(cx+d)e}{x}\right)e^{(-1)}\right)}{x}-bd^2e\sin\left(\frac{cx+d}{x}\right)-ad^2e}{\left(dfe^2-ce^3+\frac{(cx+d)e^3}{x}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -(b*d^3*f*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)
*e^(-1)) - b*c*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x
+ d)*e/x)*e^(-1))*e + b*d^3*f*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f -
 c*e + (c*x + d)*e/x)*e^(-1)) - b*c*d^2*e*sin(-(d*f - c*e)*e^(-1))*sin_inte
gral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + (c*x + d)*b*d^2*cos(-(d*f - c*e
)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e/x + (c*x + d)*
b*d^2*e*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*
e^(-1))/x - b*d^2*e*sin((c*x + d)/x) - a*d^2*e)/((d*f*e^2 - c*e^3 + (c*x +
d)*e^3/x)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{{\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))/(e + f*x)^2,x)
[Out] int((a + b*sin(c + d/x))/(e + f*x)^2, x)
```

3.293 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx$

Optimal. Leaf size=233

$$-\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{bd^2f\operatorname{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right.}{2e^4}$$

```
[Out] -1/2*a*f/e^2/(f+e/x)^2+a/e^2/(f+e/x)-b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^3-1
/2*b*d*f*cos(c+d/x)/e^3/(f+e/x)-1/2*b*d^2*f*cos(c-d*f/e)*Si(d*(f/e+1/x))/e^
4-1/2*b*d^2*f*Ci(d*(f/e+1/x))*sin(c-d*f/e)/e^4+b*d*Si(d*(f/e+1/x))*sin(c-d*
f/e)/e^3-1/2*b*f*sin(c+d/x)/e^2/(f+e/x)^2+b*sin(c+d/x)/e^2/(f+e/x)
```

Rubi [A]
time = 0.30, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383}

$$\frac{a}{e^2\left(\frac{e}{x}+f\right)}-\frac{af}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{bd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{bd^2f\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4}+\frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(\frac{e}{x}+f\right)}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(\frac{e}{x}+f\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

```
[Out] -1/2*(a*f)/(e^2*(f + e/x)^2) + a/(e^2*(f + e/x)) - (b*d*f*Cos[c + d/x])/(2*
e^3*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^3 -
 (b*d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/(2*e^4) - (b*f*Si
n[c + d/x])/(2*e^2*(f + e/x)^2) + (b*Sin[c + d/x])/(e^2*(f + e/x)) - (b*d^2
*f*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/(2*e^4) + (b*d*Sin[c - (
d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = -\text{Subst}\left(\int\left(-\frac{f(a+b\sin(c+dx))}{e(f+ex)^3}+\frac{a+b\sin(c+dx)}{e(f+ex)^2}\right)\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int\frac{a+b\sin(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e}+\frac{f\text{Subst}\left(\int\frac{a+b\sin(c+dx)}{(f+ex)^3}\,dx,x,\frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a}{(f+ex)^2}+\frac{b\sin(c+dx)}{(f+ex)^2}\right)\,dx,x,\frac{1}{x}\right)}{e}+\frac{f\text{Subst}\left(\int\left(\frac{a}{(f+ex)^3}+\frac{b\sin(c+dx)}{(f+ex)^3}\right)\,dx,\right.}{e}$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{b\text{Subst}\left(\int\frac{\sin(c+dx)}{(f+ex)^2}\,dx,x,\frac{1}{x}\right)}{e}+\frac{(bf)\text{Subst}\left(\int\frac{\sin(c+}{(f+ex}\right.}{e}$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{(bd)\text{Subst}\left(\int\frac{\cos(c}{f-}\right.}{e^2}$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-$$

$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-$$

Mathematica [A]
time = 1.22, size = 151, normalized size = 0.65

$$-\frac{bd\text{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\left(2e\cos\left(c-\frac{df}{e}\right)+df\sin\left(c-\frac{df}{e}\right)\right)+\frac{e\left(ae^3+bdf^2x(e+fx)\cos\left(c+\frac{d}{x}\right)-befx(2e+fx)\sin\left(c+\frac{d}{x}\right)\right)}{f(e+fx)^2}+bd\left(df\cos\left(c-\frac{df}{e}\right)-2e\sin\left(c-\frac{df}{e}\right)\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

```
[Out] -1/2*(b*d*CosIntegral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c -
 (d*f)/e]) + (e*(a*e^3 + b*d*f^2*x*(e + f*x)*Cos[c + d/x] - b*e*f*x*(2*e +
f*x)*Sin[c + d/x]))/(f*(e + f*x)^2) + b*d*(d*f*Cos[c - (d*f)/e] - 2*e*Sin[c
 - (d*f)/e])*SinIntegral[d*(f/e + x^(-1))])/e^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(223) = 446.
time = 0.12, size = 542, normalized size = 2.33

method	result
risch	$-\frac{a}{2f(fx+e)^2}+\frac{ib\,d^2\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}\left(1,\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)f}{4e^4}+\frac{bd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}\left(1,\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{2e^3}$
derivativedivides	$-d\left(-\frac{a}{e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}-\frac{(ce-df)a}{2e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^2}+\frac{(ce-df)b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^2e}+\frac{-\frac{\cos\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c-\right.\right.}}{}\right.}{}\right.$
default	$-d\left(-\frac{a}{e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}-\frac{(ce-df)a}{2e^2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^2}+\frac{(ce-df)b\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{2\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)^2e}+\frac{-\frac{\cos\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c-\right.\right.}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-a/e^2/(-c*e+d*f+e*(c+d/x))-1/2*(c*e-d*f)/e^2*a/(-c*e+d*f+e*(c+d/x))^2+
(c*e-d*f)/e*b*(-1/2*sin(c+d/x)/(-c*e+d*f+e*(c+d/x))^2/e+1/2*(-cos(c+d/x)/(-
c*e+d*f+e*(c+d/x))/e-(-Si(-d/x-c-(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c
+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)/e)/e)+b/e*(-sin(c+d/x)/(-c*e+d*f+e*(c+d
/x))/e+(-Si(-d/x-c-(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)
*cos((-c*e+d*f)/e)/e)/e)+1/2*c*a/(-c*e+d*f+e*(c+d/x))^2/e-c*b*(-1/2*sin(c+d
/x)/(-c*e+d*f+e*(c+d/x))^2/e+1/2*(-cos(c+d/x)/(-c*e+d*f+e*(c+d/x))/e-(-Si(-
d/x-c-(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*
f)/e)/e)/e)/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/(f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2 + e^3)
, x) + integrate(1/2*sin((c*x + d)/x)/((f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2 +
```

```
 e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2 + e^3)*sin((c
*x + d)/x)^2), x)) - 1/2*a/(f^3*x^2 + 2*f^2*x*e + f*e^2)
```

Fricas [**A**]
time = 0.39, size = 419, normalized size = 1.80

$$-\frac{2\left(\left(bdf^3x^2e+2\,bdf^2xe^2+bdfe^3\right)\operatorname{Ci}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+\left(bdf^3x^2e+2\,bdf^2xe^2+bdfe^3\right)\operatorname{Ci}\left(-\frac{(dfx+de)e^{(-1)}}{x}\right)+\left(bd^2f^4x^2+2\,bd^2f^3xe+bd^2f^2e^2\right)\operatorname{Si}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)\right)\cos\left(-(df-ce)e^{(-1)}\right)+2\left(bdf^3x^2e+bdf^2xe^2\right)\cos\left(\frac{cx+d}{x}\right)+2\,ae^4+\left(\left(bd^2f^4x^2+2\,bd^2f^3xe+bd^2f^2e^2\right)\operatorname{Ci}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+\left(bd^2f^4x^2+2\,bd^2f^3xe+bd^2f^2e^2\right)\operatorname{Ci}\left(-\frac{(dfx+de)e^{(-1)}}{x}\right)-4\left(bdf^3x^2e+2\,bdf^2xe^2+bdfe^3\right)\operatorname{Si}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)\right)\sin\left(-(df-ce)e^{(-1)}\right)-2\left(bf^2x^2e^2+2\,bfxe^3\right)\sin\left(\frac{cx+d}{x}\right)}{4\left(f^3x^2e^4+2\,f^2xe^5+fe^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*((b*d*f^3*x^2*e + 2*b*d*f^2*x*e^2 + b*d*f*e^3)*cos_integral((d*f*x
+ d*e)*e^(-1)/x) + (b*d*f^3*x^2*e + 2*b*d*f^2*x*e^2 + b*d*f*e^3)*cos_integr
al(-(d*f*x + d*e)*e^(-1)/x) + (b*d^2*f^4*x^2 + 2*b*d^2*f^3*x*e + b*d^2*f^2*
e^2)*sin_integral((d*f*x + d*e)*e^(-1)/x))*cos(-(d*f - c*e)*e^(-1)) + 2*(b*
d*f^3*x^2*e + b*d*f^2*x*e^2)*cos((c*x + d)/x) + 2*a*e^4 + ((b*d^2*f^4*x^2 +
 2*b*d^2*f^3*x*e + b*d^2*f^2*e^2)*cos_integral((d*f*x + d*e)*e^(-1)/x) + (b
*d^2*f^4*x^2 + 2*b*d^2*f^3*x*e + b*d^2*f^2*e^2)*cos_integral(-(d*f*x + d*e)
*e^(-1)/x) - 4*(b*d*f^3*x^2*e + 2*b*d*f^2*x*e^2 + b*d*f*e^3)*sin_integral((
d*f*x + d*e)*e^(-1)/x))*sin(-(d*f - c*e)*e^(-1)) - 2*(b*f^2*x^2*e^2 + 2*b*f
*x*e^3)*sin((c*x + d)/x))/(f^3*x^2*e^4 + 2*f^2*x*e^5 + f*e^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{\left(e+fx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(211) = 422$.
time = 7.65, size = 1502, normalized size = 6.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*d^5*f^3*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*sin(-(d*f
- c*e)*e^(-1)) - 2*b*c*d^4*f^2*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(
-1))*e*sin(-(d*f - c*e)*e^(-1)) - b*d^5*f^3*cos(-(d*f - c*e)*e^(-1))*sin_in
tegral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 2*b*c*d^4*f^2*cos(-(d*f - c*e
```

```
)*e^(-1))*e*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 2*b*d^4*f^2
*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*
e + b*c^2*d^3*f*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2*sin(-(
d*f - c*e)*e^(-1)) + 2*(c*x + d)*b*d^4*f^2*cos_integral((d*f - c*e + (c*x +
 d)*e/x)*e^(-1))*e*sin(-(d*f - c*e)*e^(-1))/x - b*c^2*d^3*f*cos(-(d*f - c*e
)*e^(-1))*e^2*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) - 2*(c*x +
d)*b*d^4*f^2*cos(-(d*f - c*e)*e^(-1))*e*sin_integral(-(d*f - c*e + (c*x + d
)*e/x)*e^(-1))/x + 2*b*d^4*f^2*e*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*
f - c*e + (c*x + d)*e/x)*e^(-1)) - 4*b*c*d^3*f*cos(-(d*f - c*e)*e^(-1))*cos
_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2 + b*d^4*f^2*cos((c*x + d)
/x)*e - 2*(c*x + d)*b*c*d^3*f*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-
1))*e^2*sin(-(d*f - c*e)*e^(-1))/x + 2*(c*x + d)*b*c*d^3*f*cos(-(d*f - c*e)
*e^(-1))*e^2*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x - 4*b*c*d^
3*f*e^2*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*
e^(-1)) + 2*b*c^2*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c
*x + d)*e/x)*e^(-1))*e^3 - b*c*d^3*f*cos((c*x + d)/x)*e^2 + 4*(c*x + d)*b*d
^3*f*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-
1))*e^2/x + (c*x + d)^2*b*d^3*f*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^
(-1))*e^2*sin(-(d*f - c*e)*e^(-1))/x^2 - (c*x + d)^2*b*d^3*f*cos(-(d*f - c*
e)*e^(-1))*e^2*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x^2 + 2*b*
c^2*d^2*e^3*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e
/x)*e^(-1)) + 4*(c*x + d)*b*d^3*f*e^2*sin(-(d*f - c*e)*e^(-1))*sin_integral
(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x - 4*(c*x + d)*b*c*d^2*cos(-(d*f - c
*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^3/x + (c*x +
 d)*b*d^3*f*cos((c*x + d)/x)*e^2/x - b*d^3*f*e^2*sin((c*x + d)/x) - 4*(c*x
+ d)*b*c*d^2*e^3*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x +
 d)*e/x)*e^(-1))/x + 2*(c*x + d)^2*b*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integ
ral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^3/x^2 - a*d^3*f*e^2 + 2*b*c*d^2*e
^3*sin((c*x + d)/x) + 2*(c*x + d)^2*b*d^2*e^3*sin(-(d*f - c*e)*e^(-1))*sin_
integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x^2 + 2*a*c*d^2*e^3 - 2*(c*x
+ d)*b*d^2*e^3*sin((c*x + d)/x)/x - 2*(c*x + d)*a*d^2*e^3/x)/((d^2*f^2*e^4
- 2*c*d*f*e^5 + c^2*e^6 + 2*(c*x + d)*d*f*e^5/x - 2*(c*x + d)*c*e^6/x + (c*
x + d)^2*e^6/x^2)*d)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\sin\left(c+\frac{d}{x}\right)}{{\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))/(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(c + d/x))/(e + f*x)^3, x)
```

3.294 $\int (e+fx) \left(a+b \sin \left(c+\frac{d}{x}\right)\right)^2 \, dx$

Optimal. Leaf size=254

$$a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\text{Ci}\left(\frac{d}{x}\right)-b^2d^2f\cos(2c)\text{Ci}\left(\frac{2d}{x}\right)+abd^2f\text{Ci}\left(\frac{d}{x}\right)\sin(c)-b^2d$$

```
[Out] a^2*e*x+1/2*a^2*f*x^2-2*a*b*d*e*Ci(d/x)*cos(c)-b^2*d^2*f*Ci(2*d/x)*cos(2*c)
+a*b*d*f*x*cos(c+d/x)+a*b*d^2*f*cos(c)*Si(d/x)-b^2*d*e*cos(2*c)*Si(2*d/x)+a
*b*d^2*f*Ci(d/x)*sin(c)+2*a*b*d*e*Si(d/x)*sin(c)-b^2*d*e*Ci(2*d/x)*sin(2*c)
+b^2*d^2*f*Si(2*d/x)*sin(2*c)+2*a*b*e*x*sin(c+d/x)+a*b*f*x^2*sin(c+d/x)+b^2
*d*f*x*cos(c+d/x)*sin(c+d/x)+b^2*e*x*sin(c+d/x)^2+1/2*b^2*f*x^2*sin(c+d/x)^
2
```

Rubi [A]
time = 0.41, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383, 3395, 29, 3393, 3394, 12}

$$a^2ex+\frac{1}{2}a^2fx^2+abd^2f\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-2abde\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+2abde\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin\left(c+\frac{d}{x}\right)+abdfx\cos\left(c+\frac{d}{x}\right)-b^2d^2f\cos(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2de\sin(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)+b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)-b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2ex\sin^2\left(c+\frac{d}{x}\right)+\frac{1}{2}b^2fx^2\sin^2\left(c+\frac{d}{x}\right)+b^2dfx\sin\left(c+\frac{d}{x}\right)\cos\left(c+\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

```
[Out] a^2*e*x + (a^2*f*x^2)/2 + a*b*d*f*x*Cos[c + d/x] - 2*a*b*d*e*Cos[c]*CosInte
gral[d/x] - b^2*d^2*f*Cos[2*c]*CosIntegral[(2*d)/x] + a*b*d^2*f*CosIntegral
[d/x]*Sin[c] - b^2*d*e*CosIntegral[(2*d)/x]*Sin[2*c] + 2*a*b*e*x*Sin[c + d/
x] + a*b*f*x^2*Sin[c + d/x] + b^2*d*f*x*Cos[c + d/x]*Sin[c + d/x] + b^2*e*x
*Sin[c + d/x]^2 + (b^2*f*x^2*Sin[c + d/x]^2)/2 + a*b*d^2*f*Cos[c]*SinIntegr
al[d/x] + 2*a*b*d*e*Sin[c]*SinIntegral[d/x] - b^2*d*e*Cos[2*c]*SinIntegral[
(2*d)/x] + b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = -\text{Subst}\left(\int \left(\frac{f(a+b\sin(c+dx))^2}{x^3}+\frac{e(a+b\sin(c+dx))^2}{x^2}\right) dx, x$$
$$= -\left(e\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x^2} dx, x, \frac{1}{x}\right)\right) - f\text{Subst}\left(\int \frac{(a+}{}\right.$$
$$= -\left(e\text{Subst}\left(\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin^2(c+dx)}{x^2}\right) dx, x, \frac{1}{x}\right.\right.$$
$$= a^2ex+\frac{1}{2}a^2fx^2-(2abe)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2} dx, x, \frac{1}{x}\right)-\left(b^2e\right)\text{Su}$$
$$= a^2ex+\frac{1}{2}a^2fx^2+2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin\left(c+\frac{d}{x}\right)+b^2dfx$$
$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)+b^2d^2f\log(x)+2abex\sin\left(c\right.$$
$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+2abe$$
$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+abd^2$$
$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\text{Ci}\left(\frac{d}{x}\right)-b^2d^2$$

Mathematica [A]
time = 0.29, size = 252, normalized size = 0.99

$$\frac{1}{4}\left(4a^2ex+2b^2ex+2a^2fx^2+b^2fx^2+4abdfx\cos\left(c+\frac{d}{x}\right)-2b^2ex\cos\left(2\left(c+\frac{d}{x}\right)\right)-b^2fx^2\cos\left(2\left(c+\frac{d}{x}\right)\right)+4abd\text{Ci}\left(\frac{d}{x}\right)(-2e\cos(c)+df\sin(c))-4b^2d\text{Ci}\left(\frac{2d}{x}\right)(df\cos(2c)+e\sin(2c))+8abex\sin\left(c+\frac{d}{x}\right)+4abfx^2\sin\left(c+\frac{d}{x}\right)+2b^2dfx\sin\left(2\left(c+\frac{d}{x}\right)\right)+4abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+8abde\sin(c)\text{Si}\left(\frac{d}{x}\right)-4b^2de\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+4b^2d^2f\sin(2c)\text{Si}\left(\frac{2d}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

```
[Out] (4*a^2*e*x + 2*b^2*e*x + 2*a^2*f*x^2 + b^2*f*x^2 + 4*a*b*d*f*x*Cos[c + d/x]
 - 2*b^2*e*x*Cos[2*(c + d/x)] - b^2*f*x^2*Cos[2*(c + d/x)] + 4*a*b*d*CosInt
egral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) - 4*b^2*d*CosIntegral[(2*d)/x]*(d*f*C
os[2*c] + e*Sin[2*c]) + 8*a*b*e*x*Sin[c + d/x] + 4*a*b*f*x^2*Sin[c + d/x] +
 2*b^2*d*f*x*Sin[2*(c + d/x)] + 4*a*b*d^2*f*Cos[c]*SinIntegral[d/x] + 8*a*b
*d*e*Sin[c]*SinIntegral[d/x] - 4*b^2*d*e*Cos[2*c]*SinIntegral[(2*d)/x] + 4*
b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x])/4
```

Maple [A]
time = 0.12, size = 265, normalized size = 1.04

method	result
derivativedivides	$-d\left(-\frac{a^2 f x^2}{2d}-\frac{a^2 e x}{d}+2abfd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\text{sinIntegral}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\text{cosineIntegral}(\ldots}{2}\right.\right.$
default	$-d\left(-\frac{a^2 f x^2}{2d}-\frac{a^2 e x}{d}+2abfd\left(-\frac{\sin\left(c+\frac{d}{x}\right)x^2}{2d^2}-\frac{\cos\left(c+\frac{d}{x}\right)x}{2d}-\frac{\text{sinIntegral}\left(\frac{d}{x}\right)\cos(c)}{2}-\frac{\text{cosineIntegral}(\ldots}{2}\right.\right.$
risch	$a^2ex+\frac{a^2 f x^2}{2}+abde\,\mathrm{e}^{-ic}\,\text{expIntegral}\left(1,\frac{id}{x}\right)+\frac{i\,\text{expIntegral}\left(1,\frac{2id}{x}\right)\mathrm{e}^{-2ic}b^2de}{2}+\frac{x b^2 e}{2}+\frac{f b^2 x^2}{4}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*sin(c+d/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-1/2*a^2*f/d*x^2-a^2*e/d*x+2*a*b*f*d*(-1/2*sin(c+d/x)/d^2*x^2-1/2*cos(c
+d/x)/d*x-1/2*Si(d/x)*cos(c)-1/2*Ci(d/x)*sin(c))+2*a*b*e*(-sin(c+d/x)/d*x-S
i(d/x)*sin(c)+Ci(d/x)*cos(c))-1/4*b^2*f/d*x^2-1/4*b^2*f*d*(-cos(2*d/x+2*c)/
d^2*x^2+2*sin(2*d/x+2*c)/d*x+4*Si(2*d/x)*sin(2*c)-4*Ci(2*d/x)*cos(2*c))-1/2
*b^2*e/d*x-1/4*b^2*e*(-2*cos(2*d/x+2*c)/d*x-4*Si(2*d/x)*cos(2*c)-4*Ci(2*d/x
)*sin(2*c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.43, size = 324, normalized size = 1.28

$$\frac{1}{2}a^2fx^2+\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)+\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d^2+2dx\cos\left(\frac{cx+d}{x}\right)+2x^2\sin\left(\frac{cx+d}{x}\right)\right)abf-\frac{1}{4}\left(2\left(\left(\mathrm{Ei}\left(\frac{2id}{x}\right)+\mathrm{Ei}\left(-\frac{2id}{x}\right)\right)\cos(2c)+\left(i\,\mathrm{Ei}\left(\frac{2id}{x}\right)-i\,\mathrm{Ei}\left(-\frac{2id}{x}\right)\right)\sin(2c)\right)d^2+x^2\cos\left(\frac{2(cx+d)}{x}\right)-2dx\sin\left(\frac{2(cx+d)}{x}\right)-x^2\right)b^2f-\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d-2x\sin\left(\frac{cx+d}{x}\right)\right)abe-\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{2id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{2id}{x}\right)\right)\cos(2c)+\left(\mathrm{Ei}\left(\frac{2id}{x}\right)+\mathrm{Ei}\left(-\frac{2id}{x}\right)\right)\sin(2c)\right)d+x\cos\left(\frac{2(cx+d)}{x}\right)-x\right)b^2e+a^2xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*f*x^2 + 1/2*(((-I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei(I*d/x) + E
i(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x + d)/x))*a
*b*f - 1/4*(2*((Ei(2*I*d/x) + Ei(-2*I*d/x))*cos(2*c) + (I*Ei(2*I*d/x) - I*E
```

```
i(-2*I*d/x))*sin(2*c))*d^2 + x^2*cos(2*(c*x + d)/x) - 2*d*x*sin(2*(c*x + d)
/x) - x^2)*b^2*f - (((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei
(-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*a*b*e - 1/2*(((-I*Ei(2*I*d/x) +
 I*Ei(-2*I*d/x))*cos(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*co
s(2*(c*x + d)/x) - x)*b^2*e + a^2*x*e
```

Fricas [A]
time = 0.42, size = 309, normalized size = 1.22

$$abdfx\cos\left(\frac{cx+d}{x}\right)+\frac{1}{2}\left(a^2+b^2\right)fx^2-\frac{1}{2}\left(b^2fx^2+2b^2xe\right)\cos\left(\frac{cx+d}{x}\right)^2+\left(a^2+b^2\right)xe-\frac{1}{2}\left(b^2d^2f\operatorname{Ci}\left(\frac{2d}{x}\right)+b^2d^2f\operatorname{Ci}\left(-\frac{2d}{x}\right)+2b^2de\operatorname{Si}\left(\frac{2d}{x}\right)\right)\cos\left(2c\right)+\left(abd^2f\operatorname{Si}\left(\frac{d}{x}\right)-abd\operatorname{Ci}\left(\frac{d}{x}\right)e-abd\operatorname{Ci}\left(-\frac{d}{x}\right)e\right)\cos\left(c\right)+\frac{1}{2}\left(2b^2d^2f\operatorname{Si}\left(\frac{2d}{x}\right)-b^2d\operatorname{Ci}\left(\frac{2d}{x}\right)e-b^2d\operatorname{Ci}\left(-\frac{2d}{x}\right)e\right)\sin\left(2c\right)+\frac{1}{2}\left(abd^2f\operatorname{Ci}\left(\frac{d}{x}\right)+abd^2f\operatorname{Ci}\left(-\frac{d}{x}\right)+4abde\operatorname{Si}\left(\frac{d}{x}\right)\right)\sin\left(c\right)+\left(b^2dfx\cos\left(\frac{cx+d}{x}\right)+abfx^2+2abxe\right)\sin\left(\frac{cx+d}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] a*b*d*f*x*cos((c*x + d)/x) + 1/2*(a^2 + b^2)*f*x^2 - 1/2*(b^2*f*x^2 + 2*b^2
*x*e)*cos((c*x + d)/x)^2 + (a^2 + b^2)*x*e - 1/2*(b^2*d^2*f*cos_integral(2*
d/x) + b^2*d^2*f*cos_integral(-2*d/x) + 2*b^2*d*e*sin_integral(2*d/x))*cos(
2*c) + (a*b*d^2*f*sin_integral(d/x) - a*b*d*cos_integral(d/x)*e - a*b*d*cos
_integral(-d/x)*e)*cos(c) + 1/2*(2*b^2*d^2*f*sin_integral(2*d/x) - b^2*d*co
s_integral(2*d/x)*e - b^2*d*cos_integral(-2*d/x)*e)*sin(2*c) + 1/2*(a*b*d^2
*f*cos_integral(d/x) + a*b*d^2*f*cos_integral(-d/x) + 4*a*b*d*e*sin_integra
l(d/x))*sin(c) + (b^2*d*f*x*cos((c*x + d)/x) + a*b*f*x^2 + 2*a*b*x*e)*sin((
c*x + d)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2*(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(257) = 514$.
time = 5.36, size = 1145, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^2*c^2*d^3*f*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x) - 4*a*b*c
^2*d^3*f*cos_integral(-c + (c*x + d)/x)*sin(c) + 4*b^2*c^2*d^3*f*sin(2*c)*s
```

```
in_integral(2*c - 2*(c*x + d)/x) + 4*a*b*c^2*d^3*f*cos(c)*sin_integral(c -
(c*x + d)/x) - 8*(c*x + d)*b^2*c*d^3*f*cos(2*c)*cos_integral(-2*c + 2*(c*x
+ d)/x)/x + 8*a*b*c^2*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e + 4*b^2*c
^2*d^2*cos_integral(-2*c + 2*(c*x + d)/x)*e*sin(2*c) + 8*(c*x + d)*a*b*c*d^
3*f*cos_integral(-c + (c*x + d)/x)*sin(c)/x - 4*b^2*c^2*d^2*cos(2*c)*e*sin_
integral(2*c - 2*(c*x + d)/x) - 8*(c*x + d)*b^2*c*d^3*f*sin(2*c)*sin_integr
al(2*c - 2*(c*x + d)/x)/x - 8*(c*x + d)*a*b*c*d^3*f*cos(c)*sin_integral(c -
 (c*x + d)/x)/x + 8*a*b*c^2*d^2*e*sin(c)*sin_integral(c - (c*x + d)/x) + 4*
a*b*c*d^3*f*cos((c*x + d)/x) + 4*(c*x + d)^2*b^2*d^3*f*cos(2*c)*cos_integra
l(-2*c + 2*(c*x + d)/x)/x^2 - 16*(c*x + d)*a*b*c*d^2*cos(c)*cos_integral(-c
 + (c*x + d)/x)*e/x - 8*(c*x + d)*b^2*c*d^2*cos_integral(-2*c + 2*(c*x + d)
/x)*e*sin(2*c)/x - 4*(c*x + d)^2*a*b*d^3*f*cos_integral(-c + (c*x + d)/x)*s
in(c)/x^2 + 2*b^2*c*d^3*f*sin(2*(c*x + d)/x) + 8*(c*x + d)*b^2*c*d^2*cos(2*
c)*e*sin_integral(2*c - 2*(c*x + d)/x)/x + 4*(c*x + d)^2*b^2*d^3*f*sin(2*c)
*sin_integral(2*c - 2*(c*x + d)/x)/x^2 + 4*(c*x + d)^2*a*b*d^3*f*cos(c)*sin
_integral(c - (c*x + d)/x)/x^2 - 16*(c*x + d)*a*b*c*d^2*e*sin(c)*sin_integr
al(c - (c*x + d)/x)/x + b^2*d^3*f*cos(2*(c*x + d)/x) - 4*(c*x + d)*a*b*d^3*
f*cos((c*x + d)/x)/x - 2*b^2*c*d^2*cos(2*(c*x + d)/x)*e + 8*(c*x + d)^2*a*b
*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)*e/x^2 + 4*(c*x + d)^2*b^2*d^2*co
s_integral(-2*c + 2*(c*x + d)/x)*e*sin(2*c)/x^2 - 2*(c*x + d)*b^2*d^3*f*sin
(2*(c*x + d)/x)/x - 4*a*b*d^3*f*sin((c*x + d)/x) + 8*a*b*c*d^2*e*sin((c*x +
 d)/x) - 4*(c*x + d)^2*b^2*d^2*cos(2*c)*e*sin_integral(2*c - 2*(c*x + d)/x)
/x^2 + 8*(c*x + d)^2*a*b*d^2*e*sin(c)*sin_integral(c - (c*x + d)/x)/x^2 - 2
*a^2*d^3*f - b^2*d^3*f + 4*a^2*c*d^2*e + 2*b^2*c*d^2*e + 2*(c*x + d)*b^2*d^
2*cos(2*(c*x + d)/x)*e/x - 8*(c*x + d)*a*b*d^2*e*sin((c*x + d)/x)/x - 4*(c*
x + d)*a^2*d^2*e/x - 2*(c*x + d)*b^2*d^2*e/x)/((c^2 - 2*(c*x + d)*c/x + (c*
x + d)^2/x^2)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)\,\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*sin(c + d/x))^2,x)
```

```
[Out] int((e + f*x)*(a + b*sin(c + d/x))^2, x)
```

3.295 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx$

Optimal. Leaf size=94

$$a^2x-2abd\cos(c)\text{Ci}\left(\frac{d}{x}\right)-b^2d\text{Ci}\left(\frac{2d}{x}\right)\sin(2c)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-b^2$$

```
[Out] a^2*x-2*a*b*d*Ci(d/x)*cos(c)-b^2*d*cos(2*c)*Si(2*d/x)+2*a*b*d*Si(d/x)*sin(c
)-b^2*d*Ci(2*d/x)*sin(2*c)+2*a*b*x*sin(c+d/x)+b^2*x*sin(c+d/x)^2
```

Rubi [A]
time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3442, 3398, 3378, 3384, 3380, 3383, 3394, 12}

$$a^2x-2abd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)-b^2d\sin(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2,x]
```

```
[Out] a^2*x - 2*a*b*d*Cos[c]*CosIntegral[d/x] - b^2*d*CosIntegral[(2*d)/x]*Sin[2*
c] + 2*a*b*x*Sin[c + d/x] + b^2*x*Sin[c + d/x]^2 + 2*a*b*d*Sin[c]*SinIntegr
al[d/x] - b^2*d*Cos[2*c]*SinIntegral[(2*d)/x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = -\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin^2(c+dx)}{x^2}\right)dx, x, \frac{1}{x}\right)$$
$$= a^2x-(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)-b^2\text{Subst}\left(\int \frac{\sin^2(c+dx)}{x^2}\,dx,\right.$$
$$= a^2x+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)-(2abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\right.$$
$$= a^2x+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)-\left(b^2d\right)\text{Subst}\left(\int \frac{\sin(2c+2d}{x}\right.$$
$$= a^2x-2abd\cos(c)\text{Ci}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)+2abd\,\text{si}$$
$$= a^2x-2abd\cos(c)\text{Ci}\left(\frac{d}{x}\right)-b^2d\text{Ci}\left(\frac{2d}{x}\right)\sin(2c)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\,\text{s}$$

Mathematica [A]
time = 0.10, size = 105, normalized size = 1.12

$$\frac{1}{2}\left(2a^2x+b^2x-b^2x\cos\left(2\left(c+\frac{d}{x}\right)\right)-4abd\cos(c)\text{Ci}\left(\frac{d}{x}\right)-2b^2d\text{Ci}\left(\frac{2d}{x}\right)\sin(2c)+4abx\sin\left(c+\frac{d}{x}\right)+4abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-2b^2d\cos(2c)\text{Si}\left(\frac{2d}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2,x]
```

```
[Out] (2*a^2*x + b^2*x - b^2*x*Cos[2*(c + d/x)] - 4*a*b*d*Cos[c]*CosIntegral[d/x]
 - 2*b^2*d*CosIntegral[(2*d)/x]*Sin[2*c] + 4*a*b*x*Sin[c + d/x] + 4*a*b*d*S
in[c]*SinIntegral[d/x] - 2*b^2*d*Cos[2*c]*SinIntegral[(2*d)/x])/2
```

Maple [A]
time = 0.07, size = 110, normalized size = 1.17

method	result
derivativedivides	$-d\left(-\frac{a^2x}{d}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{sinIntegral}\left(\frac{d}{x}\right)\sin(c)+\text{cosineIntegral}\left(\frac{d}{x}\right)\cos(c)\right)-\right.$
default	$-d\left(-\frac{a^2x}{d}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)x}{d}-\text{sinIntegral}\left(\frac{d}{x}\right)\sin(c)+\text{cosineIntegral}\left(\frac{d}{x}\right)\cos(c)\right)-\right.$

risch	$\frac{\pi \operatorname{csgn}\left(\frac{d}{x}\right) \mathrm{e}^{-2ic} b^2 d}{2} - \operatorname{sinIntegral}\left(\frac{2d}{x}\right) \mathrm{e}^{-2ic} b^2 d + \frac{i \operatorname{expIntegral}\left(1, -\frac{2id}{x}\right) \mathrm{e}^{-2ic} b^2 d}{2} - \frac{i d b^2 \operatorname{expIntegral}\left(1, -\frac{2}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-a^2/d*x+2*a*b*(-sin(c+d/x)/d*x-Si(d/x)*sin(c)+Ci(d/x)*cos(c))-1/2*b^2/
d*x-1/4*b^2*(-2*cos(2*d/x+2*c)/d*x-4*Si(2*d/x)*cos(2*c)-4*Ci(2*d/x)*sin(2*c
)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 137, normalized size = 1.46

$$-\left(\left(\left(\left(\operatorname{Ei}\left(\frac{id}{x}\right) + \operatorname{Ei}\left(-\frac{id}{x}\right)\right)\cos(c) - \left(-i\operatorname{Ei}\left(\frac{id}{x}\right) + i\operatorname{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d - 2x\sin\left(\frac{cx+d}{x}\right)\right)ab - \frac{1}{2}\left(\left(\left(-i\operatorname{Ei}\left(\frac{2id}{x}\right) + i\operatorname{Ei}\left(-\frac{2id}{x}\right)\right)\cos(2c) + \left(\operatorname{Ei}\left(\frac{2id}{x}\right) + \operatorname{Ei}\left(-\frac{2id}{x}\right)\right)\sin(2c)\right)d + x\cos\left(\frac{2(cx+d)}{x}\right) - x\right)b^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(c))*
d - 2*x*sin((c*x + d)/x))*a*b - 1/2*(((-I*Ei(2*I*d/x) + I*Ei(-2*I*d/x))*cos
(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*cos(2*(c*x + d)/x) - x
)*b^2 + a^2*x
```

Fricas [A]
time = 0.38, size = 130, normalized size = 1.38

$$-b^2x\cos\left(\frac{cx+d}{x}\right)^2 - b^2d\cos(2c)\operatorname{Si}\left(\frac{2d}{x}\right) + 2abd\sin(c)\operatorname{Si}\left(\frac{d}{x}\right) + 2abx\sin\left(\frac{cx+d}{x}\right) + (a^2+b^2)x - \left(abd\operatorname{Ci}\left(\frac{d}{x}\right) + abd\operatorname{Ci}\left(-\frac{d}{x}\right)\right)\cos(c) - \frac{1}{2}\left(b^2d\operatorname{Ci}\left(\frac{2d}{x}\right) + b^2d\operatorname{Ci}\left(-\frac{2d}{x}\right)\right)\sin(2c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] -b^2*x*cos((c*x + d)/x)^2 - b^2*d*cos(2*c)*sin_integral(2*d/x) + 2*a*b*d*si
n(c)*sin_integral(d/x) + 2*a*b*x*sin((c*x + d)/x) + (a^2 + b^2)*x - (a*b*d*
cos_integral(d/x) + a*b*d*cos_integral(-d/x))*cos(c) - 1/2*(b^2*d*cos_integ
ral(2*d/x) + b^2*d*cos_integral(-2*d/x))*sin(2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c + \frac{d}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(94) = 188.

time = 4.84, size = 305, normalized size = 3.24

$$-\frac{4\,abcd^2\cos\left(c\right)\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)+2\,b^2cd^2\operatorname{Ci}\left(-2\,c+\frac{2\,(cx+d)}{x}\right)\sin\left(2\,c\right)-2\,b^2cd^2\cos\left(2\,c\right)\operatorname{Si}\left(2\,c-\frac{2\,(cx+d)}{x}\right)+4\,abcd^2\sin\left(c\right)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)-\frac{4\,(cx+d)abd^2\cos(c)\operatorname{Ci}\left(-c+\frac{cx+d}{x}\right)}{x}-\frac{2\,(cx+d)b^2d^2\operatorname{Ci}\left(-2\,c+\frac{2\,(cx+d)}{x}\right)\sin(2\,c)}{x}+\frac{2\,(cx+d)b^2d^2\cos(2\,c)\operatorname{Si}\left(2\,c-\frac{2\,(cx+d)}{x}\right)}{x}-\frac{4\,(cx+d)abd^2\sin(c)\operatorname{Si}\left(c-\frac{cx+d}{x}\right)}{x}-b^2d^2\cos\left(\frac{2\,(cx+d)}{x}\right)+4\,abd^2\sin\left(\frac{cx+d}{x}\right)+2\,a^2d^2+b^2d^2}{2\left(c-\frac{cx+d}{x}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*c*d^2*cos(c)*cos_integral(-c + (c*x + d)/x) + 2*b^2*c*d^2*cos_i
ntegral(-2*c + 2*(c*x + d)/x)*sin(2*c) - 2*b^2*c*d^2*cos(2*c)*sin_integral(
2*c - 2*(c*x + d)/x) + 4*a*b*c*d^2*sin(c)*sin_integral(c - (c*x + d)/x) - 4
*(c*x + d)*a*b*d^2*cos(c)*cos_integral(-c + (c*x + d)/x)/x - 2*(c*x + d)*b^
2*d^2*cos_integral(-2*c + 2*(c*x + d)/x)*sin(2*c)/x + 2*(c*x + d)*b^2*d^2*c
os(2*c)*sin_integral(2*c - 2*(c*x + d)/x)/x - 4*(c*x + d)*a*b*d^2*sin(c)*si
n_integral(c - (c*x + d)/x)/x - b^2*d^2*cos(2*(c*x + d)/x) + 4*a*b*d^2*sin(
(c*x + d)/x) + 2*a^2*d^2 + b^2*d^2)/((c - (c*x + d)/x)*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))^2,x)
```

```
[Out] int((a + b*sin(c + d/x))^2, x)
```

3.296 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=255

$$-\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{Ci}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f}+\frac{b^2\cos(2c)\text{Ci}\left(\frac{2d}{x}\right)}{2f}+\frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a^2\log(x)}{f}+\frac{b^2\log(x)}{2f}-$$

```
[Out] 1/2*b^2*Ci(2*d/x)*cos(2*c)/f-1/2*b^2*Ci(2*d*(f/e+1/x))*cos(2*c-2*d*f/e)/f+a
^2*ln(f+e/x)/f+1/2*b^2*ln(f+e/x)/f+a^2*ln(x)/f+1/2*b^2*ln(x)/f+2*a*b*cos(c-
d*f/e)*Si(d*(f/e+1/x))/f-2*a*b*cos(c)*Si(d/x)/f-2*a*b*Ci(d/x)*sin(c)/f-1/2*
b^2*Si(2*d/x)*sin(2*c)/f+1/2*b^2*Si(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/f+2*a*b
*Ci(d*(f/e+1/x))*sin(c-d*f/e)/f
```

Rubi [A]
time = 0.45, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3512, 3398, 3384, 3380, 3383, 3393}

$$\frac{a^2\log\left(\frac{e}{x}+f\right)}{f}+\frac{a^2\log(x)}{f}+\frac{2ab\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{2ab\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{2ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{2ab\cos(c)\text{Si}\left(\frac{d}{x}\right)}{f}-\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f}+\frac{b^2\cos(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)}{2f}+\frac{b^2\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2f}-\frac{b^2\sin(2c)\text{Si}\left(\frac{2d}{x}\right)}{2f}+\frac{b^2\log\left(\frac{e}{x}+f\right)}{2f}+\frac{b^2\log(x)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

```
[Out] -1/2*(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/f + (b^2*Co
s[2*c]*CosIntegral[(2*d)/x])/(2*f) + (a^2*Log[f + e/x])/f + (b^2*Log[f + e/
x])/(2*f) + (a^2*Log[x])/f + (b^2*Log[x])/(2*f) - (2*a*b*CosIntegral[d/x]*S
in[c])/f + (2*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (2*a*
b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/f + (b^2*Sin[2*c - (2*d*f
)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(2*f) - (2*a*b*Cos[c]*SinIntegral[d/x
])/f - (b^2*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = -\text{Subst}\left(\int \left(\frac{(a+b\sin(c+dx))^2}{fx}-\frac{e(a+b\sin(c+dx))^2}{f(f+ex)}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{e\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{f+ex}\,dx,x,\frac{1}{x}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x}+\frac{2ab\sin(c+dx)}{x}+\frac{b^2\sin^2(c+dx)}{x}\right)dx,x,\frac{1}{x}\right)}{f}+\frac{e\text{Subst}\left(\int \left(\frac{a^2}{f+ex}+\frac{2ab\,\text{si}}{f}\right.\right.}{}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{a^2\log(x)}{f}-\frac{(2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}-\frac{b^2\text{Subst}\left(\int \frac{\sin^2(}{}\right.}{f}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{a^2\log(x)}{f}-\frac{b^2\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2c+2dx)}{2x}\right)dx,x,\frac{1}{x}\right)}{f}+\frac{(b^2e)\,\text{Su}}{}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a^2\log(x)}{f}+\frac{b^2\log(x)}{2f}-\frac{2ab\text{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{2a}{}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a^2\log(x)}{f}+\frac{b^2\log(x)}{2f}-\frac{2ab\text{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{2a}{}$$

$$= -\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{Ci}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{2f}+\frac{b^2\cos(2c)\text{Ci}\left(\frac{2d}{x}\right)}{2f}+\frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f\right.}{2f}$$

Mathematica [A]
time = 0.24, size = 195, normalized size = 0.76

$$\frac{-b^2\cos\left(2c-\frac{2df}{e}\right)\text{Ci}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+b^2\cos(2c)\text{Ci}\left(\frac{2d}{x}\right)+2a^2\log(e+fx)+b^2\log(e+fx)-4ab\text{Ci}\left(\frac{d}{x}\right)\sin(c)+4ab\text{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(c-\frac{df}{e}\right)+4ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+b^2\sin\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)-4ab\cos(c)\text{Si}\left(\frac{d}{x}\right)-b^2\sin(2c)\text{Si}\left(\frac{2d}{x}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

```
[Out] (-(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))]) + b^2*Cos[2*c]
*CosIntegral[(2*d)/x] + 2*a^2*Log[e + f*x] + b^2*Log[e + f*x] - 4*a*b*CosIn
tegral[d/x]*Sin[c] + 4*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e] +
 4*a*b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + b^2*Sin[2*c - (2*d*
f)/e]*SinIntegral[2*d*(f/e + x^(-1))] - 4*a*b*Cos[c]*SinIntegral[d/x] - b^2
*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Maple [A]
time = 0.13, size = 354, normalized size = 1.39

method	result
risch	$-\frac{iab\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\mathrm{expIntegral}\left(1,\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{f}+\frac{iab\,\mathrm{expIntegral}\left(1,\frac{id}{x}\right)\mathrm{e}^{-ic}}{f}+\frac{\ln(fx+e)a^2}{f}+\frac{\ln(fx+e)b^2}{2f}+$
derivativedivides	$-d\left(\frac{a^2\ln\left(\frac{d}{x}\right)}{fd}-\frac{a^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}+\frac{2ab\left(\mathrm{sinIntegral}\left(\frac{d}{x}\right)\cos(c)+\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\sin(c)\right)}{fd}-\frac{2abe\left(-\right.}{}\right.$
default	$-d\left(\frac{a^2\ln\left(\frac{d}{x}\right)}{fd}-\frac{a^2\ln\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)}{fd}+\frac{2ab\left(\mathrm{sinIntegral}\left(\frac{d}{x}\right)\cos(c)+\mathrm{cosineIntegral}\left(\frac{d}{x}\right)\sin(c)\right)}{fd}-\frac{2abe\left(-\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(a^2/f/d*ln(d/x)-a^2/f/d*ln(-c*e+d*f+e*(c+d/x))+2*a*b/f/d*(Si(d/x)*cos(c
)+Ci(d/x)*sin(c))-2*a*b*e/f/d*(-Si(-d/x-c-(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e
-Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)+1/2*b^2/f/d*ln(d/x)-1/2*b^2/f/
d*ln(-c*e+d*f+e*(c+d/x))-1/4*b^2/f/d*(-2*Si(2*d/x)*sin(2*c)+2*Ci(2*d/x)*cos
(2*c))+1/4*b^2*e/f/d*(-2*Si(-2*d/x-2*c-2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/
e+2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f - 1/2*(2*b^2*f*integrate(1/4*cos(2*(c*x + d)/x)/((f*x +
e)*cos(2*(c*x + d)/x)^2 + (f*x + e)*sin(2*(c*x + d)/x)^2), x) + 2*b^2*f*int
egrate(1/4*cos(2*(c*x + d)/x)/(f*x + e), x) - 2*a*b*f*integrate(sin((c*x +
d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e)*sin((c*x + d)/x)^2), x) - 2
*a*b*f*integrate(sin((c*x + d)/x)/(f*x + e), x) - b^2*log(f*x + e))/f
```

Fricas [A]
time = 0.39, size = 289, normalized size = 1.13

$$\frac{2\,b^2\sin\left(-2\,(df-ce)e^{(-1)}\right)\mathrm{Si}\left(\frac{2\,(dfx+de)e^{(-1)}}{x}\right)+8\,ab\cos\left(-(df-ce)e^{(-1)}\right)\mathrm{Si}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)-2\,b^2\sin(2\,c)\,\mathrm{Si}\left(\frac{2\,d}{x}\right)-8\,ab\cos(c)\,\mathrm{Si}\left(\frac{d}{x}\right)-\left(b^2\,\mathrm{Ci}\left(\frac{2\,(dfx+de)e^{(-1)}}{x}\right)+b^2\,\mathrm{Ci}\left(-\frac{2\,(dfx+de)e^{(-1)}}{x}\right)\right)\cos\left(-2\,(df-ce)e^{(-1)}\right)+\left(b^2\,\mathrm{Ci}\left(\frac{2\,d}{x}\right)+b^2\,\mathrm{Ci}\left(-\frac{2\,d}{x}\right)\right)\cos(2\,c)+2\,(2\,a^2+b^2)\log(fx+e)+4\left(ab\,\mathrm{Ci}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+ab\,\mathrm{Ci}\left(-\frac{(dfx+de)e^{(-1)}}{x}\right)\right)\sin\left(-(df-ce)e^{(-1)}\right)-4\left(ab\,\mathrm{Ci}\left(\frac{d}{x}\right)+ab\,\mathrm{Ci}\left(-\frac{d}{x}\right)\right)\sin(c)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*sin(-2*(d*f - c*e)*e^(-1))*sin_integral(2*(d*f*x + d*e)*e^(-1)/x
) + 8*a*b*cos(-(d*f - c*e)*e^(-1))*sin_integral((d*f*x + d*e)*e^(-1)/x) - 2
*b^2*sin(2*c)*sin_integral(2*d/x) - 8*a*b*cos(c)*sin_integral(d/x) - (b^2*c
os_integral(2*(d*f*x + d*e)*e^(-1)/x) + b^2*cos_integral(-2*(d*f*x + d*e)*e
^(-1)/x))*cos(-2*(d*f - c*e)*e^(-1)) + (b^2*cos_integral(2*d/x) + b^2*cos_i
ntegral(-2*d/x))*cos(2*c) + 2*(2*a^2 + b^2)*log(f*x + e) + 4*(a*b*cos_integ
ral((d*f*x + d*e)*e^(-1)/x) + a*b*cos_integral(-(d*f*x + d*e)*e^(-1)/x))*si
n(-(d*f - c*e)*e^(-1)) - 4*(a*b*cos_integral(d/x) + a*b*cos_integral(-d/x))
*sin(c))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sin \left(c + \frac{d}{x}\right)\right)^{2}}{e + f x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2/(e + f*x), x)
```

Giac [A]
time = 3.88, size = 368, normalized size = 1.44

$$-\frac{b^{2} d \cos\left(-2\,(df - ce)e^{(-1)}\right) \operatorname{Ci}\left(2\left(df - ce + \frac{(cx+d)e}{x}\right)e^{(-1)}\right) - b^{2} d \cos\left(2\,c\right) \operatorname{Ci}\left(-2\,c + \frac{2\,(cx+d)}{x}\right) - 4\,abd \operatorname{Ci}\left(\left(df - ce + \frac{(cx+d)e}{x}\right)e^{(-1)}\right) \sin\left(-(df - ce)e^{(-1)}\right) + 4\,abd \operatorname{Ci}\left(-c + \frac{cx+d}{x}\right) \sin\left(c\right) + 4\,abd \cos\left(-(df - ce)e^{(-1)}\right) \operatorname{Si}\left(-\left(df - ce + \frac{(cx+d)e}{x}\right)e^{(-1)}\right) + b^{2} d \sin\left(-2\,(df - ce)e^{(-1)}\right) \operatorname{Si}\left(-2\left(df - ce + \frac{(cx+d)e}{x}\right)e^{(-1)}\right) - b^{2} d \sin\left(2\,c\right) \operatorname{Si}\left(2\,c - \frac{2\,(cx+d)}{x}\right) - 4\,abd \cos\left(c\right) \operatorname{Si}\left(c - \frac{cx+d}{x}\right) - 2\,a^{2} d \log\left(-df + ce - \frac{(cx+d)e}{x}\right) - b^{2} d \log\left(-df + ce - \frac{(cx+d)e}{x}\right) + 2\,a^{2} d \log\left(c - \frac{cx+d}{x}\right) + b^{2} d \log\left(c - \frac{cx+d}{x}\right)}{2\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*cos(-2*(d*f - c*e)*e^(-1))*cos_integral(2*(d*f - c*e + (c*x + d
)*e/x)*e^(-1)) - b^2*d*cos(2*c)*cos_integral(-2*c + 2*(c*x + d)/x) - 4*a*b*
d*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*sin(-(d*f - c*e)*e^(-1))
 + 4*a*b*d*cos_integral(-c + (c*x + d)/x)*sin(c) + 4*a*b*d*cos(-(d*f - c*e)
*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + b^2*d*sin(-2*(
d*f - c*e)*e^(-1))*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) - b^
2*d*sin(2*c)*sin_integral(2*c - 2*(c*x + d)/x) - 4*a*b*d*cos(c)*sin_integra
l(c - (c*x + d)/x) - 2*a^2*d*log(-d*f + c*e - (c*x + d)*e/x) - b^2*d*log(-d
*f + c*e - (c*x + d)*e/x) + 2*a^2*d*log(c - (c*x + d)/x) + b^2*d*log(c - (c
*x + d)/x))/(d*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)}^2}{e+f\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))^2/(e + f*x),x)
```

```
[Out] int((a + b*sin(c + d/x))^2/(e + f*x), x)
```

3.297 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=195

$$\frac{a^2}{e\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\mathrm{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}-\frac{b^2d\mathrm{Ci}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(2c-\frac{2df}{e}\right)}{e^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}+\frac{b^2\sin^2\left(c+\right.}{e\left(f+\frac{e}{x}\right.}$$

```
[Out] a^2/e/(f+e/x)-2*a*b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^2-b^2*d*cos(2*c-2*d*f/
e)*Si(2*d*(f/e+1/x))/e^2-b^2*d*Ci(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/e^2+2*a*b
*d*Si(d*(f/e+1/x))*sin(c-d*f/e)/e^2+2*a*b*sin(c+d/x)/e/(f+e/x)+b^2*sin(c+d/
x)^2/e/(f+e/x)
```

Rubi [A]

time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383, 3394, 12}

$$\frac{a^2}{e\left(\frac{e}{x}+f\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}-\frac{b^2d\sin\left(2c-\frac{2df}{e}\right)\text{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}-\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

```
[Out] a^2/(e*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]
)/e^2 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^2 +
(2*a*b*Sin[c + d/x])/(e*(f + e/x)) + (b^2*Sin[c + d/x]^2)/(e*(f + e/x)) + (
2*a*b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^2 - (b^2*d*Cos[2*
c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = -\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a^2}{(f+ex)^2} + \frac{2ab\sin(c+dx)}{(f+ex)^2} + \frac{b^2\sin^2(c+dx)}{(f+ex)^2}\right)\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - (2ab)\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right) - b^2\text{Subst}\left(\int \frac{\sin^2(c+dx)}{(f+ex)^2}\right.$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(2abd)\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x,\right.}{e}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(b^2d)\,\text{Subst}\left(\int \frac{\sin(2c+2dx)}{f+ex}\,dx, x\right.}{e}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} - \frac{b^2d\text{Ci}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^2} +$$

Mathematica [A]
time = 0.94, size = 263, normalized size = 1.35

$$-\frac{2a^2e^2+b^2e^2+b^2efx\cos\left(2\left(c+\frac{d}{x}\right)\right)+4abdf(e+fx)\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+2b^2df(e+fx)\text{Ci}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\sin\left(2c-\frac{2df}{e}\right)-4abefx\sin\left(c+\frac{d}{x}\right)-4abdef\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)-4abdf^2x\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+2b^2def\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)+2b^2df^2x\cos\left(2c-\frac{2df}{e}\right)\text{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^2f(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

```
[Out] -1/2*(2*a^2*e^2 + b^2*e^2 + b^2*e*f*x*Cos[2*(c + d/x)] + 4*a*b*d*f*(e + f*x
)*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))] + 2*b^2*d*f*(e + f*x)*CosI
ntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e] - 4*a*b*e*f*x*Sin[c + d/x]
 - 4*a*b*d*e*f*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - 4*a*b*d*f^2
*x*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 2*b^2*d*e*f*Cos[2*c - (
2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 2*b^2*d*f^2*x*Cos[2*c - (2*d*f)
/e]*SinIntegral[2*d*(f/e + x^(-1))])/(e^2*f*(e + f*x))
```

Maple [A]
time = 0.14, size = 313, normalized size = 1.61

method	result

derivativedivides	$-d\left(-\frac{a^2}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\text{sinIntegral}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\cos}{}}{e}\right.\right.$
default	$-d\left(-\frac{a^2}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+2ab\left(-\frac{\sin\left(c+\frac{d}{x}\right)}{\left(-ce+df+e\left(c+\frac{d}{x}\right)\right)e}+\frac{-\frac{\text{sinIntegral}\left(-\frac{d}{x}-c-\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+df}{e}\right)}{e}+\frac{\cos}{}}{e}\right.\right.$
risch	$\frac{abd\,\mathrm{e}^{-\frac{i(ce-df)}{e}}\,\text{expIntegral}\left(1,\frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{e^2}-\frac{a^2}{f(fx+e)}-\frac{b^2}{2f(fx+e)}+\frac{id\,b^2\mathrm{e}^{-\frac{2i(ce-df)}{e}}\,\text{expIntegral}\left(1,\frac{2id}{x}+2ic\right.}{2e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-a^2/(-c*e+d*f+e*(c+d/x))/e+2*a*b*(-sin(c+d/x)/(-c*e+d*f+e*(c+d/x))/e+(
-Si(-d/x-c-(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c
*e+d*f)/e)/e)/e)-1/2*b^2/(-c*e+d*f+e*(c+d/x))/e-1/4*b^2*(-2*cos(2*d/x+2*c)/
(-c*e+d*f+e*(c+d/x))/e-2*(-2*Si(-2*d/x-2*c-2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)
/e)/e-2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e)/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(f^2*x + f*e) - 1/2*(b^2 + 2*(b^2*f^2*x + b^2*f*e)*integrate(1/4*cos(2
*(c*x + d)/x)/(f^2*x^2 + 2*f*x*e + e^2), x) + 2*(b^2*f^2*x + b^2*f*e)*integ
rate(1/4*cos(2*(c*x + d)/x)/((f^2*x^2 + 2*f*x*e + e^2)*cos(2*(c*x + d)/x)^2
 + (f^2*x^2 + 2*f*x*e + e^2)*sin(2*(c*x + d)/x)^2), x) - 2*(a*b*f^2*x + a*b
*f*e)*integrate(sin((c*x + d)/x)/(f^2*x^2 + 2*f*x*e + e^2), x) - 2*(a*b*f^2
*x + a*b*f*e)*integrate(sin((c*x + d)/x)/((f^2*x^2 + 2*f*x*e + e^2)*cos((c*
```

```
x + d)/x)^2 + (f^2*x^2 + 2*f*x*e + e^2)*sin((c*x + d)/x)^2), x))/(f^2*x + f
*e)
```

Fricas [A]
time = 0.42, size = 348, normalized size = 1.78

$$-\frac{2\,b^{2}fx\cos\left(\frac{cx+d}{x}\right)^{2}e-4\,abfxe\sin\left(\frac{cx+d}{x}\right)-b^{2}fxe+2\,(b^{2}df^{2}x+b^{2}dfe)\cos\left(-2\,(df-ce)e^{(-1)}\right)\mathrm{Si}\left(\frac{2\,(dfx+de)e^{(-1)}}{x}\right)-4\,(abdf^{2}x+abdfe)\sin\left(-(df-ce)e^{(-1)}\right)\mathrm{Si}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+2\left((abdf^{2}x+abdfe)\,\mathrm{Ci}\left(\frac{(dfx+de)e^{(-1)}}{x}\right)+(abdf^{2}x+abdfe)\,\mathrm{Ci}\left(-\frac{(dfx+de)e^{(-1)}}{x}\right)\right)\cos\left(-(df-ce)e^{(-1)}\right)+(2\,a^{2}+b^{2})e^{2}+\left((b^{2}df^{2}x+b^{2}dfe)\,\mathrm{Ci}\left(\frac{2\,(dfx+de)e^{(-1)}}{x}\right)+(b^{2}df^{2}x+b^{2}dfe)\,\mathrm{Ci}\left(-\frac{2\,(dfx+de)e^{(-1)}}{x}\right)\right)\sin\left(-2\,(df-ce)e^{(-1)}\right)}{2\,(f^{2}xe^{2}+fe^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*f*x*cos((c*x + d)/x)^2*e - 4*a*b*f*x*e*sin((c*x + d)/x) - b^2*f
*x*e + 2*(b^2*d*f^2*x + b^2*d*f*e)*cos(-2*(d*f - c*e)*e^(-1))*sin_integral(
2*(d*f*x + d*e)*e^(-1)/x) - 4*(a*b*d*f^2*x + a*b*d*f*e)*sin(-(d*f - c*e)*e^
(-1))*sin_integral((d*f*x + d*e)*e^(-1)/x) + 2*((a*b*d*f^2*x + a*b*d*f*e)*c
os_integral((d*f*x + d*e)*e^(-1)/x) + (a*b*d*f^2*x + a*b*d*f*e)*cos_integra
l(-(d*f*x + d*e)*e^(-1)/x))*cos(-(d*f - c*e)*e^(-1)) + (2*a^2 + b^2)*e^2 +
((b^2*d*f^2*x + b^2*d*f*e)*cos_integral(2*(d*f*x + d*e)*e^(-1)/x) + (b^2*d*
f^2*x + b^2*d*f*e)*cos_integral(-2*(d*f*x + d*e)*e^(-1)/x))*sin(-2*(d*f - c
*e)*e^(-1)))/(f^2*x*e^2 + f*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sin\left(c + \frac{d}{x}\right)\right)^{2}}{\left(e + fx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2/(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(183) = 366.
time = 7.18, size = 700, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*d^3*f*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x +
 d)*e/x)*e^(-1)) - 4*a*b*c*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f -
 c*e + (c*x + d)*e/x)*e^(-1))*e + 2*b^2*d^3*f*cos_integral(2*(d*f - c*e + (
c*x + d)*e/x)*e^(-1))*sin(-2*(d*f - c*e)*e^(-1)) - 2*b^2*c*d^2*cos_integral
(2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e*sin(-2*(d*f - c*e)*e^(-1)) + 4*a*b
```

```
*d^3*f*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e
^(-1)) - 4*a*b*c*d^2*e*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e +
(c*x + d)*e/x)*e^(-1)) - 2*b^2*d^3*f*cos(-2*(d*f - c*e)*e^(-1))*sin_integra
l(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 2*b^2*c*d^2*cos(-2*(d*f - c*e)*e
^(-1))*e*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 4*(c*x + d)*
a*b*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e
^(-1))*e/x + 2*(c*x + d)*b^2*d^2*cos_integral(2*(d*f - c*e + (c*x + d)*e/x)
*e^(-1))*e*sin(-2*(d*f - c*e)*e^(-1))/x + 4*(c*x + d)*a*b*d^2*e*sin(-(d*f -
 c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x - 2*(c*x
+ d)*b^2*d^2*cos(-2*(d*f - c*e)*e^(-1))*e*sin_integral(-2*(d*f - c*e + (c*x
 + d)*e/x)*e^(-1))/x + b^2*d^2*cos(2*(c*x + d)/x)*e - 4*a*b*d^2*e*sin((c*x
+ d)/x) - 2*a^2*d^2*e - b^2*d^2*e)/((d*f*e^2 - c*e^3 + (c*x + d)*e^3/x)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{{\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))^2/(e + f*x)^2,x)
[Out] int((a + b*sin(c + d/x))^2/(e + f*x)^2, x)
```

3.298 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx$

Optimal. Leaf size=470

$$-\frac{a^2 f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{Ci}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}+\frac{b^2d^2f\cos\left(2c-\frac{2df}{e}\right)\operatorname{Ci}\left(2d\right.}{e^4}$$

```
[Out] -1/2*a^2*f/e^2/(f+e/x)^2+a^2/e^2/(f+e/x)+b^2*d^2*f*Ci(2*d*(f/e+1/x))*cos(2*
c-2*d*f/e)/e^4-2*a*b*d*Ci(d*(f/e+1/x))*cos(c-d*f/e)/e^3-a*b*d*f*cos(c+d/x)/
e^3/(f+e/x)-a*b*d^2*f*cos(c-d*f/e)*Si(d*(f/e+1/x))/e^4-b^2*d*cos(2*c-2*d*f/
e)*Si(2*d*(f/e+1/x))/e^3-b^2*d*Ci(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/e^3-b^2*d
^2*f*Si(2*d*(f/e+1/x))*sin(2*c-2*d*f/e)/e^4-a*b*d^2*f*Ci(d*(f/e+1/x))*sin(c
-d*f/e)/e^4+2*a*b*d*Si(d*(f/e+1/x))*sin(c-d*f/e)/e^3-a*b*f*sin(c+d/x)/e^2/(
f+e/x)^2+2*a*b*sin(c+d/x)/e^2/(f+e/x)-b^2*d*f*cos(c+d/x)*sin(c+d/x)/e^3/(f+
e/x)-1/2*b^2*f*sin(c+d/x)^2/e^2/(f+e/x)^2+b^2*sin(c+d/x)^2/e^2/(f+e/x)
```

Rubi [A]
time = 0.64, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3512, 3398, 3378, 3384, 3380, 3383, 3395, 31, 3393, 3394, 12}

$$\frac{a^2}{e^2\left(\frac{e}{x}+f\right)}-\frac{a^2 f}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{abd^2f\sin\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{abd^2f\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}+\frac{2abd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(\frac{e}{x}+f\right)}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)^2}+\frac{b^2d^2f\cos\left(2c-\frac{2df}{e}\right)\operatorname{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}-\frac{b^2d\sin\left(2c-\frac{2df}{e}\right)\operatorname{CosIntegral}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{b^2d^2f\sin\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}-\frac{b^2d\cos\left(2c-\frac{2df}{e}\right)\operatorname{Si}\left(2d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{b^2df\sin\left(c+\frac{d}{x}\right)\cos\left(c+\frac{d}{x}\right)}{e^3\left(\frac{e}{x}+f\right)}+\frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e^2\left(\frac{e}{x}+f\right)}-\frac{b^2f\sin^2\left(c+\frac{d}{x}\right)}{2e^2\left(\frac{e}{x}+f\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

```
[Out] -1/2*(a^2*f)/(e^2*(f + e/x)^2) + a^2/(e^2*(f + e/x)) - (a*b*d*f*Cos[c + d/x
])/(e^3*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))
])/e^3 + (b^2*d^2*f*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/e
^4 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^3 - (a*
b*d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/e^4 - (a*b*f*Sin[c
+ d/x])/(e^2*(f + e/x)^2) + (2*a*b*Sin[c + d/x])/(e^2*(f + e/x)) - (b^2*d*f
*Cos[c + d/x]*Sin[c + d/x])/(e^3*(f + e/x)) - (b^2*f*Sin[c + d/x]^2)/(2*e^2
*(f + e/x)^2) + (b^2*Sin[c + d/x]^2)/(e^2*(f + e/x)) - (a*b*d^2*f*Cos[c - (
d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^4 + (2*a*b*d*Sin[c - (d*f)/e]*SinI
ntegral[d*(f/e + x^(-1))])/e^3 - (b^2*d*Cos[2*c - (2*d*f)/e]*SinIntegral[2*
d*(f/e + x^(-1))])/e^3 - (b^2*d^2*f*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f
/e + x^(-1))])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
```

```
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = -\text{Subst}\left(\int\left(-\frac{f(a+b\sin(c+dx))^2}{e(f+ex)^3}+\frac{(a+b\sin(c+dx))^2}{e(f+ex)^2}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{(f+ex)^2}dx,x,\frac{1}{x}\right)}{e}+\frac{f\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{(f+ex)^3}dx,x,\frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a^2}{(f+ex)^2}+\frac{2ab\sin(c+dx)}{(f+ex)^2}+\frac{b^2\sin^2(c+dx)}{(f+ex)^2}\right)dx,x,\frac{1}{x}\right)}{e}+\frac{f\text{Subst}\left(\int\left(\frac{a^2}{(f+ex)^3}\right.\right.}{}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{(2ab)\text{Subst}\left(\int\frac{\sin(c+dx)}{(f+ex)^2}dx,x,\frac{1}{x}\right)}{e}-\frac{b^2\text{Subst}\left(\int\frac{\sin}{(}\right.}{}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{b^2df\cos\left(c\right.}{e^3\left(\right.}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}+\frac{b^2d^2f\log\left(f+\frac{e}{x}\right)}{e^4}-\frac{abf\sin}{e^2\left(f\right.}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}$$

Mathematica [A]
time = 2.22, size = 740, normalized size = 1.57

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

```
[Out] -1/4*(2*a^2*e^4 + b^2*e^4 + 4*a*b*d*e^2*f^2*x*Cos[c + d/x] + 4*a*b*d*e*f^3*
x^2*Cos[c + d/x] + 2*b^2*e^3*f*x*Cos[2*(c + d/x)] + b^2*e^2*f^2*x^2*Cos[2*(
c + d/x)] - 4*b^2*d*f*(e + f*x)^2*CosIntegral[2*d*(f/e + x^(-1))]*(d*f*Cos[
2*c - (2*d*f)/e] - e*Sin[2*c - (2*d*f)/e]) + 4*a*b*d*f*(e + f*x)^2*CosInteg
ral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c - (d*f)/e]) - 8*a*b
```

```
*e^3*f*x*Sin[c + d/x] - 4*a*b*e^2*f^2*x^2*Sin[c + d/x] + 2*b^2*d*e^2*f^2*x*
Sin[2*(c + d/x)] + 2*b^2*d*e*f^3*x^2*Sin[2*(c + d/x)] + 4*a*b*d^2*e^2*f^2*C
os[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 8*a*b*d^2*e*f^3*x*Cos[c - (
d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 4*a*b*d^2*f^4*x^2*Cos[c - (d*f)/e]*
SinIntegral[d*(f/e + x^(-1))] - 8*a*b*d*e^3*f*Sin[c - (d*f)/e]*SinIntegral[
d*(f/e + x^(-1))] - 16*a*b*d*e^2*f^2*x*Sin[c - (d*f)/e]*SinIntegral[d*(f/e
+ x^(-1))] - 8*a*b*d*e*f^3*x^2*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1)
)] + 4*b^2*d*e^3*f*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 8
*b^2*d*e^2*f^2*x*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b
^2*d*e*f^3*x^2*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2
*d^2*e^2*f^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 8*b^2*d
^2*e*f^3*x*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2*d^2
*f^4*x^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(e^4*f*(e +
f*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1138 vs. $2(466) = 932$.

time = 0.20, size = 1139, normalized size = 2.42

method	result
risch	$\frac{iab\,d^2 \mathrm{e}^{-\frac{i(ce-df)}{e}} \operatorname{expIntegral}\left(1, \frac{id}{x}+ic-\frac{i(ce-df)}{e}\right) f}{2e^4}+\frac{abd\,\mathrm{e}^{-\frac{i(ce-df)}{e}} \operatorname{expIntegral}\left(1, \frac{id}{x}+ic-\frac{i(ce-df)}{e}\right)}{e^3}-\frac{a^2}{2f(fx+e)}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d*(-a^2/e^2/(-c*e+d*f+e*(c+d/x))-1/2*(c*e-d*f)/e^2*a^2/(-c*e+d*f+e*(c+d/x)
)^2+2*a*b/e*(-sin(c+d/x)/(-c*e+d*f+e*(c+d/x))/e+(-Si(-d/x-c-(-c*e+d*f)/e)*s
in((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e)/e)+2*(c*e-d*
f)/e*a*b*(-1/2*sin(c+d/x)/(-c*e+d*f+e*(c+d/x))^2/e+1/2*(-cos(c+d/x)/(-c*e+d
*f+e*(c+d/x))/e-(-Si(-d/x-c-(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*
e+d*f)/e)*sin((-c*e+d*f)/e)/e)/e)/e)-1/2*b^2/e^2/(-c*e+d*f+e*(c+d/x))-1/4*(
c*e-d*f)/e^2*b^2/(-c*e+d*f+e*(c+d/x))^2-1/4*b^2/e*(-2*cos(2*d/x+2*c)/(-c*e+
d*f+e*(c+d/x))/e-2*(-2*Si(-2*d/x-2*c-2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e-
2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e)/e)-1/4*(c*e-d*f)/e*b^
2*(-cos(2*d/x+2*c)/(-c*e+d*f+e*(c+d/x))^2/e-(-2*sin(2*d/x+2*c)/(-c*e+d*f+e*
(c+d/x))/e+2*(-2*Si(-2*d/x-2*c-2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e+2*Ci(2
*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e)/e)/e)+1/2*c*a^2/(-c*e+d*f+e
*(c+d/x))^2/e-2*c*a*b*(-1/2*sin(c+d/x)/(-c*e+d*f+e*(c+d/x))^2/e+1/2*(-cos(c
+d/x)/(-c*e+d*f+e*(c+d/x))/e-(-Si(-d/x-c-(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-
Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)/e)/e)+1/4*c*b^2/(-c*e+d*f+e*(c+
d/x))^2/e+1/4*c*b^2*(-cos(2*d/x+2*c)/(-c*e+d*f+e*(c+d/x))^2/e-(-2*sin(2*d/x
```

```
+2*c)/(-c*e+d*f+e*(c+d/x))/e+2*(-2*Si(-2*d/x-2*c-2*(-c*e+d*f)/e)*sin(2*(-c*
e+d*f)/e)/e+2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e)/e)/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(f^3*x^2 + 2*f^2*x*e + f*e^2) - 1/4*(b^2 + 4*(b^2*f^3*x^2 + 2*b^2*
f^2*x*e + b^2*f*e^2)*integrate(1/4*cos(2*(c*x + d)/x)/(f^3*x^3 + 3*f^2*x^2*
e + 3*f*x*e^2 + e^3), x) + 4*(b^2*f^3*x^2 + 2*b^2*f^2*x*e + b^2*f*e^2)*inte
grate(1/4*cos(2*(c*x + d)/x)/((f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2 + e^3)*cos
(2*(c*x + d)/x)^2 + (f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2 + e^3)*sin(2*(c*x +
d)/x)^2), x) - 4*(a*b*f^3*x^2 + 2*a*b*f^2*x*e + a*b*f*e^2)*integrate(sin((c
*x + d)/x)/(f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2 + e^3), x) - 4*(a*b*f^3*x^2 +
 2*a*b*f^2*x*e + a*b*f*e^2)*integrate(sin((c*x + d)/x)/((f^3*x^3 + 3*f^2*x^
2*e + 3*f*x*e^2 + e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*f^2*x^2*e + 3*f*x*
e^2 + e^3)*sin((c*x + d)/x)^2), x))/(f^3*x^2 + 2*f^2*x*e + f*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(442) = 884$.
time = 0.44, size = 910, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*f^2*x^2*e^2 + 2*b^2*f*x*e^3 - 2*(b^2*f^2*x^2*e^2 + 2*b^2*f*x*e^3)*
cos((c*x + d)/x)^2 - 4*((a*b*d*f^3*x^2*e + 2*a*b*d*f^2*x*e^2 + a*b*d*f*e^3)
*cos_integral((d*f*x + d*e)*e^(-1)/x) + (a*b*d*f^3*x^2*e + 2*a*b*d*f^2*x*e^
2 + a*b*d*f*e^3)*cos_integral(-(d*f*x + d*e)*e^(-1)/x) + (a*b*d^2*f^4*x^2 +
 2*a*b*d^2*f^3*x*e + a*b*d^2*f^2*e^2)*sin_integral((d*f*x + d*e)*e^(-1)/x))
*cos(-(d*f - c*e)*e^(-1)) + 2*((b^2*d^2*f^4*x^2 + 2*b^2*d^2*f^3*x*e + b^2*d
^2*f^2*e^2)*cos_integral(2*(d*f*x + d*e)*e^(-1)/x) + (b^2*d^2*f^4*x^2 + 2*b
^2*d^2*f^3*x*e + b^2*d^2*f^2*e^2)*cos_integral(-2*(d*f*x + d*e)*e^(-1)/x) -
 2*(b^2*d*f^3*x^2*e + 2*b^2*d*f^2*x*e^2 + b^2*d*f*e^3)*sin_integral(2*(d*f*
x + d*e)*e^(-1)/x))*cos(-2*(d*f - c*e)*e^(-1)) - 4*(a*b*d*f^3*x^2*e + a*b*d
*f^2*x*e^2)*cos((c*x + d)/x) - (2*a^2 + b^2)*e^4 - 2*((a*b*d^2*f^4*x^2 + 2*
a*b*d^2*f^3*x*e + a*b*d^2*f^2*e^2)*cos_integral((d*f*x + d*e)*e^(-1)/x) + (
a*b*d^2*f^4*x^2 + 2*a*b*d^2*f^3*x*e + a*b*d^2*f^2*e^2)*cos_integral(-(d*f*x
 + d*e)*e^(-1)/x) - 4*(a*b*d*f^3*x^2*e + 2*a*b*d*f^2*x*e^2 + a*b*d*f*e^3)*s
in_integral((d*f*x + d*e)*e^(-1)/x))*sin(-(d*f - c*e)*e^(-1)) - 2*((b^2*d*f
```

```
^3*x^2*e + 2*b^2*d*f^2*x*e^2 + b^2*d*f*e^3)*cos_integral(2*(d*f*x + d*e)*e^
(-1)/x) + (b^2*d*f^3*x^2*e + 2*b^2*d*f^2*x*e^2 + b^2*d*f*e^3)*cos_integral(
-2*(d*f*x + d*e)*e^(-1)/x) + 2*(b^2*d^2*f^4*x^2 + 2*b^2*d^2*f^3*x*e + b^2*d
^2*f^2*e^2)*sin_integral(2*(d*f*x + d*e)*e^(-1)/x))*sin(-2*(d*f - c*e)*e^(-
1)) + 4*(a*b*f^2*x^2*e^2 + 2*a*b*f*x*e^3 - (b^2*d*f^3*x^2*e + b^2*d*f^2*x*e
^2)*cos((c*x + d)/x))*sin((c*x + d)/x))/(f^3*x^2*e^4 + 2*f^2*x*e^5 + f*e^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3062 vs. 2(442) = 884.
time = 7.92, size = 3062, normalized size = 6.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*b^2*d^5*f^3*cos(-2*(d*f - c*e)*e^(-1))*cos_integral(2*(d*f - c*e + (
c*x + d)*e/x)*e^(-1)) - 8*b^2*c*d^4*f^2*cos(-2*(d*f - c*e)*e^(-1))*cos_inte
gral(2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e - 4*a*b*d^5*f^3*cos_integral((
d*f - c*e + (c*x + d)*e/x)*e^(-1))*sin(-(d*f - c*e)*e^(-1)) + 8*a*b*c*d^4*f
^2*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e*sin(-(d*f - c*e)*e^(-
1)) + 4*a*b*d^5*f^3*cos(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*
x + d)*e/x)*e^(-1)) - 8*a*b*c*d^4*f^2*cos(-(d*f - c*e)*e^(-1))*e*sin_integr
al(-(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 4*b^2*d^5*f^3*sin(-2*(d*f - c*e)*
e^(-1))*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) - 8*b^2*c*d^4*f
^2*e*sin(-2*(d*f - c*e)*e^(-1))*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)
*e^(-1)) + 4*b^2*c^2*d^3*f*cos(-2*(d*f - c*e)*e^(-1))*cos_integral(2*(d*f -
 c*e + (c*x + d)*e/x)*e^(-1))*e^2 + 8*(c*x + d)*b^2*d^4*f^2*cos(-2*(d*f - c
*e)*e^(-1))*cos_integral(2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e/x - 8*a*b*
d^4*f^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e
^(-1))*e - 4*a*b*c^2*d^3*f*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))
*e^2*sin(-(d*f - c*e)*e^(-1)) - 8*(c*x + d)*a*b*d^4*f^2*cos_integral((d*f -
 c*e + (c*x + d)*e/x)*e^(-1))*e*sin(-(d*f - c*e)*e^(-1))/x - 4*b^2*d^4*f^2*
cos_integral(2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e*sin(-2*(d*f - c*e)*e^(
-1)) + 4*a*b*c^2*d^3*f*cos(-(d*f - c*e)*e^(-1))*e^2*sin_integral(-(d*f - c*
e + (c*x + d)*e/x)*e^(-1)) + 8*(c*x + d)*a*b*d^4*f^2*cos(-(d*f - c*e)*e^(-1
```

))*e*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x - 8*a*b*d^4*f^2*e*
sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))
+ 4*b^2*d^4*f^2*cos(-2*(d*f - c*e)*e^(-1))*e*sin_integral(-2*(d*f - c*e + (
c*x + d)*e/x)*e^(-1)) + 4*b^2*c^2*d^3*f*e^2*sin(-2*(d*f - c*e)*e^(-1))*sin_
integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 8*(c*x + d)*b^2*d^4*f^2*e
sin(-2(d*f - c*e)*e^(-1))*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(
-1))/x - 8*(c*x + d)*b^2*c*d^3*f*cos(-2*(d*f - c*e)*e^(-1))*cos_integral(2*
(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2/x + 16*a*b*c*d^3*f*cos(-(d*f - c*e)
*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2 - 4*a*b*d^4*f
^2*cos((c*x + d)/x)*e + 8*(c*x + d)*a*b*c*d^3*f*cos_integral((d*f - c*e + (
c*x + d)*e/x)*e^(-1))*e^2*sin(-(d*f - c*e)*e^(-1))/x + 8*b^2*c*d^3*f*cos_in
tegral(2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2*sin(-2*(d*f - c*e)*e^(-1))
 - 2*b^2*d^4*f^2*e*sin(2*(c*x + d)/x) - 8*(c*x + d)*a*b*c*d^3*f*cos(-(d*f -
 c*e)*e^(-1))*e^2*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x + 16*
a*b*c*d^3*f*e^2*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x +
d)*e/x)*e^(-1)) - 8*b^2*c*d^3*f*cos(-2*(d*f - c*e)*e^(-1))*e^2*sin_integral
(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) - 8*(c*x + d)*b^2*c*d^3*f*e^2*sin(-
2*(d*f - c*e)*e^(-1))*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x
 - 8*a*b*c^2*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x +
d)*e/x)*e^(-1))*e^3 + 4*a*b*c*d^3*f*cos((c*x + d)/x)*e^2 + 4*(c*x + d)^2*b^
2*d^3*f*cos(-2*(d*f - c*e)*e^(-1))*cos_integral(2*(d*f - c*e + (c*x + d)*e/
x)*e^(-1))*e^2/x^2 - 16*(c*x + d)*a*b*d^3*f*cos(-(d*f - c*e)*e^(-1))*cos_in
tegral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2/x - 4*(c*x + d)^2*a*b*d^3*f*
cos_integral((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^2*sin(-(d*f - c*e)*e^(-1
))/x^2 - 4*b^2*c^2*d^2*cos_integral(2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))*e
^3*sin(-2*(d*f - c*e)*e^(-1)) - 8*(c*x + d)*b^2*d^3*f*cos_integral(2*(d*f -
 c*e + (c*x + d)*e/x)*e^(-1))*e^2*sin(-2*(d*f - c*e)*e^(-1))/x + 2*b^2*c*d^
3*f*e^2*sin(2*(c*x + d)/x) + 4*(c*x + d)^2*a*b*d^3*f*cos(-(d*f - c*e)*e^(-1
))*e^2*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x^2 - 8*a*b*c^2*d^
2*e^3*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x + d)*e/x)*e^
(-1)) - 16*(c*x + d)*a*b*d^3*f*e^2*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(
d*f - c*e + (c*x + d)*e/x)*e^(-1))/x + 4*b^2*c^2*d^2*cos(-2*(d*f - c*e)*e^(
-1))*e^3*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1)) + 8*(c*x + d)*
b^2*d^3*f*cos(-2*(d*f - c*e)*e^(-1))*e^2*sin_integral(-2*(d*f - c*e + (c*x
+ d)*e/x)*e^(-1))/x + 4*(c*x + d)^2*b^2*d^3*f*e^2*sin(-2*(d*f - c*e)*e^(-1)
)*sin_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x^2 + 16*(c*x + d)*a*
b*c*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral((d*f - c*e + (c*x + d)*e/x)*e
^(-1))*e^3/x - b^2*d^3*f*cos(2*(c*x + d)/x)*e^2 - 4*(c*x + d)*a*b*d^3*f*cos
((c*x + d)/x)*e^2/x + 8*(c*x + d)*b^2*c*d^2*cos_integral(2*(d*f - c*e + (c*
x + d)*e/x)*e^(-1))*e^3*sin(-2*(d*f - c*e)*e^(-1))/x - 2*(c*x + d)*b^2*d^3*
f*e^2*sin(2*(c*x + d)/x)/x + 4*a*b*d^3*f*e^2*sin((c*x + d)/x) + 16*(c*x + d
)*a*b*c*d^2*e^3*sin(-(d*f - c*e)*e^(-1))*sin_integral(-(d*f - c*e + (c*x +
d)*e/x)*e^(-1))/x - 8*(c*x + d)*b^2*c*d^2*cos(-2*(d*f - c*e)*e^(-1))*e^3*si
n_integral(-2*(d*f - c*e + (c*x + d)*e/x)*e^(-1))/x + 2*b^2*c*d^2*cos(2*(c*
x + d)/x)*e^3 - 8*(c*x + d)^2*a*b*d^2*cos(-(d*f - c*e)*e^(-1))*cos_integral

```
((d*f - c*e + (c*x + d)*e/x)*e^(-1))*e^3/x^2 + 2*a^2*d^3*f*e^2 + b^2*d^3*f*
e^2 - 4*(c*x + d)^2*b^2*d^2*cos_integral(2*(d*f...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}{\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d/x))^2/(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(c + d/x))^2/(e + f*x)^3, x)
```

3.299 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

```
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

```
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f^2*x^2 + 2*f*x*e + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^2}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)^2/(a + b*sin(c + d/x)), x)
```

3.300 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((e + f*x)/(a + b*sin(c + d/x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e+f\,x}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)/(a + b*sin(c + d/x)), x)
```

3.301 $\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{1}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d/x)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d/x])^(-1),x]
[Out] Defer[Int][(a + b*Sin[c + d/x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d/x])^(-1),x]
[Out] Integrate[(a + b*Sin[c + d/x])^(-1), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(c+d/x)),x)
[Out] int(1/(a+b*sin(c+d/x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="maxima")
[Out] integrate(1/(b*sin(c + d/x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="fricas")
[Out] integral(1/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin\left(c + \frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x)
[Out] Integral(1/(a + b*sin(c + d/x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="giac")
[Out] integrate(1/(b*sin(c + d/x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{a + b \sin\left(c + \frac{d}{x}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d/x)),x)
[Out] int(1/(a + b*sin(c + d/x)), x)
```

3.302 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((e + f*x)/(a + b*sin(c + d/x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e+f\,x}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)/(a + b*sin(c + d/x)), x)
```

3.303 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]

[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x]), x]

Rubi steps

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx = \int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]

[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f^2*x^2 + 2*f*x*e + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^2}{a+b\,\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x)),x)
[Out] int((e + f*x)^2/(a + b*sin(c + d/x)), x)
```

3.304 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A]
time = 74.51, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f^2*x^4 + 2*a*b*f*x^3*e + a*b*x^2*e^2)*cos(2*(c*x + d)/x)*cos((c*x
 + d)/x) + 2*(a*b*f^2*x^4 + 2*a*b*f*x^3*e + a*b*x^2*e^2)*cos((c*x + d)/x) +
 ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2
 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4
*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b
^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrate(-
2*(2*(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2)*cos((c*x + d)/x)^2 + 2*(a^
2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2)*sin((c*x + d)/x)^2 + (2*(2*a*b*f^2
*x^3 + 3*a*b*f*x^2*e + a*b*x*e^2)*cos((c*x + d)/x) - (a*b*d*f^2*x^2 + 2*a*b
*d*f*x*e + a*b*d*e^2)*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + 2*(2*a*b*f^2*x
^3 + 3*a*b*f*x^2*e + a*b*x*e^2)*cos((c*x + d)/x) + (4*b^2*f^2*x^3 + 6*b^2*f
*x^2*e + 2*b^2*x*e^2 + (a*b*d*f^2*x^2 + 2*a*b*d*f*x*e + a*b*d*e^2)*cos((c*x
 + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*f*x^2*e + a*b*x*e^2)*sin((c*x + d)/x))*
sin(2*(c*x + d)/x) + (a*b*d*f^2*x^2 + 2*a*b*d*f*x*e + a*b*d*e^2)*sin((c*x +
 d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((
c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (
a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x
)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*
b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x)
+ 2*(b^2*f^2*x^4 + 2*b^2*f*x^3*e + b^2*x^2*e^2 + (a*b*f^2*x^4 + 2*a*b*f*x^3
*e + a*b*x^2*e^2)*sin((c*x + d)/x))*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*
cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b -
a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x
 + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*s
in((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/
x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f^2*x^2 + 2*f*x*e + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*
x + d)/x) - a^2 - b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^2}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x))^2,x)
```

```
[Out] int((e + f*x)^2/(a + b*sin(c + d/x))^2, x)
```

3.305 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A]
time = 16.55, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Maple [A]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f*x^3 + a*b*x^2*e)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f*
x^3 + a*b*x^2*e)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x +
d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4
- a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a
^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)
*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c*x + d)
/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2 + 2*a*b*
x*e)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*cos(2*(c*x
+ d)/x) + (3*a*b*f*x^2 + 2*a*b*x*e)*cos((c*x + d)/x) + (3*b^2*f*x^2 + 2*b^2
*x*e + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*x*e)*s
in((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x
))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x +
 d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b
^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 +
 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a
*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(
b^2*f*x^3 + b^2*x^2*e + (a*b*f*x^3 + a*b*x^2*e)*sin((c*x + d)/x))*sin(2*(c*
x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*co
s((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x)
+ (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a
^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
- b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e+f\,x}{{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(a + b*sin(c + d/x))^2,x)
```

```
[Out] int((e + f*x)/(a + b*sin(c + d/x))^2, x)
```

3.306 $\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(c+d/x))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d/x])^(-2),x]
[Out] Defer[Int][(a + b*Sin[c + d/x])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A]
time = 2.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d/x])^(-2),x]
[Out] Integrate[(a + b*Sin[c + d/x])^(-2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sin(c+d/x))^2,x)

[Out] int(1/(a+b*sin(c+d/x))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="maxima")

[Out] -(2*a*b*x^2*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*a*b*x^2*cos((c*x + d)/x
) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x
+ d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*
b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2
+ 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b -
a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrat
e(-2*(2*a^2*d*cos((c*x + d)/x)^2 + 2*a^2*d*sin((c*x + d)/x)^2 + 2*a*b*x*cos
((c*x + d)/x) + a*b*d*sin((c*x + d)/x) + (2*a*b*x*cos((c*x + d)/x) - a*b*d*
sin((c*x + d)/x))*cos(2*(c*x + d)/x) + (a*b*d*cos((c*x + d)/x) + 2*a*b*x*si
n((c*x + d)/x) + 2*b^2*x)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x
 + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*c
os((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x +
d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*
b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(a*b*x^2*sin((c*x + d)/x) + b^2*x
^2)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 -
a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*
(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d
*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4
)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c
*x + d)/x))

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="fricas")

[Out] integral(-1/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2 - b^2),
x)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)^(-2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d/x))^2,x)
```

```
[Out] int(1/(a + b*sin(c + d/x))^2, x)
```

3.307 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} \, dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} \, dx = \int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} \, dx$$

Mathematica [A]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f*x^3 + a*b*x^2*e)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f*
x^3 + a*b*x^2*e)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x +
d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4
- a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a
^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)
*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c*x + d)
/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2 + 2*a*b*
x*e)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*cos(2*(c*x
+ d)/x) + (3*a*b*f*x^2 + 2*a*b*x*e)*cos((c*x + d)/x) + (3*b^2*f*x^2 + 2*b^2
*x*e + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*x*e)*s
in((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x
))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x +
 d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b
^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 +
 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a
*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(
b^2*f*x^3 + b^2*x^2*e + (a*b*f*x^3 + a*b*x^2*e)*sin((c*x + d)/x))*sin(2*(c*
x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*co
s((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x)
+ (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a
^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
- b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e+f\,x}{{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(a + b*sin(c + d/x))^2,x)
```

```
[Out] int((e + f*x)/(a + b*sin(c + d/x))^2, x)
```

3.308 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A]
time = 66.46, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)

[Out] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")

[Out] -(2*(a*b*f^2*x^4 + 2*a*b*f*x^3*e + a*b*x^2*e^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f^2*x^4 + 2*a*b*f*x^3*e + a*b*x^2*e^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2)*cos((c*x + d)/x)^2 + 2*(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2)*sin((c*x + d)/x)^2 + (2*(2*a*b*f^2*x^3 + 3*a*b*f*x^2*e + a*b*x*e^2)*cos((c*x + d)/x) - (a*b*d*f^2*x^2 + 2*a*b*d*f*x*e + a*b*d*e^2)*sin((c*x + d)/x))*cos(2*(c*x + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*f*x^2*e + a*b*x*e^2)*cos((c*x + d)/x) + (4*b^2*f^2*x^3 + 6*b^2*f*x^2*e + 2*b^2*x*e^2 + (a*b*d*f^2*x^2 + 2*a*b*d*f*x*e + a*b*d*e^2)*cos((c*x + d)/x) + 2*(2*a*b*f^2*x^3 + 3*a*b*f*x^2*e + a*b*x*e^2)*sin((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f^2*x^2 + 2*a*b*d*f*x*e + a*b*d*e^2)*sin((c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(b^2*f^2*x^4 + 2*b^2*f*x^3*e + b^2*x^2*e^2 + (a*b*f^2*x^4 + 2*a*b*f*x^3*e + a*b*x^2*e^2)*sin((c*x + d)/x))*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f^2*x^2 + 2*f*x*e + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*
x + d)/x) - a^2 - b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e+f\,x)^2}{\left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(a + b*sin(c + d/x))^2,x)
```

```
[Out] int((e + f*x)^2/(a + b*sin(c + d/x))^2, x)
```

3.309 $\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left((e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p,x\right)$$

[Out] Unintegrable((f*x+e)^m*(a+b*sin(c+d/x))^p,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Verification is not applicable to the result.

[In] Int[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]

[Out] Defer[Int][(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]

Rubi steps

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Mathematica [A]
time = 1.00, size = 0, normalized size = 0.00

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Verification is not applicable to the result.

[In] Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]

[Out] Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (fx+e)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

```
[Out] int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*(b*sin((c*x + d)/x) + a)^p, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*sin(c+d/x))**p,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e+f\,x)^m \left(a+b\,\sin\left(c+\frac{d}{x}\right)\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*sin(c + d/x))^p,x)
[Out] int((e + f*x)^m*(a + b*sin(c + d/x))^p, x)
```

3.310 $\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal. Leaf size=115

$$-\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(1+m,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\Gamma(1+m,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

```
[Out] -1/2*exp(I*a)*x^m*csc(b*x+a)*GAMMA(1+m,-I*b*x)*(c*sin(b*x+a)^3)^(1/3)/b/((-
I*b*x)^m)-1/2*x^m*csc(b*x+a)*GAMMA(1+m,I*b*x)*(c*sin(b*x+a)^3)^(1/3)/b/exp(
I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.20, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6852, 3389, 2212}

$$-\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\text{Gamma}(m+1,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\text{Gamma}(m+1,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Csc[a + b*x]*Gamma[1 + m, (-I)*b*x]*(c*Sin[a + b*x]^3)^(1
/3))/(b*((-I)*b*x)^m) - (x^m*Csc[a + b*x]*Gamma[1 + m, I*b*x]*(c*Sin[a + b*
x]^3)^(1/3))/(2*b*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^m \sqrt[3]{c \sin^3(a+bx)}\, dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^m \sin(a+bx)\, dx$$
$$= \frac{1}{2}\left(i\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int e^{-i(a+bx)}x^m\, dx - \frac{1}{2}\left(i\csc(a+bx)\sqrt[3]{c\sin^3(\right.$$
$$= -\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(1+m,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}}{}$$

Mathematica [**A**]
time = 0.08, size = 94, normalized size = 0.82

$$-\frac{e^{-ia}x^m(b^2x^2)^{-m}\csc(a+bx)\left(e^{2ia}(ibx)^m\Gamma(1+m,-ibx)+(-ibx)^m\Gamma(1+m,ibx)\right)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
[Out] -1/2*(x^m*Csc[a + b*x]*(E^((2*I)*a)*(I*b*x)^m*Gamma[1 + m, (-I)*b*x] + ((-I
)*b*x)^m*Gamma[1 + m, I*b*x])*(c*Sin[a + b*x]^3)^(1/3))/(b*E^(I*a)*(b^2*x^2
)^m)
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^m\left(c\left(\sin^3(bx+a)\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
[Out] int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Fricas [A]
time = 0.10, size = 80, normalized size = 0.70

$$-\frac{\left(e^{\left(-m\log\left(i\,b\right)-i\,a\right)}\Gamma\left(m+1,i\,bx\right)+e^{\left(-m\log\left(-i\,b\right)+i\,a\right)}\Gamma\left(m+1,-i\,bx\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{2\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*ga
mma(m + 1, -I*b*x))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b*sin(b*x
 + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sqrt[3]{c\sin^{3}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m}\,{\left(c\,{\sin\left(a+b\,x\right)}^{3}\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(a + b*x)^3)^(1/3), x)
```

3.311 $\int x^3 \sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal. Leaf size=96

$$-\frac{6\sqrt[3]{c\sin^3(a+bx)}}{b^4}+\frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] -6*(c*sin(b*x+a)^3)^(1/3)/b^4+3*x^2*(c*sin(b*x+a)^3)^(1/3)/b^2+6*x*cot(b*x+
a)*(c*sin(b*x+a)^3)^(1/3)/b^3-x^3*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6852, 3377, 2717}

$$-\frac{6\sqrt[3]{c\sin^3(a+bx)}}{b^4}+\frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}+\frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (-6*(c*Sin[a + b*x]^3)^(1/3))/b^4 + (3*x^2*(c*Sin[a + b*x]^3)^(1/3))/b^2 +
(6*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b^3 - (x^3*Cot[a + b*x]*(c*Sin[
a + b*x]^3)^(1/3))/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3 \sqrt[3]{c \sin^3(a+bx)}\, dx = \left(\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)}\right) \int x^3 \sin(a+bx)\, dx$$
$$= -\frac{x^3 \cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b} + \frac{\left(3\csc(a+bx)\sqrt[3]{c \sin^3(a+bx)}\right) \int x^2 \cos(}{b}$$
$$= \frac{3x^2 \sqrt[3]{c \sin^3(a+bx)}}{b^2} - \frac{x^3 \cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b} - \frac{\left(6\csc(a+bx)\sqrt[3]{c \sin}\right.}{}$$
$$= \frac{3x^2 \sqrt[3]{c \sin^3(a+bx)}}{b^2} + \frac{6x \cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b^3} - \frac{x^3 \cot(a+bx)\sqrt[3]{c \sin}}{b}$$
$$= -\frac{6\sqrt[3]{c \sin^3(a+bx)}}{b^4} + \frac{3x^2 \sqrt[3]{c \sin^3(a+bx)}}{b^2} + \frac{6x \cot(a+bx)\sqrt[3]{c \sin^3(a+bx)}}{b^3}$$

Mathematica [A]
time = 0.14, size = 47, normalized size = 0.49

$$-\frac{(6 - 3b^2x^2 + bx(-6 + b^2x^2)\cot(a+bx))\sqrt[3]{c \sin^3(a+bx)}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -(((6 - 3*b^2*x^2 + b*x*(-6 + b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/
3))/b^4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 151, normalized size = 1.57

method	result
risch	$-\frac{i(b^3x^3+3ib^2x^2-6bx-6i)\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} e^{2i(bx+a)}}{2b^4\left(e^{2i(bx+a)}-1\right)} - \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}}(b^3x^3-3ib^2x^2-6bx+6i}{2\left(e^{2i(bx+a)}-1\right)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^4*(b^3*x^3+3*I*b^2*x^2-6*b*x-6*I)/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2
*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(
2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*(b^3*x^3-3*
I*b^2*x^2-6*b*x+6*I)/b^4
```

Maxima [A]
time = 0.56, size = 146, normalized size = 1.52

$$\frac{3\left((bx+a)\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)a^{2}c^{\frac{1}{3}}-3\left(\left((bx+a)^{2}-2\right)\cos\left(bx+a\right)-2\left(bx+a\right)\sin\left(bx+a\right)\right)ac^{\frac{1}{3}}+\frac{4\,a^{3}c^{\frac{1}{3}}}{\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+1}+\left(\left((bx+a)^{3}-6\,bx-6\,a\right)\cos\left(bx+a\right)-3\left((bx+a)^{2}-2\right)\sin\left(bx+a\right)\right)c^{\frac{1}{3}}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*(3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2*c^(1/3) - 3*(((b*x + a)^
2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c^(1/3) + 4*a^3*c^(1/3)/(
sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1) + (((b*x + a)^3 - 6*b*x - 6*a)*cos
(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*c^(1/3))/b^4
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.77

$$-\frac{\left(\left(b^{3}x^{3}-6\,bx\right)\cos\left(bx+a\right)-3\left(b^{2}x^{2}-2\right)\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{b^{4}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -((b^3*x^3 - 6*b*x)*cos(b*x + a) - 3*(b^2*x^2 - 2)*sin(b*x + a))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b^4*sin(b*x + a))
```

Sympy [A]
time = 2.49, size = 129, normalized size = 1.34

$$\begin{cases}\frac{x^{4}\sqrt[3]{c\sin^{3}\left(a\right)}}{4} & \text{for}\: b=0\\0 & \text{for}\: a=-bx\vee a=-bx+\pi\\-\frac{x^{3}\sqrt[3]{c\sin^{3}\left(a+bx\right)}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)}+\frac{3x^{2}\sqrt[3]{c\sin^{3}\left(a+bx\right)}}{b^{2}}+\frac{6x\sqrt[3]{c\sin^{3}\left(a+bx\right)}\cos\left(a+bx\right)}{b^{3}\sin\left(a+bx\right)}-\frac{6\sqrt[3]{c\sin^{3}\left(a+bx\right)}}{b^{4}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-x**3*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)
) + 3*x**2*(c*sin(a + b*x)**3)**(1/3)/b**2 + 6*x*(c*sin(a + b*x)**3)**(1/3)
*cos(a + b*x)/(b**3*sin(a + b*x)) - 6*(c*sin(a + b*x)**3)**(1/3)/b**4, True
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")

[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^3, x)

Mupad [B]
time = 5.71, size = 109, normalized size = 1.14

$$\frac{2^{1/3}\left(c\left(3\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}\left(3\,b^2\,x^2-12\sin\left(a+b\,x\right)^2+6\,b\,x\,\sin\left(2\,a+2\,b\,x\right)+3\,b^2\,x^2\,\left(2\sin\left(a+b\,x\right)^2-1\right)-b^3\,x^3\,\sin\left(2\,a+2\,b\,x\right)\right)}{4\,b^4\sin\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*sin(a + b*x)^3)^(1/3),x)

[Out] (2^(1/3)*(c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3)*(3*b^2*x^2 - 12*sin(a + b*x)^2 + 6*b*x*sin(2*a + 2*b*x) + 3*b^2*x^2*(2*sin(a + b*x)^2 - 1) - b^3*x^3*sin(2*a + 2*b*x)))/(4*b^4*sin(a + b*x)^2)

3.312 $\int x^2 \sqrt[3]{c \sin^3(a+bx)}\, dx$

Optimal. Leaf size=74

$$\frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] 2*x*(c*sin(b*x+a)^3)^(1/3)/b^2+2*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b^3-x^2*
cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A]
time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6852, 3377, 2718}

$$\frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}+\frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (2*x*(c*Sin[a + b*x]^3)^(1/3))/b^2 + (2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/
3))/b^3 - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sqrt[3]{c\sin^3(a+bx)}\,dx &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^2\sin(a+bx)\,dx \\
&= -\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} + \frac{\left(2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\cos(a}{b} \\
&= \frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2} - \frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} - \frac{\left(2\csc(a+bx)\sqrt[3]{c\sin}\right.}{} \\
&= \frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2} + \frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3} - \frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(}}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 40, normalized size = 0.54

$$\frac{(2bx+(2-b^2x^2)\cot(a+bx))\sqrt[3]{c\sin^3(a+bx)}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] ((2*b*x + (2 - b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^3
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 133, normalized size = 1.80

method	result	size
risch	$-\frac{i(x^2b^2+2ibx-2)\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} e^{2i(bx+a)}}{2b^3\left(e^{2i(bx+a)}-1\right)} - \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}}(x^2b^2-2ibx-2)}{2\left(e^{2i(bx+a)}-1\right)b^3}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^3*(x^2*b^2+2*I*b*x-2)/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a))-
1)^3*exp(-3*I*(b*x+a)))^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(2*I*(b*x+a))
-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*(x^2*b^2-2*I*b*x-2)/b^3
```

Maxima [A]
time = 0.54, size = 99, normalized size = 1.34

$$-\frac{2\left((bx+a)\cos(bx+a)-\sin(bx+a)\right)ac^{\frac{1}{3}} - \left(\left((bx+a)^2-2\right)\cos(bx+a) - 2(bx+a)\sin(bx+a)\right)c^{\frac{1}{3}} + \frac{4a^2c^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/2*(2*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c^(1/3) - (((b*x + a)^2 -
 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^(1/3) + 4*a^2*c^(1/3)/(sin(b
*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^3
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.86

$$\frac{(2\,bx\sin{(bx+a)}-(b^2x^2-2)\cos{(bx+a)})\left(-\left(c\cos{(bx+a)}^2-c\right)\sin{(bx+a)}\right)^{\frac{1}{3}}}{b^3\sin{(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] (2*b*x*sin(b*x + a) - (b^2*x^2 - 2)*cos(b*x + a))*(-(c*cos(b*x + a)^2 - c)*
sin(b*x + a))^(1/3)/(b^3*sin(b*x + a))
```

Sympy [A]
time = 1.31, size = 107, normalized size = 1.45

$$\begin{cases} \frac{x^3\sqrt[3]{c\sin^3{(a)}}}{3} & \text{for}\ b=0 \\ 0 & \text{for}\ a=-bx\vee a=-bx+\pi \\ -\frac{x^2\sqrt[3]{c\sin^3{(a+bx)}}\cos{(a+bx)}}{b\sin{(a+bx)}}+\frac{2x\sqrt[3]{c\sin^3{(a+bx)}}}{b^2}+\frac{2\sqrt[3]{c\sin^3{(a+bx)}}\cos{(a+bx)}}{b^3\sin{(a+bx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**3*(c*sin(a)**3)**(1/3)/3, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-x**2*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)
) + 2*x*(c*sin(a + b*x)**3)**(1/3)/b**2 + 2*(c*sin(a + b*x)**3)**(1/3)*cos(
a + b*x)/(b**3*sin(a + b*x)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^2, x)
```

Mupad [B]
time = 5.49, size = 88, normalized size = 1.19

$$-\frac{\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}\,\left(\sin\left(2\,a+2\,b\,x\right)+b\,x-\frac{b^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{2}-b\,x\,\cos\left(2\,a+2\,b\,x\right)\right)}{b^3\,\left(\cos\left(2\,a+2\,b\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] -((2*c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3)*(sin(2*a + 2*b*x) + b*x -
 (b^2*x^2*sin(2*a + 2*b*x))/2 - b*x*cos(2*a + 2*b*x)))/(b^3*(cos(2*a + 2*b*
x) - 1))
```

3.313 $\int x\sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] (c*sin(b*x+a)^3)^(1/3)/b^2-x*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A]
time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {6852, 3377, 2717}

$$\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (c*Sin[a + b*x]^3)^(1/3)/b^2 - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\sqrt[3]{c\sin^3(a+bx)}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\sin(a+bx)\,dx$$
$$= -\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} + \frac{\left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\cos(a+bx$$
$$= \frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2} - \frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Mathematica [A]
time = 0.10, size = 30, normalized size = 0.67

$$\frac{(1-bx\cot(a+bx))\sqrt[3]{c\sin^3(a+bx)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] ((1 - b*x*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 117, normalized size = 2.60

method	result	size
risch	$-\frac{i(bx+i)\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} e^{2i(bx+a)}}{2b^2\left(e^{2i(bx+a)}-1\right)} - \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}}(bx-i)}{2\left(e^{2i(bx+a)}-1\right)b^2}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^2*(b*x+I)/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*
I*(b*x+a)))^(1/3)*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3
*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*(b*x-I)/b^2
```

Maxima [A]
time = 0.55, size = 60, normalized size = 1.33

$$\frac{((bx+a)\cos(bx+a)-\sin(bx+a))c^{\frac{1}{3}} + \frac{4\,ac^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)*cos(b*x + a) - sin(b*x + a))*c^(1/3) + 4*a*c^(1/3)/(sin(b*x
 + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^2
```

Fricas [A]
time = 0.35, size = 55, normalized size = 1.22

$$-\frac{(bx\cos(bx+a)-\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{b^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(b*x*cos(b*x + a) - sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(
1/3)/(b^2*sin(b*x + a))
```

Sympy [A]
time = 0.77, size = 70, normalized size = 1.56

$$\begin{cases} \frac{x^2\sqrt[3]{c\sin^3(a)}}{2} & \text{for } b=0 \\ 0 & \text{for } a=-bx\vee a=-bx+\pi \\ -\frac{x\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)}+\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-x*(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)) +
 (c*sin(a + b*x)**3)**(1/3)/b**2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x, x)
```

Mupad [B]
time = 5.14, size = 63, normalized size = 1.40

$$\frac{\left(\frac{\sin(a+b\,x)^2}{2}-\frac{b\,x\,\sin(2\,a+2\,b\,x)}{4}\right)\,\left(2\,c\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\right)^{1/3}}{b^2\,\sin\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] ((sin(a + b*x)^2/2 - (b*x*sin(2*a + 2*b*x))/4)*(2*c*(3*sin(a + b*x) - sin(3
*a + 3*b*x)))^(1/3))/(b^2*sin(a + b*x)^2)
```

3.314 $\int \sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] -cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3286, 2718}

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c\sin^3(a+bx)}\,dx &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \sin(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.00

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3),x]
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 105, normalized size = 4.20

method	result	size
risch	$-\frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} e^{2i(bx+a)}}{2b\left(e^{2i(bx+a)}-1\right)}-\frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}}}{2b\left(e^{2i(bx+a)}-1\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] -1/2*I/b/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a))
)^(1/3)*exp(2*I*(b*x+a))-1/2*I/b/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a)
)-1)^3*exp(-3*I*(b*x+a)))^(1/3)
```

Maxima [A]
time = 0.74, size = 31, normalized size = 1.24

$$-\frac{2\,c^{\frac{1}{3}}}{b\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
[Out] -2*c^(1/3)/(b*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))
```

Fricas [A]
time = 0.36, size = 43, normalized size = 1.72

$$-\frac{\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}\cos\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.
time = 0.46, size = 49, normalized size = 1.96

$$\begin{cases} x\sqrt[3]{c\sin^3(a)} & \text{for } b = 0 \\ 0 & \text{for } a = -bx \vee a = -bx + \pi \\ -\frac{\sqrt[3]{c\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x*(c*sin(a)**3)**(1/3), Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a, -b*x
+ pi)), (-(c*sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3), x)
```

Mupad [B]
time = 4.85, size = 49, normalized size = 1.96

$$-\frac{\sin(2\,a+2\,b\,x)\,\left(2\,c\,\left(3\,\sin(a+b\,x)-\sin(3\,a+3\,b\,x)\right)\right)^{1/3}}{4\,b\sin\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(1/3),x)
```

```
[Out] -(sin(2*a + 2*b*x)*(2*c*(3*sin(a + b*x) - sin(3*a + 3*b*x)))^(1/3))/(4*b*si
n(a + b*x)^2)
```

3.315 $\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x} \, dx$

Optimal. Leaf size=55

$$\text{Ci}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,\text{Si}(bx)$$

```
[Out] cos(a)*csc(b*x+a)*Si(b*x)*(c*sin(b*x+a)^3)^(1/3)+Ci(b*x)*csc(b*x+a)*sin(a)*
(c*sin(b*x+a)^3)^(1/3)
```

Rubi [A]

time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6852, 3384, 3380, 3383}

$$\sin(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

```
[Out] CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3) + Cos[a]*Csc[
a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x}\,dx$$
$$= \left(\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(bx)}{x}\,dx + \left(\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3\right.$$
$$= \text{Ci}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,\text{Si}($$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.65

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,(\text{Ci}(bx)\sin(a)+\cos(a)\text{Si}(bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x,x]
[Out] Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(CosIntegral[b*x]*Sin[a] + Cos[a]*Sin
Integral[b*x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 228, normalized size = 4.15

method	result
risch	$-\frac{\text{expIntegral}(1,-ibx)\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} e^{i(bx+2a)}}{2\left(e^{2i(bx+a)}-1\right)} - \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} e^{ibx}\pi\,\text{csgn}(bx)}{2\left(e^{2i(bx+a)}-1\right)} + \frac{i\left(ic\left(e^{2i(bx-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*Ei(1,-I*b*x)/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I
*(b*x+a)))^(1/3)*exp(I*(b*x+2*a))-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*
I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*exp(I*b*x)*Pi*csgn(b*x)+I*(I*c*(exp(
2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*exp(I*b*x)*
Si(b*x)+1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(
b*x+a))-1)*exp(I*b*x)*Ei(1,-I*b*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.58, size = 42, normalized size = 0.76

$$\frac{1}{4}\left(\left(i\,E_1(i\,bx) - i\,E_1(-i\,bx)\right)\cos(a) + \left(E_1(i\,bx)+E_1(-i\,bx)\right)\sin(a)\right)c^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] 1/4*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x))*cos(a) + (e
xp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x))*sin(a))*c^(1/3)
```

Fricas [A]
time = 0.35, size = 80, normalized size = 1.45

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(a\right)\operatorname{Si}\left(bx\right)+\left(4^{\frac{2}{3}}\operatorname{Ci}\left(bx\right)+4^{\frac{2}{3}}\operatorname{Ci}\left(-bx\right)\right)\sin\left(a\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}\sin\left(bx+a\right)}{8\left(\cos\left(bx+a\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(2*4^(2/3)*cos(a)*sin_integral(b*x) + (4^(2/3)*cos_integral(b*
x) + 4^(2/3)*cos_integral(-b*x))*sin(a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x +
 a))^(1/3)*sin(b*x + a)/(cos(b*x + a)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{c\sin^{3}\left(a+bx\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(c\,{\sin\left(a+b\,x\right)}^{3}\right)}^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(1/3)/x,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(1/3)/x, x)
```

3.316 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt[3]{c\sin^3(a+bx)}}{x}+b\cos(a)\text{Ci}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}-b\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\,\text{Si}(bx)$$

```
[Out] -(c*sin(b*x+a)^3)^(1/3)/x+b*Ci(b*x)*cos(a)*csc(b*x+a)*(c*sin(b*x+a)^3)^(1/3
)-b*csc(b*x+a)*Si(b*x)*sin(a)*(c*sin(b*x+a)^3)^(1/3)
```

Rubi [A]
time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6852, 3378, 3384, 3380, 3383}

$$b\cos(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}-b\sin(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}-\frac{\sqrt[3]{c\sin^3(a+bx)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x]^3)^(1/3)/x) + b*Cos[a]*CosIntegral[b*x]*Csc[a + b*x]*(c*S
in[a + b*x]^3)^(1/3) - b*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3)*SinIn
tegral[b*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x^2}\,dx\\ &= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + \left(b\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(a+bx)}{x}\,dx\\ &= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + \left(b\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(bx)}{x}\,dx - \Big(\\ &= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + b\cos(a)\mathrm{Ci}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\csc(a+bx)\end{aligned}$$

Mathematica [A]
time = 0.13, size = 51, normalized size = 0.66

$$\frac{\sqrt[3]{c\sin^3(a+bx)}\,(-1+bx\cos(a)\mathrm{Ci}(bx)\csc(a+bx)-bx\csc(a+bx)\sin(a)\mathrm{Si}(bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

```
[Out] ((c*Sin[a + b*x]^3)^(1/3)*(-1 + b*x*Cos[a]*CosIntegral[b*x]*Csc[a + b*x] -
b*x*Csc[a + b*x]*Sin[a]*SinIntegral[b*x]))/x
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 155, normalized size = 2.01

method	result
risch	$\frac{ib\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}}\left(\frac{ie^{2i(bx+a)}}{bx}-\text{expIntegral}(1,-ibx)e^{i(bx+2a)}\right)}{2e^{2i(bx+a)}-2} - \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{1}{3}} b\left(\frac{i}{bx}+e^{ibx}\,\text{expI}\right.}{2\left(e^{2i(bx+a)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*b/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))
^(1/3)*(I/b/x*exp(2*I*(b*x+a))-Ei(1,-I*b*x)*exp(I*(b*x+2*a)))-1/2*I*(I*c*(e
xp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*b*(I/b/x
+exp(I*b*x)*Ei(1,I*b*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.69, size = 229, normalized size = 2.97

$$\frac{\left(\left(\left(\sqrt{3}-i\right)E_2(ibx)+\left(\sqrt{3}+i\right)E_2(-ibx)\right)\cos(a)^3+\left(\left(\sqrt{3}-i\right)E_2(ibx)+\left(\sqrt{3}+i\right)E_2(-ibx)\right)\cos(a)\sin(a)^2+\left(\left(-i\sqrt{3}-1\right)E_2(ibx)+\left(i\sqrt{3}-1\right)E_2(-ibx)\right)\sin(a)^3-\left(\left(\sqrt{3}+i\right)E_2(ibx)+\left(\sqrt{3}-i\right)E_2(-ibx)\right)\cos(a)+\left(\left(\left(-i\sqrt{3}-1\right)E_2(ibx)+\left(i\sqrt{3}-1\right)E_2(-ibx)\right)\cos(a)^2+\left(i\sqrt{3}-1\right)E_2(ibx)+\left(-i\sqrt{3}-1\right)E_2(-ibx)\right)\sin(a)\right)bc^{\frac{1}{3}}}{8\left(a\cos(a)^2+a\sin(a)^2-(bx+a)\left(\cos(a)^2+\sin(a)^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(((sqrt(3) - I)*exp_integral_e(2, I*b*x) + (sqrt(3) + I)*exp_integral_e
(2, -I*b*x))*cos(a)^3 + ((sqrt(3) - I)*exp_integral_e(2, I*b*x) + (sqrt(3)
+ I)*exp_integral_e(2, -I*b*x))*cos(a)*sin(a)^2 + ((-I*sqrt(3) - 1)*exp_int
egral_e(2, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*sin(a)^3 - (
(sqrt(3) + I)*exp_integral_e(2, I*b*x) + (sqrt(3) - I)*exp_integral_e(2, -I
*b*x))*cos(a) + (((-I*sqrt(3) - 1)*exp_integral_e(2, I*b*x) + (I*sqrt(3) -
1)*exp_integral_e(2, -I*b*x))*cos(a)^2 + (I*sqrt(3) - 1)*exp_integral_e(2,
I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*sin(a))*b*c^(1/3)/(a*c
os(a)^2 + a*sin(a)^2 - (b*x + a)*(cos(a)^2 + sin(a)^2))
```

Fricas [A]
time = 0.36, size = 112, normalized size = 1.45

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos(bx+a)^2-\left(2\cdot 4^{\frac{2}{3}}bx\sin(a)\operatorname{Si}(bx)-\left(4^{\frac{2}{3}}bx\operatorname{Ci}(bx)+4^{\frac{2}{3}}bx\operatorname{Ci}(-bx)\right)\cos(a)\right)\sin(bx+a)-2\cdot 4^{\frac{2}{3}}\right)\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{8\left(x\cos(bx+a)^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(2*4^(2/3)*cos(b*x + a)^2 - (2*4^(2/3)*b*x*sin(a)*sin_integral
(b*x) - (4^(2/3)*b*x*cos_integral(b*x) + 4^(2/3)*b*x*cos_integral(-b*x))*co
s(a))*sin(b*x + a) - 2*4^(2/3))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3
)/(x*cos(b*x + a)^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sin\left(a+b\,x\right)^3\right)^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(1/3)/x^2,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(1/3)/x^2, x)
```

3.317 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2}-\frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x}-\frac{1}{2}b^2\mathrm{Ci}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}-\frac{1}{2}b^2\cos(a$$

```
[Out] -1/2*(c*sin(b*x+a)^3)^(1/3)/x^2-1/2*b*cot(b*x+a)*(c*sin(b*x+a)^3)^(1/3)/x-1
/2*b^2*cos(a)*csc(b*x+a)*Si(b*x)*(c*sin(b*x+a)^3)^(1/3)-1/2*b^2*Ci(b*x)*csc
(b*x+a)*sin(a)*(c*sin(b*x+a)^3)^(1/3)
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6852, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}b^2\sin(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}-\frac{1}{2}b^2\cos(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}-\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2}-\frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(c*Sin[a + b*x]^3)^(1/3)/x^2 - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3
))/(2*x) - (b^2*CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/
3))/2 - (b^2*Cos[a]*Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x])
/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x^3}\,dx$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} + \frac{1}{2}\left(b\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(a+bx)}{x^2}\,dx$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}\left(b^2\csc(a+bx)\sqrt[3]{c\sin^3(\right.$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}\left(b^2\cos(a)\csc(a+bx)\sqrt[3]{\right.$$

$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}b^2\text{Ci}(bx)\csc(a+bx)\sin$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.59

$$-\frac{\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\,(bx\cos(a+bx)+b^2x^2\text{Ci}(bx)\sin(a)+\sin(a+bx)+b^2x^2\cos(a)\text{Si}(bx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(b*x*Cos[a + b*x] + b^2*x^2*Cos
Integral[b*x]*Sin[a] + Sin[a + b*x] + b^2*x^2*Cos[a]*SinIntegral[b*x]))/x^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 183, normalized size = 1.58

method	result
risch	$-\frac{b^{2}\left(ic\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3} \mathrm{e}^{-3i(bx+a)}\right)^{\frac{1}{3}}\left(\frac{\mathrm{e}^{2i(bx+a)}}{2x^{2}b^{2}}+\frac{i\mathrm{e}^{2i(bx+a)}}{2bx}-\frac{\mathrm{expIntegral}(1,-ibx)\mathrm{e}^{i(bx+2a)}}{2}\right)}{2\left(\mathrm{e}^{2i(bx+a)}-1\right)}+\frac{\left(ic\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\mathrm{e}^{-3i(bx+a)}\right)^{\frac{1}{3}} b^{2}\left(\right.}{2\,\mathrm{e}^{2i(bx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/(exp(2*I*(b*x+a))-1)*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a))
)^(1/3)*(1/2/x^2/b^2*exp(2*I*(b*x+a))+1/2*I/b/x*exp(2*I*(b*x+a))-1/2*Ei(1,-
I*b*x)*exp(I*(b*x+2*a)))+1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))
^(1/3)/(exp(2*I*(b*x+a))-1)*b^2*(1/2/x^2/b^2-1/2*I/b/x-1/2*exp(I*b*x)*Ei(1,
I*b*x))
```

<u>Maxima</u> [C] Result contains complex when optimal does not.
time = 0.58, size = 256, normalized size = 2.21

$$-\frac{\left(\left(\left(\left(\sqrt{3}-i\right)E_3(i\,bx)+\left(\sqrt{3}+i\right)E_3(-i\,bx)\right)\cos\left(a\right)^3+\left(\left(\sqrt{3}-i\right)E_3(i\,bx)+\left(\sqrt{3}+i\right)E_3(-i\,bx)\right)\cos\left(a\right)\sin\left(a\right)^2+\left(\left(-i\sqrt{3}-1\right)E_3(i\,bx)+\left(i\sqrt{3}-1\right)E_3(-i\,bx)\right)\sin\left(a\right)^3-\left(\left(\sqrt{3}+i\right)E_3(i\,bx)+\left(\sqrt{3}-i\right)E_3(-i\,bx)\right)\cos\left(a\right)+\left(\left(\left(-i\sqrt{3}-1\right)E_3(i\,bx)+\left(i\sqrt{3}-1\right)E_3(-i\,bx)\right)\cos\left(a\right)^2+\left(i\sqrt{3}-1\right)E_3(i\,bx)+\left(-i\sqrt{3}-1\right)E_3(-i\,bx)\right)\sin\left(a\right)\right)b^2c^{\frac{1}{3}}}{8\left(a^2\cos\left(a\right)^2+a^2\sin\left(a\right)^2+\left(bx+a\right)^2\left(\cos\left(a\right)^2+\sin\left(a\right)^2\right)-2\left(a\cos\left(a\right)^2+a\sin\left(a\right)^2\right)\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((sqrt(3) - I)*exp_integral_e(3, I*b*x) + (sqrt(3) + I)*exp_integral_
e(3, -I*b*x))*cos(a)^3 + ((sqrt(3) - I)*exp_integral_e(3, I*b*x) + (sqrt(3)
 + I)*exp_integral_e(3, -I*b*x))*cos(a)*sin(a)^2 + ((-I*sqrt(3) - 1)*exp_in
tegral_e(3, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*sin(a)^3 -
((sqrt(3) + I)*exp_integral_e(3, I*b*x) + (sqrt(3) - I)*exp_integral_e(3, -
I*b*x))*cos(a) + (((-I*sqrt(3) - 1)*exp_integral_e(3, I*b*x) + (I*sqrt(3) -
 1)*exp_integral_e(3, -I*b*x))*cos(a)^2 + (I*sqrt(3) - 1)*exp_integral_e(3,
 I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*sin(a))*b^2*c^(1/3)/(
a^2*cos(a)^2 + a^2*sin(a)^2 + (b*x + a)^2*(cos(a)^2 + sin(a)^2) - 2*(a*cos(
a)^2 + a*sin(a)^2)*(b*x + a))
```

<u>Fricas</u> [A]
time = 0.36, size = 140, normalized size = 1.21

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(bx+a\right)^2-\left(2\cdot 4^{\frac{2}{3}}b^2x^2\cos\left(a\right)\mathrm{Si}\left(bx\right)+2\cdot 4^{\frac{2}{3}}bx\cos\left(bx+a\right)+\left(4^{\frac{2}{3}}b^2x^2\,\mathrm{Ci}\left(bx\right)+4^{\frac{2}{3}}b^2x^2\,\mathrm{Ci}\left(-bx\right)\right)\sin\left(a\right)\right)\sin\left(bx+a\right)-2\cdot 4^{\frac{2}{3}}\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{16\left(x^2\cos\left(bx+a\right)^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(2*4^(2/3)*cos(b*x + a)^2 - (2*4^(2/3)*b^2*x^2*cos(a)*sin_int
egral(b*x) + 2*4^(2/3)*b*x*cos(b*x + a) + (4^(2/3)*b^2*x^2*cos_integral(b*x
) + 4^(2/3)*b^2*x^2*cos_integral(-b*x))*sin(a))*sin(b*x + a) - 2*4^(2/3))*(
-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(x^2*cos(b*x + a)^2 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sin(a+b\,x)^3\right)^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(1/3)/x^3,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(1/3)/x^3, x)
```

3.318 $\int x^m \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx$

Optimal. Leaf size=153

$$\frac{1}{4} i e^{ia} x^{1+m} \left(-ibx^2\right)^{\frac{1}{2}(-1-m)} \csc\left(a+bx^2\right) \Gamma\left(\frac{1+m}{2}, -ibx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} - \frac{1}{4} i e^{-ia} x^{1+m} \left(ibx^2\right)^{\frac{1}{2}(-1-m)} \csc$$

```
[Out] 1/4*I*exp(I*a)*x^(1+m)*(-I*b*x^2)^(-1/2-1/2*m)*csc(b*x^2+a)*GAMMA(1/2+1/2*m
,-I*b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)-1/4*I*x^(1+m)*(I*b*x^2)^(-1/2-1/2*m)*cs
c(b*x^2+a)*GAMMA(1/2+1/2*m,I*b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)/exp(I*a)
```

Rubi [A]
time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6852, 3470, 2250}

$$\frac{1}{4} i e^{ia} x^{m+1} \left(-ibx^2\right)^{\frac{1}{2}(-m-1)} \csc\left(a+bx^2\right) \text{Gamma}\left(\frac{m+1}{2}, -ibx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)} - \frac{1}{4} i e^{-ia} x^{m+1} \left(ibx^2\right)^{\frac{1}{2}(-m-1)} \csc\left(a+bx^2\right) \text{Gamma}\left(\frac{m+1}{2}, ibx^2\right) \sqrt[3]{c \sin^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (I/4)*E^(I*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 +
 m)/2, (-I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3) - ((I/4)*x^(1 + m)*(I*b*x^2)^
((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 + m)/2, I*b*x^2]*(c*Sin[a + b*x^2]^3)^
(1/3))/E^(I*a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^m \sin\left(a+bx^2\right)\,dx\\ &= \frac{1}{2}\left(i\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int e^{-ia-ibx^2}x^m\,dx - \frac{1}{2}\left(i\csc\left(a+bx^2\right)\right.\\ &= \frac{1}{4}ie^{ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},-ibx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\end{aligned}$$

Mathematica [**A**]
time = 0.20, size = 138, normalized size = 0.90

$$\frac{1}{4}ix^{1+m}\left(b^2x^4\right)^{\frac{1}{2}(-1-m)}\csc\left(a+bx^2\right)\left(-\left(-ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},ibx^2\right)\left(\cos(a)-i\sin(a)\right)+\left(ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-ibx^2\right)\left(\cos(a)+i\sin(a)\right)\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (I/4)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]*(-(((-I)*b*x^2)^((1 +
 m)/2)*Gamma[(1 + m)/2, I*b*x^2]*(Cos[a] - I*Sin[a])) + (I*b*x^2)^((1 + m)/
2)*Gamma[(1 + m)/2, (-I)*b*x^2]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^2]^3)^(
1/3)
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int x^m\left(c\left(\sin^3\left(b\,x^2+a\right)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Fricas [A]
time = 0.10, size = 98, normalized size = 0.64

$$-\frac{\left(e^{\left(-\frac{1}{2}\,(m-1)\log(i\,b)-i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},i\,bx^2\right)+e^{\left(-\frac{1}{2}\,(m-1)\log(-i\,b)+i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-i\,bx^2\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{4\,b\sin\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(e^(-1/2*(m - 1)*log(I*b) - I*a)*gamma(1/2*m + 1/2, I*b*x^2) + e^(-1/2
*(m - 1)*log(-I*b) + I*a)*gamma(1/2*m + 1/2, -I*b*x^2))*(-(c*cos(b*x^2 + a)
^2 - c)*sin(b*x^2 + a))^(1/3)/(b*sin(b*x^2 + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\left(c\,{\sin\left(b\,x^2+a\right)}^3\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(a + b*x^2)^3)^(1/3), x)
```

3.319 $\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt[3]{c \sin^3(a+bx^2)}}{2b^2} - \frac{x^2 \cot(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)}}{2b}$$

```
[Out] 1/2*(c*sin(b*x^2+a)^3)^(1/3)/b^2-1/2*x^2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1
/3)/b
```

Rubi [A]
time = 0.13, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3460, 3377, 2717}

$$\frac{\sqrt[3]{c \sin^3(a+bx^2)}}{2b^2} - \frac{x^2 \cot(a+bx^2) \sqrt[3]{c \sin^3(a+bx^2)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (c*Sin[a + b*x^2]^3)^(1/3)/(2*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(1/3))/(2*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt[3]{c\sin^3(a+bx^2)}\,dx &= \left(\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\int x^3\sin(a+bx^2)\,dx\\ &= \frac{1}{2}\left(\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,x^2\right)\\ &= -\frac{x^2\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}+\frac{\left(\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\text{Subs}}{2b}\\ &= \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{2b^2}-\frac{x^2\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 38, normalized size = 0.66

$$-\frac{(-1+bx^2\cot(a+bx^2))\sqrt[3]{c\sin^3(a+bx^2)}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -1/2*((-1 + b*x^2*Cot[a + b*x^2])*(c*Sin[a + b*x^2]^3)^(1/3))/b^2
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.16, size = 135, normalized size = 2.33

method	result	size
risch	$-\frac{i(b\,x^2+i)\left(ic\left(e^{2i(b\,x^2+a)}-1\right)^3 e^{-3i(b\,x^2+a)}\right)^{\frac{1}{3}} e^{2i(b\,x^2+a)}}{4b^2\left(e^{2i(b\,x^2+a)}-1\right)}-\frac{i\left(ic\left(e^{2i(b\,x^2+a)}-1\right)^3 e^{-3i(b\,x^2+a)}\right)^{\frac{1}{3}}(b\,x^2-i)}{4\left(e^{2i(b\,x^2+a)}-1\right)b^2}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/b^2*(b*x^2+I)/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*e
xp(-3*I*(b*x^2+a)))^(1/3)*exp(2*I*(b*x^2+a))-1/4*I*(I*c*(exp(2*I*(b*x^2+a))
-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*(b*x^2-I)/b^2
```

Maxima [**A**]
time = 0.53, size = 32, normalized size = 0.55

$$\frac{(bx^2\cos(bx^2+a)-\sin(bx^2+a))c^{\frac{1}{3}}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*c^(1/3)/b^2
```

Fricas [**A**]
time = 0.35, size = 67, normalized size = 1.16

$$-\frac{(bx^2\cos(bx^2+a)-\sin(bx^2+a))\left(-\left(c\cos(bx^2+a)^2-c\right)\sin(bx^2+a)\right)^{\frac{1}{3}}}{2\,b^2\sin(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin
(b*x^2 + a))^(1/3)/(b^2*sin(b*x^2 + a))
```

Sympy [**A**]
time = 2.37, size = 85, normalized size = 1.47

$$\begin{cases}\frac{x^4\sqrt[3]{c\sin^3(a)}}{4} & \text{for } b=0\\ 0 & \text{for } a=-bx^2\vee a=-bx^2+\pi\\ -\frac{x^2\sqrt[3]{c\sin^3(a+bx^2)}\cos(a+bx^2)}{2b\sin(a+bx^2)}+\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{2b^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x**2) | Eq(
a, -b*x**2 + pi)), (-x**2*(c*sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(2*
b*sin(a + b*x**2)) + (c*sin(a + b*x**2)**3)**(1/3)/(2*b**2), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^3, x)
```

Mupad [B]
time = 5.19, size = 71, normalized size = 1.22

$$\frac{\left(\frac{\sin\left(b\,x^2+a\right)^2}{4}-\frac{b\,x^2\,\sin\left(2\,b\,x^2+2\,a\right)}{8}\right)\,{\left(-2\,c\,\left(\sin\left(3\,b\,x^2+3\,a\right)-3\,\sin\left(b\,x^2+a\right)\right)\right)}^{1/3}}{b^2\,\sin\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] ((sin(a + b*x^2)^2/4 - (b*x^2*sin(2*a + 2*b*x^2))/8)*(-2*c*(sin(3*a + 3*b*x
^2) - 3*sin(a + b*x^2)))^(1/3))/(b^2*sin(a + b*x^2)^2)
```

3.320 $\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\, dx$

Optimal. Leaf size=155

$$-\frac{x\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc(a+bx^2)\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\ \mathrm{c}}{}$$

[Out] -1/2*x*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)/b+1/4*cos(a)*csc(b*x^2+a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*csc(b*x^2+a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(3/2)

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6852, 3466, 3435, 3433, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,x\right)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b^{3/2}}-\frac{x\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]

[Out] -1/2*(x*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b + (Sqrt[Pi/2]*Cos[a]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b^(3/2)) - (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b^(3/2))

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2 \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^2\sin\left(a+bx^2\right)\,dx$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \cos}{2b}$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right.}{2b}$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right.}{2b^{3/2}}$$

Mathematica [A]
time = 0.18, size = 105, normalized size = 0.68

$$-\frac{\csc\left(a+bx^2\right)\left(2\sqrt{b}\,x\cos\left(a+bx^2\right)-\sqrt{2\pi}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\sqrt{2\pi}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -1/4*(Csc[a + b*x^2]*(2*Sqrt[b]*x*Cos[a + b*x^2] - Sqrt[2*Pi]*Cos[a]*Fresne
lC[Sqrt[b]*Sqrt[2/Pi]*x] + Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]
)*(c*Sin[a + b*x^2]^3)^(1/3))/b^(3/2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 240, normalized size = 1.55

method	result
risch	$\frac{\left(ic\left(e^{2i(bx^2+a)}-1\right)^3 e^{-3i(bx^2+a)}\right)^{\frac{1}{3}}\left(-\frac{ix\,e^{2i(bx^2+a)}}{2b}+\frac{i\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)e^{i(bx^2+2a)}}{4b\sqrt{-ib}}\right)}{2\,e^{2i(bx^2+a)}-2}-\frac{ix\left(ic\left(e^{2i(bx^2+a)}-1\right)^3 e^{-3i(\right.}{4b\left(e^{2i(bx^2+a)}-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)
))^(1/3)*(-1/2*I/b*x*exp(2*I*(b*x^2+a))+1/4*I/b*Pi^(1/2)/(-I*b)^(1/2)*erf((
-I*b)^(1/2)*x)*exp(I*(b*x^2+2*a)))-1/4*I*x/b/(exp(2*I*(b*x^2+a))-1)*(I*c*(e
xp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)+1/8*I*(I*c*(exp(2*I*(b*x^
2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)/b
*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 73, normalized size = 0.47

$$\frac{8\,b^2c^{\frac{1}{3}}x\cos\left(bx^2+a\right)+\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i-1\right)\,\cos\left(a\right)+\left(i+1\right)\,\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{i\,b}\,x\right)+\left(-\left(i+1\right)\,\cos\left(a\right)-\left(i-1\right)\,\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{-i\,b}\,x\right)\right)b^{\frac{3}{2}}c^{\frac{1}{3}}}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/32*(8*b^2*c^(1/3)*x*cos(b*x^2 + a) + sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) +
(I + 1)*sin(a))*erf(sqrt(I*b)*x) + (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf(s
qrt(-I*b)*x))*b^(3/2)*c^(1/3))/b^3
```

Fricas [A]
time = 0.36, size = 156, normalized size = 1.01

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(a\right)\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)-4^{\frac{2}{3}}\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)\sin\left(a\right)-2\cdot4^{\frac{2}{3}}bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{16\left(b^2\cos\left(bx^2+a\right)^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*s
qrt(b/pi))*sin(b*x^2 + a) - 4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(
2)*x*sqrt(b/pi))*sin(b*x^2 + a)*sin(a) - 2*4^(2/3)*b*x*cos(b*x^2 + a)*sin(b
*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(b^2*cos(b*x^2
+ a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[3]{c \sin^3 (a + bx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c \sin\left(b\, x^2 + a\right)^3\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] int(x^2*(c*sin(a + b*x^2)^3)^(1/3), x)
```

3.321 $\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx$

Optimal. Leaf size=31

$$-\frac{\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

```
[Out] -1/2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(1/3)/b
```

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 3286, 2718}

$$-\frac{\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -1/2*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x\sqrt[3]{c\sin^3(a+bx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \sqrt[3]{c\sin^3(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\left(\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$-\frac{\cot(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
[Out] -1/2*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 119, normalized size = 3.84

method	result	size
risch	$-\frac{i\left(ic\left(e^{2i(b x^2+a)}-1\right)^3 e^{-3i(b x^2+a)}\right)^{\frac{1}{3}} e^{2i(b x^2+a)}}{4b\left(e^{2i(b x^2+a)}-1\right)}-\frac{i\left(ic\left(e^{2i(b x^2+a)}-1\right)^3 e^{-3i(b x^2+a)}\right)^{\frac{1}{3}}}{4b\left(e^{2i(b x^2+a)}-1\right)}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] -1/4*I/b/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x
^2+a)))^(1/3)*exp(2*I*(b*x^2+a))-1/4*I/b/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2
*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)
```

Maxima [A]
time = 0.53, size = 16, normalized size = 0.52

$$\frac{c^{\frac{1}{3}}\cos(bx^2+a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*c^(1/3)*cos(b*x^2 + a)/b
```

Fricas [A]
time = 0.38, size = 51, normalized size = 1.65

$$-\frac{\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}\cos\left(bx^2+a\right)}{2\,b\sin\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)*cos(b*x^2 + a)/(b*sin
(b*x^2 + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.
time = 0.81, size = 63, normalized size = 2.03

$$\begin{cases} \frac{x^2\sqrt[3]{c\sin^3\left(a\right)}}{2} & \text{for } b=0 \\ 0 & \text{for } a=-bx^2 \vee a=-bx^2+\pi \\ -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}\cos\left(a+bx^2\right)}{2b\sin\left(a+bx^2\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x**2) | Eq(
a, -b*x**2 + pi)), (-(c*sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(2*b*sin
(a + b*x**2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x, x)
```

Mupad [B]
time = 4.82, size = 53, normalized size = 1.71

$$-\frac{\sin\left(2\,b\,x^2+2\,a\right)\,{\left(-2\,c\,\left(\sin\left(3\,b\,x^2+3\,a\right)-3\,\sin\left(b\,x^2+a\right)\right)\right)}^{1/3}}{8\,b\,{\sin\left(b\,x^2+a\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] -(sin(2*a + 2*b*x^2)*(-2*c*(sin(3*a + 3*b*x^2) - 3*sin(a + b*x^2)))^(1/3))/
(8*b*sin(a + b*x^2)^2)
```

3.322 $\int \sqrt[3]{c \sin^3\left(a+bx^2\right)}\, dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\sin(a)\sqrt[3]{c\cdots}}{\sqrt{b}}$$

```
[Out] 1/2*cos(a)*csc(b*x^2+a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a
)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(1/2)+1/2*csc(b*x^2+a)*FresnelC(x*b^(1/2)*2^(
1/2)/Pi^(1/2))*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6852, 3434, 3433, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\,x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a +
 b*x^2]^3)^(1/3))/Sqrt[b] + (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqr
t[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \sin\left(a+bx^2\right)\,dx\\ &= \left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \sin\left(bx^2\right)\,dx + \left(\csc\left(a+bx^2\right)\sin(a)\right.\\ &= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.68

$$\frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)\left(\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (Sqrt[Pi/2]*Csc[a + b*x^2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Fresnel
C[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 157, normalized size = 1.34

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-ib}\,x\right)\sqrt{\pi}\,\left(ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3 e^{-3i\left(bx^2+a\right)}\right)^{\frac{1}{3}} e^{i\left(bx^2+2a\right)}}{4\sqrt{-ib}\,\left(e^{2i\left(bx^2+a\right)}-1\right)} - \frac{\left(ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3 e^{-3i\left(bx^2+a\right)}\right)^{\frac{1}{3}} e^{ix^2b}\sqrt{\pi}\,\operatorname{erf}}{4\left(e^{2i\left(bx^2+a\right)}-1\right)\sqrt{ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*erf((-I*b)^(1/2)*x)/(-I*b)^(1/2)*Pi^(1/2)/(exp(2*I*(b*x^2+a))-1)*(I*c*(
exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)*exp(I*(b*x^2+2*a))-1/4*(
I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))
-1)*exp(I*b*x^2)*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 51, normalized size = 0.44

$$\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\,\cos\left(a\right)+(i-1)\,\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{i\,b}\,x\right)+\left((i-1)\,\cos\left(a\right)-(i+1)\,\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{-i\,b}\,x\right)\right)c^{\frac{1}{3}}}{16\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(I*b)*x)
+ ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*c^(1/3)/sqrt(b)
```

Fricas [A]
time = 0.37, size = 128, normalized size = 1.09

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(a\right)\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^{2}+a\right)+4^{\frac{2}{3}}\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^{2}+a\right)\sin\left(a\right)\right)\left(-\left(c\cos\left(bx^{2}+a\right)^{2}-c\right)\sin\left(bx^{2}+a\right)\right)^{\frac{1}{3}}}{8\left(b\cos\left(bx^{2}+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sq
rt(b/pi))*sin(b*x^2 + a) + 4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2
)*x*sqrt(b/pi))*sin(b*x^2 + a)*sin(a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2
 + a))^(1/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[3]{c\sin^{3}\left(a+bx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\sin\left(b\,x^2+a\right)^3\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(1/3),x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(1/3), x)
```

3.323 $\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x} \, dx$

Optimal. Leaf size=73

$$\frac{1}{2}\text{Ci}(bx^2)\csc(a+bx^2)\sin(a)\sqrt[3]{c\sin^3(a+bx^2)}+\frac{1}{2}\cos(a)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\,\text{Si}(bx^2)$$

```
[Out] 1/2*cos(a)*csc(b*x^2+a)*Si(b*x^2)*(c*sin(b*x^2+a)^3)^(1/3)+1/2*Ci(b*x^2)*cs
c(b*x^2+a)*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3458, 3457, 3456}

$$\frac{1}{2}\sin(a)\text{CosIntegral}(bx^2)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}+\frac{1}{2}\cos(a)\text{Si}(bx^2)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

```
[Out] (CosIntegral[b*x^2]*Csc[a + b*x^2]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/2 + (
Cos[a]*Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx = \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(a+bx^2\right)}{x}\,dx$$
$$= \left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(bx^2\right)}{x}\,dx + \left(\csc\left(a+bx^2\right)\sin(a)\right.$$
$$= \frac{1}{2}\mathrm{Ci}\left(bx^2\right)\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)} + \frac{1}{2}\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.64

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\mathrm{Ci}\left(bx^2\right)\sin(a)+\cos(a)\mathrm{Si}\left(bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

```
[Out] (Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(CosIntegral[b*x^2]*Sin[a] + Cos
[a]*SinIntegral[b*x^2]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 268, normalized size = 3.67

method	result
risch	$-\frac{\mathrm{expIntegral}(1,-ix^2b)\left(ic\left(\mathrm{e}^{2i\left(bx^2+a\right)}-1\right)^3\mathrm{e}^{-3i\left(bx^2+a\right)}\right)^{\frac{1}{3}}\mathrm{e}^{i\left(bx^2+2a\right)}}{4\left(\mathrm{e}^{2i\left(bx^2+a\right)}-1\right)}-\frac{i\left(ic\left(\mathrm{e}^{2i\left(bx^2+a\right)}-1\right)^3\mathrm{e}^{-3i\left(bx^2+a\right)}\right)^{\frac{1}{3}}\mathrm{e}^{ix^2b}\pi\,\mathrm{csgn}(b}{4\left(\mathrm{e}^{2i\left(bx^2+a\right)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Ei(1,-I*b*x^2)/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*ex
p(-3*I*(b*x^2+a)))^(1/3)*exp(I*(b*x^2+2*a))-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-
1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*Pi*csgn
(b*x^2)+1/2*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp
(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*Si(b*x^2)+1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3
*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*Ei(1,-I*b*x
^2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.60, size = 47, normalized size = 0.64

$$\frac{1}{8}\left(\left(i\,\mathrm{Ei}\left(i\,bx^2\right)-i\,\mathrm{Ei}\left(-i\,bx^2\right)\right)\cos\left(a\right)-\left(\mathrm{Ei}\left(i\,bx^2\right)+\mathrm{Ei}\left(-i\,bx^2\right)\right)\sin\left(a\right)\right)c^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] 1/8*((I*Ei(I*b*x^2) - I*Ei(-I*b*x^2))*cos(a) - (Ei(I*b*x^2) + Ei(-I*b*x^2))
*sin(a))*c^(1/3)
```

Fricas [A]
time = 0.35, size = 94, normalized size = 1.29

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(a\right)\operatorname{Si}\left(bx^{2}\right)+\left(4^{\frac{2}{3}}\operatorname{Ci}\left(bx^{2}\right)+4^{\frac{2}{3}}\operatorname{Ci}\left(-bx^{2}\right)\right)\sin\left(a\right)\right)\left(-\left(c\cos\left(bx^{2}+a\right)^{2}-c\right)\sin\left(bx^{2}+a\right)\right)^{\frac{1}{3}}\sin\left(bx^{2}+a\right)}{16\left(\cos\left(bx^{2}+a\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(2*4^(2/3)*cos(a)*sin_integral(b*x^2) + (4^(2/3)*cos_integral
(b*x^2) + 4^(2/3)*cos_integral(-b*x^2))*sin(a))*(-(c*cos(b*x^2 + a)^2 - c)*
sin(b*x^2 + a))^(1/3)*sin(b*x^2 + a)/(cos(b*x^2 + a)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{2}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,\sin\left(b\,x^2+a\right)^3\right)}^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(1/3)/x,x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(1/3)/x, x)
```

3.324 $\int \frac{\sqrt[3]{c \sin^3(a+bx^2)}}{x^2} dx$

Optimal. Leaf size=135

$$-\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}+\sqrt{b}\,\sqrt{2\pi}\,\cos(a)\csc(a+bx^2)\,C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)\sqrt[3]{c\sin^3(a+bx^2)}-\sqrt{b}\,\sqrt{2\pi}\,\csc(a+b$$

```
[Out] -(c*sin(b*x^2+a)^3)^(1/3)/x+cos(a)*csc(b*x^2+a)*FresnelC(x*b^(1/2)*2^(1/2)/
Pi^(1/2))*(c*sin(b*x^2+a)^3)^(1/3)*b^(1/2)*2^(1/2)*Pi^(1/2)-csc(b*x^2+a)*Fr
esnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*(c*sin(b*x^2+a)^3)^(1/3)*b^(1/2)*
2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6852, 3468, 3435, 3433, 3432}

$$\sqrt{2\pi}\,\sqrt{b}\,\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,x\right)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}-\sqrt{2\pi}\,\sqrt{b}\,\sin(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}-\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x^2]^3)^(1/3)/x) + Sqrt[b]*Sqrt[2*Pi]*Cos[a]*Csc[a + b*x^2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3) - Sqrt[b]*Sqrt[2*
Pi]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^
3)^(1/3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^2}\,dx &= \left(\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\int \frac{\sin(a+bx^2)}{x^2}\,dx \\
&= -\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x} + \left(2b\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\int \cos(a+bx^2)\,d \\
&= -\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x} + \left(2b\cos(a)\csc(a+bx^2)\sqrt[3]{c\sin^3(a+bx^2)}\right)\int \cos(bx^2 \\
&= -\frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x} + \sqrt{b}\sqrt{2\pi}\cos(a)\csc(a+bx^2)\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3(
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 105, normalized size = 0.78

$$\frac{\left(-1+\sqrt{b}\sqrt{2\pi}\,x\cos(a)\csc(a+bx^2)\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{b}\sqrt{2\pi}\,x\csc(a+bx^2)\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right)\sqrt[3]{c\sin^3(a+bx^2)}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

```
[Out] ((-1 + Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/P
i]*x] - Sqrt[b]*Sqrt[2*Pi]*x*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*
Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/x
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 232, normalized size = 1.72

method	result
risch	$\frac{\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{1}{3}}\left(-\frac{e^{2i\left(b\,x^2+a\right)}}{x}+\frac{ib\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-ib}\,x\right)e^{i\left(b\,x^2+2a\right)}}{\sqrt{-ib}}\right)}{2\,e^{2i\left(b\,x^2+a\right)}-2}+\frac{\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+\right.}\right.}{2x\left(e^{2i\left(b\,x^2+a\right)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sin(b*x^2+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)
))^(1/3)*(-1/x*exp(2*I*(b*x^2+a))+I*b*Pi^(1/2)/(-I*b)^(1/2)*erf((-I*b)^(1/2
)*x)*exp(I*(b*x^2+2*a)))+1/2/x/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+
a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)+1/2*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp
(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*b*Pi^(1/2)/(I*b
)^(1/2)*erf((I*b)^(1/2)*x)

Maxima [C] Result contains complex when optimal does not.
time = 0.61, size = 76, normalized size = 0.56

$$\frac{\sqrt{bx^2}\left(\left(\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,bx^2\right)-\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-i\,bx^2\right)\right)\cos\left(a\right)+\left(\left(i+1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,bx^2\right)-\left(i-1\right)\sqrt{2}\,\Gamma\left(-\frac{1}{2},-i\,bx^2\right)\right)\sin\left(a\right)\right)c^{\frac{1}{3}}}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="maxima")

[Out] 1/16*sqrt(b*x^2)*(((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I + 1)*sqrt(2)*g
amma(-1/2, -I*b*x^2))*cos(a) + ((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I -
 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a))*c^(1/3)/x

Fricas [A]
time = 0.37, size = 150, normalized size = 1.11

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)-4^{\frac{2}{3}}\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)\sin\left(a\right)+4^{\frac{2}{3}}\cos\left(bx^2+a\right)^2-4^{\frac{2}{3}}\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{4\left(x\cos\left(bx^2+a\right)^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="fricas")

[Out] -1/4*4^(1/3)*(4^(2/3)*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*
sqrt(b/pi))*sin(b*x^2 + a) - 4^(2/3)*sqrt(2)*pi*x*sqrt(b/pi)*fresnel_sin(sq
rt(2)*x*sqrt(b/pi))*sin(b*x^2 + a)*sin(a) + 4^(2/3)*cos(b*x^2 + a)^2 - 4^(2
/3))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(x*cos(b*x^2 + a)^2 -
 x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(b\,x^2+a\right)}^3\right)}^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(1/3)/x^2,x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(1/3)/x^2, x)
```

3.325 $\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}b\cos(a)\mathrm{Ci}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{2}b\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+\right.}$$

```
[Out] -1/2*(c*sin(b*x^2+a)^3)^(1/3)/x^2+1/2*b*Ci(b*x^2)*cos(a)*csc(b*x^2+a)*(c*si
n(b*x^2+a)^3)^(1/3)-1/2*b*csc(b*x^2+a)*Si(b*x^2)*sin(a)*(c*sin(b*x^2+a)^3)^
(1/3)
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6852, 3460, 3378, 3384, 3380, 3383}

$$\frac{1}{2}b\cos(a)\mathrm{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{2}b\sin(a)\mathrm{Si}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(c*Sin[a + b*x^2]^3)^(1/3)/x^2 + (b*Cos[a]*CosIntegral[b*x^2]*Csc[a +
b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/2 - (b*Csc[a + b*x^2]*Sin[a]*(c*Sin[a +
b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c \sin^3\left(a+bx^2\right)}}{x^3}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(a+bx^2\right)}{x^3}\,dx \\
&= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}\left(b\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int \frac{\cos(a-}{x} \right. \\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}\left(b\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\right. \\
&= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}b\cos(a)\text{Ci}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{2}b\,\text{c}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.68

$$-\frac{\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(-bx^2\cos(a)\text{Ci}(bx^2)+\sin\left(a+bx^2\right)+bx^2\sin(a)\text{Si}(bx^2)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(-(b*x^2*Cos[a]*CosIntegral
[b*x^2]) + Sin[a + b*x^2] + b*x^2*Sin[a]*SinIntegral[b*x^2]))/x^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 214, normalized size = 2.18

method	result
risch	$\frac{\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{1}{3}}\left(-\frac{e^{2i\left(b\,x^2+a\right)}}{2x^2}-\frac{ib\,\mathrm{expIntegral}\left(1,-ix^2b\right)e^{i\left(b\,x^2+2a\right)}}{2}\right)}{2\,e^{2i\left(b\,x^2+a\right)}-2}+\frac{(ic(e^{2i(b\,x^2+a)}-1)^3 e^{-3i(b\,x^2+a)}}{4x^2\left(e^{2i\left(b\,x^2+a\right)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)
))^(1/3)*(-1/2/x^2*exp(2*I*(b*x^2+a))-1/2*I*b*Ei(1,-I*b*x^2)*exp(I*(b*x^2+2
*a)))+1/4/x^2/(exp(2*I*(b*x^2+a))-1)*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I
*(b*x^2+a)))^(1/3)-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))
^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*b*Ei(1,I*b*x^2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.61, size = 52, normalized size = 0.53

$$-\frac{1}{8}\left(\left(\Gamma\left(-1, i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos\left(a\right)-\left(i\,\Gamma\left(-1, i\,bx^2\right)-i\,\Gamma\left(-1,-i\,bx^2\right)\right)\sin\left(a\right)\right)bc^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*((gamma(-1, I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) - (I*gamma(-1, I*b*
x^2) - I*gamma(-1, -I*b*x^2))*sin(a))*b*c^(1/3)
```

Fricas [A]
time = 0.37, size = 138, normalized size = 1.41

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(bx^2+a\right)^2-\left(2\cdot 4^{\frac{2}{3}}bx^2\sin\left(a\right)\mathrm{Si}\left(bx^2\right)-\left(4^{\frac{2}{3}}bx^2\,\mathrm{Ci}\left(bx^2\right)+4^{\frac{2}{3}}bx^2\,\mathrm{Ci}\left(-bx^2\right)\right)\cos\left(a\right)\right)\sin\left(bx^2+a\right)-2\cdot 4^{\frac{2}{3}}\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{16\left(x^2\cos\left(bx^2+a\right)^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(2*4^(2/3)*cos(b*x^2 + a)^2 - (2*4^(2/3)*b*x^2*sin(a)*sin_int
egral(b*x^2) - (4^(2/3)*b*x^2*cos_integral(b*x^2) + 4^(2/3)*b*x^2*cos_integ
```

```
ral(-b*x^2))*cos(a))*sin(b*x^2 + a) - 2*4^(2/3))*(-(c*cos(b*x^2 + a)^2 - c)
*sin(b*x^2 + a))^(1/3)/(x^2*cos(b*x^2 + a)^2 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x**3,x)
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(b\,x^2+a\right)}^3\right)}^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(1/3)/x^3,x)
[Out] int((c*sin(a + b*x^2)^3)^(1/3)/x^3, x)
```

3.326 $\int x^m \sqrt[3]{c\sin^3(a+bx^n)}\,dx$

Optimal. Leaf size=157

$$\frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\Gamma\left(\frac{1+m}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\Gamma(}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^(1+m)*csc(a+b*x^n)*GAMMA((1+m)/n,-I*b*x^n)*(c*sin(a+b*x^n)
^3)^(1/3)/n/((-I*b*x^n)^((1+m)/n))-1/2*I*x^(1+m)*csc(a+b*x^n)*GAMMA((1+m)/n
,I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.25, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6852, 3504, 2250}

$$\frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Csc[a + b*x^n]*Gamma[(1 + m)/n, (-I)*b*x^n]*(c*Sin
[a + b*x^n]^3)^(1/3))/(n*((-I)*b*x^n)^((1 + m)/n)) - ((I/2)*x^(1 + m)*Csc[a
 + b*x^n]*Gamma[(1 + m)/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*
(I*b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^m \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x^m\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x^m\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\right.$$
$$= \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\Gamma\left(\frac{1+m}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-i}}{}$$

Mathematica [**A**]
time = 0.22, size = 142, normalized size = 0.90

$$\frac{ix^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\csc(a+bx^n)\left(-(-ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
[Out] ((I/2)*x^(1 + m)*Csc[a + b*x^n]*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n
, I*b*x^n]*(Cos[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-
I)*b*x^n]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))
^((1 + m)/n))
```

Maple [**F**]
time = 0.19, size = 0, normalized size = 0.00

$$\int x^m\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a + b*x^n)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.327 $\int x^3 \sqrt[3]{c \sin^3(a+bx^n)}\, dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}x^4(-ibx^n)^{-4/n}\csc(a+bx^n)\,\Gamma\left(\frac{4}{n}, -ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\csc(a+bx^n)\,\Gamma\left(\frac{4}{n}, ibx^n\right)}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^4*csc(a+b*x^n)*GAMMA(4/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3
)/n/((-I*b*x^n)^(4/n))-1/2*I*x^4*csc(a+b*x^n)*GAMMA(4/n,I*b*x^n)*(c*sin(a+b
*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(4/n))
```

Rubi [A]
time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6852, 3504, 2250}

$$\frac{ie^{ia}x^4(-ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n}, -ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n}, ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^4*Csc[a + b*x^n]*Gamma[4/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(4/n)) - ((I/2)*x^4*Csc[a + b*x^n]*Gamma[4/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(4/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x^3\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x^3\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\right.$$
$$= \frac{ie^{ia}x^4(-ibx^n)^{-4/n}\csc(a+bx^n)\Gamma\left(\frac{4}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n}{}$$

Mathematica [A]
time = 0.13, size = 129, normalized size = 0.90

$$\frac{ix^4(b^2x^{2n})^{-4/n}\csc(a+bx^n)\left(-(-ibx^n)^{4/n}\Gamma\left(\frac{4}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{4/n}\Gamma\left(\frac{4}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
[Out] ((I/2)*x^4*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(4/n)*Gamma[4/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(4/n)*Gamma[4/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(4/n))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int x^3\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt[3]{c \sin^3 (a + bx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(a+b*x**n)**3)**(1/3),x)
[Out] Integral(x**3*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(c \sin\left(a + b\, x^n\right)^3\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x^3*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.328 $\int x^2 \sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}x^3(-ibx^n)^{-3/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{3}{n},-ibx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\csc\left(a+bx^n\right)\Gamma\left(\frac{3}{n},ibx^n\right)\sqrt[3]{\cdots}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^3*csc(a+b*x^n)*GAMMA(3/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3
)/n/((-I*b*x^n)^(3/n))-1/2*I*x^3*csc(a+b*x^n)*GAMMA(3/n,I*b*x^n)*(c*sin(a+b
*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(3/n))
```

Rubi [A]
time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6852, 3504, 2250}

$$\frac{ie^{ia}x^3(-ibx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^3*Csc[a + b*x^n]*Gamma[3/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(3/n)) - ((I/2)*x^3*Csc[a + b*x^n]*Gamma[3/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x^2\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x^2\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{\right.$$
$$= \frac{ie^{ia}x^3(-ibx^n)^{-3/n}\csc(a+bx^n)\Gamma\left(\frac{3}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^3(ibx}{}$$

Mathematica [A]
time = 0.13, size = 129, normalized size = 0.90

$$\frac{ix^3(b^2x^{2n})^{-3/n}\csc(a+bx^n)\left(-(-ibx^n)^{3/n}\Gamma\left(\frac{3}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{3/n}\Gamma\left(\frac{3}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
[Out] ((I/2)*x^3*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(3/n)*Gamma[3/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(3/n)*Gamma[3/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(3/n))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^2\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[3]{c \sin^3(a + bx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(a+b*x**n)**3)**(1/3),x)
[Out] Integral(x**2*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c \sin\left(a + b\,x^n\right)^3\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x^2*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.329 $\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}x^2(-ibx^n)^{-2/n}\csc(a+bx^n)\,\Gamma\left(\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^2(ibx^n)^{-2/n}\csc(a+bx^n)\,\Gamma\left(\frac{2}{n},ibx^n\right)}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x^2*csc(a+b*x^n)*GAMMA(2/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3
)/n/((-I*b*x^n)^(2/n))-1/2*I*x^2*csc(a+b*x^n)*GAMMA(2/n,I*b*x^n)*(c*sin(a+b
*x^n)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(2/n))
```

Rubi [A]
time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6852, 3504, 2250}

$$\frac{ie^{ia}x^2(-ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^2(ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^2*Csc[a + b*x^n]*Gamma[2/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(2/n)) - ((I/2)*x^2*Csc[a + b*x^n]*Gamma[2/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(2/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\text{s}}\right.$$
$$= \frac{ie^{ia}x^2(-ibx^n)^{-2/n}\csc(a+bx^n)\Gamma\left(\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^2(ibx^n)}{}$$

Mathematica [A]
time = 0.12, size = 129, normalized size = 0.90

$$\frac{ix^2(b^2x^{2n})^{-2/n}\csc(a+bx^n)\left(-(-ibx^n)^{2/n}\Gamma\left(\frac{2}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{2/n}\Gamma\left(\frac{2}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^2*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(2/n)*Gamma[2/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(2/n)*Gamma[2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(2/n))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int x\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x**n)**3)**(1/3),x)
[Out] Integral(x*(c*sin(a + b*x**n)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a + b*x^n)^3)^(1/3),x)
[Out] int(x*(c*sin(a + b*x^n)^3)^(1/3), x)
```

3.330 $\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx$

Optimal. Leaf size=135

$$\frac{ie^{ia}x(-ibx^n)^{-1/n}\csc(a+bx^n)\,\Gamma\left(\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\csc(a+bx^n)\,\Gamma\left(\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

```
[Out] 1/2*I*exp(I*a)*x*csc(a+b*x^n)*GAMMA(1/n,-I*b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/
n/((-I*b*x^n)^(1/n))-1/2*I*x*csc(a+b*x^n)*GAMMA(1/n,I*b*x^n)*(c*sin(a+b*x^n
)^3)^(1/3)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6852, 3446, 2239}

$$\frac{ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x*Csc[a + b*x^n]*Gamma[n^(-1), (-I)*b*x^n]*(c*Sin[a + b*x^n]
^3)^(1/3))/(n*((-I)*b*x^n)^n^(-1)) - ((I/2)*x*Csc[a + b*x^n]*Gamma[n^(-1),
I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin$$
$$= \frac{ie^{ia}x(-ibx^n)^{-1/n}\csc(a+bx^n)\Gamma\left(\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x(ibx^n)^{-1}}{}$$

Mathematica [A]
time = 0.09, size = 119, normalized size = 0.88

$$\frac{ix(b^2x^{2n})^{-1/n}\csc(a+bx^n)\left(-(-ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3),x]
[Out] ((I/2)*x*Csc[a + b*x^n]*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[
a] - I*Sin[a])) + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Si
n[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^n^(-1))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3),x)
[Out] int((c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c\sin^3{(a+bx^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(1/3),x)
```

```
[Out] int((c*sin(a + b*x^n)^3)^(1/3), x)
```

3.331 $\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x} dx$

Optimal. Leaf size=73

$$\frac{\text{Ci}(bx^n)\csc(a+bx^n)\sin(a)\sqrt[3]{c\sin^3(a+bx^n)}}{n}+\frac{\cos(a)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\,\text{Si}(bx^n)}{n}$$

```
[Out] cos(a)*csc(a+b*x^n)*Si(b*x^n)*(c*sin(a+b*x^n)^3)^(1/3)/n+Ci(b*x^n)*csc(a+b*
x^n)*sin(a)*(c*sin(a+b*x^n)^3)^(1/3)/n
```

Rubi [A]
time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3458, 3457, 3456}

$$\frac{\sin(a)\text{CosIntegral}(bx^n)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{n}+\frac{\cos(a)\text{Si}(bx^n)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

```
[Out] (CosIntegral[b*x^n]*Csc[a + b*x^n]*Sin[a]*(c*Sin[a + b*x^n]^3)^(1/3))/n + (
Cos[a]*Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*SinIntegral[b*x^n])/n
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3458

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx &= \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(a+bx^n)}{x}\,dx\\ &= \left(\cos(a)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(bx^n)}{x}\,dx + \left(\csc(a+bx^n)\sin(a)\right.\\ &= \frac{\mathrm{Ci}(bx^n)\csc(a+bx^n)\sin(a)\sqrt[3]{c\sin^3(a+bx^n)}}{n} + \frac{\cos(a)\csc(a+bx^n)\sqrt[3]{c\sin^3(a-}}{n}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 0.64

$$\frac{\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\,(\mathrm{Ci}(bx^n)\sin(a)+\cos(a)\mathrm{Si}(bx^n))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

```
[Out] (Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*(CosIntegral[b*x^n]*Sin[a] + Cos
[a]*SinIntegral[b*x^n]))/n
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 280, normalized size = 3.84

method	result
risch	$-\frac{\mathrm{expIntegral}(1,-ib\,x^n)\left(ic\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^3\mathrm{e}^{-3i(a+b\,x^n)}\right)^{\frac{1}{3}}\mathrm{e}^{i(b\,x^n+2a)}}{2n\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)}-\frac{i\left(ic\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^3\mathrm{e}^{-3i(a+b\,x^n)}\right)^{\frac{1}{3}}\mathrm{e}^{ib\,x^n}\pi\,\mathrm{csgn}(b\,x}{2\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Ei(1,-I*b*x^n)/n/(exp(2*I*(a+b*x^n))-1)*(I*c*(exp(2*I*(a+b*x^n))-1)^3*
exp(-3*I*(a+b*x^n)))^(1/3)*exp(I*(b*x^n+2*a))-1/2*I*(I*c*(exp(2*I*(a+b*x^n)
)-1)^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp(2*I*(a+b*x^n))-1)*exp(I*b*x^n)/n*Pi*
csgn(b*x^n)+I*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp
(2*I*(a+b*x^n))-1)*exp(I*b*x^n)/n*Si(b*x^n)+1/2*(I*c*(exp(2*I*(a+b*x^n))-1)
^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp(2*I*(a+b*x^n))-1)*exp(I*b*x^n)/n*Ei(1,-I
*b*x^n)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 144, normalized size = 1.97

$$\frac{\left(\left(\left(\sqrt{3}+i\right)\mathrm{Ei}(i\,bx^{n})-\left(\sqrt{3}+i\right)\mathrm{Ei}(-i\,bx^{n})-\left(\sqrt{3}-i\right)\mathrm{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)+\left(\sqrt{3}-i\right)\mathrm{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a)-\left(\left(-i\sqrt{3}+1\right)\mathrm{Ei}(i\,bx^{n})+\left(-i\sqrt{3}+1\right)\mathrm{Ei}(-i\,bx^{n})+\left(i\sqrt{3}+1\right)\mathrm{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)+\left(i\sqrt{3}+1\right)\mathrm{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(a)\right)c^{\frac{1}{3}}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] 1/8*(((sqrt(3) + I)*Ei(I*b*x^n) - (sqrt(3) + I)*Ei(-I*b*x^n) - (sqrt(3) - I
)*Ei(I*b*e^(n*conjugate(log(x)))) + (sqrt(3) - I)*Ei(-I*b*e^(n*conjugate(lo
g(x)))))*cos(a) - ((-I*sqrt(3) + 1)*Ei(I*b*x^n) + (-I*sqrt(3) + 1)*Ei(-I*b*
x^n) + (I*sqrt(3) + 1)*Ei(I*b*e^(n*conjugate(log(x)))) + (I*sqrt(3) + 1)*Ei
(-I*b*e^(n*conjugate(log(x)))))*sin(a))*c^(1/3)/n
```

Fricas [A]
time = 0.35, size = 98, normalized size = 1.34

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\,\mathrm{Ci}\left(bx^{n}\right)\sin\left(a\right)+4^{\frac{2}{3}}\,\mathrm{Ci}\left(-bx^{n}\right)\sin\left(a\right)+2\cdot 4^{\frac{2}{3}}\cos\left(a\right)\mathrm{Si}\left(bx^{n}\right)\right)\left(-\left(c\cos\left(bx^{n}+a\right)^{2}-c\right)\sin\left(bx^{n}+a\right)\right)^{\frac{1}{3}}\sin\left(bx^{n}+a\right)}{8\left(n\cos\left(bx^{n}+a\right)^{2}-n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(4^(2/3)*cos_integral(b*x^n)*sin(a) + 4^(2/3)*cos_integral(-b*
x^n)*sin(a) + 2*4^(2/3)*cos(a)*sin_integral(b*x^n))*(-(c*cos(b*x^n + a)^2 -
 c)*sin(b*x^n + a))^(1/3)*sin(b*x^n + a)/(n*cos(b*x^n + a)^2 - n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(1/3)/x,x)
```

```
[Out] int((c*sin(a + b*x^n)^3)^(1/3)/x, x)
```

3.332 $\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x^2}\,dx$

Optimal. Leaf size=139

$$\frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}-\frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}$$

```
[Out] 1/2*I*exp(I*a)*(-I*b*x^n)^(1/n)*csc(a+b*x^n)*GAMMA(-1/n,-I*b*x^n)*(c*sin(a+
b*x^n)^3)^(1/3)/n/x-1/2*I*(I*b*x^n)^(1/n)*csc(a+b*x^n)*GAMMA(-1/n,I*b*x^n)*
(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/x
```

Rubi [A]

time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6852, 3504, 2250}

$$\frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}-\frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]*Gamma[-n^(-1), (-I)*b*x^n
]*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x) - ((I/2)*(I*b*x^n)^n^(-1)*Csc[a + b*x^n
]*Gamma[-n^(-1), I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(a+bx^n)}{x^2}\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{e^{-ia-ibx^n}}{x^2}\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin$$
$$= \frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\,\Gamma\left(-\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx} - \frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\csc($$

Mathematica [A]
time = 0.11, size = 110, normalized size = 0.79

$$\frac{i\csc(a+bx^n)\left(-(ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(-ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

```
[Out] ((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^n^(-1)*Gamma[-n^(-1), I*b*x^n]*(Cos[a] -
 I*Sin[a])) + ((-I)*b*x^n)^n^(-1)*Gamma[-n^(-1), (-I)*b*x^n]*(Cos[a] + I*Si
n[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(1/3)/x^2,x)
```

```
[Out] int((c*sin(a + b*x^n)^3)^(1/3)/x^2, x)
```

3.333 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}(-ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}-\frac{ie^{-ia}(ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},ibx^n\right)\sqrt[3]{c\,\mathrm{s}}}{2nx^2}$$

```
[Out] 1/2*I*exp(I*a)*(-I*b*x^n)^(2/n)*csc(a+b*x^n)*GAMMA(-2/n,-I*b*x^n)*(c*sin(a+
b*x^n)^3)^(1/3)/n/x^2-1/2*I*(I*b*x^n)^(2/n)*csc(a+b*x^n)*GAMMA(-2/n,I*b*x^n
)*(c*sin(a+b*x^n)^3)^(1/3)/exp(I*a)/n/x^2
```

Rubi [A]

time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6852, 3504, 2250}

$$\frac{ie^{ia}(-ibx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}-\frac{ie^{-ia}(ibx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]*Gamma[-2/n, (-I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(1/3))/(n*x^2) - ((I/2)*(I*b*x^n)^(2/n)*Csc[a + b*x^n]*G
amma[-2/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x^2)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(a+bx^n)}{x^3}\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{e^{-ia-ibx^n}}{x^3}\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\mathrm{s}\right.$$
$$= \frac{ie^{ia}(-ibx^n)^{2/n}\csc(a+bx^n)\,\Gamma\left(-\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2} - \frac{ie^{-ia}(ibx^n)^{2/n}\,\mathrm{c}}{}$$

Mathematica [A]
time = 0.11, size = 114, normalized size = 0.80

$$\frac{i\csc(a+bx^n)\left(-(ibx^n)^{2/n}\Gamma\left(-\frac{2}{n},ibx^n\right)(\cos(a)-i\sin(a))+(-ibx^n)^{2/n}\Gamma\left(-\frac{2}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

```
[Out] ((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^(2/n)*Gamma[-2/n, I*b*x^n]*(Cos[a] - I*S
in[a])) + ((-I)*b*x^n)^(2/n)*Gamma[-2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))*(
c*Sin[a + b*x^n]^3)^(1/3))/(n*x^2)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c\sin^3{(a + bx^n)}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(1/3)/x^3,x)
```

```
[Out] int((c*sin(a + b*x^n)^3)^(1/3)/x^3, x)
```

3.334 $\int x^m (c\sin^3(a+bx))^{2/3}\, dx$

Optimal. Leaf size=169

$$\frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)}+\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,-2ibx)\left(c\sin^3(a+bx)\right)}{b}$$

```
[Out] 1/2*x^(1+m)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)/(1+m)+I*2^(-3-m)*exp(2*I*a)
*x^m*csc(b*x+a)^2*GAMMA(1+m,-2*I*b*x)*(c*sin(b*x+a)^3)^(2/3)/b/((-I*b*x)^m)
-I*2^(-3-m)*x^m*csc(b*x+a)^2*GAMMA(1+m,2*I*b*x)*(c*sin(b*x+a)^3)^(2/3)/b/ex
p(2*I*a)/((I*b*x)^m)
```

Rubi [A]
time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6852, 3393, 3388, 2212}

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\csc^2(a+bx)\text{Gamma}(m+1,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\csc^2(a+bx)\text{Gamma}(m+1,2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}+\frac{x^{m+1}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(2*(1 + m)) + (I*2^(-3
- m)*E^((2*I)*a)*x^m*Csc[a + b*x]^2*Gamma[1 + m, (-2*I)*b*x]*(c*Sin[a + b*x
]^3)^(2/3))/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Csc[a + b*x]^2*Gamma[1 + m
, (2*I)*b*x]*(c*Sin[a + b*x]^3)^(2/3))/(b*E^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^m\sin^2(a+bx)\,dx\\ &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx\\ &= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)}-\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\\ &= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)}-\frac{1}{4}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int\\ &= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)}+\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(}{b}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 142, normalized size = 0.84

$$\frac{2^{-3-m}x^m(b^2x^2)^{-m}\csc^2(a+bx)\left(2^{2+m}bx(b^2x^2)^m-i(1+m)(-ibx)^m\Gamma(1+m,2ibx)(\cos(a)-i\sin(a))^2+i(1+m)(ibx)^m\Gamma(1+m,-2ibx)(\cos(a)+i\sin(a))^2\right)\left(c\sin^3(a+bx)\right)^{2/3}}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (2^(-3 - m)*x^m*Csc[a + b*x]^2*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I)
*b*x)^m*Gamma[1 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)^m
*Gamma[1 + m, (-2*I)*b*x]*(Cos[a] + I*Sin[a])^2)*(c*Sin[a + b*x]^3)^(2/3))/
(b*(1 + m)*(b^2*x^2)^m)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int x^m\left(c\left(\sin^3(bx+a)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/4*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))*c^
(2/3)/(m + 1)
```

Fricas [A]
time = 0.10, size = 112, normalized size = 0.66

$$-\frac{\left(4\,bxx^{m}-(i\,m+i)e^{(-m\log(2i\,b)-2i\,a)}\Gamma(m+1,2i\,bx)-(-i\,m-i)e^{(-m\log(-2i\,b)+2i\,a)}\Gamma(m+1,-2i\,bx)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{8\left((bm+b)\cos\left(bx+a\right)^{2}-bm-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*x^m - (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
 - (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(2/3)/((b*m + b)*cos(b*x + a)^2 - b*m - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(c\sin\left(a+b\,x\right)^3\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a + b*x)^3)^(2/3),x)
[Out] int(x^m*(c*sin(a + b*x)^3)^(2/3), x)
```

3.335 $\int x^3 \left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal. Leaf size=165

$$-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^3\cot(a+bx)\left(c\sin^3(a+\right.}{2b}$$

```
[Out] -3/8*(c*sin(b*x+a)^3)^(2/3)/b^4+3/4*x^2*(c*sin(b*x+a)^3)^(2/3)/b^2+3/4*x*co
t(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b^3-1/2*x^3*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/
3)/b-3/8*x^2*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)/b^2+1/8*x^4*csc(b*x+a)^2*(
c*sin(b*x+a)^3)^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6852, 3392, 30, 3391}

$$-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}+\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{3x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{8b^2}+\frac{1}{8}x^4\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (-3*(c*Sin[a + b*x]^3)^(2/3))/(8*b^4) + (3*x^2*(c*Sin[a + b*x]^3)^(2/3))/(4
*b^2) + (3*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(4*b^3) - (x^3*Cot[a +
b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (3*x^2*Csc[a + b*x]^2*(c*Sin[a + b*x
]^3)^(2/3))/(8*b^2) + (x^4*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^3\sin^2(a+bx)\,dx$$

$$=\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}\Big(\csc^2(a+bx)\,(c$$

$$=-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx\right)}{4b^3}$$

$$=-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx\right)}{4b^3}$$

Mathematica [A]
time = 0.20, size = 79, normalized size = 0.48

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(2b^4x^4+(3-6b^2x^2)\cos(2(a+bx))+(6bx-4b^3x^3)\sin(2(a+bx))\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*b^4*x^4 + (3 - 6*b^2*x^2)*Cos[2
*(a + b*x)] + (6*b*x - 4*b^3*x^3)*Sin[2*(a + b*x)]))/(16*b^4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 208, normalized size = 1.26

method	result
risch	$-\frac{x^4\left(ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}\right)^{\frac{2}{3}}e^{2i(bx+a)}}{8\left(e^{2i(bx+a)}-1\right)^2}-\frac{i\left(4b^3x^3+6ib^2x^2-6bx-3i\right)\left(ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}\right)^{\frac{2}{3}}e^{4i(bx+a)}}{32b^4\left(e^{2i(bx+a)}-1\right)^2}+\frac{i\Big(ic\big(e^{2i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^4/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a
)))^(2/3)*exp(2*I*(b*x+a))-1/32*I/b^4*(4*b^3*x^3+6*I*b^2*x^2-6*b*x-3*I)/(ex
p(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*ex
p(4*I*(b*x+a))+1/32*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/
(exp(2*I*(b*x+a))-1)^2*(4*b^3*x^3-6*I*b^2*x^2-6*b*x+3*I)/b^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(141) = 282.
time = 0.57, size = 286, normalized size = 1.73

$$-\frac{32\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^{3}}{(\cos(bx+a)+1)^{3}}}{\frac{2\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{\sin(bx+a)^{4}}{(\cos(bx+a)+1)^{4}}+1}\right)a^{3}+6\left(2\,(bx+a)^{2}-2\,(bx+a)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\right)a^{2}c^{\frac{2}{3}}-2\left(4\,(bx+a)^{3}-6\,(bx+a)\cos\left(2\,bx+2\,a\right)-3\left(2\,(bx+a)^{2}-1\right)\sin\left(2\,bx+2\,a\right)\right)ac^{\frac{2}{3}}+\left(2\,(bx+a)^{4}-3\left(2\,(bx+a)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-2\left(2\,(bx+a)^{3}-3\,bx-3\,a\right)\sin\left(2\,bx+2\,a\right)\right)c^{\frac{2}{3}}}{32\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/32*(32*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(
2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
 + 1))*a^3 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
2*a))*a^2*c^(2/3) - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*
(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*c^(2/3) + (2*(b*x + a)^4 - 3*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2
*a))*c^(2/3))/b^4
```

Fricas [A]
time = 0.36, size = 111, normalized size = 0.67

$$-\frac{\left(2\,b^{4}x^{4}+6\,b^{2}x^{2}-6\left(2\,b^{2}x^{2}-1\right)\cos\left(bx+a\right)^{2}-4\left(2\,b^{3}x^{3}-3\,bx\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-3\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{16\left(b^{4}\cos\left(bx+a\right)^{2}-b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4*x^4 + 6*b^2*x^2 - 6*(2*b^2*x^2 - 1)*cos(b*x + a)^2 - 4*(2*b^3*
x^3 - 3*b*x)*cos(b*x + a)*sin(b*x + a) - 3)*(-(c*cos(b*x + a)^2 - c)*sin(b*
x + a))^(2/3)/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x**3*(c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(c\sin\left(a+b\,x\right)^3\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a + b*x)^3)^(2/3),x)
```

```
[Out] int(x^3*(c*sin(a + b*x)^3)^(2/3), x)
```

3.336 $\int x^2(c\sin^3(a+bx))^{2/3}\,dx$

Optimal. Leaf size=139

$$\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}+\frac{\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^3}-\frac{x^2\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b}-\frac{x\csc^2(a+bx)\,(c\,\mathrm{si}}{4b^2}$$

```
[Out] 1/2*x*(c*sin(b*x+a)^3)^(2/3)/b^2+1/4*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b^3-
1/2*x^2*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b-1/4*x*csc(b*x+a)^2*(c*sin(b*x+a
)^3)^(2/3)/b^2+1/6*x^3*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A]
time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6852, 3392, 30, 2715, 8}

$$\frac{\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^3}+\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}-\frac{x\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}}{4b^2}+\frac{1}{6}x^3\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}-\frac{x^2\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (x*(c*Sin[a + b*x]^3)^(2/3))/(2*b^2) + (Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/
3))/(4*b^3) - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (x*Csc[a
+ b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(4*b^2) + (x^3*Csc[a + b*x]^2*(c*Sin[a +
 b*x]^3)^(2/3))/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\left(c\sin^3(a+bx)\right)^{2/3}\,dx=\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^2\sin^2(a+bx)\,dx$$

$$=\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}\Big(\csc^2(a+bx)\left(c\,\mathrm{si}\right.$$

$$=\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}+\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^2\cot(a+bx)\left(c\sin^3(a\right.}{2b}$$

$$=\frac{x(c\sin^3(a+bx))^{2/3}}{2b^2}+\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^2\cot(a+bx)\left(c\sin^3(a\right.}{2b}$$

Mathematica [A]
time = 0.19, size = 69, normalized size = 0.50

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(4b^3x^3-6bx\cos(2(a+bx))+(3-6b^2x^2)\sin(2(a+bx))\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)
] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)]))/(24*b^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 190, normalized size = 1.37

method	result

risch	$-\frac{x^{3}\left(ic\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\mathrm{e}^{-3i(bx+a)}\right)^{\frac{2}{3}}\mathrm{e}^{2i(bx+a)}}{6\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}-\frac{i(2x^{2}b^{2}+2ibx-1)\left(ic\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\mathrm{e}^{-3i(bx+a)}\right)^{\frac{2}{3}}\mathrm{e}^{4i(bx+a)}}{16b^{3}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}+\frac{i\left(ic\left(\mathrm{e}^{2i(bx+a)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x^3/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a
)))^(2/3)*exp(2*I*(b*x+a))-1/16*I/b^3*(2*x^2*b^2+2*I*b*x-1)/(exp(2*I*(b*x+a
))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*exp(4*I*(b*x+a
))+1/16*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*
x+a))-1)^2*(2*x^2*b^2-2*I*b*x-1)/b^3
```

Maxima [A]
time = 0.56, size = 219, normalized size = 1.58

$$\frac{48\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^{3}}{(\cos(bx+a)+1)^{3}}}{\frac{2\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{\sin(bx+a)^{4}}{(\cos(bx+a)+1)^{4}}+1}\right)a^{2}+6\left(2(bx+a)^{2}-2(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\right)ac^{\frac{2}{3}}-\left(4(bx+a)^{3}-6(bx+a)\cos(2bx+2a)-3\left(2(bx+a)^{2}-1\right)\sin(2bx+2a)\right)c^{\frac{2}{3}}}{48b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/48*(48*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*
x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2
*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
+ 1))*a^2 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a))*a*c^(2/3) - (4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*c^(2/3))/b^3
```

Fricas [A]
time = 0.35, size = 95, normalized size = 0.68

$$-\frac{\left(2\,b^{3}x^{3}-6\,bx\cos\left(bx+a\right)^{2}-3\left(2\,b^{2}x^{2}-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,bx\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{12\left(b^{3}\cos\left(bx+a\right)^{2}-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*si
n(b*x + a) + 3*b*x)*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^3*cos(b
*x + a)^2 - b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a + b*x)^3)^(2/3),x)
```

```
[Out] int(x^2*(c*sin(a + b*x)^3)^(2/3), x)
```

3.337 $\int x(c\sin^3(a+bx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{(c\sin^3(a+bx))^{2/3}}{4b^2}-\frac{x\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[Out] 1/4*(c*sin(b*x+a)^3)^(2/3)/b^2-1/2*x*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b+1/
4*x^2*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6852, 3391, 30}

$$\frac{(c\sin^3(a+bx))^{2/3}}{4b^2}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{x\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (c*Sin[a + b*x]^3)^(2/3)/(4*b^2) - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3)
)/(2*b) + (x^2*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx = \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x\sin^2(a+bx)\,dx$$
$$= \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2} - \frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{2}\Big(\csc^2(a+bx)\,\big(c\sin^3($$
$$= \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2} - \frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{4}x^2\csc^2(a+bx)\,\big(c\sin^3$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.70

$$-\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\cos(2(a+bx))+2bx(-bx+\sin(2(a+bx)))\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] -1/8*(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(Cos[2*(a + b*x)] + 2*b*x*(-(
b*x) + Sin[2*(a + b*x)])))/b^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 174, normalized size = 2.20

method	result
risch	$-\frac{x^2\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} e^{2i(bx+a)}}{4\left(e^{2i(bx+a)}-1\right)^2} - \frac{i(2bx+i)\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} e^{4i(bx+a)}}{16b^2\left(e^{2i(bx+a)}-1\right)^2} + \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(b}\right.}{16\left(e^{2i(bx+a)}-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a
)))^(2/3)*exp(2*I*(b*x+a))-1/16*I/b^2*(2*b*x+I)/(exp(2*I*(b*x+a))-1)^2*(I*c
*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*exp(4*I*(b*x+a))+1/16*I*(I
*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*(
2*b*x-I)/b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(67) = 134.
time = 0.53, size = 162, normalized size = 2.05

$$-\frac{16\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right) - \frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1} - \frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a + \left(2(bx+a)^2 - 2(bx+a)\sin(2bx+2a) - \cos(2bx+2a)\right)c^{\frac{2}{3}}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/16*(16*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(
2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
 + 1))*a + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
)*c^(2/3))/b^2
```

Fricas [A]
time = 0.36, size = 82, normalized size = 1.04

$$-\frac{\left(2\,b^2x^2-4\,bx\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,\cos\left(bx+a\right)^2+1\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{8\,\left(b^2\cos\left(bx+a\right)^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^2*x^2 - 4*b*x*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 1)*(
-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x*(c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a + b*x)^3)^(2/3),x)
```

```
[Out] int(x*(c*sin(a + b*x)^3)^(2/3), x)
```

3.338 $\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal. Leaf size=55

$$-\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

```
[Out] -1/2*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/b+1/2*x*csc(b*x+a)^2*(c*sin(b*x+a)^3
)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3286, 2715, 8}

$$\frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] -1/2*(Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/b + (x*Csc[a + b*x]^2*(c*Sin[a
 + b*x]^3)^(2/3))/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(c\sin^3(a+bx)\right)^{2/3}\,dx = \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \sin^2(a+bx)\,dx$$
$$= -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int 1\,dx$$
$$= -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

Mathematica [A]
time = 0.07, size = 47, normalized size = 0.85

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(2(a+bx)-\sin(2(a+bx)))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*(a + b*x) - Sin[2*(a + b*x)]))/
(4*b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 158, normalized size = 2.87

method	result
risch	$-\frac{x\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} e^{2i(bx+a)}}{2\left(e^{2i(bx+a)}-1\right)^2} - \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} e^{4i(bx+a)}}{8b\left(e^{2i(bx+a)}-1\right)^2} + \frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}}{8\left(e^{2i(bx+a)}-1\right)^2 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a))
)^(2/3)*exp(2*I*(b*x+a))-1/8*I/b/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+
a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*exp(4*I*(b*x+a))+1/8*I*(I*c*(exp(2*I*(b*x
+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(47) = 94$.
time = 0.53, size = 116, normalized size = 2.11

$$\frac{c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right) - \frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1} - \frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4} + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*x + a)/(c
os(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2*sin(b*x
+ a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4 + 1))/b
```

Fricas [A]
time = 0.35, size = 60, normalized size = 1.09

$$-\frac{(bx-\cos(bx+a)\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{2}{3}}}{2\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - cos(b*x + a)*sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a
))^(2/3)/(b*cos(b*x + a)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin^3(a+bx)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(2/3),x)
```

```
[Out] int((c*sin(a + b*x)^3)^(2/3), x)
```

3.339 $\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx$

Optimal. Leaf size=99

$$-\frac{1}{2}\cos(2a)\text{Ci}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+\frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}+\frac{1}{2}\csc^2(a+bx)\,\text{si}$$

```
[Out] -1/2*Ci(2*b*x)*cos(2*a)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)+1/2*csc(b*x+a)^
2*ln(x)*(c*sin(b*x+a)^3)^(2/3)+1/2*csc(b*x+a)^2*Si(2*b*x)*sin(2*a)*(c*sin(b
*x+a)^3)^(2/3)
```

Rubi [A]
time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6852, 3393, 3384, 3380, 3383}

$$-\frac{1}{2}\cos(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+\frac{1}{2}\sin(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+\frac{1}{2}\log(x)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))
+ (Csc[a + b*x]^2*Log[x]*(c*Sin[a + b*x]^3)^(2/3))/2 + (Csc[a + b*x]^2*Sin[
2*a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*x])/2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x}\,dx\\ &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \left(\frac{1}{2x}-\frac{\cos(2a+2bx)}{2x}\right)dx\\ &= \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{1}{2}\Big(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\Big)\int\\ &= \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{1}{2}\Big(\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right.\\ &= -\frac{1}{2}\cos(2a)\text{Ci}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+\frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.51

$$\frac{1}{2}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-\cos(2a)\text{Ci}(2bx)+\log(x)+\sin(2a)\text{Si}(2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x,x]
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x]) +
Log[x] + Sin[2*a]*SinIntegral[2*b*x]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 283, normalized size = 2.86

method	result
risch	$\frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} e^{2ibx}\pi\,\text{csgn}(bx)}{4\left(e^{2i(bx+a)}-1\right)^2}-\frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} e^{2ibx}\,\text{sinIntegral}(2bx)}{2\left(e^{2i(bx+a)}-1\right)^2}-\frac{\left(ic\left(e^{2i(bx+a)}-1\right)^3 e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a)
)-1)^2*exp(2*I*b*x)*Pi*csgn(b*x)-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I
*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*exp(2*I*b*x)*Si(2*b*x)-1/4*(I*c*(ex
p(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*exp(2*I
*b*x)*Ei(1,-2*I*b*x)-1/4*Ei(1,-2*I*b*x)/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*
I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*exp(2*I*(b*x+2*a))-1/2*ln(x)/(exp(
2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*exp(
2*I*(b*x+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.57, size = 52, normalized size = 0.53

$$-\frac{1}{8}\left((E_1(2i\,bx)+E_1(-2i\,bx))\cos\left(2\,a\right)+(-i\,E_1(2i\,bx)+i\,E_1(-2i\,bx))\sin\left(2\,a\right)+2\,\log\left(bx\right)\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/8*((exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a) +
 (-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))*sin(2*a) +
 2*log(b*x))*c^(2/3)
```

Fricas [A]
time = 0.38, size = 88, normalized size = 0.89

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx\right)-\left(4^{\frac{1}{3}}\operatorname{Ci}\left(2\,bx\right)+4^{\frac{1}{3}}\operatorname{Ci}\left(-2\,bx\right)\right)\cos\left(2\,a\right)+2\cdot 4^{\frac{1}{3}}\log\left(x\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{16\left(\cos\left(bx+a\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/16*4^(2/3)*(2*4^(1/3)*sin(2*a)*sin_integral(2*b*x) - (4^(1/3)*cos_integr
al(2*b*x) + 4^(1/3)*cos_integral(-2*b*x))*cos(2*a) + 2*4^(1/3)*log(x))*(-(c
*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(cos(b*x + a)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\sin\left(a+b\,x\right)^3\right)}^{2/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(2/3)/x,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(2/3)/x, x)
```

3.340 $\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=86

$$-\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+b\text{Ci}(2bx)\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}+b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)$$

```
[Out] -(c*sin(b*x+a)^3)^(2/3)/x+b*cos(2*a)*csc(b*x+a)^2*Si(2*b*x)*(c*sin(b*x+a)^3
)^(2/3)+b*Ci(2*b*x)*csc(b*x+a)^2*sin(2*a)*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6852, 3394, 12, 3384, 3380, 3383}

$$b\sin(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+b\cos(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x]^3)^(2/3)/x) + b*CosIntegral[2*b*x]*Csc[a + b*x]^2*Sin[2*a
]*(c*Sin[a + b*x]^3)^(2/3) + b*Cos[2*a]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(
2/3)*SinIntegral[2*b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x^2}\,dx \\
&= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(2b\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2a+2bx)}{2x}\,dx \\
&= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2a+2bx)}{x}\,dx \\
&= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2bx)}{x}\,dx \\
&= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b\mathrm{Ci}(2bx)\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3} + b\cos(2a
\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 65, normalized size = 0.76

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-1+\cos(2(a+bx))+2bx\mathrm{Ci}(2bx)\sin(2a)+2bx\cos(2a)\mathrm{Si}(2bx)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 2*b*x*Cos
Integral[2*b*x]*Sin[2*a] + 2*b*x*Cos[2*a]*SinIntegral[2*b*x]))/(2*x)
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 211, normalized size = 2.45

method	result
risch	$\frac{i\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} b\left(\frac{i}{bx}+2\,e^{2ibx}\,\mathrm{expIntegral}(1,2ibx)\right)}{4\left(e^{2i(bx+a)}-1\right)^2}+\frac{ib\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}\left(\frac{ie^{4i(bx+a)}}{xb}-2\,\mathrm{expInteg}\right.}{4\left(e^{2i(bx+a)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a)
)-1)^2*b*(I/b/x+2*exp(2*I*b*x)*Ei(1,2*I*b*x))+1/4*I*b/(exp(2*I*(b*x+a))-1)^
2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*(I/x/b*exp(4*I*(b*x+
a))-2*Ei(1,-2*I*b*x)*exp(2*I*(b*x+2*a)))+1/2/x/(exp(2*I*(b*x+a))-1)^2*(I*c*
(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*exp(2*I*(b*x+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.57, size = 265, normalized size = 3.08

$$\frac{\left(\left(\left(-i\sqrt{3}+1\right)E_2(2i\,bx)+\left(i\sqrt{3}+1\right)E_2(-2i\,bx)\right)\cos(2a)^3-\left(\left(\sqrt{3}+i\right)E_2(2i\,bx)+\left(\sqrt{3}-i\right)E_2(-2i\,bx)\right)\sin(2a)^3+\left(\left(\left(-i\sqrt{3}+1\right)E_2(2i\,bx)+\left(i\sqrt{3}+1\right)E_2(-2i\,bx)\right)\cos(2a)-4\right)\sin(2a)^2+\left(\left(i\sqrt{3}+1\right)E_2(2i\,bx)+\left(-i\sqrt{3}+1\right)E_2(-2i\,bx)\right)\cos(2a)-4\cos(2a)^2-\left(\left(\left(\sqrt{3}+i\right)E_2(2i\,bx)+\left(\sqrt{3}-i\right)E_2(-2i\,bx)\right)\cos(2a)^2-\left(\sqrt{3}-i\right)E_2(2i\,bx)-\left(\sqrt{3}+i\right)E_2(-2i\,bx)\right)\sin(2a)\right)bc^{\frac{2}{3}}}{16\left(a\cos(2a)^2+a\sin(2a)^2-(bx+a)\left(\cos(2a)^2+\sin(2a)^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(((-I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_in
tegral_e(2, -2*I*b*x))*cos(2*a)^3 - ((sqrt(3) + I)*exp_integral_e(2, 2*I*b*
x) + (sqrt(3) - I)*exp_integral_e(2, -2*I*b*x))*sin(2*a)^3 + (((-I*sqrt(3)
+ 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_integral_e(2, -2*I*b*
x))*cos(2*a) - 4)*sin(2*a)^2 + ((I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x)
+ (-I*sqrt(3) + 1)*exp_integral_e(2, -2*I*b*x))*cos(2*a) - 4*cos(2*a)^2 - (
((sqrt(3) + I)*exp_integral_e(2, 2*I*b*x) + (sqrt(3) - I)*exp_integral_e(2,
 -2*I*b*x))*cos(2*a)^2 - (sqrt(3) - I)*exp_integral_e(2, 2*I*b*x) - (sqrt(3
) + I)*exp_integral_e(2, -2*I*b*x))*sin(2*a))*b*c^(2/3)/(a*cos(2*a)^2 + a*s
in(2*a)^2 - (b*x + a)*(cos(2*a)^2 + sin(2*a)^2))
```

Fricas [A]
time = 0.37, size = 108, normalized size = 1.26

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}bx\cos(2a)\,\mathrm{Si}(2bx)+2\cdot 4^{\frac{1}{3}}\cos(bx+a)^2+\left(4^{\frac{1}{3}}bx\,\mathrm{Ci}(2bx)+4^{\frac{1}{3}}bx\,\mathrm{Ci}(-2bx)\right)\sin(2a)-2\cdot 4^{\frac{1}{3}}\right)\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{2}{3}}}{8\left(x\cos(bx+a)^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*4^(2/3)*(2*4^(1/3)*b*x*cos(2*a)*sin_integral(2*b*x) + 2*4^(1/3)*cos(b*
x + a)^2 + (4^(1/3)*b*x*cos_integral(2*b*x) + 4^(1/3)*b*x*cos_integral(-2*b
*x))*sin(2*a) - 2*4^(1/3))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(x*
cos(b*x + a)^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^{2/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(2/3)/x^2,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(2/3)/x^2, x)
```

3.341 $\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx$

Optimal. Leaf size=119

$$-\frac{(c\sin^3(a+bx))^{2/3}}{2x^2}-\frac{b\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{x}+b^2\cos(2a)\text{Ci}(2bx)\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}-b^2$$

```
[Out] -1/2*(c*sin(b*x+a)^3)^(2/3)/x^2-b*cot(b*x+a)*(c*sin(b*x+a)^3)^(2/3)/x+b^2*C
i(2*b*x)*cos(2*a)*csc(b*x+a)^2*(c*sin(b*x+a)^3)^(2/3)-b^2*csc(b*x+a)^2*Si(2
*b*x)*sin(2*a)*(c*sin(b*x+a)^3)^(2/3)
```

Rubi [A]
time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6852, 3395, 29, 3393, 3384, 3380, 3383}

$$b^2\cos(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}-b^2\sin(2a)\text{Si}(2bx)\csc^2(a+bx)\,(c\sin^3(a+bx))^{2/3}-\frac{(c\sin^3(a+bx))^{2/3}}{2x^2}-\frac{b\cot(a+bx)\,(c\sin^3(a+bx))^{2/3}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*(c*Sin[a + b*x]^3)^(2/3)/x^2 - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3
))/x + b^2*Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2
/3) - b^2*Csc[a + b*x]^2*Sin[2*a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*
x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x^3}\,dx$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\csc^2(a+bx)\left(c\sin^3\right.\right.$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b^2\csc^2(a+bx)\log(x)\,($$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\csc^2(a+bx)\left(c\sin^3\right.\right.$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\cos(2a)\csc^2(a+bx\right.$$

$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b^2\cos(2a)\mathrm{Ci}(2bx)\csc^2($$

Mathematica [A]

time = 0.14, size = 85, normalized size = 0.71

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-1+\cos(2(a+bx))+4b^2x^2\cos(2a)\mathrm{Ci}(2bx)-2bx\sin(2(a+bx))-4b^2x^2\sin(2a)\mathrm{Si}(2bx)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 4*b^2*x^2
*Cos[2*a]*CosIntegral[2*b*x] - 2*b*x*Sin[2*(a + b*x)] - 4*b^2*x^2*Sin[2*a]*
SinIntegral[2*b*x]))/(4*x^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 238, normalized size = 2.00

method	result
risch	$-\frac{\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} b^2\left(\frac{1}{2x^2b^2}-\frac{i}{bx}-2\,e^{2ibx}\,\mathrm{expIntegral}(1,2ibx)\right)}{4\left(e^{2i(bx+a)}-1\right)^2}-\frac{b^2\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}\left(\frac{e^{4i(bx+a)}}{2x^2b^2}+\right.}{4\left(e^{2i(bx\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))
-1)^2*b^2*(1/2/x^2/b^2-I/b/x-2*exp(2*I*b*x)*Ei(1,2*I*b*x))-1/4*b^2/(exp(2*I
*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)*(1/2/x^
2/b^2*exp(4*I*(b*x+a))+I/x/b*exp(4*I*(b*x+a))-2*Ei(1,-2*I*b*x)*exp(2*I*(b*x
+2*a)))+1/4/x^2/(exp(2*I*(b*x+a))-1)^2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I
*(b*x+a)))^(2/3)*exp(2*I*(b*x+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.63, size = 296, normalized size = 2.49

$$-\frac{\left(\left(\left(-i\sqrt{3}+1\right)E_3(2ibx)+\left(i\sqrt{3}+1\right)E_3(-2ibx)\right)\cos(2a)^3-\left(\left(\sqrt{3}+i\right)E_3(2ibx)+\left(\sqrt{3}-i\right)E_3(-2ibx)\right)\sin(2a)^3+\left(\left(\left(-i\sqrt{3}+1\right)E_3(2ibx)+\left(i\sqrt{3}+1\right)E_3(-2ibx)\right)\cos(2a)-2\right)\sin(2a)^2+\left(\left(i\sqrt{3}+1\right)E_3(2ibx)+\left(-i\sqrt{3}+1\right)E_3(-2ibx)\right)\cos(2a)-2\cos(2a)^2-\left(\left(\left(\sqrt{3}+i\right)E_3(2ibx)+\left(\sqrt{3}-i\right)E_3(-2ibx)\right)\cos(2a)^2-\left(\sqrt{3}-i\right)E_3(2ibx)-\left(\sqrt{3}+i\right)E_3(-2ibx)\right)\sin(2a)\right)b^2c^{\frac{2}{3}}}{16\left(a^2\cos(2a)^2+a^2\sin(2a)^2+(bx+a)^2\left(\cos(2a)^2+\sin(2a)^2\right)-2\left(a\cos(2a)^2+a\sin(2a)^2\right)(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(((-I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)*exp_i
ntegral_e(3, -2*I*b*x))*cos(2*a)^3 - ((sqrt(3) + I)*exp_integral_e(3, 2*I*b
*x) + (sqrt(3) - I)*exp_integral_e(3, -2*I*b*x))*sin(2*a)^3 + (((-I*sqrt(3)
 + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)*exp_integral_e(3, -2*I*b
*x))*cos(2*a) - 2)*sin(2*a)^2 + ((I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x)
 + (-I*sqrt(3) + 1)*exp_integral_e(3, -2*I*b*x))*cos(2*a) - 2*cos(2*a)^2 -
(((sqrt(3) + I)*exp_integral_e(3, 2*I*b*x) + (sqrt(3) - I)*exp_integral_e(3
, -2*I*b*x))*cos(2*a)^2 - (sqrt(3) - I)*exp_integral_e(3, 2*I*b*x) - (sqrt(
```

```
3) + I)*exp_integral_e(3, -2*I*b*x))*sin(2*a))*b^2*c^(2/3)/(a^2*cos(2*a)^2
+ a^2*sin(2*a)^2 + (b*x + a)^2*(cos(2*a)^2 + sin(2*a)^2) - 2*(a*cos(2*a)^2
+ a*sin(2*a)^2)*(b*x + a))
```

Fricas [A]
time = 0.39, size = 142, normalized size = 1.19

$$\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}b^2x^2\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx\right)+2\cdot 4^{\frac{1}{3}}bx\cos\left(bx+a\right)\sin\left(bx+a\right)-4^{\frac{1}{3}}\cos\left(bx+a\right)^2-\left(4^{\frac{1}{3}}b^2x^2\,\mathrm{Ci}\left(2\,bx\right)+4^{\frac{1}{3}}b^2x^2\,\mathrm{Ci}\left(-2\,bx\right)\right)\cos\left(2\,a\right)+4^{\frac{1}{3}}\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{8\left(x^2\cos\left(bx+a\right)^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*4^(2/3)*(2*4^(1/3)*b^2*x^2*sin(2*a)*sin_integral(2*b*x) + 2*4^(1/3)*b*x
*cos(b*x + a)*sin(b*x + a) - 4^(1/3)*cos(b*x + a)^2 - (4^(1/3)*b^2*x^2*cos_
integral(2*b*x) + 4^(1/3)*b^2*x^2*cos_integral(-2*b*x))*cos(2*a) + 4^(1/3))
*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(x^2*cos(b*x + a)^2 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin^3\left(a+bx\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^{2/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^(2/3)/x^3,x)
```

```
[Out] int((c*sin(a + b*x)^3)^(2/3)/x^3, x)
```

3.342 $\int x^m(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal. Leaf size=209

$$\frac{x^{1+m}\csc^2(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{2(1+m)}+2^{-\frac{7}{2}-\frac{m}{2}}e^{2ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},-2ibx^2\right)$$

```
[Out] 1/2*x^(1+m)*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/(1+m)+2^(-7/2-1/2*m)*ex
p(2*I*a)*x^(1+m)*(-I*b*x^2)^(-1/2-1/2*m)*csc(b*x^2+a)^2*GAMMA(1/2+1/2*m,-2*
I*b*x^2)*(c*sin(b*x^2+a)^3)^(2/3)+2^(-7/2-1/2*m)*x^(1+m)*(I*b*x^2)^(-1/2-1/
2*m)*csc(b*x^2+a)^2*GAMMA(1/2+1/2*m,2*I*b*x^2)*(c*sin(b*x^2+a)^3)^(2/3)/exp
(2*I*a)
```

Rubi [A]

time = 0.22, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3484, 3471, 2250}

$$e^{2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\text{Gamma}\left(\frac{m+1}{2},-2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+e^{-2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\text{Gamma}\left(\frac{m+1}{2},2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{x^{m+1}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(2*(1 + m)) + 2^(-7
/2 - m/2)*E^((2*I)*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*
Gamma[(1 + m)/2, (-2*I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3) + (2^(-7/2 - m/2)
*x^(1 + m)*(I*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*Gamma[(1 + m)/2, (2*I)*b
*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/E^((2*I)*a)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^m\sin^2\left(a+bx^2\right)\,dx\\ &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^2\right)\right)dx\\ &= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)}-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right.\right.\\ &= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)}-\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right.\right.\\ &= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)}+2^{-\frac{7}{2}-\frac{m}{2}}e^{2ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\text{cs}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 189, normalized size = 0.90

$$\frac{2^{\frac{1}{2}(-7-m)}x^{1+m}(b^2x^4)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx^2\right)\left(2^{\frac{5+m}{2}}(b^2x^4)^{\frac{1+m}{2}}+(1+m)\left(-ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},2ibx^2\right)(\cos(2a)-i\sin(2a))+(1+m)\left(ibx^2\right)^{\frac{1+m}{2}}\Gamma\left(\frac{1+m}{2},-2ibx^2\right)(\cos(2a)+i\sin(2a))\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (2^((-7 - m)/2)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]^2*(2^((5 +
m)/2)*(b^2*x^4)^((1 + m)/2) + (1 + m)*((-I)*b*x^2)^((1 + m)/2)*Gamma[(1 + m
)/2, (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) + (1 + m)*(I*b*x^2)^((1 + m)/2)*G
amma[(1 + m)/2, (-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a]))*(c*Sin[a + b*x^2]^3)
^(2/3))/(1 + m)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^m\left(c\left(\sin^3\left(b\,x^2+a\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^2 + 2*a), x))*c^(2/3)/(m + 1)
```

Fricas [A]
time = 0.11, size = 130, normalized size = 0.62

$$-\frac{\left(8\,bxx^{m}-(i\,m+i)e^{\left(-\frac{1}{2}\,(m-1)\log(2i\,b)-2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2i\,bx^{2}\right)-(-i\,m-i)e^{\left(-\frac{1}{2}\,(m-1)\log(-2i\,b)+2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-2i\,bx^{2}\right)\right)\left(-\left(c\cos\left(bx^{2}+a\right)^{2}-c\right)\sin\left(bx^{2}+a\right)\right)^{\frac{2}{3}}}{16\left((bm+b)\cos\left(bx^{2}+a\right)^{2}-bm-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(8*b*x*x^m - (I*m + I)*e^(-1/2*(m - 1)*log(2*I*b) - 2*I*a)*gamma(1/2*
m + 1/2, 2*I*b*x^2) - (-I*m - I)*e^(-1/2*(m - 1)*log(-2*I*b) + 2*I*a)*gamma
(1/2*m + 1/2, -2*I*b*x^2))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)
/((b*m + b)*cos(b*x^2 + a)^2 - b*m - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\left(c\sin^{3}\left(a+bx^{2}\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a + b*x^2)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.343 $\int x^3(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal. Leaf size=91

$$\frac{(c\sin^3(a+bx^2))^{2/3}}{8b^2}-\frac{x^2\cot(a+bx^2)\,(c\sin^3(a+bx^2))^{2/3}}{4b}+\frac{1}{8}x^4\csc^2(a+bx^2)\,\left(c\sin^3(a+bx^2)\right)^{2/3}$$

```
[Out] 1/8*(c*sin(b*x^2+a)^3)^(2/3)/b^2-1/4*x^2*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(2
/3)/b+1/8*x^4*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3460, 3391, 30}

$$\frac{(c\sin^3(a+bx^2))^{2/3}}{8b^2}-\frac{x^2\cot(a+bx^2)\,(c\sin^3(a+bx^2))^{2/3}}{4b}+\frac{1}{8}x^4\csc^2(a+bx^2)\,\left(c\sin^3(a+bx^2)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (c*Sin[a + b*x^2]^3)^(2/3)/(8*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(2/3))/(4*b) + (x^4*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^3 \sin^2\left(a+bx^2\right) dx \\
&= \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int x\sin^2(a+bx)\,dx, x, x^2\right) \\
&= \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2} - \frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b} + \frac{1}{4}\Big(\csc^2\left(a+bx\right. \\
&= \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2} - \frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b} + \frac{1}{8}x^4\csc^2\left(a+b\right.
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 67, normalized size = 0.74

$$-\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(\cos\left(2\left(a+bx^2\right)\right)+2bx^2\left(-bx^2+\sin\left(2\left(a+bx^2\right)\right)\right)\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] -1/16*(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(Cos[2*(a + b*x^2)] + 2*
b*x^2*(-(b*x^2) + Sin[2*(a + b*x^2)])))/b^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 200, normalized size = 2.20

method	result
risch	$-\frac{x^4\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}} e^{2i\left(b\,x^2+a\right)}}{8\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2} - \frac{i(2b\,x^2+i)\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}} e^{4i\left(b\,x^2+a\right)}}{32b^2\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2} + \frac{i\Big(ic\Big(\mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b
*x^2+a)))^(2/3)*exp(2*I*(b*x^2+a))-1/32*I/b^2*(2*b*x^2+I)/(exp(2*I*(b*x^2+a
```

```
))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*exp(4*I*(b
*x^2+a))+1/32*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(e
xp(2*I*(b*x^2+a))-1)^2*(2*b*x^2-I)/b^2
```

Maxima [A]
time = 0.57, size = 47, normalized size = 0.52

$$-\frac{(2\,b^2x^4-2\,bx^2\sin{(2\,bx^2+2\,a)}-\cos{(2\,bx^2+2\,a)})c^{\frac{2}{3}}}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^2*x^4 - 2*b*x^2*sin(2*b*x^2 + 2*a) - cos(2*b*x^2 + 2*a))*c^(2/3)
/b^2
```

Fricas [A]
time = 0.36, size = 96, normalized size = 1.05

$$-\frac{\left(2\,b^2x^4-4\,bx^2\cos{(bx^2+a)}\sin{(bx^2+a)}-2\,\cos{(bx^2+a)}^2+1\right)\left(-\left(c\cos{(bx^2+a)}^2-c\right)\sin{(bx^2+a)}\right)^{\frac{2}{3}}}{16\,\left(b^2\cos{(bx^2+a)}^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^2*x^4 - 4*b*x^2*cos(b*x^2 + a)*sin(b*x^2 + a) - 2*cos(b*x^2 + a)
^2 + 1)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a
)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c\sin^3{\left(a+bx^2\right)}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x**3*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(c \sin\left(b x^2+a\right)^3\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a + b*x^2)^3)^(2/3),x)
[Out] int(x^3*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.344 $\int x^2(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal. Leaf size=195

$$\frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{\sqrt{\pi}\,\cos(2a)\csc^2\left(a+bx^2\right)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}+\frac{\sqrt{\pi}\,\mathrm{cs}}{}$$

```
[Out] 1/6*x^3*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)-1/8*x*csc(b*x^2+a)^2*(c*sin
(b*x^2+a)^3)^(2/3)*sin(2*b*x^2+2*a)/b+1/16*cos(2*a)*csc(b*x^2+a)^2*FresnelS
(2*x*b^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)*Pi^(1/2)/b^(3/2)+1/16*csc(b
*x^2+a)^2*FresnelC(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^(2/3)*
Pi^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$,
Rules used = {6852, 3484, 3467, 3434, 3433, 3432}

$$\frac{\sqrt{\pi}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}+\frac{\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}-\frac{x\sin\left(2a+2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b}+\frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x^3*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/6 + (Sqrt[Pi]*Cos[2*a]*Cs
c[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))
/(16*b^(3/2)) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]
*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(16*b^(3/2)) - (x*Csc[a + b*x^2]^2*(c
*Sin[a + b*x^2]^3)^(2/3)*Sin[2*a + 2*b*x^2])/(8*b)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^2\sin^2\left(a+bx^2\right)\,dx\\
&= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+2bx^2\right)\right)dx\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\Bigl(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b}\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b}\\
&= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)S\Bigl(\frac{2\sqrt{b}}{\sqrt{\pi}}}{16b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 113, normalized size = 0.58

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(3\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+3\sqrt{\pi}C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)+2\sqrt{b}x\left(4bx^2-3\sin\left(2\left(a+bx^2\right)\right)\right)\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(3*Sqrt[Pi]*Cos[2*a]*FresnelS[
(2*Sqrt[b]*x)/Sqrt[Pi]] + 3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2
*a] + 2*Sqrt[b]*x*(4*b*x^2 - 3*Sin[2*(a + b*x^2)])))/(48*b^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 309, normalized size = 1.58

method	result
risch	$\frac{ix\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}}}{16b\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}-\frac{i\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}} e^{2ix^2b}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{64\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2 b\sqrt{ib}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*
(b*x^2+a)))^(2/3)-1/64*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))
^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)/b*Pi^(1/2)*2^(1/2)/(I*b)^(1/
2)*erf(2^(1/2)*(I*b)^(1/2)*x)+1/4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b
*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*(-1/4*I*x/b*exp(4*I*(b*x^2+a))+1/8
*I/b*Pi^(1/2)/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))-1/
6*x^3/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^
2+a)))^(2/3)*exp(2*I*(b*x^2+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 99, normalized size = 0.51

$$-\frac{3\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i+1\right)\,\cos\left(2\,a\right)-\left(i-1\right)\,\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}\,x\right)+\left(-\left(i-1\right)\,\cos\left(2\,a\right)+\left(i+1\right)\,\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{-2i\,b}\,x\right)\right)b^{\frac{3}{2}}c^{\frac{2}{3}}+16\left(4\,b^3x^3-3\,b^2x\sin\left(2\,bx^2+2\,a\right)\right)c^{\frac{2}{3}}}{768\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/768*(3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*e
rf(sqrt(2*I*b)*x) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)
*x))*b^(3/2)*c^(2/3) + 16*(4*b^3*x^3 - 3*b^2*x*sin(2*b*x^2 + 2*a))*c^(2/3))
/b^3
```

Fricas [A]
time = 0.39, size = 146, normalized size = 0.75

$$-\frac{4^{\frac{2}{3}}\left(8\cdot 4^{\frac{1}{3}}b^2x^3-12\cdot 4^{\frac{1}{3}}bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)+3\cdot 4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+3\cdot 4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{192\left(b^2\cos\left(bx^2+a\right)^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/192*4^(2/3)*(8*4^(1/3)*b^2*x^3 - 12*4^(1/3)*b*x*cos(b*x^2 + a)*sin(b*x^2
 + a) + 3*4^(1/3)*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi)) + 3*4^
(1/3)*pi*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))*sin(2*a))*(-(c*cos(b*x^2 +
a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c \sin^3 \left(a + bx^2\right)\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c \sin\left(b x^2 + a\right)^3\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a + b*x^2)^3)^(2/3),x)
```

```
[Out] int(x^2*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.345 $\int x(c\sin^3(a+bx^2))^{2/3}\,dx$

Optimal. Leaf size=65

$$-\frac{\cot(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{4b}+\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[Out] -1/4*cot(b*x^2+a)*(c*sin(b*x^2+a)^3)^(2/3)/b+1/4*x^2*csc(b*x^2+a)^2*(c*sin(
b*x^2+a)^3)^(2/3)
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6847, 3286, 2715, 8}

$$\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\cot(a+bx^2)\left(c\sin^3(a+bx^2)\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] -1/4*(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/b + (x^2*Csc[a + b*x^2]^2*
(c*Sin[a + b*x^2]^3)^(2/3))/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int\left(c\sin^3(a+bx)\right)^{2/3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int\sin^2(a+bx)\,dx,x,x^2\right)\\
&= -\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{S}\\
&= -\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.85

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2(a+bx^2)-\sin\left(2(a+bx^2)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(2*(a + b*x^2) - Sin[2*(a + b*
x^2)]))/(8*b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 182, normalized size = 2.80

method	result
risch	$-\frac{x^2\left(ic\left(e^{2i(bx^2+a)}-1\right)^3e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}e^{2i(bx^2+a)}}{4\left(e^{2i(bx^2+a)}-1\right)^2}-\frac{i\left(ic\left(e^{2i(bx^2+a)}-1\right)^3e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}e^{4i(bx^2+a)}}{16b\left(e^{2i(bx^2+a)}-1\right)^2}+\frac{i\left(ic\left(e^{2i(bx^2+a}\right.\right.}{16\left(e^{2i(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b
*x^2+a)))^(2/3)*exp(2*I*(b*x^2+a))-1/16*I/b/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(
```

```
exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*exp(4*I*(b*x^2+a))+1/16*
I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+
a))-1)^2/b
```

Maxima [**A**]
time = 0.57, size = 28, normalized size = 0.43

$$-\frac{(2\,bx^2 - \sin{(2\,bx^2 + 2\,a)})c^{\frac{2}{3}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/16*(2*b*x^2 - sin(2*b*x^2 + 2*a))*c^(2/3)/b
```

Fricas [**A**]
time = 0.35, size = 72, normalized size = 1.11

$$-\frac{(bx^2 - \cos{(bx^2 + a)}\sin{(bx^2 + a)})\Big(-\Big(c\cos{(bx^2 + a)}^2 - c\Big)\sin{(bx^2 + a)}\Big)^{\frac{2}{3}}}{4\,\Big(b\cos{(bx^2 + a)}^2 - b\Big)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^2 - cos(b*x^2 + a)*sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin
(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\Big(c\sin^3{(a + bx^2)}\Big)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a + b*x^2)^3)^(2/3),x)
```

```
[Out] int(x*(c*sin(a + b*x^2)^3)^(2/3), x)
```

3.346 $\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx$

Optimal. Leaf size=148

$$\frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\sqrt{\pi}\,\cos(2a)\csc^2\left(a+bx^2\right)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{\sqrt{\pi}\,\mathrm{cs}}{}$$

```
[Out] 1/2*x*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)-1/4*cos(2*a)*csc(b*x^2+a)^2*F
resnelC(2*x*b^(1/2)/Pi^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)*Pi^(1/2)/b^(1/2)+1/4
*csc(b*x^2+a)^2*FresnelS(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^
(2/3)*Pi^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6852, 3438, 3435, 3433, 3432}

$$-\frac{\sqrt{\pi}\,\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{\sqrt{\pi}\,\sin(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/2 - (Sqrt[Pi]*Cos[2*a]*Csc[
a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))/(
4*Sqrt[b]) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Si
n[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx = \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin^2\left(a+bx^2\right)\,dx$$

$$= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(2a+2bx^2\right)\right)\,dx$$

$$= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right.$$

$$= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+b\right.\right.\right.$$

$$= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)}{4\sqrt{b}}$$

Mathematica [A]
time = 0.07, size = 93, normalized size = 0.63

$$\frac{\csc^2\left(a+bx^2\right)\left(2\sqrt{b}\,x-\sqrt{\pi}\,\cos(2a)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)+\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\sin(2a)\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(2*Sqrt[b]*x - Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/S
qrt[Pi]] + Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b
*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 224, normalized size = 1.51

method	result
risch	$\frac{\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}} e^{2ix^2b}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{16\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2\sqrt{ib}}+\frac{\operatorname{erf}\left(\sqrt{-2ib}\,x\right)\sqrt{\pi}\,\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3}{8\sqrt{-2ib}\,\left(e^{2i\left(b\,x^2+a\right)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2*exp(2*I*b*x^2)*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*erf(2^(1/2)*(I*b)^(
1/2)*x)+1/8*erf((-2*I*b)^(1/2)*x)/(-2*I*b)^(1/2)*Pi^(1/2)/(exp(2*I*(b*x^2+a
))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*exp(2*I*(b
*x^2+2*a))-1/2*x/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp
(-3*I*(b*x^2+a)))^(2/3)*exp(2*I*(b*x^2+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.62, size = 76, normalized size = 0.51

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,\left(\left(\left(i-1\right)\,\cos\left(2\,a\right)+\left(i+1\right)\,\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}\,x\right)+\left(-\left(i+1\right)\,\cos\left(2\,a\right)-\left(i-1\right)\,\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{-2i\,b}\,x\right)\right)b^{\frac{3}{2}}c^{\frac{2}{3}}+16\,b^2c^{\frac{2}{3}}x}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/64*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(
sqrt(2*I*b)*x) + (-(I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(-2*I*b)*x)
)*b^(3/2)*c^(2/3) + 16*b^2*c^(2/3)*x)/b^2
```

Fricas [A]
time = 0.38, size = 114, normalized size = 0.77

$$\frac{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)-2\cdot 4^{\frac{1}{3}}bx\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{16\,\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/16*4^(2/3)*(4^(1/3)*pi*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x*sqrt(b/pi)) -
4^(1/3)*pi*sqrt(b/pi)*fresnel_sin(2*x*sqrt(b/pi))*sin(2*a) - 2*4^(1/3)*b*x)
*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(b\,x^2+a\right)}^3\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(2/3),x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(2/3), x)
```

3.347 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{4}\cos(2a)\text{Ci}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{4}\text{csc}$$

```
[Out] -1/4*Ci(2*b*x^2)*cos(2*a)*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)+1/2*csc(b
*x^2+a)^2*ln(x)*(c*sin(b*x^2+a)^3)^(2/3)+1/4*csc(b*x^2+a)^2*Si(2*b*x^2)*sin
(2*a)*(c*sin(b*x^2+a)^3)^(2/3)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6852, 3484, 3459, 3457, 3456}

$$-\frac{1}{4}\cos(2a)\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}\log(x)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

```
[Out] -1/4*(Cos[2*a]*CosIntegral[2*b*x^2]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(
2/3)) + (Csc[a + b*x^2]^2*Log[x]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (Csc[a + b
*x^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/4
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x}\,dx\\ &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2x}-\frac{\cos\left(2a+2bx^2\right)}{2x}\right)dx\\ &= \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right.\right.\\ &= \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(\right.\right.\right.\\ &= -\frac{1}{4}\cos(2a)\mathrm{Ci}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}\csc^2\left(a+bx^2\right)\log(x\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 60, normalized size = 0.52

$$\frac{1}{4}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\cos(2a)\mathrm{Ci}\left(2bx^2\right)+2\log(x)+\sin(2a)\mathrm{Si}\left(2bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x^
2]) + 2*Log[x] + Sin[2*a]*SinIntegral[2*b*x^2]))/4
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 331, normalized size = 2.88

method	result
risch	$\frac{i\left(ic\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\mathrm{e}^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}}\mathrm{e}^{2ix^2b}\pi\,\mathrm{csgn}\left(b\,x^2\right)}{8\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2}-\frac{i\left(ic\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^3\mathrm{e}^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}}\mathrm{e}^{2ix^2b}\,\mathrm{sinIntegral}\left(2b\,x^2\right)}{4\left(\mathrm{e}^{2i\left(b\,x^2+a\right)}-1\right)^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*
x^2+a))-1)^2*exp(2*I*b*x^2)*Pi*csgn(b*x^2)-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1
)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*Si(2
*b*x^2)-1/8*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2
*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*Ei(1,-2*I*b*x^2)-1/8*Ei(1,-2*I*b*x^2)/(ex
p(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2
/3)*exp(2*I*(b*x^2+2*a))-1/2*ln(x)/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(
b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*exp(2*I*(b*x^2+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.66, size = 55, normalized size = 0.48

$$\frac{1}{16}\left(\left(\mathrm{Ei}\left(2i\,bx^2\right)+\mathrm{Ei}\left(-2i\,bx^2\right)\right)\cos\left(2\,a\right)-\left(-i\,\mathrm{Ei}\left(2i\,bx^2\right)+i\,\mathrm{Ei}\left(-2i\,bx^2\right)\right)\sin\left(2\,a\right)-4\,\log\left(x\right)\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/16*((Ei(2*I*b*x^2) + Ei(-2*I*b*x^2))*cos(2*a) - (-I*Ei(2*I*b*x^2) + I*Ei(
-2*I*b*x^2))*sin(2*a) - 4*log(x))*c^(2/3)
```

Fricas [A]
time = 0.38, size = 100, normalized size = 0.87

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx^2\right)-\left(4^{\frac{1}{3}}\,\mathrm{Ci}\left(2\,bx^2\right)+4^{\frac{1}{3}}\,\mathrm{Ci}\left(-2\,bx^2\right)\right)\cos\left(2\,a\right)+4\cdot 4^{\frac{1}{3}}\log\left(x\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{32\left(\cos\left(bx^2+a\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/32*4^(2/3)*(2*4^(1/3)*sin(2*a)*sin_integral(2*b*x^2) - (4^(1/3)*cos_inte
gral(2*b*x^2) + 4^(1/3)*cos_integral(-2*b*x^2))*cos(2*a) + 4*4^(1/3)*log(x)
)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(cos(b*x^2 + a)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(2/3)/x,x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(2/3)/x, x)
```

3.348 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=132

$$-\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\sqrt{b}\,\sqrt{\pi}\,\cos(2a)\csc^2\left(a+bx^2\right)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\sqrt{b}\,\sqrt{\pi}\,\csc^2\left(a+\right.$$

```
[Out] -(c*sin(b*x^2+a)^3)^(2/3)/x+cos(2*a)*csc(b*x^2+a)^2*FresnelS(2*x*b^(1/2)/Pi
^(1/2))*(c*sin(b*x^2+a)^3)^(2/3)*b^(1/2)*Pi^(1/2)+csc(b*x^2+a)^2*FresnelC(2
*x*b^(1/2)/Pi^(1/2))*sin(2*a)*(c*sin(b*x^2+a)^3)^(2/3)*b^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6852, 3474, 4669, 3454, 3434, 3433, 3432}

$$\sqrt{\pi}\,\sqrt{b}\,\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\sqrt{\pi}\,\sqrt{b}\,\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x^2]^3)^(2/3)/x) + Sqrt[b]*Sqrt[Pi]*Cos[2*a]*Csc[a + b*x^2]^
2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3) + Sqrt[b]*Sqr
t[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a]*(c*Sin[a +
 b*x^2]^3)^(2/3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3454

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
```

```
[u, x]
```

Rule 3474

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[x^(m +
1)*(Sin[a + b*x^n]^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x^2}\,dx\\ &= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\left(4b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \cos\left(a+bx^2\right.\\ &= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\left(2b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(2(a+b\right.\\ &= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\left(2b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(2a+2b\right.\\ &= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\left(2b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(\right.\\ &= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}+\sqrt{b}\,\sqrt{\pi}\,\cos(2a)\csc^2\left(a+bx^2\right)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.14, size = 107, normalized size = 0.81

$$\frac{\csc^2\left(a+bx^2\right)\left(-1+\cos\left(2\left(a+bx^2\right)\right)+2\sqrt{b}\,\sqrt{\pi}\,x\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)+2\sqrt{b}\,\sqrt{\pi}\,xC\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\sin(2a)\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x^2]^2*(-1 + Cos[2*(a + b*x^2)] + 2*Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]*
FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] + 2*Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[b]
*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b*x^2]^3)^(2/3))/(2*x)
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 301, normalized size = 2.28

method	result
risch	$-\frac{\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}}}{4x\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}-\frac{i\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}} e^{2ix^2b}b\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{4\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2\sqrt{ib}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x
^2+a)))^(2/3)-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3
)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*b*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*er
f(2^(1/2)*(I*b)^(1/2)*x)+1/4/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+
a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*(-1/x*exp(4*I*(b*x^2+a))+2*I*b*Pi^(1/2)
/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))+1/2/x/(exp(2*I*
(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*ex
p(2*I*(b*x^2+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.62, size = 90, normalized size = 0.68

$$\frac{\sqrt{2}\,\sqrt{bx^2}\left(\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},2i\,bx^2\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-2i\,bx^2\right)\right)\cos\left(2\,a\right)+\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},2i\,bx^2\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-2i\,bx^2\right)\right)\sin\left(2\,a\right)\right)c^{\frac{2}{3}}+8\,c^{\frac{2}{3}}}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(2)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) + (I -
1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*cos(2*a) + ((I - 1)*sqrt(2)*gamma(-1/2,
 2*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*sin(2*a))*c^(2/3) +
8*c^(2/3))/x
```

Fricas [A]
time = 0.36, size = 127, normalized size = 0.96

$$-\frac{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\pi x\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+4^{\frac{1}{3}}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)+4^{\frac{1}{3}}\cos\left(bx^2+a\right)^2-4^{\frac{1}{3}}\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{4\left(x\cos\left(bx^2+a\right)^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*4^(2/3)*(4^(1/3)*pi*x*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi))
+ 4^(1/3)*pi*x*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))*sin(2*a) + 4^(1/3)*co
s(b*x^2 + a)^2 - 4^(1/3))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/
(x*cos(b*x^2 + a)^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(b\,x^2+a\right)}^3\right)}^{2/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(2/3)/x^2,x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(2/3)/x^2, x)
```

3.349 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2\left(a+bx^2\right)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{1}{2}b\text{Ci}\left(2bx^2\right)\csc^2\left(\right.$$

```
[Out] -1/4*csc(b*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/x^2+1/4*cos(2*b*x^2+2*a)*csc(b
*x^2+a)^2*(c*sin(b*x^2+a)^3)^(2/3)/x^2+1/2*b*cos(2*a)*csc(b*x^2+a)^2*Si(2*b
*x^2)*(c*sin(b*x^2+a)^3)^(2/3)+1/2*b*Ci(2*b*x^2)*csc(b*x^2+a)^2*sin(2*a)*(c
*sin(b*x^2+a)^3)^(2/3)
```

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6852, 3484, 3461, 3378, 3384, 3380, 3383}

$$\frac{1}{2}b\sin(2a)\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2\left(a+bx^2\right)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

```
[Out] -1/4*(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/x^2 + (Cos[2*(a + b*x^2)
]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(4*x^2) + (b*CosIntegral[2*b
*x^2]*Csc[a + b*x^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (b*Cos[2*a]
*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3484

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x^3}\,dx$$
$$= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2x^3}-\frac{\cos\left(2a+2bx^2\right)}{2x^3}\right)dx$$
$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)$$
$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}-\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)$$
$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a\right.\right.}{4x^2}$$
$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a\right.\right.}{4x^2}$$
$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2}+\frac{\cos\left(2(a+bx^2)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a\right.\right.}{4x^2}$$

Mathematica [A]
time = 0.10, size = 79, normalized size = 0.49

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-1+\cos\left(2(a+bx^2)\right)+2bx^2\text{Ci}(2bx^2)\sin(2a)+2bx^2\cos(2a)\text{Si}(2bx^2)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-1 + Cos[2*(a + b*x^2)] + 2*b
*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + 2*b*x^2*Cos[2*a]*SinIntegral[2*b*x^2])
)/(4*x^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 277, normalized size = 1.72

method	result
risch	$-\frac{\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}}}{8x^2\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}+\frac{i\left(ic\left(e^{2i\left(b\,x^2+a\right)}-1\right)^3 e^{-3i\left(b\,x^2+a\right)}\right)^{\frac{2}{3}} e^{2ix^2b}b\,\text{expIntegral}(1,2ix^2b)}{4\left(e^{2i\left(b\,x^2+a\right)}-1\right)^2}+\frac{\left(ic\left(e^{2i\left(b\,x\right.\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x^2/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b
*x^2+a)))^(2/3)+1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2
/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*b*Ei(1,2*I*b*x^2)+1/4/(exp(2*I*
(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)*(-
1/2/x^2*exp(4*I*(b*x^2+a))-I*b*Ei(1,-2*I*b*x^2)*exp(2*I*(b*x^2+2*a)))+1/4/x
^2/(exp(2*I*(b*x^2+a))-1)^2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a
)))^(2/3)*exp(2*I*(b*x^2+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.61, size = 64, normalized size = 0.40

$$-\frac{(((i\,\Gamma(-1,2i\,bx^2)-i\,\Gamma(-1,-2i\,bx^2))\cos{(2\,a)}+(\Gamma(-1,2i\,bx^2)+\Gamma(-1,-2i\,bx^2))\sin{(2\,a)})bx^2-1)c^{\frac{2}{3}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((I*gamma(-1, 2*I*b*x^2) - I*gamma(-1, -2*I*b*x^2))*cos(2*a) + (gamma
(-1, 2*I*b*x^2) + gamma(-1, -2*I*b*x^2))*sin(2*a))*b*x^2 - 1)*c^(2/3)/x^2
```

Fricas [A]
time = 0.38, size = 132, normalized size = 0.82

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}bx^2\cos{(2\,a)}\operatorname{Si}{(2\,bx^2)}+2\cdot 4^{\frac{1}{3}}\cos{(bx^2+a)}^2+\left(4^{\frac{1}{3}}bx^2\operatorname{Ci}{(2\,bx^2)}+4^{\frac{1}{3}}bx^2\operatorname{Ci}{(-2\,bx^2)}\right)\sin{(2\,a)}-2\cdot 4^{\frac{1}{3}}\right)\left(-\left(c\cos{(bx^2+a)}^2-c\right)\sin{(bx^2+a)}\right)^{\frac{2}{3}}}{16\left(x^2\cos{(bx^2+a)}^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*4^(2/3)*(2*4^(1/3)*b*x^2*cos(2*a)*sin_integral(2*b*x^2) + 2*4^(1/3)*c
os(b*x^2 + a)^2 + (4^(1/3)*b*x^2*cos_integral(2*b*x^2) + 4^(1/3)*b*x^2*cos_
integral(-2*b*x^2))*sin(2*a) - 2*4^(1/3))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*
x^2 + a))^(2/3)/(x^2*cos(b*x^2 + a)^2 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin^3{(a+bx^2)}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sin\left(b\,x^2+a\right)^3\right)^{2/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^2)^3)^(2/3)/x^3,x)
```

```
[Out] int((c*sin(a + b*x^2)^3)^(2/3)/x^3, x)
```

3.350 $\int x^m (c \sin^3(a+bx^n))^{2/3}\, dx$

Optimal. Leaf size=217

$$\frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc^2(a+bx^n)\,\Gamma\left(\frac{1+m}{n},-2ibx^n\right)(c\sin$$

```
[Out] 1/2*x^(1+m)*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/(1+m)+exp(2*I*a)*x^(1+m
)*csc(a+b*x^n)^2*GAMMA((1+m)/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/(2^((1+
m+2*n)/n))/n/((-I*b*x^n)^((1+m)/n))+x^(1+m)*csc(a+b*x^n)^2*GAMMA((1+m)/n,2*
I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/(2^((1+m+2*n)/n))/exp(2*I*a)/n/((I*b*x^n)
^((1+m)/n))
```

Rubi [A]
time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3506, 3505, 2250}

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{x^{m+1}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(2*(1 + m)) + (E^((
2*I)*a)*x^(1 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (-2*I)*b*x^n]*(c*Sin[a
+ b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1
 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^m \left(c \sin^3\left(a+bx^n\right)\right)^{2/3} dx = \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^m \sin^2\left(a+bx^n\right) dx$$

$$= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^n\right)\right) dx$$

$$= \frac{x^{1+m}\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2(1+m)} - \frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right.\right.\right.$$

$$= \frac{x^{1+m}\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2(1+m)} - \frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right.\right.\right.$$

$$= \frac{x^{1+m}\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2(1+m)} + \frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}\left(-ibx^n\right)^{-\frac{1+m}{n}}\ \mathrm{c}}{}$$

Mathematica [**A**]
time = 0.53, size = 194, normalized size = 0.89

$$\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\csc^2\left(a+bx^n\right)\left(2^{\frac{1+m+n}{n}}e^{2ia}n(b^2x^{2n})^{\frac{1+m}{n}}+e^{4ia}(1+m)\left(ibx^n\right)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)+(1+m)\left(-ibx^n\right)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2ibx^n\right)\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{(1+m)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(2^((1 + m + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n))^
((1 + m)/n) + E^((4*I)*a)*(1 + m)*(I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-
2*I)*b*x^n] + (1 + m)*((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (2*I)*b*x^n
])*(c*Sin[a + b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*(1 + m)*n*(
b^2*x^(2*n))^((1 + m)/n))
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^m \left(c\left(\sin^3\left(a+b\,x^n\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))*c^(2/3)/(m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(a+b*x**n)**3)**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \left(c \sin \left(a + b x^n\right)^3\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x^m*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.351 $\int x^3(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal. Leaf size=188

$$\frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{4^{-1-\frac{2}{n}}e^{2ia}x^4(-ibx^n)^{-4/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{4}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n\right.}{n}$$

```
[Out] 1/8*x^4*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+4^(-1-2/n)*exp(2*I*a)*x^4*c
sc(a+b*x^n)^2*GAMMA(4/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^
(4/n))+4^(-1-2/n)*x^4*csc(a+b*x^n)^2*GAMMA(4/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3
)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(4/n))
```

Rubi [A]
time = 0.23, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3506, 3505, 2250}

$$\frac{e^{2ia}4^{-\frac{2}{n}-1}x^4(-ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{2}{n}-1}x^4(ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n},2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^4*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/8 + (4^(-1 - 2/n)*E^((2*I
)*a)*x^4*Csc[a + b*x^n]^2*Gamma[4/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(4/n)) + (4^(-1 - 2/n)*x^4*Csc[a + b*x^n]^2*Gamma[4/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(4/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^3\sin^2\left(a+bx^n\right)\,dx\\ &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{x^3}{2}-\frac{1}{2}x^3\cos\left(2a+2bx^n\right)\right)\,dx\\ &= \frac{1}{8}x^4\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)\right.\\ &= \frac{1}{8}x^4\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)\right.\\ &= \frac{1}{8}x^4\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}+\frac{4^{-1-\frac{2}{n}}e^{2ia}x^4\left(-ibx^n\right)^{-4/n}\csc^2\left(a+\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 161, normalized size = 0.86

$$\frac{2^{-3-\frac{4}{n}}e^{-2ia}x^4\left(b^2x^{2n}\right)^{-4/n}\csc^2\left(a+bx^n\right)\left(16^{\frac{1}{n}}e^{2ia}n\left(b^2x^{2n}\right)^{4/n}+2e^{4ia}\left(ibx^n\right)^{4/n}\Gamma\left(\frac{4}{n},-2ibx^n\right)+2\left(-ibx^n\right)^{4/n}\Gamma\left(\frac{4}{n},2ibx^n\right)\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-3 - 4/n)*x^4*Csc[a + b*x^n]^2*(16^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(
4/n) + 2*E^((4*I)*a)*(I*b*x^n)^(4/n)*Gamma[4/n, (-2*I)*b*x^n] + 2*((-I)*b*x
^n)^(4/n)*Gamma[4/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)
*n*(b^2*x^(2*n))^(4/n))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^{3}\left(c\left(\sin^{3}\left(a+b\,x^{n}\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/16*(x^4 - 4*integrate(x^3*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(c\sin\left(a+b\,x^n\right)^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x^3*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.352 $\int x^2(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal. Leaf size=188

$$\frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{2^{-2-\frac{3}{n}}e^{2ia}x^3(-ibx^n)^{-3/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{3}{n},-2ibx^n\right)\left(c\sin^3(a+bx\right.}{n}$$

```
[Out] 1/6*x^3*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+2^(-2-3/n)*exp(2*I*a)*x^3*c
sc(a+b*x^n)^2*GAMMA(3/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^
(3/n))+2^(-2-3/n)*x^3*csc(a+b*x^n)^2*GAMMA(3/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3
)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(3/n))
```

Rubi [A]
time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3506, 3505, 2250}

$$\frac{e^{2ia}2^{-\frac{3}{n}-2}x^3(-ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{3}{n}-2}x^3(ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^3*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/6 + (2^(-2 - 3/n)*E^((2*I
)*a)*x^3*Csc[a + b*x^n]^2*Gamma[3/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(3/n)) + (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*Gamma[3/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(3/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx = \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int x^2\sin^2\left(a+bx^n\right)\,dx$$

$$= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+2bx^n\right)\right)\,dx$$

$$= \frac{1}{6}x^3\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2}\right.$$

$$= \frac{1}{6}x^3\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2}\right.$$

$$= \frac{1}{6}x^3\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}+\frac{2^{-2-\frac{3}{n}}e^{2ia}x^3(-ibx^n)^{-3/n}\csc^2\left(a+b\right.}{}$$

Mathematica [A]
time = 0.38, size = 168, normalized size = 0.89

$$\frac{2^{-2-\frac{3}{n}}e^{-2ia}x^3(b^2x^{2n})^{-3/n}\csc^2\left(a+bx^n\right)\left(2^{\frac{3+n}{n}}e^{2ia}n(b^2x^{2n})^{3/n}+3e^{4ia}(ibx^n)^{3/n}\Gamma\left(\frac{3}{n},-2ibx^n\right)+3(-ibx^n)^{3/n}\Gamma\left(\frac{3}{n},2ibx^n\right)\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*(2^((3 + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n
))^(3/n) + 3*E^((4*I)*a)*(I*b*x^n)^(3/n)*Gamma[3/n, (-2*I)*b*x^n] + 3*((-I)
*b*x^n)^(3/n)*Gamma[3/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(3*E^((2
*I)*a)*n*(b^2*x^(2*n))^(3/n))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^2\left(c\left(\sin^3\left(a+b\,x^n\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/12*(x^3 - 3*integrate(x^2*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x^2*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.353 $\int x(c\sin^3(a+bx^n))^{2/3}\,dx$

Optimal. Leaf size=188

$$\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{4^{-1-\frac{1}{n}}e^{2ia}x^2(-ibx^n)^{-2/n}\csc^2(a+bx^n)\,\Gamma\left(\frac{2}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n\right.}{n}$$

```
[Out] 1/4*x^2*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+4^(-1-1/n)*exp(2*I*a)*x^2*c
sc(a+b*x^n)^2*GAMMA(2/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^
(2/n))+4^(-1-1/n)*x^2*csc(a+b*x^n)^2*GAMMA(2/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3
)^(2/3)/exp(2*I*a)/n/((I*b*x^n)^(2/n))
```

Rubi [A]
time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6852, 3506, 3505, 2250}

$$\frac{e^{2ia}4^{-\frac{1}{n}-1}x^2(-ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{1}{n}-1}x^2(ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^2*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/4 + (4^(-1 - n^(-1))*E^((
2*I)*a)*x^2*Csc[a + b*x^n]^2*Gamma[2/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^
(2/3))/(n*((-I)*b*x^n)^(2/n)) + (4^(-1 - n^(-1))*x^2*Csc[a + b*x^n]^2*Gamma
[2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(2/
n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c\sin^3(a+bx^n)\right)^{2/3}\,dx &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x\sin^2(a+bx^n)\,dx\\ &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\left(\frac{x}{2}-\frac{1}{2}x\cos(2a+2bx^n)\right)dx\\ &= \frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{2}\Big(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2}\\ &= \frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{4}\Big(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2}\\ &= \frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{4^{-1-\frac{1}{n}}e^{2ia}x^2(-ibx^n)^{-2/n}\csc^2(a+b}{}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 160, normalized size = 0.85

$$\frac{4^{-\frac{1+n}{n}}e^{-2ia}x^2(b^2x^{2n})^{-2/n}\csc^2(a+bx^n)\left(4^{\frac{1}{n}}e^{2ia}n(b^2x^{2n})^{2/n}+e^{4ia}(ibx^n)^{2/n}\,\Gamma\left(\frac{2}{n},-2ibx^n\right)+(-ibx^n)^{2/n}\,\Gamma\left(\frac{2}{n},2ibx^n\right)\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
[Out] (x^2*Csc[a + b*x^n]^2*(4^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(2/n) + E^((4*I
)*a)*(I*b*x^n)^(2/n)*Gamma[2/n, (-2*I)*b*x^n] + ((-I)*b*x^n)^(2/n)*Gamma[2/
n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4^((1 + n)/n)*E^((2*I)*a)*n*(
b^2*x^(2*n))^(2/n))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x\left(c\left(\sin^3(a+b\,x^n)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
[Out] -1/8*(x^2 - 2*integrate(x*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\sin^{3}\left(a+bx^{n}\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x**n)**3)**(2/3),x)
[Out] Integral(x*(c*sin(a + b*x**n)**3)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a + b*x^n)^3)^(2/3),x)
[Out] int(x*(c*sin(a + b*x^n)^3)^(2/3), x)
```

3.354 $\int \left(c \sin^3\left(a+b x^n\right)\right)^{2/3} \, dx$

Optimal. Leaf size=178

$$\frac{1}{2} x \csc^2\left(a+b x^n\right) \left(c \sin^3\left(a+b x^n\right)\right)^{2/3}+\frac{2^{-2-\frac{1}{n}} e^{2 i a} x \left(-i b x^n\right)^{-1/n} \csc^2\left(a+b x^n\right) \Gamma\left(\frac{1}{n},-2 i b x^n\right) \left(c \sin^3\left(a+b x^n\right)\right.}{n}$$

```
[Out] 1/2*x*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)+2^(-2-1/n)*exp(2*I*a)*x*csc(a
+b*x^n)^2*GAMMA(1/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n/((-I*b*x^n)^(1/n
))+2^(-2-1/n)*x*csc(a+b*x^n)^2*GAMMA(1/n,2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3
)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A]
time = 0.07, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6852, 3448, 3447, 2239}

$$\frac{e^{2 i a} 2^{-\frac{1}{n}-2} x \left(-i b x^n\right)^{-1/n} \text{Gamma}\left(\frac{1}{n},-2 i b x^n\right) \csc^2\left(a+b x^n\right) \left(c \sin^3\left(a+b x^n\right)\right)^{2/3}}{n}+\frac{e^{-2 i a} 2^{-\frac{1}{n}-2} x \left(i b x^n\right)^{-1/n} \text{Gamma}\left(\frac{1}{n},2 i b x^n\right) \csc^2\left(a+b x^n\right) \left(c \sin^3\left(a+b x^n\right)\right)^{2/3}}{n}+\frac{1}{2} x \csc^2\left(a+b x^n\right) \left(c \sin^3\left(a+b x^n\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (2^(-2 - n^(-1))*E^((2*
I)*a)*x*Csc[a + b*x^n]^2*Gamma[n^(-1), (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(n*((-I)*b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*Gamma[n
^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3448

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\,dx &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \sin^2\left(a+bx^n\right)\,dx\\ &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(2a+2bx^n\right)\right)\,dx\\ &= \frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\\ &= \frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\\ &= \frac{1}{2}x\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x\left(-ibx^n\right)^{-1/n}\csc^2\left(a+bx^n\right)\mathrm{I}}{n}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 149, normalized size = 0.84

$$\frac{2^{-2-\frac{1}{n}}e^{-2ia}x\left(b^2x^{2n}\right)^{-1/n}\csc^2\left(a+bx^n\right)\left(2^{1+\frac{1}{n}}e^{2ia}n\left(b^2x^{2n}\right)^{\frac{1}{n}}+e^{4ia}\left(ibx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-2ibx^n\right)+\left(-ibx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},2ibx^n\right)\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*(2^(1 + n^(-1))*E^((2*I)*a)*n*(b^2*x^(2
*n))^n^(-1) + E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-2*I)*b*x^n] + ((
-I)*b*x^n)^n^(-1)*Gamma[n^(-1), (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(
E^((2*I)*a)*n*(b^2*x^(2*n))^n^(-1))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(c\left(\sin^3\left(a+b\,x^n\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3),x)
```

[Out] int((c*sin(a+b*x^n)^3)^(2/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")

[Out] -1/4*c^(2/3)*(x - integrate(cos(2*b*x^n + 2*a), x))

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")

[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin^{3}\left(a+bx^{n}\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(a+b*x**n)**3)**(2/3),x)

[Out] Integral((c*sin(a + b*x**n)**3)**(2/3), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")

[Out] integrate((c*sin(b*x^n + a)^3)^(2/3), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sin(a + b*x^n)^3)^(2/3),x)

[Out] int((c*sin(a + b*x^n)^3)^(2/3), x)

3.355 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx$

Optimal. Leaf size=121

$$-\frac{\cos(2a)\text{Ci}(2bx^n)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}+\frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}+\frac{\csc^2\left(a+\right.}{}$$

```
[Out] -1/2*Ci(2*b*x^n)*cos(2*a)*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/n+1/2*csc
(a+b*x^n)^2*ln(x)*(c*sin(a+b*x^n)^3)^(2/3)+1/2*csc(a+b*x^n)^2*Si(2*b*x^n)*s
in(2*a)*(c*sin(a+b*x^n)^3)^(2/3)/n
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6852, 3506, 3459, 3457, 3456}

$$-\frac{\cos(2a)\text{CosIntegral}(2bx^n)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}+\frac{\sin(2a)\text{Si}(2bx^n)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}+\frac{1}{2}\log(x)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

```
[Out] -1/2*(Cos[2*a]*CosIntegral[2*b*x^n]*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(
2/3))/n + (Csc[a + b*x^n]^2*Log[x]*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (Csc[a +
 b*x^n]^2*Sin[2*a]*(c*Sin[a + b*x^n]^3)^(2/3)*SinIntegral[2*b*x^n])/(2*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x}\,dx &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^n\right)}{x}\,dx\\ &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \left(\frac{1}{2x}-\frac{\cos\left(2a+2bx^n\right)}{2x}\right)dx\\ &= \frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right.\right.\right.\\ &= \frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}-\frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(\right.\right.\right.\\ &= -\frac{\cos(2a)\mathrm{Ci}(2bx^n)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}+\frac{1}{2}\csc^2\left(a+bx^n\right)\log(x)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.52

$$\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\left(-\cos(2a)\mathrm{Ci}(2bx^n)+n\log(x)+\sin(2a)\mathrm{Si}(2bx^n)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x^
n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n]))/(2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 343, normalized size = 2.83

method	result
risch	$\frac{i\left(ic\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^3\mathrm{e}^{-3i(a+b\,x^n)}\right)^{\frac{2}{3}}\mathrm{e}^{2ib\,x^n}\pi\,\mathrm{csgn}(b\,x^n)}{4\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^2n}-\frac{i\left(ic\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^3\mathrm{e}^{-3i(a+b\,x^n)}\right)^{\frac{2}{3}}\mathrm{e}^{2ib\,x^n}\,\mathrm{sinIntegral}(2b\,x^n)}{2\left(\mathrm{e}^{2i(a+b\,x^n)}-1\right)^2n}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(exp(2*I*(a+
b*x^n))-1)^2*exp(2*I*b*x^n)/n*Pi*csgn(b*x^n)-1/2*I*(I*c*(exp(2*I*(a+b*x^n))
-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(exp(2*I*(a+b*x^n))-1)^2*exp(2*I*b*x^n)/n*
Si(2*b*x^n)-1/4*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(e
xp(2*I*(a+b*x^n))-1)^2*exp(2*I*b*x^n)/n*Ei(1,-2*I*b*x^n)-1/4*Ei(1,-2*I*b*x^
n)/n/(exp(2*I*(a+b*x^n))-1)^2*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x
^n)))^(2/3)*exp(2*I*(b*x^n+2*a))-1/2*ln(x)/(exp(2*I*(a+b*x^n))-1)^2*(I*c*(e
xp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)*exp(2*I*(a+b*x^n))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 153, normalized size = 1.26

$$\frac{\left(\left(\left(i\sqrt{3}+1\right)\mathrm{Ei}(2i\,bx^n)+\left(i\sqrt{3}+1\right)\mathrm{Ei}(-2i\,bx^n)+\left(-i\sqrt{3}+1\right)\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\left(-i\sqrt{3}+1\right)\mathrm{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(2a)-4n\log(x)-\left(\left(\sqrt{3}-i\right)\mathrm{Ei}(2i\,bx^n)-\left(\sqrt{3}-i\right)\mathrm{Ei}(-2i\,bx^n)-\left(\sqrt{3}+i\right)\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\sin(2a)\right)c^{\frac{2}{3}}}{16n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/16*(((I*sqrt(3) + 1)*Ei(2*I*b*x^n) + (I*sqrt(3) + 1)*Ei(-2*I*b*x^n) + (-I
*sqrt(3) + 1)*Ei(2*I*b*e^(n*conjugate(log(x)))) + (-I*sqrt(3) + 1)*Ei(-2*I*
b*e^(n*conjugate(log(x)))))*cos(2*a) - 4*n*log(x) - ((sqrt(3) - I)*Ei(2*I*b
*x^n) - (sqrt(3) - I)*Ei(-2*I*b*x^n) - (sqrt(3) + I)*Ei(2*I*b*e^(n*conjugat
e(log(x)))) + (sqrt(3) + I)*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))*c
^(2/3)/n
```

Fricas [A]
time = 0.38, size = 106, normalized size = 0.88

$$\frac{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\cos(2a)\,\mathrm{Ci}(2bx^n)+4^{\frac{1}{3}}\cos(2a)\,\mathrm{Ci}(-2bx^n)-2\cdot 4^{\frac{1}{3}}n\log(x)-2\cdot 4^{\frac{1}{3}}\sin(2a)\,\mathrm{Si}(2bx^n)\right)\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{2}{3}}}{16\left(n\cos(bx^n+a)^2-n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/16*4^(2/3)*(4^(1/3)*cos(2*a)*cos_integral(2*b*x^n) + 4^(1/3)*cos(2*a)*cos
_integral(-2*b*x^n) - 2*4^(1/3)*n*log(x) - 2*4^(1/3)*sin(2*a)*sin_integral(
2*b*x^n))*(-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/(n*cos(b*x^n + a
)^2 - n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sin\left(a+b\,x^n\right)^3\right)^{2/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(2/3)/x,x)
```

```
[Out] int((c*sin(a + b*x^n)^3)^(2/3)/x, x)
```

3.356 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=180

$$-\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2x}+\frac{2^{-2+\frac{1}{n}}e^{2ia}\left(-ibx^n\right)^{\frac{1}{n}}\csc^2\left(a+bx^n\right)\Gamma\left(-\frac{1}{n},-2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx}+$$

```
[Out] -1/2*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/x+2^(-2+1/n)*exp(2*I*a)*(-I*b*
x^n)^(1/n)*csc(a+b*x^n)^2*GAMMA(-1/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)/n
/x+2^(-2+1/n)*(I*b*x^n)^(1/n)*csc(a+b*x^n)^2*GAMMA(-1/n,2*I*b*x^n)*(c*sin(a
+b*x^n)^3)^(2/3)/exp(2*I*a)/n/x
```

Rubi [A]

time = 0.19, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3506, 3505, 2250}

$$\frac{e^{2ia}2^{\frac{1}{n}-2}(-ibx^n)^{\frac{1}{n}}\,\mathrm{Gamma}\left(-\frac{1}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx}+\frac{e^{-2ia}2^{\frac{1}{n}-2}(ibx^n)^{\frac{1}{n}}\,\mathrm{Gamma}\left(-\frac{1}{n},2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx}-\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

```
[Out] -1/2*(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/x + (2^(-2 + n^(-1))*E^(
(2*I)*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]^2*Gamma[-n^(-1), (-2*I)*b*x^n]*
(c*Sin[a + b*x^n]^3)^(2/3))/(n*x) + (2^(-2 + n^(-1))*(I*b*x^n)^n^(-1)*Csc[a
 + b*x^n]^2*Gamma[-n^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*
I)*a)*n*x)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{(c\sin^3(a+bx^n))^{2/3}}{x^2}\,dx = \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\sin^2(a+bx^n)}{x^2}\,dx$$
$$= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \left(\frac{1}{2x^2}-\frac{\cos(2a+2bx^n)}{2x^2}\right)dx$$
$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)$$
$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)$$
$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x}+\frac{2^{-2+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\,\Gamma\left(-\right.}{nx}$$

Mathematica [A]
time = 0.25, size = 125, normalized size = 0.69

$$\frac{e^{-2ia}\csc^2(a+bx^n)\left(-2e^{2ia}n+2^{\frac{1}{n}}e^{4ia}(-ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-2ibx^n\right)+2^{\frac{1}{n}}(ibx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},2ibx^n\right)\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
[Out] (Csc[a + b*x^n]^2*(-2*E^((2*I)*a)*n + 2^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^n^(
-1)*Gamma[-n^(-1), (-2*I)*b*x^n] + 2^n^(-1)*(I*b*x^n)^n^(-1)*Gamma[-n^(-1),
 (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(c(\sin^3(a+b\,x^n))\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
[Out] int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="maxima")
[Out] 1/4*(x*integrate(cos(2*b*x^n + 2*a)/x^2, x) + 1)*c^(2/3)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="fricas")
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x**2,x)
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="giac")
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{2/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(2/3)/x^2,x)
[Out] int((c*sin(a + b*x^n)^3)^(2/3)/x^2, x)
```

3.357 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=184

$$-\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4x^2}+\frac{4^{-1+\frac{1}{n}}e^{2ia}\left(-ibx^n\right)^{2/n}\csc^2\left(a+bx^n\right)\Gamma\left(-\frac{2}{n},-2ibx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx^2}$$

```
[Out] -1/4*csc(a+b*x^n)^2*(c*sin(a+b*x^n)^3)^(2/3)/x^2+4^(-1+1/n)*exp(2*I*a)*(-I*
b*x^n)^(2/n)*csc(a+b*x^n)^2*GAMMA(-2/n,-2*I*b*x^n)*(c*sin(a+b*x^n)^3)^(2/3)
/n/x^2+4^(-1+1/n)*(I*b*x^n)^(2/n)*csc(a+b*x^n)^2*GAMMA(-2/n,2*I*b*x^n)*(c*s
in(a+b*x^n)^3)^(2/3)/exp(2*I*a)/n/x^2
```

Rubi [A]
time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 3506, 3505, 2250}

$$\frac{e^{2ia}4^{\frac{1}{n}-1}\left(-ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx^2}+\frac{e^{-2ia}4^{\frac{1}{n}-1}\left(ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx^2}-\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

```
[Out] -1/4*(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/x^2 + (4^(-1 + n^(-1))*E
^((2*I)*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]^2*Gamma[-2/n, (-2*I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(2/3))/(n*x^2) + (4^(-1 + n^(-1))*(I*b*x^n)^(2/n)*Csc[a
+ b*x^n]^2*Gamma[-2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a
)*n*x^2)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3506

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{x^3}\,dx &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^n\right)}{x^3}\,dx \\ &= \left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right)\int \left(\frac{1}{2x^3}-\frac{\cos\left(2a+2bx^n\right)}{2x^3}\right)dx \\ &= -\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4x^2}-\frac{1}{2}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right) \\ &= -\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4x^2}-\frac{1}{4}\left(\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}\right) \\ &= -\frac{\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4x^2}+\frac{4^{-1+\frac{1}{n}}e^{2ia}\left(-ibx^n\right)^{2/n}\csc^2\left(a+bx^n\right)\Gamma\left(-\right.}{nx^2}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 129, normalized size = 0.70

$$\frac{e^{-2ia}\csc^2\left(a+bx^n\right)\left(-e^{2ia}n+4^{\frac{1}{n}}e^{4ia}\left(-ibx^n\right)^{2/n}\Gamma\left(-\frac{2}{n},-2ibx^n\right)+4^{\frac{1}{n}}\left(ibx^n\right)^{2/n}\Gamma\left(-\frac{2}{n},2ibx^n\right)\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{4nx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
[Out] (Csc[a + b*x^n]^2*(-(E^((2*I)*a)*n) + 4^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^(2/
n)*Gamma[-2/n, (-2*I)*b*x^n] + 4^n^(-1)*(I*b*x^n)^(2/n)*Gamma[-2/n, (2*I)*b
*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x^2)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin^3\left(a+b\,x^n\right)\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*x^2*integrate(cos(2*b*x^n + 2*a)/x^3, x) + 1)*c^(2/3)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin^3\left(a+bx^n\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(2/3)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\sin\left(a+b\,x^n\right)}^3\right)}^{2/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x^n)^3)^(2/3)/x^3,x)
[Out] int((c*sin(a + b*x^n)^3)^(2/3)/x^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```